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A MATHEMATICAL MODEL TO STUDY THE SIMILARITIES OF BLOOD FLUID MODELS THROUGH INCLINED MULTI-STENOSED ARTERY

Sapna Ratan Shah*, S. U. Siddiqui & Anuradha Singh****
* School of Computational and Integrative Sciences, Jawaharlal Nehru University, New Delhi
** Department of Mathematics, Harcourt Butler Technical University, Kanpur, Uttar Pradesh

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Abstract:

A mathematical model is presented to comparative steady of the flow behavior of Casson's and Bingham Plastic fluid model through an inclined tube of non-uniform cross-section with multiple stenoses. The equation describing the flow has been solved and the expressions parameters on flow variables have been studied. The present study may be helpful for better understanding the flow characteristics of blood having multiple stenoses. The graphical representations have been made to validate the analytical findings with a view of its applicability to stenotic diseases. It is found that the flow of resistance increases with the height of the stenosis but decreases with the angle of inclination. The flow characteristics namely, velocity, pressure gradient, flow rate, resistance to flow have been derived. It is shown that the resistance to flow increases with the height of the secondary stenosis as well as with the yield stress. The results are compared with the available data presented by previous researchers.

Key Words: Blood Flow, Blood Vessels, Axially Symmetric Stenosis, Casson's Fluid Model, Bingham Fluid Model & Inclined Tube

Introduction:

In medicine, one of the major health hazards is atherosclerosis, which is the leading cause of death in many countries. Atherosclerosis or stenosis is a cardiovascular disease, which refers to the narrowing of arterial lumen. i.e. the inner open space or cavity of an artery due to deposition of fatty substances. Stenosis leads to an increase in the resistance to the flow and associated reduction in blood supply in the downstream which causes hypertension, myocardial infarction and cerebral strokes etc. (Biswas and Ali [3], Kumar and Diwakar [7]). Hence it is essential to study the blood flow through a stenosed artery to prevent the arterial diseases.

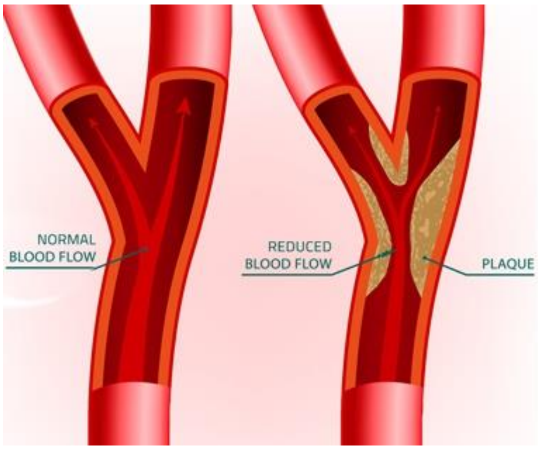


Figure 1: An inclined Blood vessel with multiple stenosis

In view of this, several theoretical and mathematical models (Agarwal and Varshney [1], Ellahi Rahman et al. [5]) have been developed to study the blood flow characteristics due to the presence of stenosis in the lumen of the blood vessel. Mohan et al. [10] studied the effect of paired stenosis through small artery. Bhatnagar et al. [2] studied the effects of an overlapping stenosis on blood flow characteristics in a narrow artery. All these investigations have considered the effects of stenosis through a tube of uniform cross-section. But, it is known that many ducts in physiological systems are not horizontal but have some inclination to the axis. Sankar et al. [13] studied the flow of Herschel-Bulkely fluid through an inclined tube of non-uniform cross section with multiple stenosis. Biswas and Chakraborty [4] have analysed mathematical models by considering blood as a Herschel-Bulkley type non- Newtonian fluid. In a recent paper Liepsch [8] have presented a mathematical model to show the effect of stenoses on Casson flow of blood. A mathematical model of blood flow through an irregular arterial mild stenoses is developed by Karimi [6] and studied that if the viscosity of fluid increases the velocity of fluid decreases in the presence of stenoses. Mekheimer and Elkot [9] developed a mathematical model for studying blood flow through a narrow artery with multiple stenoses and they have observed that stenoses height and axial velocity of flow very much influence the shear stress in a stenosed artery. In the present study we propose to discuss the effects of inclined multi-stenoses arteries on blood flow characteristics using Casson's and Bingham plastic fluid model. Results have been comparing for the both fluid model. Mathematical equations have been solved with the help of numerical technique.

Formulation of the Problem:

Let us consider an axially symmetric, laminar, fully develop flow of the blood through a tube of non-uniform cross-section and with two stenoses (fig.1).

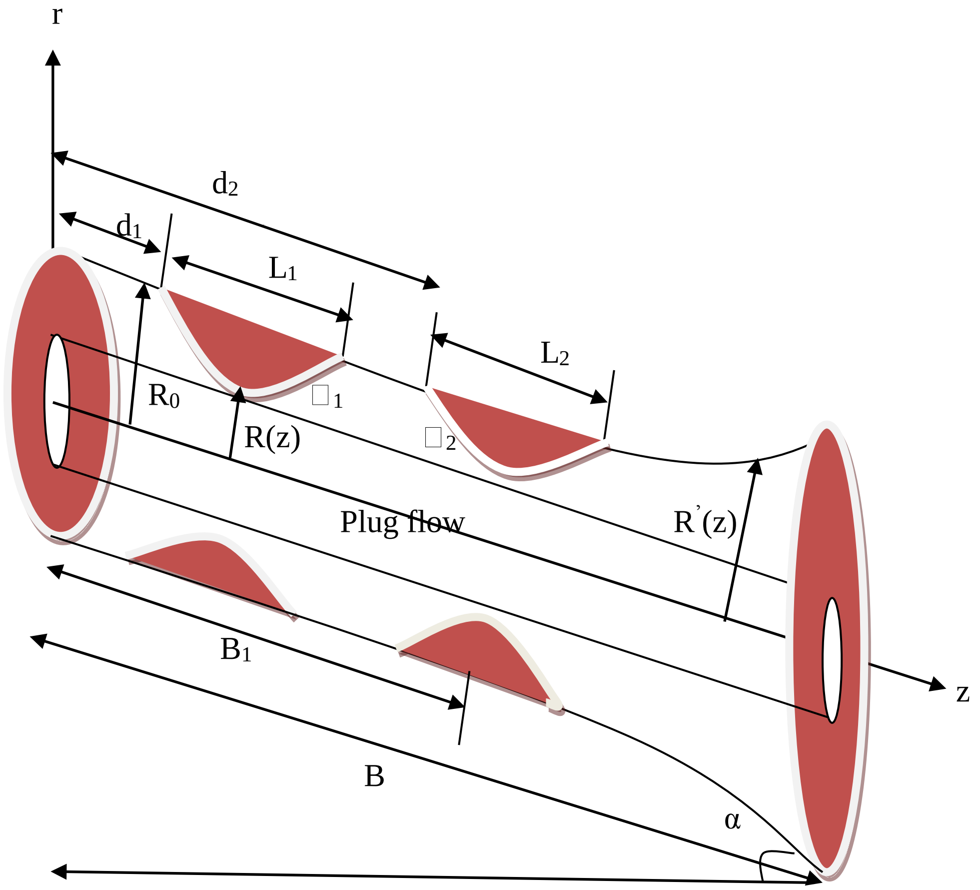


Figure 2: Geometry of an inclined tube with multiple stenosis

The momentum equation is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = -\frac{\partial p}{\partial z} + \frac{sin\alpha}{F}, \tag{1}$$

$$F = \frac{\mu U^n}{\rho g R_0^{n+1}} \tag{2}$$

Where τ_{rz} is the shear stress for the fluid,
The Casson Fluid model:

$$\tau_{rz}^{\frac{1}{2}} = \left[-\mu\frac{\partial u}{\partial r}\right]^{\frac{1}{2}} + \tau_0^{\frac{1}{2}}, \qquad \tau_{rz} > \tau_0 \tag{3}$$

$$\frac{\partial u}{\partial r} = 0, \tau_{rz} < \tau_0, \tag{4}$$

The Bingham Plastic Fluid model:

$$\tau_{rz} = -\mu\frac{\partial u}{\partial r} + \tau_0, \qquad \tau_{rz} > \tau_0 \tag{5}$$

$$\frac{\partial u}{\partial r} = 0, \ \tau_{rz} < \tau_0, \tag{6}$$

Here u is the axial velocity, p is the pressure, τ_0 is the yield stress, μ is the fluid viscosity, U is some characteristic velocity, ρ is the density, g is the acceleration due to gravity and R_0 is the radius of the tube. When

$\tau_{rz} < \tau_0$ i.e. the shear stress is less than the yield stress, there is a core region which flows as a plug (Fig. 1), and Eq. (2.4), (2.6) corresponds to vanishing velocity gradient in that region. However, the fluid behavior is indicated whenever $\tau_{rz} > \tau_0$.

The boundary conditions are:

$$\tau \text{ is finite at } r=0, \quad (7)$$

$$u=0 \text{ at } r=h(z). \quad (8)$$

For the analysis presented in the sequel, we use the following non-dimensional variable

$$\bar{z} = \frac{z}{L}, \bar{\delta} = \frac{\delta}{R_0}, \bar{R}(z) = \frac{R(z)}{R_0}, \bar{P} = \frac{p}{\left(\frac{\mu U L}{R_0^2}\right)}, \bar{\tau}_0 = \frac{\tau_0}{\mu\left(\frac{U}{R_0}\right)}, \bar{\tau}_{rz} = \frac{\tau_{rz}}{\mu\left(\frac{U}{R_0}\right)}, \bar{Q} = \frac{Q}{\pi R_0^2} \quad \bar{F} = \frac{F}{\mu U \lambda} \quad (9)$$

The geometry of the stenoses in non-dimensional form is given by

$$h = R(z) = \begin{cases} R_0 & : 0 \le z \le d_1, \\ R_0 - \frac{\delta_1}{2}\left(1 + \cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right) & : d_1 \le z \le L_1, \\ R_0 & : d_1 + L_1 \le z \le B_1 - \frac{L_2}{2}, \\ R_0 - \frac{\delta_1}{L_2}\left(1 + \cos\frac{2\pi}{L_2}(z - B_1)\right) & : B_1 - \frac{L_2}{2} \le z \le B_1, \\ R^*(z) - \frac{\delta_1}{2}\left(1 + \cos\frac{2\pi}{L_2}(z - B_1)\right) & : B_1 \le z \le B_1 + \frac{L_2}{2}, \\ R^*(z) & : B_1 + \frac{L_2}{2} \le z \le B, \end{cases} \quad (10)$$

The following restrictions for mild stenoses are supposed to be satisfied:

$$\delta_i \ll \min(R_0, R_{\text{out}}),$$

$$\delta_i \ll L_i, \quad \text{where } R_{\text{out}} = R(z) \text{ at } z = B.$$

Here L_i and δ_i are the lengths and maximum heights of two stenoses.

Analytical Solution of the Problem:

Solving Eqs. (1),(3) and (5) under the boundary conditions (7) and (8), we obtain the velocity, when $P = -\frac{\partial p}{\partial z}$, and $f = \frac{\sin\alpha}{F}$.

$$u = \frac{(P+f)}{\mu}\left[\frac{r^2-h^2}{4} + \frac{\tau_0}{(P+h)}(r-h) - \frac{2^2\sqrt{2}}{3}\left(r^{3/2} - h^{3/2}\right)\left(\frac{\tau_0}{(P+f)}\right)^{1/2}\right] \quad (11)$$

$$u = \frac{(P+f)}{\mu}\left[\frac{r^2-h^2}{4} + \frac{\tau_0}{(P+h)}(r-h)\right] \quad \text{for } r_0 \le r \le h \quad (12)$$

Velocity for the Casson's fluid model is given by Eq. (11) and the velocity for the Bingham Plastic fluid model is given by Eq. (12). Using the condition (14) and (16), we finally get the upper limit of the plug flow region (i.e the region between r = 0 and r = r_0 for which $|\tau_{rz}| < \tau_0$) as

$$r_0 = \frac{2\tau_0}{(P+f)} \quad (13)$$

And using the condition $\tau_{rz} = \tau_h$ at r = h, we obtain

$$\frac{r_0}{h} = \frac{\tau_0}{\tau_h} = \tau, \quad 0 < \tau < 1. \quad (14)$$

Taking r = r_0 in Eqs. (3.1), (3.2) we get the plug core velocity as

$$u_p = \frac{(P+f)}{\mu}\left[\frac{r_0^2}{12} - \frac{hr_0}{2} + \frac{2}{3}h^{3/2}r_0^{1/2} - \frac{h^2}{4}\right] \quad \text{for} \quad 0 \le r \le r_0. \quad (15)$$

$$u_p = \frac{(P+f)h^2}{\mu}\left[\frac{1}{4} + \frac{\tau_0^2}{2} - \frac{\tau_0}{2}\right] \quad \text{for } 0 \le r \le r_0. \quad (16)$$

Plug core velocity for the Casson's fluid model is given by Eq. (15) and the plug core velocity for the Bingham Plastic fluid model is given by Eq. (16).

The volumetric flow rate is defined by

$$Q = 2r\left[\int_0^{r_0} u_p r dr + \int_{r_0}^{h} u r dr\right] \quad (17)$$

Substituting Eq. (11, 12) and Eq. (15, 16) in Eq. (17) and integrating, we finally get

$$Q = \frac{h^3}{2\mu}(P+f)\left[-\frac{1}{12h}(\tau_0^2+1) + \tau_0^{\frac{5}{2}}h\left(4 - \tau_0^{\frac{1}{2}}\right) + \frac{\tau_0}{(P+f)}\left(\frac{4}{3}(1-\tau_0^3) - 2(1-\tau_0^2)\right) - \frac{\sqrt[2]{2}}{3}\left(\frac{\tau_0}{(P+f)}\right)^{\frac{1}{2}} h^{\frac{1}{2}}\left(\frac{1}{7}\left(1 - \tau_0^{\frac{7}{2}}\right) - \frac{1}{4}(1-\tau_0^2)\right)\right] \quad (18)$$

$$Q = \frac{h^3}{\mu}(P+f)\left[\tau_0^2 h\left(\frac{1}{4} + \frac{1}{4}\tau_0^2 - \frac{\tau_0}{2}\right) + \frac{h}{4(1-\tau_0)} - \frac{h}{8}(1-\tau_0^4) + \frac{2\tau_0}{(P+f)}\left(\frac{1}{3}(1-\tau_0^3) - \frac{1}{2}(1-\tau_0^2)\right)\right] \quad (19)$$

The volume flow rate for the Casson's fluid model is given by Eq. (18) and the volume flow rate for the Bingham Plastic fluid model is given by Eq. (19).

Using Eq. (18):

$$\frac{dp}{dz}=\left[\left(\frac{2\sqrt[2]{2}}{3}(\tau_0 h)^{\frac{1}{2}}\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)+\left(\left(\frac{2}{9}\tau_0 h\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)^2-4\left(-\frac{1}{12h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}h(4-\tau_0)\right)\left(\frac{\tau_0}{2}\left(\frac{4}{3}(1-\tau_0^3)-2(1-\tau_0^2)\right)-\frac{2\mu Q}{h^3}\right)\right)^{\frac{1}{2}}\right)/\left(-\frac{1}{6h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}\frac{h}{2}(4-\tau_0)\right)\right]^2-f \quad (20)$$

The pressure drop Δp across the stenosis between the cross-sections z = $\pm$ L/2 can be obtained by integrating Eq. (20) as

$$\Delta p=-\int_{-L/2}^{L/2}\left(\left[\left(\frac{2\sqrt[2]{2}}{3}(\tau_0 h)^{\frac{1}{2}}\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)+\left(\left(\frac{2}{9}\tau_0 h\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)^2-4\left(-\frac{1}{12h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}h(4-\tau_0)\right)\left(\frac{\tau_0}{2}\left(\frac{4}{3}(1-\tau_0^3)-2(1-\tau_0^2)\right)-\frac{2\mu Q}{h^3}\right)\right)^{\frac{1}{2}}\right)/\left(-\frac{1}{6h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}\frac{h}{2}(4-\tau_0)\right)\right]^2-f\right)dz \quad (21)$$

Using non-dimensional scheme in Eq. (21)

$$\Delta p=-\int_{-1}^{1}\left(\left[\left(\frac{2\sqrt[2]{2}}{3}(\tau_0 h)^{\frac{1}{2}}\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)+\left(\left(\frac{2}{9}\tau_0 h\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)^2-4\left(-\frac{1}{12h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}h(4-\tau_0)\right)\left(\frac{\tau_0}{2}\left(\frac{4}{3}(1-\tau_0^3)-2(1-\tau_0^2)\right)-\frac{2\mu Q}{h^3}\right)\right)^{\frac{1}{2}}\right)/\left(-\frac{1}{6h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}\frac{h}{2}(4-\tau_0)\right)\right]^2-f\right)dz \quad (22)$$

Using Eq. (19):

$$\frac{dp}{dz}=-\text{p}=\left[-Q\mu+2\tau_0\left(\frac{1}{3}h^3(1-\tau_0^3)-\frac{h^3}{2}(1-\tau_0^2)\right)/\tau_0^2h^4\left(\frac{1}{4}+\frac{1}{4}\tau_0^2-\frac{\tau_0}{2}+\frac{h^4}{4}(1-\tau_0^2)-\frac{h^4}{8}(1-\tau_0^4)\right)\right]+f \quad (23)$$

The pressure drop Δp across the stenosis between the cross-sections z = $\pm$ L/2 can be obtained by integrating Eq. (23) as

$$\Delta p=-\int_{-L/2}^{L/2}\left(\left[\left(\frac{2\sqrt[2]{2}}{3}(\tau_0 h)^{\frac{1}{2}}\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)+\left(\left(\frac{2}{9}\tau_0 h\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)^2-4\left(-\frac{1}{12h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}h(4-\tau_0)\right)\left(\frac{\tau_0}{2}\left(\frac{4}{3}(1-\tau_0^3)-2(1-\tau_0^2)\right)-\frac{2\mu Q}{h^3}\right)\right)^{\frac{1}{2}}\right)/\left(-\frac{1}{6h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}\frac{h}{2}(4-\tau_0)\right)\right]^2-f\right)dz \quad (24)$$

Using non-dimensional scheme in Eq.(24)

$$\Delta p=-\int_{-1}^{1}\left[-Q\mu+2\tau_0\left(\frac{1}{3}h^3(1-\tau_0^3)-\frac{h^3}{2}(1-\tau_0^2)\right)/\tau_0^2h^4\left(\frac{1}{4}+\frac{1}{4}\tau_0^2-\frac{\tau_0}{2}+\frac{h^4}{4}(1-\tau_0^2)-\frac{h^4}{8}(1-\tau_0^4)\right)\right]dz+f\,dz \quad (25)$$

The resistance to flow λ, is defined by

$$\lambda=\frac{\Delta p}{Q}. \quad (26)$$

Using Eqs. (22) and (25) in Eq. (26), we get

$$\lambda = -\frac{1}{Q}\int_{-1}^{1}\Bigg(\Bigg[\Bigg(\frac{2\sqrt{2}}{3}(\tau_0 h)^{\frac{1}{2}}\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)+\left(\frac{2}{9}\tau_0 h\left(\frac{1}{7}\left(1-\tau_0^{\frac{7}{2}}\right)-\frac{1}{4}(1-\tau_0^2)\right)^2-4\left(-\frac{1}{12h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}h(4-\tau_0)\right)\left(\frac{\tau_0}{2}\left(\frac{4}{3}(1-\tau_0^3)-2(1-\tau_0^2)\right)-\frac{2\mu Q}{h^3}\right)\right)^{\frac{1}{2}}\Bigg)/\left(-\frac{1}{6h}(\tau_0^2+1)+\tau_0^{\frac{3}{2}}\frac{h}{2}(4-\tau_0)\right)\Bigg]^2-f\Bigg)dz \quad (27)$$

$$\lambda = -\frac{1}{Q}\int_{-1}^{1}\left[-Q\mu+2\tau_0\left(\frac{1}{3}h^3(1-\tau_0^3)-\frac{h^3}{2}(1-\tau_0^2)\right)/\tau_0^2h^4\left(\frac{1}{4}+\frac{1}{4}\tau_0^2-\frac{\tau_0}{2}+\frac{h^4}{4}(1-\tau_0^2)-\frac{h^4}{8}(1-\tau_0^4)\right)\right]+f\,dz \quad (28)$$

The resistance to flow for the Casson's fluid model is given by Eq. (3.17) and the resistance to flow for the Bingham Plastic fluid model is given by Eq. (28).

Results and Discussion:

Stenosis is a serious cardiovascular disease. The irregular growth of stenosis affects the flow of blood in the arteries and which leads to serious circulatory disorders. Stenoses are formed by the accumulation of fats/ lipids on the inner wall of the arteries. Stenosis developed in the arteries can cause several diseases like blood pressure, atherosclerosis, heart attack and brain hemorrhage. The effects of various parameters on the resistance to flow are computed numerically by taking

$$\frac{R^*(z)}{R_0} = \exp[\beta B^2 (z - B_1)^2]$$

And $d_1 = L_1 = L_2 = 0.2$, $B_1 = 0.8$, $B = 1$, $\beta_1 = 0.01$.

It is observed that the resistance to flow increases with the height of both the primary and secondary stenosis (δ_1, δ_2). Figs.3-4 shows the variation of resistance to flow with respect to height of secondary stenosis for different values of inclination (α) and height of primary and secondary stenosis δ_1 and δ_2. Fig. shows that the resistance to flow increases with the height of secondary stenosis. It is also notice that the resistance to flow also decreases with the decreasing value of inclination (α). Fig. 5 shows the variation to resistance to flow with respect to the height of the secondary stenosis for different values of parameter F. the resistance to flow increases for the increasing values of height of the secondary stenosis for F = 0.5. Fig. 6 shows the variation of resistance to flow with respect to height of secondary stenoses. Figure shows that the resistance to flow increases with the increasing values of height of secondary stenoses. It is also shown that the resistance to flow increases with the yield stress. Fig. 7 shows the variation of resistance to flow with respect to height of secondary stenoses for different values of inclination (α) and height of primary stenoses taken $\delta 1 = 0$. Fig. shows that the resistance to flow increases with the height of secondary stenoses. It is also notice that the resistance to flow decreases with the increasing value of inclination (α). Fig. 8 shows the variation of resistance to flow with respect to height of secondary stenoses for different values of inclination (α) and height of primary stenoses taken $\delta_1 = 0.1$.

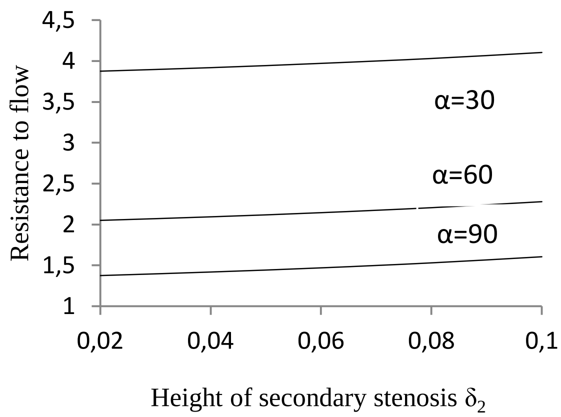


Figure 3: varition of resistance to flow with respect to height of secondary stenoses when F=0.1 and δ_1=0

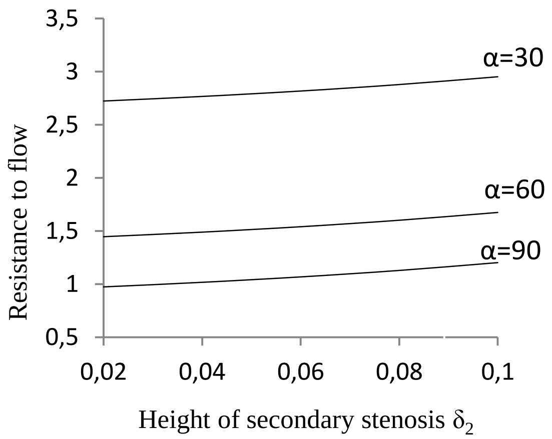
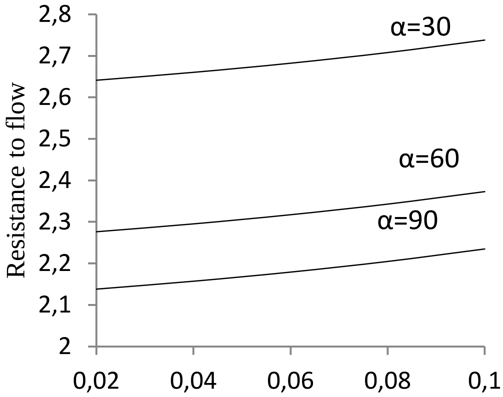
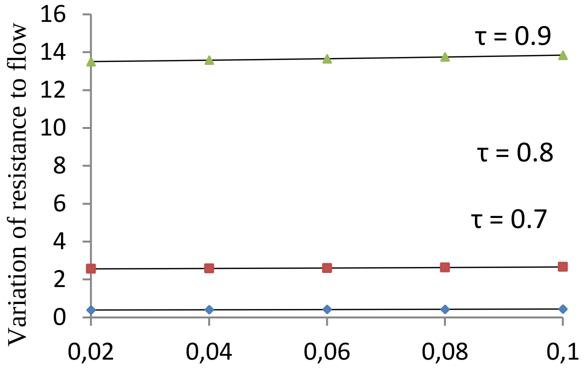


Figure 4: Variation of resistance to flow with respect to height of secondary stenosis when F=0.1 and δ_1=0.1



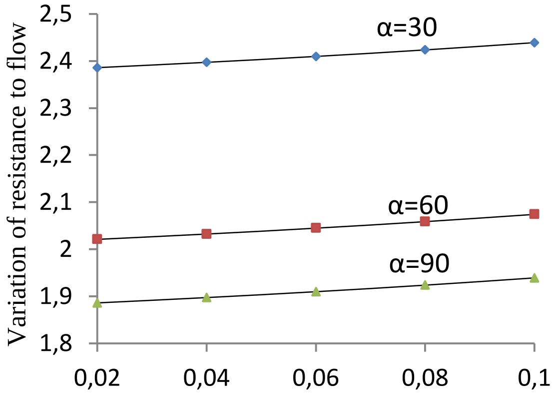
Height of secondary stenosis δ_2

Figure 5: Variation of resistence to flow with respect to heightof secondary stenosis when F = 0.5 and δ_1= 0



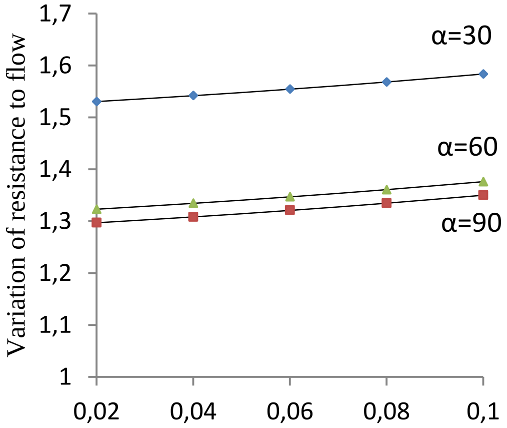
Height of secondary stenosis δ_2 when δ_1=0

Figure 6: Variation of resistance to flow with respect to δ_2



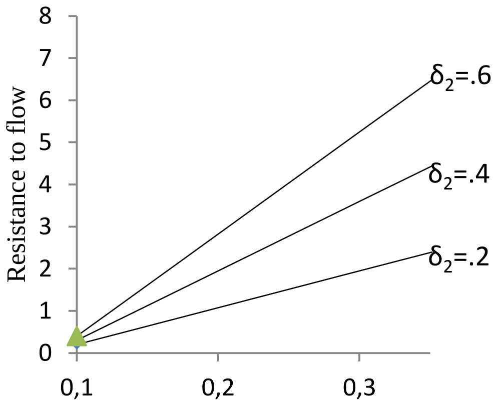
Height of secondary stenoses δ_2 when δ_1=0

Figure 7: Variation of resistance to flow with respect to δ_2



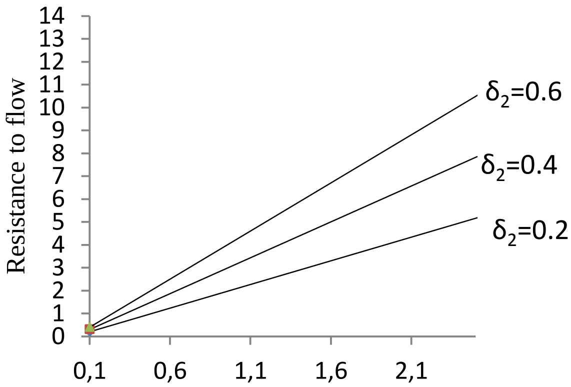
Height of secondary stenoses δ_2 when δ_1=0.1

Fig.ure 8: Variation of resistance to flow with respect to δ_2



Viscosity of blood

Figure 9: Variation of resistance to flow with respact to μ



Viscosity of blood

Figure 10: Variation of resistant to flow with respect to viscosity of blood

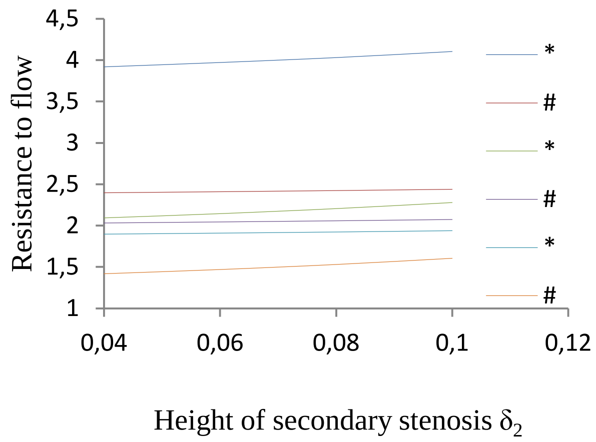


Figure 11: Variation of resistance to flow wit respect to δ_2 for different values of α When F=0.1, δ_1 = 0. * for Casson fluid and # for Bingham Plastic fluid

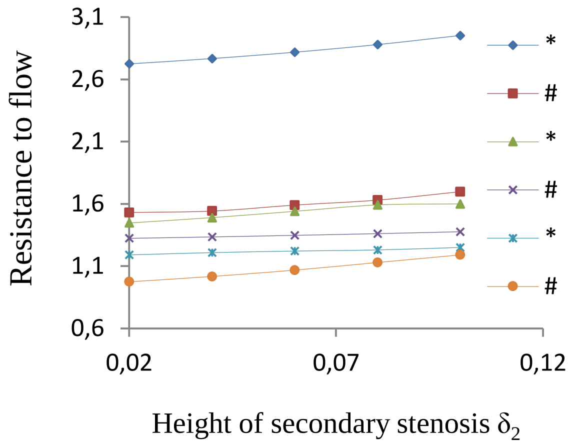


Figure 12: Variation of resistance to flow respect to δ_2 for different values of α, When δ_1 = 0.1. * for Casson fluid and # for Bing Plastic fluid.

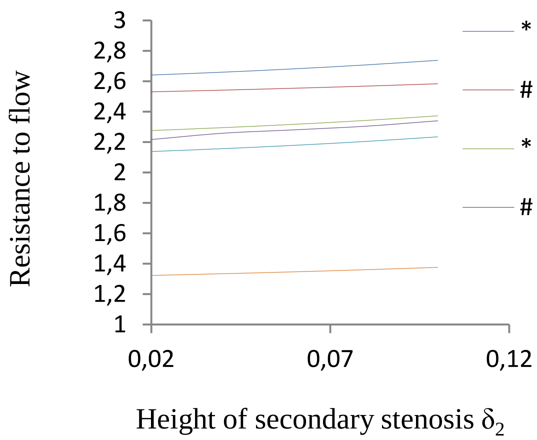


Fig.ure 13: Variation of resistance to flow respect to δ_2 for different values of α, When I δ_1 = 0. * for Casson fluid and # for Bingh Plastic fluid.

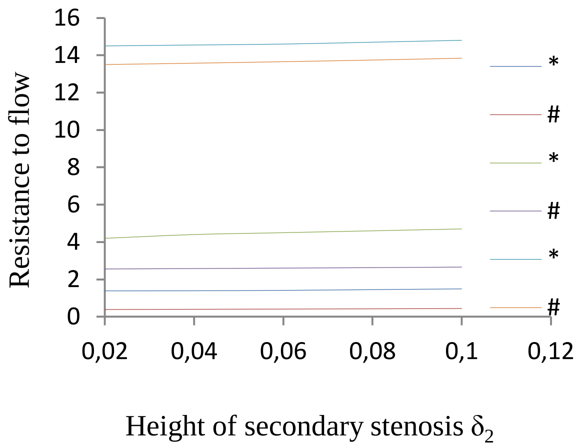


Fig.ure 14: Variation of resistance to flow with respect to δ_2 for different values of τ, when, δ_1 = 0. * for Casson fluid and # for Bingham Plastic fluid.

Figure shows that the resistance to flow increases with the height of secondary stenoses. It is also notice that the resistance to flow decreases with the increasing value of inclination (α). Figs. 9 Depicts the results for the resistance to flow with different values of viscosity of the blood for the Bingham Plastic fluid. Fig. 10 depicts the results for the resistance to flow with different values of viscosity of the blood for Casson's fluid. The resistance to flow increases with the increasing value of viscosity of blood. It is also shown in this figure that the resistance to flow increases for the increasing value of the height of the secondary stenoses. In is also found that the results for Casson's Fluid give higher results in comparison of Bingham Plastic fluid. Figs. 11-12, gives the results for resistance to flow with the height of the secondary stenosis for different values of α. It is found that the casson's fluid model gives higher results in comparison to Bingham Plastic fluid. It is found that resistance to flow increases with respect to increasing values of δ_2 and increases for the decreasing values of α for the both fluid models. Figs. 13-14, gives the results for resistance to flow with the height of the secondary stenosis for different values of α and τ. It is found that the casson's fluid model gives higher results in comparison to Bingham Plastic fluid. It is found that resistance to flow increases with respect to increasing values of δ_2 and τ. It is also found that the resistance to flow increses for the decreasing values of α for the both fluid models.

Conclusion:

Stenoses developed in the arteries can cause several diseases like blood pressure, atherosclerosis, heart attack and brain hemorrhage. A comparative study of Casson's and Bingham Plastic fluid through an inclined tube of non-uniform cross-section with multiple stenoses has been presented. Solutions have been obtained for primary and secondary stenoses. The resistance to flow increases as the secondary stenoses and viscosity of blood increases but the resistance to flow also decreases with the decreasing value of angle of inclination. It is found that the Casson's Fluid model gives more appropriate resuls in comparison of Bingham Plastic fluid model. This present study may be helpful for better understanding the flow characteristics of blood having multi-stenoses.

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