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SUB-TRIDENT FORM USING FUZZY AGGREGATION

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ABSTRACT

This Paper deals with the solution to find the Optimal Path and the Optimal Solution with the help of Fuzzy Aggregation Operations such as Arithmetic Mean and Geometric Mean and by using Trapezoidal Fuzzy Numbers through Pascal's Triangle Graded Mean Approach. Here the results are obtained as Fuzzy Sub-Triangular Form and this form in turn converted to Sub-Triangular Form. The Minimum value of Sub-Trident Form gives the Shortest Path and the Optimum Solution is obtained by giving a suitable numerical example.

KEYWORDS: Fuzzy Aggregation, Fuzzy Numbers, Pascal's Triangle, Optimal solution and Sub-Trident Form

INTRODUCTION

The Shortest Path Problem is one of the most fundamental optimization problems to find the shortest path. Dubois and Prade introduced this shortest path problem in the year 1980[2]. The same shortest path problem was worked by Okada and Soper [5] using fuzzy numbers.In the year 1998, Chen and Hsieh [6] and [7] give the Graded Mean Integration Representation for generalized fuzzy numbers. Then in the year 2013, S.K.Kadhar Babu and B.Rajesh[11] introduced Pascal's Triangle Graded Mean in Statistical Optimization. Fuzzy Set Theory is introduced by Lotfi.A.Zadeh in the year 1965[1]. Aggregation Operation on fuzzy numbers by which several fuzzy numbers is combined to produce a single fuzzy number is introduced by George.J.Klir and Tina.A.Fogler [4]. In this paper the Shortest Path using Sub-Trident Form for Trapezoidalfuzzy numbers through Aggregation Operations such as Arithmetic Mean and Geometric Mean is calculated by giving a suitable numerical example[3]. This paper consists of seven sections: Introduction part is in the first section, the second section deals with the preliminaries, the third section deals with the methodologies used in this paper, fourth section deals with the working rule or the algorithm, illustrative example in the fifth section, Calculation Part in the sixth section and finally the conclusion based on our study.

PRELIMINARIES

In this section, some basic definition of fuzzy set theory and fuzzy number is discussed [11].

Definition 2.1.A fuzzyset $\tilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}}(x)$ represents the grade of membership of $x \in \mu_{\tilde{A}}(x)$. More general representation for a fuzzy set is given by $\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) / x \in X \right\}$

.**Definition 2.2.**A fuzzy set $\tilde{A}$ defined on the set of real numbers $\Re$ is said to be a fuzzy number if its membership function $\tilde{A} : \Re \rightarrow [0,1]$ has the following characteristics.

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a) $\tilde{A}$ is convex if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in X, \lambda \in [0,1]$$

b) $\tilde{A}$ is normal if there exists an $x \in \Re$ such that if $\max \mu_{\tilde{A}}(x) = 1.$

c) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

2.1 Representation of Generalized (Trapezoidal) Fuzzy Number

In general, a generalized fuzzy number A is described at any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions:

- μ_A is a continuous mapping from R to [0,1],
- μ_A(x)=0, $-\infty \prec x \leq c,$
- $\mu_A(x) = L(x)$ is strictly increasing on [c,a]
- $\mu_A(x) = w, a \leq x \leq b,$
- $\mu_A(x) = R(x)$ is strictly decreasing on [b,d],
- $\mu_A(x) = 0, d \leq x \prec \infty$ Where $0 \prec w \leq 1$ and a, b, c and d are real numbers.

Here denote this type of generalized fuzzy number as $A = (c, a, b, d; w)_{LR}$. When w=1, denote this type of generalized fuzzy number as $A = (c, a, b, d)_{LR}$. When L(x) and R(x) are straight line, then A is Trapezoidal fuzzy number and denote it as (c, a, b, d).

2.2 Graded Mean Integration Representation

In 1998, Chen and Hsieh [6] and [7] proposed the graded mean integration representation for representing generalized fuzzy number. Suppose L^{-1}, R^{-1} are inverse functions of L and R respectively, and the graded mean h-level value of generalized fuzzy number $A = (c, a, b, d; w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)]/2$.Then the graded mean integration representation of generalized fuzzy number based on the integral value of graded mean h-level is

$$P(A) = \frac{\int_0^w h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right)dh}{\int_0^w h dh} = \frac{c + 2a + 2b + d}{6}$$

Where h is between 0 and w, 0<w ≤1;

2.3 Pascal's Triangle Graded Mean Approach

The Graded Mean Integration Representation for generalized fuzzy number by Chen and Hsieh [6] - [8].Later Sk.Kadhar Babu and B.Rajesh Anand introduces Pascal's Triangle Graded Mean in Statistical Optimization [10].But the present approach is a very simple one for analyzing fuzzy variables to get the optimum shortest path. This procedure is taken from the following Pascal's triangle. Here take the coefficients of fuzzy variables as Pascal's triangle numbers. Then just add and divide by the total of Pascal's number and call it as Pascal's Triangle Graded Mean Approach.

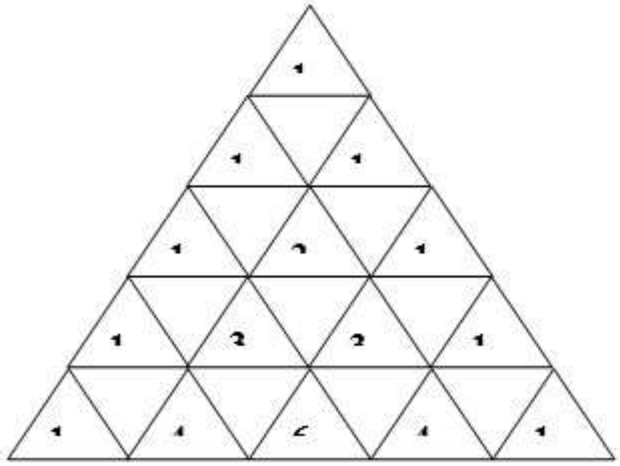

Figure: 1 Pascal's Triangle

The following are the Pascal's triangular approach:

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers then take the coefficient of fuzzy numbers from Pascal's triangles and apply the approach to get the following formula:

$$P(A) = \frac{a_1 + 3a_2 + 3a_3 + a_4}{8}; P(B) = \frac{b_1 + 3b_2 + 3b_3 + b_4}{8};$$

The coefficients of a_1, a_2, a_3, a_4 and b_1, b_2, b_3, b_4 are 1, 3, 3, 1.This approach can be extended fir n-dimensional Pascal's Triangular fuzzy order also.

PROPOSED METHOD

3.1 Fuzzy Sub-Triangular Form of Pascal's Triangle

Trapezoidal Fuzzy Numbers:

The Pascal's Triangle for Trapezoidal Fuzzy Number is given in figure: 2 and the Sub -Triangles for Trapezoidal Fuzzy Number is given in Figure: 3(*Set: I*) as follows:

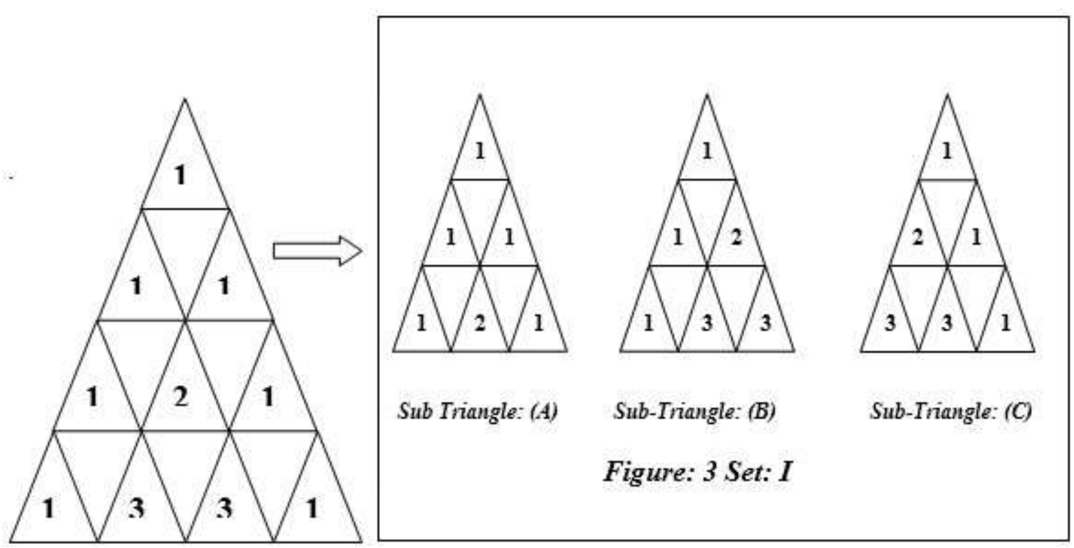

Figure: 2

$P_1 = P(A) = \frac{a_1 + a_2 + a_3}{3}$, $q_1 = P(B) = \frac{a_1 + 2a_2 + a_3}{4}$, $r_1 = P(C) = \frac{a_1 + a_2 + a_3}{3}$.

$P_2 = P(A) = \frac{a_1 + a_2 + a_3}{3}$, $q_2 = P(B) = \frac{a_1 + 3a_2 + 3a_3}{7}$, $r_2 = P(C) = \frac{3a_1 + 2a_2 + a_3}{6}$.

$P_3 = P(A) = \frac{a_1 + 2a_2 + 3a_3}{6}$, $q_3 = P(B) = \frac{3a_1 + 3a_2 + a_3}{7}$, $r_3 = P(C) = \frac{a_1 + a_2 + a_3}{3}$.

The Fuzzy Sub-Triangular Form for Trapezoidal Fuzzy Number is given by

$FST_f = (p_p, q_q, r_r)$, where $p_p = \frac{p_1 + p_2 + p_3}{3}, q_q = \frac{q_1 + q_2 + q_3}{3}, r_r = \frac{r_1 + r_2 + r_3}{3}$

3.2 Sub-Trident Form

The Sub-Trident Form of Fuzzy Number is given by $ST_{ri} = \frac{1}{3}\left[p_p^{1/3} + q_q^{1/3} + r_r^{1/3}\right]$, where p_p, q_q, r_r are the Graded Means of the Pascal's Triangle from the Fuzzy Triangular Form.

3.3 Fuzzy Aggregation

Aggregation operations on fuzzy numbers are done by the combination of several fuzzy numbers to form a single fuzzy number [4]. The aggregation operation is given as follows[10]:

- *Arithmetic Mean:* The arithmetic mean aggregation operator defined on *n* trapezoidal fuzzynumbers $\langle a_1,b_1,c_1d_1\rangle, \langle a_2,b_2,c_2d_2\rangle, \langle a_3,b_3,c_3d_3\rangle......\langle a_i,b_i,c_id_i\rangle......\langle a_n,b_n,c_nd_n\rangle$, is $\langle \bar{a},\bar{b},\bar{c},\bar{d}\rangle$ where

$$\bar{a} = \frac{1}{n}\sum_1^n a_i, \bar{b} = \frac{1}{n}\sum_1^n b_i, \bar{c} = \frac{1}{n}\sum_1^n c_i \, and \, \bar{d} = \frac{1}{n}\sum_1^n d_i$$

- *Geometric Mean*: The Geometric mean aggregation operator defined on n trapezoidal fuzzy numbers $\langle a_1,b_1,c_1d_1\rangle, \langle a_2,b_2,c_2d_2\rangle, \langle a_3,b_3,c_3d_3\rangle......\langle a_i,b_i,c_id_i\rangle......\langle a_n,b_n,c_nd_n\rangle$, is $\langle \bar{a},\bar{b},\bar{c},\bar{d}\rangle$ where

$$\bar{a} = \left(\prod_1^n a_i\right)^{\frac{1}{n}}, \bar{b} = \left(\prod_1^n b_i\right)^{\frac{1}{n}}, \bar{c} = \left(\prod_1^n c_i\right)^{\frac{1}{n}} \, and \quad \bar{d} = \left(\prod_1^n d_i\right)^{\frac{1}{n}}$$

ALGORITHM

The Working Rule for the Sub-Trident Form to find the shortest path and the optimum solution is given by the following algorithm:

Step: 1Choose all possible paths

Step: 2 Input the Trapezoidal fuzzy number as edge weight.

Step: 2apply aggregation operation such as arithmetic mean and geometric mean for each path

Step: 3 Calculate fuzzy sub-triangular form (FST_f) and converting to Sub-Trident Form (ST_{ri}).

Step: 4 find the minimum value of the Sub-Trident Form (ST_{ri}).

Step: 5 Repeat Step: 4 for all the adjacent edges and the minimum of all adjacent edges arrive at the shortest path.

Step: 6 Optimum Solution is obtained by $optsol = \left(\sum \min ST_{ri}\right) * 100.$

ILLUSTRATIVE EXAMPLE

In order to illustrate the above procedure consider a network shown in figure: 8 where each arc length is represented as a trapezoidal fuzzy number to identify the shortest path using Sub-Trident Form [9]:

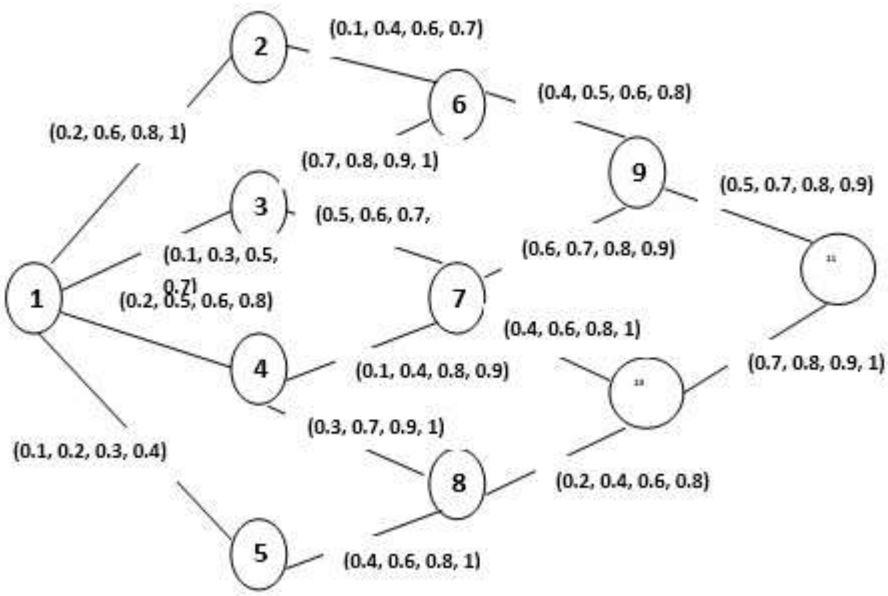

Figure: 4 Illustrative Example

CALCULATION PART

The Calculation to find the optimal path through Sub-Trident Form using Fuzzy Aggregation such as Arithmetic Mean and Geometric Mean is given by the following tables:

TABLE I. Aggregation Operation: Arithmetic Mean

Possible Paths	Arithmetic Mean $(\bar{a},\bar{b},\bar{c},\bar{d})$	$p_p=\frac{p_1+p_2+p_3}{3}$	$q_q=\frac{q_1+q_2+q_3}{3}$	$r_r=\frac{r_1+r_2+r_3}{3}$	Sub-Trident Form $ST_{T_{\tilde{A}}}=\frac{1}{3}\left(p_p^{1/3}+q_q^{1/3}+r_r^{1/3}\right)$
1→2→6→9→11	(0.3,0.55,0.7,0.85)	0.5389	0.5226	0.4944	0.8033
1→3→6→9→11	(0.425,0.575,0.7,0.85)	0.5820	0.5682	0.5514	0.8277
1→3→7→9→11	(0.425,0.575,0.7,0.825)	0.5820	0.5682	0.5514	0.8277
1→3→7→10→11	(0.425,0.575,0.725,0.875)	0.5917	0.5893	0.5583	0.8338
1→4→7→9→11	(0.35,0.575,0.75,0.875)	0.5805	0.5613	0.5361	0.8238
1→4→7→10→11	(0.35,0.575,0.775,0.925)	0.5903	0.5682	0.5431	0.8277
1→4→8→10→11	(0.35,0.6,0.75,0.9)	0.5889	0.5726	0.5445	0.8284
1→5→8→10→11	(0.35,0.5,0.65,0.8)	0.4444	0.5	0.4833	***0.7805***

TABLE II. Aggregation Operation: Geometric Mean

Possible Paths	Geometric Mean $(\bar{a},\bar{b},\bar{c},\bar{d})$	$p_p=\frac{p_1+p_2+p_3}{3}$	$q_q=\frac{q_1+q_2+q_3}{3}$	$r_r=\frac{r_1+r_2+r_3}{3}$	Sub-Trident Form $ST_{T_n}=\frac{1}{3}(p_p^{1/3}+q_q^{1/3}+r_r^{1/3})$
1→2→6→9→11	(0.2514,0.5384,0.6928,0.8426)	0.5187	0.5086	0.4697	0.7930
1→3→6→9→11	(0.3440,0.5384,0.6817,0.8426)	0.5402	0.5244	0.5026	0.8053
1→3→7→9→11	(0.3807,0.5450,0.6880,0.8207)	0.5550	0.5392	0.5208	0.8134
1→3→7→10→11	(0.3440,0.5422,0.7085,0.8651)	0.5518	0.5335	0.5113	0.8103
1→4→7→9→11	(0.2783,0.5595,0.7445,0.8739)	0.5533	0.5332	0.5015	0.8088
1→4→7→10→11	(0.2736,0.5566,0.7667.0.9212)	0.5597	0.5366	0.5049	0.8110
1→4→8→10→11	(0.3027,0.5785,0.7348,0.8944)	0.5627	0.5458	0.5147	0.8147
1→5→8→10→11	(0.2736,0.4427,0.6,0.7521)	0.4569	0.4395	0.4207	***0.7599***

Thus in both Arithmetic and Geometric Mean the minimum value of Sub-Trident Form occurs in the path 1→5→8→10→11.The minimum value among arithmetic and geometric mean is the geometric mean that is 0.7599. Thus Geometric Mean is comparatively better than the Arithmetic Mean. Thus the optimum solution is given by 0.7599*100=759.9.

CONCLUSION
This method is simple when comparing to other existing methods for finding the shortest path. In both aggregation operations, bothin the arithmetic mean and geometric mean, the minimum value of the Sub-Trident Formsgive the shortest path as1→5→8→10→11. Also the minimum value among arithmetic and geometric mean is 0.7599. Thus Geometric Mean is comparatively better than the Arithmetic Mean.Thus the optimum solution is given by 0.7599*100=759.9.

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