14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

BUILDING SPACE TIME BLOCK CODES WITH SET PARTITIONING FOR THREE TRANSMIT ANTENNAS: APPLICATION TO STTC CODES

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ABSTRACT

This paper introduces new variations about the codes recently introduced by Jafarkhani & al named Super Orthogonal Space Time Trellis Codes (SOSTTC). Using powerful set partitioning rules, these codes are able to combine the coding advantage of STTC's together with the advantage diversity of STBC. This partitioning is based mainly on the determinant criterion introduced first by Tarokh. In this paper we propose a new application field of these codes in the difficult context of a three transmit antenna system. The new obtained STTC codes enables to improve significantly the performances of the best existing STTC codes.

1. INTRODUCTION

Since the first studies presented by Tarokh *& al* in [1] who gave the main criteria to optimize the construction of Space Time Codes, particularly the rank and determinant criteria, there has been a great deal of research aiming at improving the designs of Space Time Trellis Codes (STTC) [2]. A recent idea provided by Jafarkhani *& al* in [3] and by Fitz *& al* in [4] is to use a combination of STBC and STTC based on Ungerboeck's partition rules [5], which offers always a maximum diversity in the case of two transmit antennas. For the construction of subsets, the classical Euclidean distance criterion is replaced here by the determinant criterion which consists in maximizing the minimum determinant of the product, of the difference of the transmission matrices for two codewords, by its transpose conjugate. This criterion is quantified by the coding gain distance (CGD). After the set partitioning step, a trellis is built, affecting a particular STBC from a set of possible candidates to transitions originating from a state. Doing this, it is always guaranteed that we get the diversity of the corresponding STBC while it is possible to find some particular STBC matrices which enable to build a trellis with the maximum coding gain. This is the other contribution of Jafarkhani who demonstrated that it was possible to expand the well known transmit STBC scheme of Alamouti [6] resulting in several possible STBC candidates for each trellis state. The resulting codes exhibit outstanding performances when compared for example to the optimized STTC in the open literature, even in the case of a moderate number of states and are named Super Orthogonal Space Time Trellis Codes (SOSTTC).

We propose in this paper a new Space Time Block Code design for the difficult case of three transmit antennas. Our goal is to show that it is possible to build powerful STTC codes with STBC designs with maximum transmission rate $r = 1$, even in the case where orthogonality is broken. The proposed STBC design is based on the coupling of two quasi-orthogonal 2×2 STBC codes to form a quasi orthogonal 3×3 STBC code and is named classically quasi-orthogonal STBC. We optimize the coding gain within each coset by maximizing the distance between codewords. Furthermore, in the case where we build trellises, we optimize the diversity gain between codewords belonging to different cosets by multiplying the basic STBC matrix with a particular unitary matrix transform, in each state. Using this method, we eventually build a flexible powerful STTC code. The design is highly flexible since, operating at different levels of set partitioning, we can build trellises with different number of states. The search of unitary matrices aims at maximizing the distance between codewords belonging to different cosets and is done with extensive computer search.

Simulation results show the efficiency of the proposed codes when we compare their performances with those of the best existing STTC codes in the open literature [7-8]. Comparing codes with the same number of states, we obtain gains of 1 dB at FER = 10^{-2}, 10^{-3} with QPSK modulations and 0.5 dB with 8PSK for the same FER levels.

The paper is organized as follows. In the second part we recall the main key parameters to design a performing space time block code based on set partitioning and then, we build our quasi orthogonal STBC design which is then expanded to obtain STTC code. Part three contains the simulation results including comparisons with some existing STTC codes. Conclusion is eventually given in section four.

2. STBC DESIGNS FOR THREE TRANSMIT ANTENNA SYSTEM

The ultimate goal of set-partitioning is to achieve a better coding gain through using a trellis code structure. The pairwise distance is a measure that can be used to achieve a better coding gain. Depending on the kind of code, the pairwise distance could be defined differently. For example for the orthogonal space-time block codes the pairwise distance is the determinant criterion. However, pairwise error probability and Euclidean distance could be other criteria for different kinds of codes. In the paper, we will always use the determinant criterion to establish our partitioning rules.

The SOSTTC concept has been first studied in [3-4] in the case of two transmit antennas. These codes combine set partitioning and a super set of orthogonal space-time block codes to provide full diversity and improved coding gain over earlier space-time trellis code construction. The super-orthogonal set is derived using constellation rotation with angles chosen so as to not (if possible) expand the transmit symbols constellation. The set partitioning is obtained using the criteria of minimum CGD. This powerful design tool is obtained as follows: let us denote the transmission matrix of the used space-time code as: $c_1 = \mathcal{G}(x_1, x_2)$ where x_1, x_2 are the transmitted symbols and the difference of the transmission matrices for codewords c_1 and c_2 as: $\boldsymbol{D}(c_1, c_2) = \mathcal{G}(x_1, x_2) - \mathcal{G}(x_1', x_2')$. Following the definitions of [3], the diversity of such a code is defined by the minimum rank of the matrix $\boldsymbol{D}(c_1, c_2)$. The code is said to be a full-diversity code when $\boldsymbol{D}(c_1, c_2)$ is full rank for every pair of codewords. The minimum of the determinant of the matrix $\boldsymbol{A}(c_1, c_2) = \boldsymbol{D}^H(c_1, c_2)\boldsymbol{D}(c_1, c_2)$ over all possible pairs of distinct codewords c_1 and c_2 corresponds to the coding gain. Using this definition, it is then straightforward to define the CGD between codewords (c_1,c_2) as:

$CGD(c_1, c_2) = d^2(c_1, c_2) = \det(\boldsymbol{A}(c_1, c_2))$.

2.1. Quasi-orthogonal STBC design

The first task is to find a simple STBC structure to derive a set partitioning rule. This structure will be the basis to obtain a powerful quasi orthogonal structure which will be incorporated into a trellis. The goal is to obtain a final STBC structure which is able to outperform the classical STTC designs when used in a trellis. We decided to use first the simple matrix code given in (1) which is made of two 2×2 STBC orthogonal designs coupled via symbol x_2. As it can be seen, symbols x_1 and x_3 only repeat twice per three symbols, so this code obviously cannot achieve full diversity and presents bad performances. However, it is possible, in this case, to obtain simple partitioning rules that will be reused for the final code design. The STBC matrix looks like :

$$\mathcal{G}(x_1, x_2, x_3) = \begin{pmatrix} -x_2^* & x_1 & 0 \\ x_1^* & x_2 & -x_3^* \\ 0 & x_3 & x_2^* \end{pmatrix} \tag{1}$$

The computation of the determinant of matrix $\boldsymbol{A}(c_1, c_2)$ yields to the following expression :

$\det(\boldsymbol{A}(c_1, c_2)) = | x_2' - x_2 |^2 \; (| x_1' - x_1 |^2 + | x_2' - x_2 |^2 + | x_3' - x_3 |^2)^2$

With : $x_1 = e^{j.l_1.\omega}, x_2 = e^{j.l_2.\omega}, x_3 = e^{j.l_3.\omega}$ and $x_1' = e^{j.k_1.\omega}$, $x_2' = e^{j.k_2.\omega}, x_3' = e^{j.k_3.\omega}$ we eventually obtain :

$$\begin{aligned} &\det(\boldsymbol{A}(c_1, c_2)) = 64.\sin^2(\frac{| k_2 - l_2 |}{2}\omega) \times \\ &(\sin^2(\frac{| k_1 - l_1 |}{2}\omega) + \sin^2(\frac{| k_2 - l_2 |}{2}\omega) + \sin^2(\frac{| k_3 - l_3 |}{2}\omega))^2 \end{aligned} \tag{2}$$

We have $\omega = 2.\pi / L$ (L is the size of the transmit constellation) and $\omega = \pi / 2$ for a QPSK constellation. The expression (2) clearly implies that symbols x_2' and x_2 have to be different in each coset to maintain a non-null CGD. This yields to a set partitioning with at least sixteen cosets and, within each coset, symbols x_1, x_1', x_3 and x_3', can be chosen either accordingly to the partitioning rules already given in [3] Fig. 3, this gives the set partitioning tree of Table I, either by filling all couples of symbols x_1, x_1' and x_3, x_3' with different values, this gives the set partitioning tree of Table II.

TABLE I
Set partioning for QPSK with 16 states and three transmit antennas

S_{0000}	S_{0001}	S_{0010}	S_{0011}	S_{0100}	S_{0101}	S_{0110}	S_{0111}	S_{1000}	S_{1001}	S_{1010}	S_{1011}	S_{1100}	S_{1101}	S_{1110}	S_{1111}
000	030	020	010	101	131	121	111	001	031	021	011	100	130	120	110
212	202	232	222	313	303	333	323	213	203	233	223	312	302	332	322
022	012	002	032	123	113	103	133	023	013	003	033	122	112	102	132
230	220	210	200	331	321	311	301	231	221	211	201	330	320	310	300

TABLE II
Set partioning for QPSK with 16 states and three transmit antennas

S_{0000}	S_{0001}	S_{0010}	S_{0011}	S_{0100}	S_{0101}	S_{0110}	S_{0111}	S_{1000}	S_{1001}	S_{1010}	S_{1011}	S_{1100}	S_{1101}	S_{1110}	S_{1111}
000	001	002	003	010	011	012	013	020	021	022	023	030	031	032	033
111	112	113	110	121	122	123	120	131	132	133	130	101	102	103	100
222	223	220	221	232	233	230	231	202	203	200	201	212	213	210	211
333	330	331	332	303	300	301	302	313	310	311	312	323	320	321	322

The minimum CGD value within each coset obtained by this code is equal to 72 with the set partitioning of Table I. This first approach induces us to think of a performing SOSTTC with transmission rate r=1. Starting from the structure (1), we add two symbols to completely fill the matrix, we obtain the form (3). Since it does not exist any simple closed form expression for $\det(\boldsymbol{A}(c_1, c_2))$, we will optimize the new proposed STBC structure by computer search. Thus, we test the following STBC matrix:

$$\mathcal{G}(x_1, x_2, x_3) = \begin{pmatrix} -x_2^* & x_1 & x_1^*.e^{j.\beta} \\ x_1^* & x_2 & -x_3^* \\ x_3^*.e^{j.\alpha} & x_3 & x_2^* \end{pmatrix} \tag{3}$$

Angles α, β belong to the constellation symbol, if we don't want to expand the constellation. They constitute new degrees of freedom to obtain a set partitioning with a lower number of cosets when compared to tables I-II. The difficulty here is to find an optimum set partitioning algorithm since it does not exist any simple closed form expression for $\det(\boldsymbol{A}(c_1, c_2))$. Recently, M. Janani and A. Nosratinia proposed a general procedure which can handle our particular problem [9]. Using their algorithm and the basic structure given in (3), we obtain the set partitioning given in Fig. 1 for QPSK constellation. The set partitioning at the first level (four cosets) corresponds to the case where there are always two different symbols in each triplet belonging to a given coset. At the first level of set partitioning, we can obtain a mathematical expression of $\det(\boldsymbol{A}(c_1, c_2))$. Denoting $z_1 = x_1 - x_1'$, $z_2 = x_2 - x_2'$ and $z_3 = x_3 - x_3'$, we have to consider three cases:

a) symbols x_1 and x_1' are equal i.e. $z_1 = 0$, this yields to :

$$\boldsymbol{A}(c_1, c_2) = \begin{pmatrix} |z_2|^2 & 0 & -(z_2)^*.(z_3).e^{-j.\alpha} \\ 0 & |z_2|^2 + |z_3|^2 & 0 \\ -(z_2).(z_3)^*.e^{j.\alpha} & 0 & 2.|z_3|^2 + |z_2|^2 \end{pmatrix}$$

and :

$$\det(\boldsymbol{A}(c_1, c_2)) = |z_2|^2.\left(|z_2|^2 + |z_3|^2\right)^2$$

b) symbols x_2 and x_2' are equal i.e. $z_2 = 0$, this yields to :

$$\boldsymbol{A}(c_1, c_2) = \begin{pmatrix} 2.|z_1|^2 & -(z_3).(z_1)^*.e^{j.\beta} & (z_3)^*.(z_1) \\ -(z_3)^*.(z_1).e^{-j.\beta} & |z_1|^2 + |z_3|^2 & (z_3).(z_1)^*.e^{-j.\alpha} \\ (z_3).(z_1)^* & (z_3)^*.(z_1).e^{j.\alpha} & 2.|z_3|^2 \end{pmatrix}$$

and :

$$\det(\boldsymbol{A}(c_1, c_2)) = |z_1|^4 |z_3|^2 + |z_1|^2 |z_3|^4 - 2.\mathrm{Re}[(z_3^*)^3 (z_1)^3.e^{j.(\alpha - \beta)}]$$

c) symbols x_3 and x_3' are equal i.e. $z_3 = 0$, this yields to :

$$A(c_1,c_2)=\begin{pmatrix} |z_2|^2+2.|z_1|^2 & 0 & (z_1)^*.(z_2).e^{j.\beta} \\ 0 & |z_1|^2+|z_2|^2 & 0 \\ (z_1).(z_2)^*.e^{-j.\beta} & 0 & |z_2|^2 \end{pmatrix}$$

and :

$$\det(A(c_1,c_2))=|z_2|^2.\left(|z_1|^2+|z_3|^2\right).\left(|z_2|^2+|z_3|^2\right)$$

Hence, considering case b), angles (α,β) are mainly needed to separate triplets of symbols which share the same second symbol.

We give the minCGD value within each coset and the corresponding couple of angles (α,β) which enables to reach this value on Fig. 1. The choice of (α,β) which maximizes the minCGD within each coset is done by computer search with a sampling rate $\pi/16$. One can remark that the obtained values for (α,β) does not correspond to QPSK constellation points. This entails that constellation expansion is needed if we want to use set partitioning with maximum minCGD values. Using a set partitioning level with 32 cosets with those given in Fig .1 enables to obtain a minCGD value equal to 256. The obtained values for 16 and 32 states are superior to those exhibited by the best STTC given in [7-8]. Without constellation expansion, using the set partitioning with sixteen cosets of Fig. 1, we found a minCGD equal to 8 with (α,β)=(0,0).

The next problem to use the STBC code (3) into a STTC design consists in the way to expand the basic structure to obtain a trellis. We have to obtain a final design with full diversity. In fact, according to [1], proving the full diversity is equivalent to showing the determinants of matrices $A(c_1,c_2)$ are nonzero over all possible codewords c_1 and c_2. Since it is clearly the case when they belong to the same coset, this entails that we have to check if matrices $A(c_1,c_2)$ are full rank when c_1 and c_2 belong to different cosets. Without any new tool to separate the coset, it is clear, for whatever kind of set partitioning we used, that we did not obtain a full diversity code. For example, we check that the minimum of $\det(A(c_1,c_2))$ is equal to zero when c_1 belongs to coset 1 and c_2 belongs to coset 2 in Fig. 1, with sixteen levels of set partitioning.

To solve this problem, we assign different unit transform matrices Θ to each state. In fact, a unitary matrix Θ_j corresponds to a rotation and preserves distances among the sent constellation points. This means that minCGD value is left unchanged by applying an unitary transform Θ_j to $\mathcal{G}(x_1,x_2,x_3)$ within each coset. The search for unitary matrices $\Theta_j, i=1,...,16$ is done using the parametrization given in [10].

With this parametrization and depending on the selected partitioning level we used, we look for unit matrices whose umber is equal to the number of cosets and with the property to obtain a maximum separation distance between the cosets. The distance is once again quantified by the determinant criterion and the distance between coset *i* and coset *j* will be defined as the minimum of $\det(A(c_1,c_2))$ when c_1 belongs to coset *i* and c_2 belongs to coset *j*.

Using the set partitioning with sixteen cosets and the set partitioning illustrated on Fig. 1, we found sixteen unitary matrices with min $\det(A(c_1,c_2))$ maximum value equal to 1.22, after 10^7 computer runs. For a set partitioning with eight cosets in Fig. 1, we obtain a min $\det(A(c_1,c_2))$ maximum value equal to 4.16. The STTC final code with sixteen cosets is illustrated on Fig. 2.

Using, the basic structure of (3) we built a set partitioning with 8-PSK modulation too. We found the first level of set partitioning by assigning triplets containing at least two different symbols to a given coset. This leads to eight cosets with sixty four elements in each coset. The minCGD value in each coset is equal to 3.10^{-2} for the eight cosets partitioning, 0.25 for the sixteen coset partitioning and 0.8 for the thirty two coset partitioning. For the first partitioning level with eight cosets we found eight unitary matrices to separate the cosets with a minimum distance equal to 4.25. This distance drastically reduces to 8.2 10^{-2} for the sixteen coset partitioning and 2.5 10^{-2} for the thirty two coset partitioning.

2.2. Decoding of quasi-orthogonal STBC based STTC codes

Using the quasi-static block Rayleigh fading model, we can write the received signal : (For simplicity reasons we only consider the case of one receive antenna)

$$Y = H.\Theta_j.\mathcal{G}(x_1,x_2,x_3)+N$$

H contains the channel coefficients, which are supposed constant for the duration of a packet. Θ_j is the 3×3 unitary matrix used in state *j*. We note : $[h_1^j\ h_2^j\ h_3^j]=H.\Theta_j$. N is the vector of additive Gaussian noise with zero mean and variance σ^2. In the case of multiple receive antennas, the considered SNR in the plotted FER curves will correspond to a SNR per receive antenna. Writing the received signal within three successive time slot intervals, we obtain:

$$y_1=-h_1^j.x_2^*+h_2^j.x_1^*+h_3^j.x_3^*.\mathrm{e}^{j.\alpha}+n_1$$

$$y_2=h_1^j.x_1+h_2^j.x_2+h_3^j.x_3+n_2$$

$$y_3=h_1^j.x_1^*.\mathrm{e}^{j.\beta}-h_2^j.x_3^*+h_3^j.x_2^*+n_3$$

In order to compute the branch metrics and in accordance with the different candidates' symbol triplets, we form the auxiliary quantities :

$$y_1'=y_1-h_3^j.x_3^*.\mathrm{e}^{j.\alpha}$$

$$y_2'=y_2-h_3^j.x_3$$

$$y_2''=y_2-h_1^j.x_1$$

$$y_3'=y_3-h_1^j.x_1^*.\mathrm{e}^{j.\beta}$$

It is then possible to use maximum ratio combining MRC technique to obtain:

$$\begin{aligned} y_1'^*.h_2^j+h_1^{j*}.y_2' &= \left(|h_1^j|^2+|h_2^j|^2\right).x_1+n_2.h_1^{j*}+n_1^*.h_2^j \\ -y_1'^*.h_1^j+h_2^{j*}.y_2' &= \left(|h_1^j|^2+|h_2^j|^2\right).x_2+n_2.h_2^{j*}-n_1^*.h_1^j \\ y_3'^*.h_3^j+h_2^{j*}.y_2'' &= \left(|h_2^j|^2+|h_3^j|^2\right).x_2+n_2.h_2^{j*}+n_3^*.h_3^j \\ -y_3'^*.h_2^j+h_3^{j*}.y_2'' &= \left(|h_2^j|^2+|h_3^j|^2\right).x_3+n_2.h_3^{j*}-n_3^*.h_2^j \end{aligned} \quad (4)$$

It is thus possible by summing lines 2 and 3 in (4) to obtain a matrix form as given in (5):

$$Z=M.X+N' \quad (5)$$

with: $Z=\begin{pmatrix} y_1^*.h_2^j+y_2.h_1^{j*} \\ -y_1^*.h_1^j+2.y_2.h_2^{j*}+y_3^*.h_3^j \\ -y_3^*.h_2^j+y_2.h_3^{j*} \end{pmatrix}, X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},$

S_{00}	S_{01}	S_{10}	S_{11}
$s_0\ s_0\ s_0$	$s_0\ s_0\ s_1$	$s_0\ s_0\ s_2$	$s_0\ s_0\ s_3$
$s_0\ s_1\ s_1$	$s_0\ s_1\ s_2$	$s_0\ s_1\ s_3$	$s_0\ s_1\ s_0$
$s_0\ s_2\ s_2$	$s_0\ s_2\ s_3$	$s_0\ s_2\ s_0$	$s_0\ s_2\ s_1$
$s_0\ s_3\ s_3$	$s_0\ s_3\ s_0$	$s_0\ s_3\ s_1$	$s_0\ s_3\ s_2$
$s_1\ s_0\ s_1$	$s_1\ s_0\ s_2$	$s_1\ s_0\ s_3$	$s_1\ s_0\ s_0$
$s_1\ s_1\ s_2$	$s_1\ s_1\ s_3$	$s_1\ s_1\ s_0$	$s_1\ s_1\ s_1$
$s_1\ s_2\ s_3$	$s_1\ s_2\ s_0$	$s_1\ s_2\ s_1$	$s_1\ s_2\ s_2$
$s_1\ s_3\ s_0$	$s_1\ s_3\ s_1$	$s_1\ s_3\ s_2$	$s_1\ s_3\ s_3$
$s_2\ s_0\ s_2$	$s_2\ s_0\ s_3$	$s_2\ s_0\ s_0$	$s_2\ s_0\ s_1$
$s_2\ s_1\ s_3$	$s_2\ s_1\ s_0$	$s_2\ s_1\ s_1$	$s_2\ s_1\ s_2$
$s_2\ s_2\ s_0$	$s_2\ s_2\ s_1$	$s_2\ s_2\ s_2$	$s_2\ s_2\ s_3$
$s_2\ s_3\ s_1$	$s_2\ s_3\ s_2$	$s_2\ s_3\ s_3$	$s_2\ s_3\ s_0$
$s_3\ s_0\ s_3$	$s_3\ s_0\ s_0$	$s_3\ s_0\ s_1$	$s_3\ s_0\ s_2$
$s_3\ s_1\ s_0$	$s_3\ s_1\ s_1$	$s_3\ s_1\ s_2$	$s_3\ s_1\ s_3$
$s_3\ s_2\ s_1$	$s_3\ s_2\ s_2$	$s_3\ s_2\ s_3$	$s_3\ s_2\ s_0$
$s_3\ s_3\ s_2$	$s_3\ s_3\ s_3$	$s_3\ s_3\ s_0$	$s_3\ s_3\ s_1$

Min CGD = 4.7

$(\alpha,\beta) = (3\pi/8, 13\pi/8)$

S_{000}	S_{001}	S_{010}	S_{011}	S_{100}	S_{101}	S_{110}	S_{111}
$s_0\ s_0\ s_0$	$s_0\ s_3\ s_0$	$s_0\ s_2\ s_0$	$s_0\ s_1\ s_0$	$s_0\ s_0\ s_1$	$s_0\ s_3\ s_1$	$s_0\ s_2\ s_1$	$s_0\ s_1\ s_1$
$s_2\ s_1\ s_2$	$s_2\ s_0\ s_2$	$s_2\ s_3\ s_2$	$s_2\ s_2\ s_2$	$s_2\ s_1\ s_3$	$s_2\ s_0\ s_3$	$s_2\ s_3\ s_3$	$s_2\ s_2\ s_3$
$s_0\ s_2\ s_2$	$s_0\ s_1\ s_2$	$s_0\ s_0\ s_2$	$s_0\ s_3\ s_2$	$s_0\ s_2\ s_3$	$s_0\ s_1\ s_3$	$s_0\ s_0\ s_3$	$s_0\ s_3\ s_3$
$s_2\ s_3\ s_0$	$s_2\ s_2\ s_0$	$s_2\ s_1\ s_0$	$s_2\ s_0\ s_0$	$s_2\ s_3\ s_1$	$s_2\ s_2\ s_1$	$s_2\ s_1\ s_1$	$s_2\ s_0\ s_1$
$s_1\ s_0\ s_1$	$s_1\ s_3\ s_1$	$s_1\ s_2\ s_1$	$s_1\ s_1\ s_1$	$s_1\ s_0\ s_0$	$s_1\ s_3\ s_0$	$s_1\ s_2\ s_0$	$s_1\ s_1\ s_0$
$s_3\ s_1\ s_3$	$s_3\ s_0\ s_3$	$s_3\ s_3\ s_3$	$s_3\ s_2\ s_3$	$s_3\ s_1\ s_2$	$s_3\ s_0\ s_2$	$s_3\ s_3\ s_2$	$s_3\ s_2\ s_2$
$s_1\ s_2\ s_3$	$s_1\ s_1\ s_3$	$s_1\ s_0\ s_3$	$s_1\ s_3\ s_3$	$s_1\ s_2\ s_2$	$s_1\ s_1\ s_2$	$s_1\ s_0\ s_2$	$s_1\ s_3\ s_2$
$s_3\ s_3\ s_1$	$s_3\ s_2\ s_1$	$s_3\ s_1\ s_1$	$s_3\ s_0\ s_1$	$s_3\ s_3\ s_0$	$s_3\ s_2\ s_0$	$s_3\ s_1\ s_0$	$s_3\ s_0\ s_0$

MinCGD=16 $(\alpha,\beta)=(\pi/8, 13\pi/8)$

S_{0000}	S_{0010}	S_{0100}	S_{0110}	S_{1000}	S_{1010}	S_{1100}	S_{1110}
$s_0\ s_0\ s_0$	$s_0\ s_3\ s_0$	$s_0\ s_2\ s_0$	$s_0\ s_1\ s_0$	$s_0\ s_0\ s_1$	$s_0\ s_3\ s_1$	$s_0\ s_2\ s_1$	$s_0\ s_1\ s_1$
$s_2\ s_1\ s_2$	$s_2\ s_0\ s_2$	$s_2\ s_3\ s_2$	$s_2\ s_2\ s_2$	$s_2\ s_1\ s_3$	$s_2\ s_0\ s_3$	$s_2\ s_3\ s_3$	$s_2\ s_2\ s_3$
$s_0\ s_2\ s_2$	$s_0\ s_1\ s_2$	$s_0\ s_0\ s_2$	$s_0\ s_3\ s_2$	$s_0\ s_2\ s_3$	$s_0\ s_1\ s_3$	$s_0\ s_0\ s_3$	$s_0\ s_3\ s_3$
$s_2\ s_3\ s_0$	$s_2\ s_2\ s_0$	$s_2\ s_1\ s_0$	$s_2\ s_0\ s_0$	$s_2\ s_3\ s_1$	$s_2\ s_2\ s_1$	$s_2\ s_1\ s_1$	$s_2\ s_0\ s_1$
$s_1\ s_0\ s_1$	$s_1\ s_3\ s_1$	$s_1\ s_2\ s_1$	$s_1\ s_1\ s_1$	$s_1\ s_0\ s_0$	$s_1\ s_3\ s_0$	$s_1\ s_2\ s_0$	$s_1\ s_1\ s_0$
$s_3\ s_1\ s_3$	$s_3\ s_0\ s_3$	$s_3\ s_3\ s_3$	$s_3\ s_2\ s_3$	$s_3\ s_1\ s_2$	$s_3\ s_0\ s_2$	$s_3\ s_3\ s_2$	$s_3\ s_2\ s_2$
$s_1\ s_2\ s_3$	$s_1\ s_1\ s_3$	$s_1\ s_0\ s_3$	$s_1\ s_3\ s_3$	$s_1\ s_2\ s_2$	$s_1\ s_1\ s_2$	$s_1\ s_0\ s_2$	$s_1\ s_3\ s_2$
$s_3\ s_3\ s_1$	$s_3\ s_2\ s_1$	$s_3\ s_1\ s_1$	$s_3\ s_0\ s_1$	$s_3\ s_3\ s_0$	$s_3\ s_2\ s_0$	$s_3\ s_1\ s_0$	$s_3\ s_0\ s_0$
S_{0001}	S_{0011}	S_{0101}	S_{0111}	S_{1001}	S_{1011}	S_{1101}	S_{1111}

MinCGD=72 $(\alpha,\beta)=(5\pi/16, 5\pi/16)$

Figure 1 – Set Partitioning for QPSK

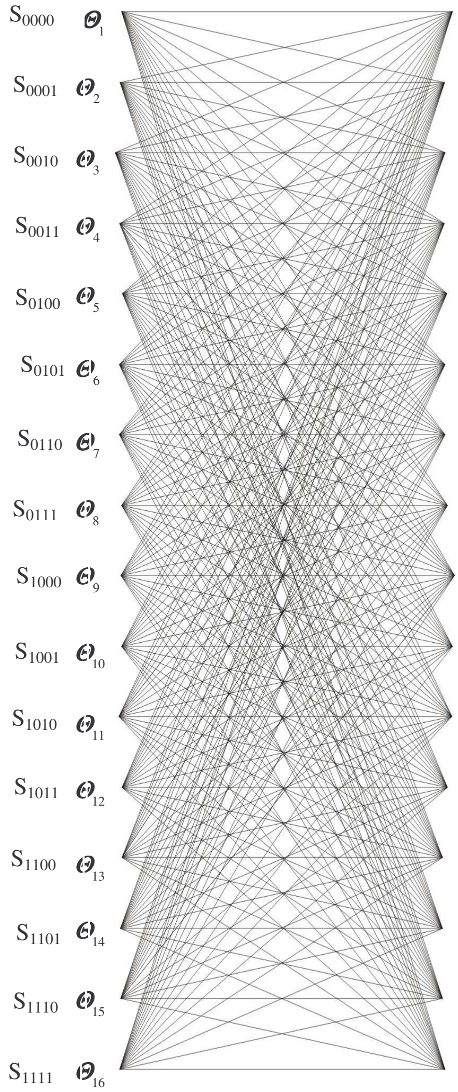

Figure 2 - 16 states STTC for three transmit antennas

$$
\boldsymbol{M} = \begin{pmatrix} |h_1|^2 + |h_2|^2 & 0 & h_3.h_1^* + h_2.h_3^*.\mathrm{e}^{-j.\alpha} \\ h_1.h_2^* + h_3.h_1^*.\mathrm{e}^{-j.\beta} & |h_1|^2 + 2.|h_2|^2 + |h_3|^2 & h_3.h_2^* - h_1.h_3^*.\mathrm{e}^{-j.\alpha} \\ h_1.h_3^* - h_2.h_1^*.\mathrm{e}^{-j.\beta} & 0 & |h_2|^2 + |h_3|^2 \end{pmatrix}
$$

$$
\boldsymbol{N}' = \begin{pmatrix} n_1^*.h_2^j + n_2.h_1^{j*} \\ -n_1^*.h_1^j + 2.n_2.h_2^{j*} + n_3^*.h_3^j \\ n_2.h_3^{j*} - n_3^*.h_2^j \end{pmatrix}
$$

and the maximum likelihood receiver has to minimize the following metric :

$$
(\boldsymbol{Z} - \boldsymbol{M}.\boldsymbol{X}).\boldsymbol{R}_{N'N'}^{-1}.(\boldsymbol{Z} - \boldsymbol{M}.\boldsymbol{X})^H \tag{6}
$$

with $\boldsymbol{R}_{N'N'}$, the autocorrelation matrix of $\boldsymbol{N}'$. The STBC based STTC code is then decoding using the classical Viterbi algorithm with metric branches given by (6).

3. SIMULATION RESULTS

Clearly, our goal is to prove that our optimized quasi-orthogonal design is able to outperform the best existing STTC codes in the open literature for three transmit antenna systems. We use as reference STTC codes, the optimized codes proposed by Vucetic *& al* in [7-8] with the rank & determinant criteria or the trace criterion.

The channel between each pair of transmit-receive antenna is a flat quasistatic Rayleigh fading channel and the channel coefficients are zero mean complex Gaussian variables with variance 0.5 per complex dimension. The packet length is taken equal to 130 PSK symbols, either from a QPSK or a 8-PSK constellation. The results are obtained by Monte-Carlo simulation runs and are averaged over 1000000 channel realizations at FER = 10^{-3}. The fading channels are considered uncorrelated. The performances of the proposed codes are given in terms of Frame Error Rate (FER) and we always give the corresponding outage capacity.

a-**QPSKmodulation:** We use optimized STTC codes with 16, 32 and 64 states. For the STBC based STTC codes, we used the set partitioning of Fig.1 and we tested two kinds of codes with 16 and 32 states. The results for the case of three transmit-one receive antennas are given on Fig. 3. We can see that our STBC quasi-ortho 32-state outperforms the STTC 64-state by approximately 0.2 dB at FER = 10^{-2}. Furthermore, when we compare codes with the same number of states, we notice that our quasi-orthogonal STBC designs enables a gain of 1.3 dB for the 16 state codes and 1 dB for the 32 state codes.

b-**8-PSKmodulation:** We use optimized STTC codes with 8, 16 and 32 states. For the STBC based STTC codes, we tested two kinds of codes with 16 and 32 states. The results are given on Fig. 4 for the case of one receive antenna. The conclusions are the same as those given for the QPSK case but the advantage of the proposed STBC designs is less obvious. In fact, working with the same number of states, our STBC designs enable a gain of 0.5 dB at FER = 10^{-2}. The case of two receive antennas is depicted of Fig. 5. The STBC based STTC codes outperform the optimized STTC by approximately 0.3 dB at FER = 10^{-2}

4. CONCLUSION

In this paper we have generalized the use of Super Orthogonal Space Time Trellis Codes (SOSTTC) in the context of three transmit antenna systems with non-orthogonal STBC designs. Based on the determinant criterion, we build new performing space-time trellis codes which exhibit high minimum CGD values within each coset. The design we found is made of the coupling of two quasi orthogonal 2 × 2 space-time block codes. To build a trellis with the chosen STBC basic structure, the cosets are separated by means of unitary transform matrices. Doing this, we are ensured that our code always exhibits maximum diversity. We compare the performances of our STBC based STTC codes with those of the best existing STTC codes and we found that our designs enables a gain of 1 dB at FER = 10^{-2} ,10^{-3} for QPSK modulation and a gain of 0.5 dB for 8-PSK modulation for the same FER levels.

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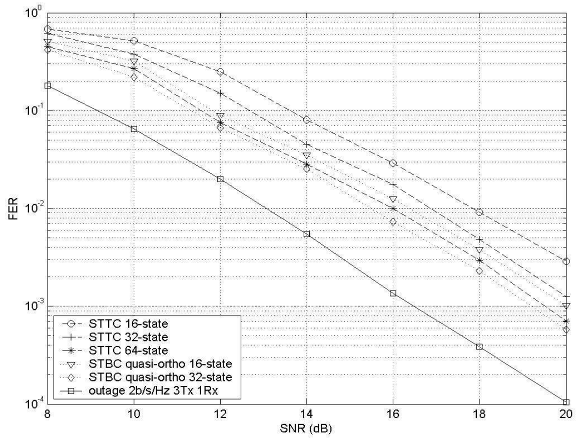

Figure 3 - Performance comparison of STTC and STBC based STTC QPSK codes for three transmit-one receive antennas

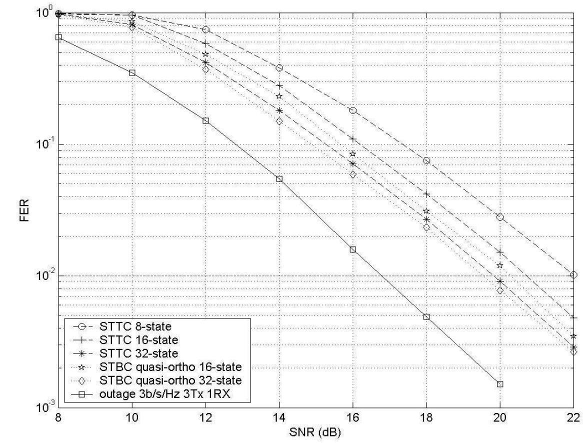

Figure 4 - Performance comparison of STTC and STBC based STTC 8-PSK codes for three transmit-one receive antennas

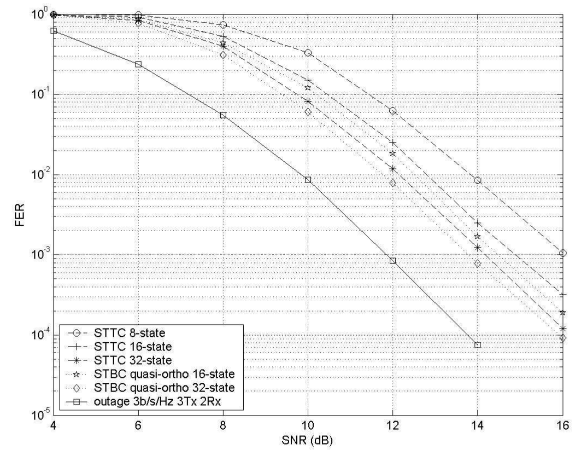

Figure 5 - Performance comparison of STTC and STBC based STTC 8-PSK codes for three transmit-two receive antennas