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On Neutrosophic Implications

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Abstract: In this paper, we firstly review the neutrosophic set, and then construct two new concepts called neutrosophic implication of type 1 and of type 2 for neutrosophic sets. Furthermore, some of their basic properties and some results associated with the two neutrosophic implications are proven.

Keywords: *Neutrosophic Implication, Neutrosophic Set, N-norm, N-conorm*.

1 Introduction

Neutrosophic set (NS) was introduced by Florentin Smarandache in 1995 [1], as a generalization of the fuzzy set proposed by Zadeh [2], interval-valued fuzzy set [3], intuitionistic fuzzy set [4], interval-valued intuitionistic fuzzy set [5], and so on. This concept represents uncertain, imprecise, incomplete and inconsistent information existing in the real world. A NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and with lies in] 0^-, 1^+ [, the non-standard unit interval.

NS has been studied and applied in different fields including decision making problems [6, 7, 8], Databases [10], Medical diagnosis problem [11], topology [12], control theory [13], image processing [14, 15, 16] and so on.

In this paper, motivated by fuzzy implication [17] and intutionistic fuzzy implication [18], we will introduce the definitions of two new concepts called neutrosophic implication for neutrosophic set.

This paper is organized as follow: In section 2 some basic definitions of neutrosophic sets are presented. In section 3, we propose some sets operations on neutrosophic sets. Then, two kind of neutrosophic implication are proposed. Finally, we conclude the paper.

2 Preliminaries

This section gives a brief overview of concepts of neutrosophic sets, single valued neutrosophic sets, neutrosophic norm and neutrosophic conorm which will be utilized in the rest of the paper.

Definition 1 (Neutrosophic set) [1]

Let X be a universe of discourse then, the neutrosophic set A is an object having the form:

$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$, where the functions $T, I, F : X \to\ ]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition.

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$, we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2 (Single-valued Neutrosophic sets) [20]

Let X be an universe of discourse with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, for each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

When X is continuous, an SVNS A can be written as

$$A= \int_X \frac{<T_A(x), I_A(x), F_A(x)>}{x}, x \in X. \qquad (2)$$

When X is discrete, an SVNS A can be written as

$$A= \sum_1^n \frac{<T_A(x_i), I_A(x_i), F_A(x_i)>}{x_i}, x_i \in X \qquad (3)$$

Definition 3 (Neutrosophic norm, n-norm) [19]

Mapping N_n: $(]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[)^2 \to]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$

$N_n(x(T_1, I_1, F_1), y(T_2, I_2, F_2)) = (N_nT(x,y), N_nI(x,y), N_nF(x,y))$, where

$N_nT(.,.), N_nI(.,.), N_nF(.,.)$

are the truth/membership, indeterminacy, and respectively falsehood/ nonmembership components.

N_n have to satisfy, for any x, y, z in the neutrosophic logic/set M of the universe of discourse X, the following axioms

a) Boundary Conditions: N_n (x, 0) = 0, N_n (x, 1) = x.
b) Commutativity: N_n (x, y) = N_n (y, x).
c) Monotonicity: If x ≤y, then N_n (x, z) ≤ N_n (y, z).
d) Associativity: N_n (N_n (x, y), z) = N_n (x, N_n (y, z)).

N_n represents the intersection operator in neutrosophic set theory.

Let J ∈ {T, I, F} be a component.

Most known N-norms, as in fuzzy logic and set the T-norms, are:

• The Algebraic Product N-norm: $N_{n-algebraic}$J(x, y) = x · y
• The Bounded N-Norm: $N_{n-bounded}$J(x, y) = max{0, x + y −1}
• The Default (min) N-norm: N_{n-min} (x, y) = min{x, y}.

A general example of N-norm would be this.

Let x(T_1, I_1, F_1) and y (T_2, I_2, F_2) be in the neutrosophic set M. Then:

N_n (x, y) = (T_1 ∧ T_2, I_1 ∨ I_2, F_1 ∨ F_2) (4)

where the "∧" operator is a N-norm (verifying the above N-norms axioms); while the "∨" operator, is a N-conorm.

For example, ∧ can be the Algebraic Product T-norm/N-norm, so $T_1 \wedge T_2 = T_1 \cdot T_2$ and ∨ can be the Algebraic Product T-conorm/N-conorm, so $T_1 \vee T_2 = T_1 + T_2 - T_1 \cdot T_2$

Or ∧ can be any T-norm/N-norm, and ∨ any T-conorm/N-conorm from the above.

Definition 4 (**Neutrosophic conorm, N-conorm) [19]**

Mapping N_c: ( ]-0,1+[×]-0,1+[×]-0,1+[ )2→]-0,1+[×]-0,1+[×]-0,1+[

N_c (x(T_1, I_1, F_1), y(T_2, I_2, F_2)) = (N_cT(x,y), N_cI(x,y), N_cF(x,y)),

where N_cT(.,.),N_cI(.,.),N_cF(.,.) are the truth/membership, indeterminacy, and respectively falsehood/non membership components.

N_c have to satisfy, for any x, y, z in the neutrosophic logic/set M of universe of discourse X, the following axioms:

a) Boundary Conditions: N_c (x, 1) = 1, N_c (x, 0) = x.
b) Commutativity: N_c (x, y) = N_c (y, x).
c) Monotonicity: if x ≤y, then N_c (x, z) ≤ N_c (y, z).
d) Associativity: N_c (N_c (x, y), z) = N_c (x, N_c (y, z))

N_c represents respectively the union operator in neutrosophic set theory.

Let J ∈ {T, I, F} be a component. Most known N-conorms, as in fuzzy logic and set the T-conorms, are:

• The Algebraic Product N-conorm: $N_{c-algebraic}$ J(x, y) = x + y −x · y
• The Bounded N-conorm: $N_{c-bounded}$ J(x, y) = min{1, x + y}
• The Default (max) N-conorm: N_{c-max} J(x, y) = max{x, y}.

A general example of N-conorm would be this.

Let x(T_1, I_1, F_1) and y(T_2, I_2, F_2) be in the neutrosophic set/logic M. Then:

N_c (x, y) = (T1∨T2, I1∧I2, F1∧F2) (5)

where the "∧" operator is a N-norm (verifying the above N-conorms axioms); while the "∨" operator, is a N-norm.

For example, ∧ can be the Algebraic Product T-norm/N-norm, so T1∧T2= T1·T2 and ∨ can be the Algebraic Product T-conorm/N-conorm, so T1∨T2= T1+T2-T1·T2.

Or ∧ can be any T-norm/N-norm, and ∨ any T-conorm/N-conorm from the above.

In 2013, A. Salama [21] introduced beside the intersection and union operations between two neutrosophic set A and B, another operations defined as follows:

Definition 5

Let A, B two neutrosophic sets

A $\cap_1$ B = min (T_A , T_B) ,max (I_A , I_B) , max(F_A , F_B)
A $\cup_1$ B = (max (T_A , T_B) , max (I_A , I_B) ,min(F_A , F_B))
A $\cap_2$ B={ min (T_A , T_B), min (I_A , I_B), max (F_A , F_B)}
A $\cup_2$ B = (max (T_A , T_B) , min (I_A , I_B) ,min(F_A , F_B))
A^C= (F_A , I_A , T_A).

Remark

For the sake of simplicity we have denoted:

$\cap_2$ = min min max, $\cup_2$ = max min min
$\cap_1$ = min max max, $\cup_1$ = max max min.

Where $\cap_1$, $\cup_2$ represent the intersection set and the union set proposed by Florentin Smarandache and $\cap_2$, $\cup_1$ represent the intersection set and the union set proposed by A.Salama.

3 Neutrosophic Implications

In this subsection, we introduce the set operations on neutrosophic set, which we will work with. Then, two neutrosophic implication are constructed on the basis of single valued neutrosophic set .The two neutrosophic implications are denoted by $_{NS1}$ and $_{NS2}$. Also, important properties of $_{NS1}$ and $_{NS2}$ are demonstrated and proved.

Definition 6 (Set Operations on Neutrosophic sets)

Let A and B two neutrosophic sets , we propose the following operations on NSs as follows:

$A\ @\ B = (\frac{T_A + T_B}{2}, \frac{I_A + I_B}{2}, \frac{F_A + F_B}{2})$ where $< T_A, I_A, F_A > \in A$,$< T_B, I_B, F_B > \in B$

$A\ \$\ B = (\overline{T_A T_B}, \overline{I_A I_B}, \overline{F_A F_B})$,where $< T_A, I_A, F_A > \in A$,$< T_B, I_B, F_B > \in B$

$A\ \#\ B = (\frac{2 T_A T_B}{T_A + T_B}, \frac{2 I_A I_B}{I_A + I_B}, \frac{2 F_A F_B}{F_A + F_B})$, where $< T_A, I_A, F_A > \in A$,$< T_B, I_B, F_B > \in B$

$A \oplus$ B=(T_A +T_B-T_A T_B , I_A I_B, F_A F_B) ,where $< T_A, I_A, F_A > \in A$,$< T_B, I_B, F_B > \in B$

$A \otimes$ B= (T_B T_A , I_A+I_B-I_A I_B, F_A +F_B-F_A F_B), where $< T_A, I_A, F_A > \in A$,$< T_B, I_B, F_B > \in B$

Obviously, for every two A and B, (A @ B), (A $ B), (A# B) , $A\oplus$ B and $A\otimes$ B are also NSs.
Based on definition of standard implication denoted by "A → B", which is equivalent to "non A or B". We extended it for neutrosophic set as follows:

Definition 7
Let A(x) ={<x, $T_A(x)$, $I_A(x)$, $F_A(x)$> | **x** ∈ **X**} and B(x) ={<x, $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X} , A, B ∈ NS(X). So, depending on how we handle the indeterminacy, we can defined two types of neutrosophic implication, then ${}_{NS1}$ is the neutrosophic type1 defined as
A ${}_{NS1}$ B ={< x, $F_A(x) \vee T_B(x)$, $I_A(x) \wedge I_B(x)$, $T_A(x) \wedge F_B(x)$ > | x ∈ X} (6)
And
${}_{NS2}$ is the neutrosophic type 2 defined as
A ${}_{NS2}$ B =={< x, $F_A(x) \vee T_B(x)$, $I_A(x) \vee I_B(x)$, $T_A(x) \wedge F_B(x)$ > | x ∈ X} (7)
by ∨ and ∧ we denote a neutrosophic norm (N-norm) and neutrosophic conorm (N-conorm).

Note: The neutrosophic implications are not unique, as this depends on the type of functions used in N-norm and N-conorm**.**
Throughout this paper, we used the function (dual) min/ max.

Theorem 1
For A, B and C ∈ NS(X),

i. A $\cup_1$ B ${}_{NS1}$ C =(A ${}_{NS1}$ C) $\cap_1$ (B ${}_{NS1}$ C)
ii. A ${}_{NS1}$ B $\cap_1$ C =(A ${}_{NS1}$ B) $\cap_1$ (A ${}_{NS1}$ C)
iii. A $\cap_1$ B ${}_{NS1}$ C = (A ${}_{NS1}$ C) $\cup_1$ (B ${}_{NS1}$ C)
iv. A ${}_{NS1}$ B $\cup_1$ C =(A ${}_{NS1}$ B)$\cup_1$ (A ${}_{NS1}$ C)

Proof
(i) From definition in (5) ,we have
A $\cup_1$ B ${}_{NS1}$ C ={<x ,Max(min(F_A , F_B), T_C) , Min(max (I_A, I_B), I_C) , Min(max (T_A , T_B), F_C) >| x ∈ X} (8)
and
(A ${}_{NS1}$ C) $\cap_1$ (B ${}_{NS1}$ C)= {<x, Min(max(F_A , T_C), max(F_B, T_C)) , Max (min (I_A , I_C), min (I_B , I_C)), Max(min (T_A , F_C), min (T_B , F_C)) >| x ∈ X} (9)
Comparing the result of (8) and (9), we get
Max(min(F_A , F_B), T_C)= Min(max(F_A , T_C), max(F_B , T_C))
Min(max (I_A , I_B), I_C)= Max (min (I_A , I_C), min (I_B , I_C))
Min(max (T_A , T_B), F_C)= Max**(**min (T_A , F_C), min (T_B , F_C))

Hence, A $\cup_1$ B ${}_{NS1}$ C = (A ${}_{NS1}$ C) $\cap_1$ (B ${}_{NS1}$ C)

(ii) From definition in (5), we have
A ${}_{NS1}$ B $\cap_1$ C={Max(F_A, min(T_B , T_C)) , Min(I_A ,max (I_B, I_C)) , Min(T_A, max (F_B , F_C) >| x ∈ X} (10)
and (A ${}_{NS1}$ B) $\cap_1$ (A ${}_{NS1}$ C) = {<x, Min (max (F_A , T_B), max(F_A , T_C)) , Max (min (I_A , I_B), min (I_A , I_C)), Max(min (T_A , F_B), min (T_A , F_C) >| x ∈ X}
(11)
Comparing the result of (10) and (11), we get
Max(F_A, min(T_B , T_C))= Min(max(F_A , T_B), max(F_A , T_C))
Min(I_A ,max (I_B , I_C))= Max (min (I_A , I_B), min (I_A , I_C))
Min(T_A, max (F_B , F_C)= Max(min (T_A , F_B), min (T_A , F_C))
Hence, A $\cap_1$ B ${}_{NS1}$ C = (A ${}_{NS1}$ C) $\cup_1$ (B ${}_{NS1}$ C)

(iii) From definition in (5), we have
A $\cap_1$ B ${}_{NS1}$ C ={< x , Max(max(F_A , F_B), T_C) , Min(min(I_A , I_B), I_C) , Min(min (T_A , T_B), F_C) >| x ∈ X} (12)
and
(A ${}_{NS1}$ C)$\cup_1$ (B ${}_{NS1}$ C) = {<x, Max(max(F_A , T_C), max(F_B , T_C)) , Max (min (I_A , I_C), min (I_B , I_C)), Min(min (T_A , F_C), min (T_B , F_C)) >| x ∈ X}
(13)
Comparing the result of (12) and (13), we get
Max(max(F_A , F_B), T_C)= Max(max(F_A , T_C), max(F_B , T_C))
Min(min(I_A , I_B), I_C)= Max (min (I_A , I_C), min (I_B, I_C))
Min(min (T_A , T_B), F_C)= Min(min (T_A , F_C), min (T_B , F_C)),
Hence, A $\cap_1$ B ${}_{NS1}$ C = (A ${}_{NS1}$ C) $\cup_1$ (B ${}_{NS1}$ C)
(iv) From definition in (5), we have
A ${}_{NS1}$ B $\cup_1$ C ={<x, Max (F_A ,Max (T_B , T_C)), Min (I_A , Max (I_B , I_C)), Min(T_A, Min(F_B , F_C)) >| x ∈ X} (14)
and
(A ${}_{NS1}$ B) $\cup_1$ (A ${}_{NS1}$ C) = {<x, Max(max (F_A, T_B), max(F_A , T_C)) , Max (min (I_A , I_B), min (I_A , I_C)), Min(min (T_A , F_B), min (T_A , F_C)) >| x ∈ X} (15)
Comparing the result of (14) and (15), we get
Max (F_A , Max (T_B , T_C)) = Max(max(F_A , T_B), max(F_A , T_C))
Min (I_A , Max (I_B , I_C)) = Max (min (I_A , I_B), min (I_A , I_C))
Min (T_A, Min(F_B , F_C)) = Min(min (T_A , F_B), min (T_A , F_C))
hence, A ${}_{NS1}$ B $\cup_1$ C = (A ${}_{NS1}$ B)$\cup_1$ (A ${}_{NS1}$ C)

In the following theorem, we use the operators: $\cap_2$ = min min max , $\cup_2$ = max min min.

Theorem 2 For A, B and C ∈ NS(X),

i. A $\cup_2$ B ${}_{NS1}$ C =(A ${}_{NS1}$ C) $\cap_2$ (B ${}_{NS1}$ C)

ii. A $_{NS1}$ B $\cap_2$ C =(A $_{NS1}$ B) $\cap_2$ (A $_{NS1}$ C)
iii. A $\cap_2$ B $_{NS1}$ C = (A $_{NS1}$ C) $\cup_2$ (B $_{NS1}$ C)
iv. A $_{NS1}$ B $\cup_2$ C =(A $_{NS1}$ B) $\cup_2$ (A $_{NS1}$ C)

Proof
The proof is straightforward.
In view of A $_{NS2}$ B ={< x, $F_A \vee T_B$, $I_A \vee I_B$, $T_A \wedge F_B$ >| x ∈ X} , we have the following theorem:

Theorem 3
For A, B and C ∈ NS(X),

i. A $\cup_1$ B $_{NS2}$ C =(A $_{NS2}$ C) $\cap_1$ (B $_{NS2}$ C)
ii. A $_{NS2}$ B $\cap_1$ C =(A $_{NS2}$ B) $\cap_1$ (A $_{NS2}$ C)
iii. A ∩ B $_{NS2}$ C = (A $_{NS2}$ C) $\cup_1$ (B $_{NS2}$ C)
iv. A $_{NS2}$ B $\cup_1$ C =(A $_{NS2}$ B)$\cup_1$ (A $_{NS2}$ C)

Proof
(i) From definition in (5), we have
A $\cup_1$ B $_{NS2}$ C ={<x, Max(min(F_A , F_B), T_C), Max(max (I_A, I_B), I_C) , Min(max (T_A , T_B), F_C) >| x ∈ X} (16)
and
(A $_{NS2}$ C)$\cap_1$ (B $_{NS2}$ C)= {<x, Min(max(F_A , T_C), max(F_B , T_C)) , Max (max (I_A , I_C), max (I_B , I_C)), Max(min (T_A , F_C), min (T_B , F_C)) >| x ∈ X} (17)
Comparing the result of (16) and (17), we get
Max(min(F_A , F_B), T_C)= Min(max(F_A , T_C), max(F_B , T_C))
Max(max (I_A , I_B), I_C)= Max (max(I_A , I_C), max (I_B , I_C))
Min(max (T_A , T_B), F_C)= Max(min (T_A , F_C), min (T_B , F_C))
hence, A $\cup_1$ B $_{NS2}$ C = (A $_{NS2}$ C) $\cap_1$ (B $_{NS2}$ C)
(ii) From definition in (5) ,we have
A $_{NS2}$ B $\cap_1$ C={<x ,Max(F_A, min(T_B , T_C)) , Max(I_A , max (I_B , I_C)) , Min(T_A, max (F_B , F_C) >| x ∈ X} (18)
and
(A $_{NS2}$ B) ∩ (A $_{NS2}$ C) = {<x,Min(max(F_A , T_B), max(F_A , T_C)) , Max (max (I_A , I_B),max (I_A , I_C)), Max(min (T_A , F_B), min (T_A , F_C)) >| x ∈ X} (19)
Comparing the result of (18) and (19), we get
Max(F_A, min(T_B , T_C))= Min(max(F_A , T_B), max(F_A , T_C))
Max(I_A ,max (I_B , I_C))= Max (max(I_A , I_B), max (I_A , I_C))
Min(T_A, max (F_B , F_C)= Max(min (T_A , F_B), min (T_A , F_C))
Hence , A $_{NS2}$ B$\cap_1$ C =(A $_{NS2}$ B) $\cap_1$ (A $_{NS2}$ C)
(iii) From definition in (5), we have
A $\cap_1$ B $_{NS2}$ C ={<x, Max(max(F_A , F_B), T_C) , Max(max (I_A, I_B), I_C) , Min(min (T_A , T_B), F_C) >| x ∈ X} (20)
and
(A $_{NS2}$ C) $\cup_1$ (B $_{NS2}$ C) = {Max(max(F_A , T_C), max(F_B, T_C)) , Max (max (I_A , I_C), max (I_B , I_C)), Min(min (T_A , F_C), min (T_B , F_C)) } (21)
Comparing the result of (20) and (21), we get
Max(max(F_A , F_B), T_C)= Max(max(F_A , T_C), max(F_B , T_C))
Max(max(I_A , I_B), I_C)= Max (max (I_A , I_C), max (I_B , I_C))
Min(min (T_A , T_B), F_C)= Min(min (T_A , F_C), min (T_B , F_C)),
hence, A $\cap_1$ B $_{NS2}$ C = (A $_{NS2}$ C) $\cup_1$ (B $_{NS2}$ C)
(iv) From definition in (5) ,we have
A $_{NS2}$ B $\cup_1$ C ={<x, Max (F_A , Max (T_B , T_C)), Max (I_A , Max (I_B , I_C)) , Min (T_A, Min(F_B , F_C))> | x ∈ X} (22)
and
(A $_{NS2}$ B) $\cup_1$ (A $_{NS2}$ C)= Max(max(F_A , T_B), max(F_A , T_C)) , Max (max (I_A , I_B), max (I_A , I_C)), Min(min (T_A , F_B), min (T_A , F_C)) (23).
Comparing the result of (22) and (23), we get
Max (F_A , Max (T_B , T_C)) = Max(max(F_A , T_B), max(F_A , T_C))
Max (I_A , Max (I_B , I_C)) = Max (max (I_A , I_B), max (I_A , I_C))
Min (T_A, Min(F_B , F_C)) = Min(min (T_A , F_B), min (T_A , F_C))
hence , A $_{NS2}$ B $\cup_1$ C =(A $_{NS2}$ B)$\cup_1$ (A $_{NS2}$ C)
Using the two operators $\cap_2$ = min min max ,
$\cup_2$ = max min min, we have

Theorem 4
For A, B and C ∈ NS(X),

i. A $\cup_2$ B $_{NS2}$ C =(A $_{NS2}$ C) $\cap_2$ (B $_{NS2}$ C)
ii. A $_{NS2}$ B $\cap_2$ C =(A $_{NS2}$ B) $\cap_2$ (A $_{NS2}$ C)
iii. A $\cap_2$ B $_{NS2}$ C = (A $_{NS2}$ C) $\cup_2$ (B $_{NS2}$ C)
iv. A $_{NS2}$ B $\cup_2$ C =(A $_{NS2}$ B)$\cup_2$ (A $_{NS2}$ C)

Proof
The proof is straightforward.

Theorem 5
For A, B ∈ NS(X),

i. A $_{NS2}$ B^C = A^C $\cup_1$ B^C
ii. (A $_{NS2}$ $B^C)^c$ =(A^C $\cup_1$ B^C $)^c$ = A $\cap_1$ B
iii. (A $_{NS1}$ $B^C)^c$ = A $\cap_2$ B
iv. A^C $_{NS1}$ B = A $\cup_2$ B
v. A $_{NS1}$ B^C = (A $\cap_2$ B $)^c$

Proof
(i) From definition in (5) ,we have

A $_{NS2}$ B^C ={<x, max (F_A , F_B) ,min (I_A , I_B) , min (T_A , T_B) | x ∈ X} (24)

and
A^C $\cup_1$ B^C ={ max (F_A , F_B) ,min (I_A , I_B) , min (T_A, T_B)} (25)

From (24) and (25), we get $A \;_{NS2}\; B^C = A^C \cup_1 B^C$

(ii) From definition in (5), we have

$A^C \cup_1 B^C$ ={<x, max (F_A , F_B) ,min (I_A , I_B) , min (T_A, T_B) > | x ∈ X} (26)

and

($A^C \cup_1 B^C$ $)^c$= {<x, min (T_A , T_B),min (I_A , I_B) ,max (F_A , F_B) > | x ∈ X}
(27)

From (26) and (27), we get $(A \;_{NS2}\; B^C)^c$ =($A^C \cup_1 B^C$ $)^c$= A $\cap_1$ B

(iii) From definition in (5) ,we have

$(A \;_{NS1}\; B^C)^c$ ={ <x, min (T_A , T_B), min (I_A , I_B), max (F_A, F_B) > | x ∈ X} (28)

and

A $\cap_2$ B={ min (T_A , T_B), min (I_A , I_B), max (F_A , F_B)} (29)

From (28) and (29), we get $(A \;_{NS1}\; B^C)^c$ = A $\cap_2$ B

(iv)

$A^C \;_{NS1}\;$ B = $A \cup_2 B$ ={ <x, max (T_A , T_B) , min (I_A , I_B), min (F_A , F_B) > | x ∈ X}

(v)

$A \;_{NS1}\; B^C$ ={<x, max (F_A , F_B),min (I_A , I_B) , max (T_A , T_B) > | x ∈ X} (30)

and

$(A \cap_2 B)^c$ ={<x, max (F_A , F_B),min (I_A , I_B) , max (T_A, T_B) > | x ∈ X} (31)

From (30) and (31), we get $A \;_{NS1}\; B^C = (A \cap_2 B)^c$

Theorem 6

For A, B ∈ NS(X),

i. $(A \quad B)^c \;_{NS1}\; (A @ B) = (A@B)^c \;_{NS1}\; (A \quad B) = (A \oplus B)$

ii. $(A \otimes B)^c \;_{NS1}\; (A @ B) = (A@B)^c \;_{NS1}\; (A \otimes B) = (A@ B)$

iii. $(A \otimes B)^c \;_{NS1}\; (A \# B) = (A\# B)^c \;_{NS1}\; (A \otimes B) = (A\# B)$

iv. $(A \oplus B)^c \;_{NS1}\; (A \$ B) = (A\$ B)^c \;_{NS1}\; (A \oplus B) = (A \oplus B)$

v. $(A \otimes B)^c \;_{NS1}\; (A \$ B) = (A\$ B)^c \;_{NS1}\; (A \otimes B) = (A \$ B)$

vi. $(A \otimes B)^c \;_{NS1}\; (A \oplus B) = (A \oplus B)^c \;_{NS1}\; (A \otimes B) = (A \oplus B)$

Proof

Let us recall following simple fact for any two real numbers a and b.

Max(a, b) +Min(a, b) = a +b.

Max(a, b) x Min(a, b) = a x b.

(i) From definition in (6), we have

$(A \oplus B)^c \;_{NS1}\; (A @ B)$ = {<x,Max(T_A +T_B-T_A T_B, $\frac{T_A+T_B}{2}$) ,Min(I_A I_B, $\frac{I_A+I_B}{2}$) ,Min(F_A F_B, $\frac{F_A+F_B}{2}$) > | x ∈ X} = (T_A +T_B-T_A T_B, I_A I_B, F_A F_B)

= $A \oplus B$ (32)

and

$(A@B)^c \;_{NS1}\; (A \oplus B)$ = ($\frac{F_A+F_B}{2}$, $\frac{I_A+I_B}{2}$, $\frac{T_A+T_B}{2}$) $_{NS1}$ (T_A +T_B-T_A T_B , I_A I_B, F_A F_B)

= {<x, Max($\frac{T_A+T_B}{2}$, T_A +T_B-T_A T_B) ,Min($\frac{I_A+I_B}{2}$,I_A I_B) ,Min($\frac{F_A+F_B}{2}$, F_A F_B) > | x ∈ X} (33)

=(T_A +T_B-T_A T_B, I_A I_B, F_A F_B)

= $A \oplus B$

From (32) and (33), we get the result (i)

(ii) From definition in (6), we have

$(A \otimes B)^c = (T_B\, T_A\,, I_A + I_B - I_A\, I_B, F_A\, + F_B - F_A\, F_B)^c = (F_A\, + F_B - F_A\, F_B, I_A + I_B - I_A\, I_B, T_B\, T_A)$

$(A \otimes B)^c \;_{NS1}\; (A @ B)$ =

$= (F_A\, + F_B - F_A\, F_B, I_A + I_B - I_A\, I_B, T_B\, T_A) \;_{NS1}\; (\frac{T_A+T_B}{2}, \frac{I_A+I_B}{2}, \frac{F_A+F_B}{2})$

={<x, Max (T_B T_A, $\frac{T_A+T_B}{2}$) ,Min($I_B + I_B - I_A\, I_B$, $\frac{I_A+I_B}{2}$) ,Min($F_A\, + F_B - F_A\, F_B$, $\frac{F_A+F_B}{2}$) > | x ∈ X}

=($\frac{T_A+T_B}{2}$, $\frac{I_A+I_B}{2}$, $\frac{F_A+F_B}{2}$) = (A @ B) (34)

and

$(A@B)^c \;_{NS1}\; (A \otimes B)$ =

=($\frac{F_A+F_B}{2}$, $\frac{I_A+I_B}{2}$, $\frac{T_A+T_B}{2}$) $_{NS1}$(T_A T_B, I_A +I_B-I_A I_B, F_A +F_B-F_A F_B)

={< x, Max($\frac{T_A+T_B}{2}$, T_A T_B), Min ($\frac{I_A+I_B}{2}$, I_A +I_B-I_A I_B), Min ($\frac{F_A+F_B}{2}$, F_A +F_B-F_A F_B) | x ∈ X}

=($\frac{T_A+T_B}{2}$, $\frac{I_A+I_B}{2}$, $\frac{F_A+F_B}{2}$) = (A @ B) (35)

From (34) and (35), we get the result (ii)

(iii) From definition in (6) ,we have

$(A \otimes B)^c \;_{NS1}\; (A \# B)$ =($F_A\, + F_B - F_A\, F_B, I_A + I_B - I_A\, I_B, T_B\, T_A$) $_{NS1}$ ($\frac{2\,T_A T_B}{T_A+T_B}$, $\frac{2\,I_A I_B}{I_A+I_B}$, $\frac{2\,F_A F_B}{F_A+F_B}$)

= {<x , Max (T_B T_A , $\frac{2\,T_A T_B}{T_A+T_B}$) ,Min($I_A + I_B - I_A\, I_B$, $\frac{2\,I_A I_B}{I_A+I_B}$), Min($F_A\, + F_B - F_A\, F_B$, $\frac{2\,F_A F_B}{F_A+F_B}$) > | x ∈ X}

=($\frac{2\,T_A T_B}{T_A+T_B}$, $\frac{2\,I_A I_B}{I_A+I_B}$, $\frac{2\,F_A F_B}{F_A+F_B}$)

=($A\# B$) (36)

and

$(A\# B)^c \;_{NS1}\; (A \otimes B)$=($\frac{2\,F_A F_B}{F_A+F_B}$, $\frac{2\,I_A I_B}{I_A+I_B}$, $\frac{2\,T_A T_B}{T_A+T_B}$) $_{NS1}$ (T_B T_A , I_A+I_B-I_A I_B, F_A +F_B-F_A F_B)

={<x, Max ($\frac{2\,T_A T_B}{T_A+T_B}$, T_B T_A) ,Min($\frac{2\,I_A I_B}{I_A+I_B}$, I_A+I_B-I_A I_B), Min($\frac{2\,F_A F_B}{F_A+F_B}$, F_A +F_B-F_A F_B) > | x ∈ X}

$= \left(\frac{2T_A T_B}{T_A + T_B}, \frac{2I_A I_B}{I_A + I_B}, \frac{2F_A F_B}{F_A + F_B}\right) = (A\#\ B)$ (37)

From (36) and (37), we get the result (iii).

(iv) From definition in (6), we have

$(A \oplus B)^c\ _{NS1}\ (A\ \$\ B) = (F_A F_B,\ , I_A I_B, T_A + T_B - T_A T_B)\ _{NS1}(\overline{T_A T_B}, \overline{I_A I_B}, \overline{F_A F_B})$

$= \{<x, \text{Max}\ (T_A + T_B - T_A T_B, \overline{T_A T_B}), \text{Min}(I_A I_B, \overline{I_A I_B},), \text{Min}(F_A F_B, \overline{F_A F_B}) > |\ x \in X\}$

$= (\overline{T_A T_B}, \overline{I_A I_B}, \overline{F_A F_B})$

$= (A\ \$\ B)$ (38)

and

$(A\$ B)^c\ _{NS1}\ (A \oplus B) = (\overline{F_A F_B}, \overline{I_A I_B}, \overline{T_A T_B})\ _{NS1}(T_A + T_B - T_A T_B, I_A I_B, F_A F_B)$

$= \{<x, \text{Max}\ (\overline{T_A T_B}, T_A + T_B - T_A T_B), \text{Min}(\overline{I_A I_B}, I_A I_B), \text{Min}(\overline{F_A F_B}, F_A F_B) > |\ x \in X\}$

$= (\overline{T_A T_B}, \overline{I_A I_B}, \overline{F_A F_B})$

$= (A\ \$\ B)$ (39)

From (38) and (39), we get the result (iv).

(v) From definition in (6), we have

$(A \otimes B)^c\ _{NS1}\ (A\ \$\ B) = (F_A + F_B - F_A F_B, I_A + I_B - I_A I_B, T_B T_A)\ _{NS1}\ (\overline{T_A T_B}, \overline{I_A I_B}, \overline{F_A F_B})$

$= \{<x, \text{Max}\ (T_B T_A, \overline{T_A T_B}), \text{Min}(I_A + I_B - I_A I_B, \overline{I_A I_B}), \text{Min}(F_A + F_B - F_A F_B, \overline{F_A F_B}) > |\ x \in X\}$

$= (\overline{T_A T_B}, \overline{I_A I_B}, \overline{F_A F_B})$

$= (A\ \$\ B)$ (40)

and

$(A\$ B)^c\ _{NS1}\ (A \otimes B) = (\overline{F_A F_B}, \overline{I_A I_B}, \overline{T_A T_B})\ _{NS1}(T_B T_A, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B)$

$= \{<x, \text{Max}\ (\overline{T_A T_B}, T_B T_A), \text{Min}(\overline{I_A I_B}, I_A + I_B - I_A I_B), \text{Min}(\overline{F_A F_B}, F_A + F_B - F_A F_B) > |\ x \in X\}$

$= (\overline{T_A T_B}, \overline{I_A I_B}, \overline{F_A F_B})$

$= (A\ \$\ B)$ (41)

From (40) and (41), we get the result (v).

(vi) From definition in (6), we have

$(A \otimes B)^c\ _{NS1}\ (A \oplus B) = (A \oplus B)^c\ _{NS1}\ (A \otimes B) = (A \oplus B)$

$(A \otimes B)^c\ _{NS1}\ (A \oplus B) = (F_A + F_B - F_A F_B, I_A + I_B - I_A I_B, T_B T_A)\ _{NS1}(T_A + T_B - T_A T_B, I_A I_B, F_A F_B)$

$= \{<x, \text{Max}\ (T_B T_A, T_A + T_B - T_A T_B), \text{Min}(I_A + I_B - I_A I_B, I_A I_B), \text{Min}(F_A + F_B - F_A F_B, F_A F_B) > |\ x \in X\}$

$= (T_A + T_B - T_A T_B, I_A I_B, F_A F_B)$

$= (A \oplus B)$ (42)

and

$(A \oplus B)^c\ _{NS1}\ (A \otimes B) = (F_A F_B, I_A I_B, T_A + T_B - T_A T_B)\ _{NS1}\ (T_B T_A, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B)$

$= \{<x, \text{Max}\ (T_A + T_B - T_A T_B, T_B T_A), \text{Min}\ (I_B I_A, I_A + I_B - I_A I_B), \text{Min}(F_A F_B, F_A + F_B - F_A F_B) > |\ x \in X\}$

$= (T_A + T_B - T_A T_B, I_A I_B, F_A F_B)$

$= (A \oplus B)$ (43)

From (42) and (43), we get the result (vi).

The following theorem is not valid.

Theorem 7

For A, B $\in$ NS(X),

i. $(A\ \ \ B)\ _{NS1}\ (A\ @\ B)^{C\ c} = (A@B)\ _{NS1}\ (A\ \ \ B)^{C\ c} = (A@B)$

ii. $(A \otimes B)\ _{NS1}\ (A\ @\ B)^{C\ c} = (A@B)\ _{NS1}\ (A \otimes B)^{C\ c} = (A \otimes B)$

iii. $(A \oplus B)\ _{NS1}\ (A\ \#\ B)^{C\ c} = (A\#\ B)\ _{NS1}\ (A \oplus B)^{C\ c} = (A\#\ B)$

iv. $(A \otimes B)\ _{NS1}\ (A\ \#\ B)^{C\ c} = (A\#\ B)\ _{NS1}\ (A \otimes B)^{C\ c} = (A \otimes B)$

v. $(A \oplus B)\ _{NS1}\ (A\ \$\ B)^{C\ c} = (A\$ B)\ _{NS1}\ (A \oplus B)^{C\ c} = (A\ \$\ B)$

vi. $(A \otimes B)\ _{NS1}\ (A\ \$\ B)^{C\ c} = (A\$ B)\ _{NS1}\ (A \otimes B)^{C\ c} = (A \otimes B)$

Proof

The proof is straightforward.

Theorem 8

For A, B $\in$ NS(X),

i. $(A\ \ \ B)\ _{NS2}\ (A\ @\ B)^{C\ c} = (A@B)\ _{NS2}\ (A\ \ \ B)^{C\ c} = (A@B)$

ii. $(A \otimes B)\ _{NS2}\ (A\ @\ B)^{C\ c} = (A@B)\ _{NS2}\ (A \otimes B)^{C\ c} = (A \otimes B)$

iii. $(A \oplus B)\ _{NS2}\ (A\ \#\ B)^{C\ c} = (A\#\ B)\ _{NS2}\ (A \oplus B)^{C\ c} = (A\#\ B)$

iv. $(A \otimes B)\ _{NS2}\ (A\ \#\ B)^{C\ c} = (A\#B)\ _{NS2}\ (A \otimes B)^{C\ c} = (A \otimes B)$

v. $(A \oplus B)\ _{NS2}\ (A\ \$\ B)^{C\ c} = (A\$ B)\ _{NS2}\ (A \oplus B)^{C\ c} = (A\ \$\ B)$

vi. $(A \otimes B)\ _{NS2}\ (A\ \$\ B)^{C\ c} = (A\$ B)\ _{NS2}\ (A \otimes B)^{C\ c} = (A \otimes B)$

Proof

(i) From definition in (6), we have

$(A \quad B)_{NS2} \quad (A @ B)^{C}\ ^{c} = (T_A + T_B - T_A\, T_B\ , I_A\, I_B\ , F_A\, F_B)_{NS2}\ (\frac{F_A + F_B}{2}, \frac{I_A + I_B}{2}, \frac{T_A + T_B}{2})$

$= \{<x, \ \text{Max}\ \ F_A\, F_B, \frac{F_A + F_B}{2}\ , \text{Max}\ \ I_A\, I_B\ , \frac{I_A + I_B}{2}\ , \ \text{Min}\ \ T_A + T_B - T_A\, T_B\ , \frac{T_A + T_B}{2}\ ^{c} > |\ x \in X\}$

$= \ \frac{F_A + F_B}{2}\ , \frac{I_A + I_B}{2}\ , \frac{T_A + T_B}{2}\ ^{c}$

$= (\frac{T_A + T_B}{2}, \frac{I_A + I_B}{2}, \frac{F_A + F_B}{2})$

$= (A @ B)$ (44)

and

$(A @ B)_{NS2} \quad (A \oplus B)^{C}\ ^{c}$

$= (\frac{T_A + T_B}{2}, \frac{I_A + I_B}{2}, \frac{F_A + F_B}{2})_{NS2}\ (F_A\, F_B, I_A\, I_B, T_A + T_B - T_A\, T_B)^{c}$

$=$

$\text{Max}\ \ \frac{F_A + F_B}{2}, F_A\, F_B\ \ , Max\ \ \frac{I_A + I_B}{2}, I_A\, I_B\ \ , Min(\frac{T_A + T_B}{2}, T_A + T_B - T_A\, T_B)^{c}$

$= \ \frac{F_A + F_B}{2}\ , \frac{I_A + I_B}{2}, \frac{T_A + T_B}{2}\ ^{c}$

$= (\frac{T_A + T_B}{2}\ , \frac{I_A + I_B}{2}\ , \frac{F_A + F_B}{2})$

$= (A @ B)$ (45)

From (44) and (45), we get the result (i).

(ii) From definition in (6) ,we have

$(A \otimes B)_{NS2} \quad (A @ B)^{C}\ ^{c} = \ \text{Max}\ \ F_A + F_B - F_A\, F_B, \frac{F_A + F_B}{2}\ \ , \text{Max}\ \ I_A + I_B - I_A\, I_B\ , \frac{I_A + I_B}{2}\ \ , \text{Min}\ \ T_B\, T_A\ , \frac{T_A + T_B}{2}\ ^{c}$

$= \ F_A + F_B - F_A\, F_B, I_A + I_B - I_A\, I_B, T_B\, T_A\ ^{c}$

$= (T_B\, T_A\ , I_A + I_B - I_A\, I_B\ , F_A + F_B - F_A\, F_B)$

$= (A \otimes B)$ (46)

and

$(A @ B)_{NS2} \quad (A \otimes B)^{C}\ ^{c} = \{<x, (\frac{T_A + T_B}{2}\ , \frac{I_A + I_B}{2}\ , \frac{F_A + F_B}{2})\ (F_A + F_B - F_A\, F_B, I_A + I_B - I_A\, I_B, T_B\, T_A) > |\ x \in X\}$

$=$

$\text{Max}\ \ \frac{F_A + F_B}{2}, F_A + F_B - F_A\, F_B\ \ , \ \text{Max}\ \ \frac{I_A + I_B}{2}\ , I_A + I_B - I_A\, I_B\ \ , \text{Min}\ \ \frac{T_A + T_B}{2}\ , T_B\, T_A\ ^{c}$

$= \ F_A + F_B - F_A\, F_B, I_A + I_B - I_A\, I_B, T_B\, T_A\ ^{c}$

$= (T_B\, T_A, I_A + I_B - I_A\, I_B\ , F_A + F_B - F_A\, F_B)$

$= (A \otimes B)$ (47)

From (46) and (47), we get the result (ii).

(iii)From definition in (6), we have

$(A \oplus B)_{NS2} \quad (A \# \ B)^{C}\ ^{c} =$

$=$

$\text{Max}\ \ F_A\, F_B, \frac{2\, F_A\, F_B}{F_A + F_B}\ \ , \text{Max}\ \ I_A\, I_B\ , \frac{2\, I_A\, I_B}{I_A + I_B}\ \ , \text{Min}\ \ T_A + T_B - T_A\, T_B\ , \ \frac{2\, T_A\, T_B}{T_A + T_B}\ ^{c}$

$= \ \frac{2\, F_A\, F_B}{F_A + F_B}, \ \frac{2\, I_A\, I_B}{I_A + I_B}, \ \frac{2\, T_A\, T_B}{T_A + T_B}\ ^{c}$

$= \ \frac{2\, T_A\, T_B}{T_A + T_B}, \ \frac{2\, I_A\, I_B}{I_A + I_B}, \ \frac{2\, F_A\, F_B}{F_A + F_B}$

$= (A \# \ B)$ (48)

and

$(A \# \ B)_{NS2} \quad (A \oplus B)^{C}\ ^{c} = \ \frac{2\, T_A\, T_B}{T_A + T_B}, \ \frac{2\, I_A\, I_B}{I_A + I_B}, \frac{2\, F_A\, F_B}{F_A + F_B}\ _{NS2}\ (F_A\, F_B, \ , I_A\, I_B, T_A + T_B - T_A\, T_B)$

$=$

$\text{Max}\ \ \frac{2\, F_A\, F_B}{F_A + F_B}, F_A\, F_B\ \ , \text{Max}\ \ \frac{2\, I_A\, I_B}{I_A + I_B}\ , I_A\, I_B\ \ , \ \text{Min}\ \ \frac{2\, T_A\, T_B}{T_A + T_B}, T_A + T_B - T_A\, T_B\ ^{c}$

$= \ \frac{2\, F_A\, F_B}{F_A + F_B}, \ \frac{2\, I_A\, I_B}{I_A + I_B}, \ \frac{2\, T_A\, T_B}{T_A + T_B}\ ^{c}$

$= \ \frac{2\, T_A\, T_B}{T_A + T_B}, \ \frac{2\, I_A\, I_B}{I_A + I_B}, \ \frac{2\, F_A\, F_B}{F_A + F_B}$

$= (A \# \ B)$ (49)

From (48) and (49), we get the result (iii).

(iv) From definition in (6), we have

$(A \otimes B)_{NS2} \quad (A \# \ B)^{C}\ ^{c} =$

$=$

$\text{Max}\ \ F_A + F_B - F_A\, F_B, \frac{2\, F_A\, F_B}{F_A + F_B}\ \ , \ \text{Max}\ \ I_A + I_B - I_A\, I_B\ , \frac{2\, I_A\, I_B}{I_A + I_B}\ \ , \text{Min}\ \ T_B\, T_A\ , \ \frac{2\, T_A\, T_B}{T_A + T_B}\ ^{c}$

$= \ F_A + F_B - F_A\, F_B, \ \ I_A + I_B - I_A\, I_B, \ T_B\, T_A\ ^{c}$

$= \ T_B\, T_A, \ \ I_A + I_B - I_A\, I_B, \ F_A + F_B - F_A\, F_B$

$= (A \otimes B)$ (50)

and

$(A \# \ B)_{NS2} \quad (A \otimes \ B)^{C}\ ^{c} =$

$= \ \text{Max}\ \ \frac{2\, F_A\, F_B}{F_A + F_B}, \ F_A + F_B - F_A\, F_B\ \ , \text{Max}\ \ \frac{2\, I_A\, I_B}{I_A + I_B}\ , I_A + I_B - I_A\, I_B\ , \ \ , \text{Min}\ \ \frac{2\, T_A\, T_B}{T_A + T_B}, T_B\, T_A\ ^{c}$

$= \ F_A + F_B - F_A\, F_B, \ \ I_A + I_B - I_A\, I_B, \ T_B\, T_A\ ^{c}$

$= \ T_B\, T_A, \ \ I_A + I_B - I_A\, I_B, \ F_A + F_B - F_A\, F_B$

$= (A \otimes B)$ (51)

From (50) and (51), we get the result (iv).

(v) From definition in (6), we have

$(A \oplus B)_{NS2} \quad (A \ \$ \ B)^{C}\ ^{c} =$

$= \ \text{Max}\ \ F_A\, F_B, \ \ \overline{F_A\, F_B}\ \ , \text{Max}\ \ I_A\, I_B\ , \ \ \overline{I_A\ I_B}\ \ , \ \text{Min}\ \ T_A + T_B - T_A\, T_B, \ \ \overline{T_A\, T_B}\ ^{c}$

$= \ \ \overline{F_A\, F_B}, \ \ \overline{I_A\ I_B}, \ \ \overline{T_A\, T_B}\ ^{c}$

$= (\ \ \overline{T_A\, T_B}\ , \ \ \overline{I_A\ I_B}\ , \ \ \overline{F_A\, F_B})$

$= (A \ \$ \ B)$ (52)

and

$(A\$B)\ _{NS2}\ (A\oplus B)^{C\ c} =$

$= \left(\text{Max}\ \overline{F_A F_B},\ F_A F_B,\ \text{Max}\ \overline{I_A\ I_B},\ I_A I_B,\ \text{Min}\ \overline{T_A T_B}, T_A + T_B - T_A T_B \right)^c$

$= \overline{F_A F_B},\ \overline{I_A\ I_B},\ \overline{T_A T_B}\ ^c$

$=(\ \overline{T_A T_B},\ \overline{I_A\ I_B},\ \overline{F_A F_B})$

$=(A\ \$\ B)$ (53)

From (52) and (53), we get the result (v).

(vi) From definition in (2), we have

$(A\otimes B)\ _{NS2}\ (A\ \$\ B)^{C\ c}$

$= \left(Max\ F_A + F_B - F_A F_B,\ \overline{F_A F_B},\ Max\ I_A + I_B - I_A I_B,\ \overline{I_A\ I_B},\ Min\ T_B T_A,\ \overline{T_A T_B} \right)^c$

$= F_A + F_B - F_A F_B, I_A + I_B - I_A I_B, T_B T_A\ ^c$

$= T_B T_A, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B$

$=(A\otimes B)$ (54)

and

$(A\$B)\ _{NS2}\ (A\otimes B)^{C\ c} =$

$= \left(\text{Max}\ \overline{F_A F_B},\ F_A + F_B - F_A F_B,\ \text{Max}\ \overline{I_A\ I_B},\ I_A + I_B - I_A I_B,\ \text{Min}\ \overline{T_A T_B},\ T_B T_A \right)^c$

$= F_A + F_B - F_A F_B, I_A + I_B - I_A I_B, T_B T_A\ ^c$

$= T_B T_A, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B$

$=(A\otimes B)$ (55)

From (54) and (55), we get the result (v).

The following are not valid.

$< T_A, F_A >$	$< T_B, F_B >$	$A\ _{NS1}\ B$	$A\ _{NS1}\ B$	V(A→ B)
< 0 ,1>	**< 0 ,1>**	**< 1 ,0>**	**< 1 ,0>**	**< 1 ,0>**
< 0 ,1>	**< 1 ,0>**	**< 1 ,0>**	**< 1 ,0>**	**< 1 ,0>**
< 1 ,0>	**< 0 ,1>**	**< 0 ,1>**	**< 0 ,1>**	**< 0 ,1>**
< 1 ,0>	**< 1 ,0>**	**< 1 ,0>**	**< 1 ,0>**	**< 1 ,0>**

Theorem 9

1- $(A\ \ B)^c\ _{NS2}\ (A\ @\ B) = (A@B)^c\ _{NS2}\ (A\ \ B) =(A\oplus B)$

2- $(A\otimes B)^c\ _{NS2}\ (A\ @\ B) =$

$(A@B)^c\ _{NS2}\ (A\otimes B) = (A@\ B)$

3- $(A\oplus B)\ _{NS2}\ (A\ \#\ B)^{C\ c} =$

$(A\#B)\ _{NS2}\ (A\oplus B)^{C\ c} = (A\#\ B)$

4- $(A\otimes B)^c\ _{NS2}\ (A\ \#\ B) =$

$(A\#\ B)^c\ _{NS2}\ (A\otimes B) = (A\#\ B)$

5- $(A\oplus B)^c\ _{NS2}\ (A\ \$\ B) = (A\$B)^c\ _{NS2}\ (A\oplus B) = (A\oplus B)$

6- $(A\otimes B)^c\ _{NS2}\ (A\ \$\ B) = (A\$B)^c\ _{NS2}\ (A\otimes B) = (A\ \$\ B)$

8- $(A\otimes B)^c\ _{NS2}\ (A\ \$\ B) = (A\$B)^c\ _{NS2}\ (A\otimes B) = (A\ \$\ B)$

9- $(A\otimes B)^c\ _{NS2}\ (A\oplus B) = (A\oplus B)^c\ _{NS2}\ (A\otimes B) = (A\oplus B)$

Example

We prove only the (i)

1- $(A\ \ B)^c\ _{NS2}\ (A\ @\ B) =$

$F_A F_B, I_A I_B, T_A + T_B - T_A T_B\ _{NS2}\ (\frac{T_A + T_B}{2}, \frac{I_A + I_B}{2}, \frac{F_A + F_B}{2})$

={<x, max $(T_A + T_B - T_A T_B, \frac{T_A + T_B}{2})$,max$(I_A I_B, \frac{I_A + I_B}{2})$,min $(F_A F_B, \frac{F_A + F_B}{2})$ > | **x** ∈ $\boldsymbol{X}$}

={<x, $T_A + T_B - T_A T_B$, $\frac{I_A + I_B}{2}$, $\frac{F_A + F_B}{2}$ > | **x** ∈ $\boldsymbol{X}$} ≠ $(A\oplus B)$

The same thing, for $(A@B)^c\ _{NS2}\ (A\ \ B)$

Then,

$(A\ \ B)^c\ _{NS2}\ (A\ @\ B) = (A@B)^c\ _{NS2}\ (A\ \ B) \neq (A\oplus B)$.

Remark

We remark that if the indeterminacy values are restricted to 0, and the membership /non-membership are restricted to 0 and 1. The results of the two neutrosophic implications $_{NS1}$ and $_{NS2}$ collapse to the fuzzy /intuitionistic fuzzy implications defined (V(A→ B)) in [17]

Table

Comparison of three kind of implications

From the table, we conclude that fuzzy /intuitionistic fuzzy implications are special case of neutrosophic implication.

Conclusion

In this paper, the neutrosophic implication is studied. The basic knowledge of the neutrosophic set is firstly reviewed, a two kind of neutrosophic implications are constructed, and its properties.

These implications may be the subject of further research, both in terms of their properties or comparison with other neutrosophic implication, and possible applications.

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