NSS
University of New Mexico

Neutrosophic Soft Graphs

Nasir Shah[1] and Asim Hussain[2]

[1] Department of Mathematics, Riphah International University, I-14, Islamabad, Pakistan. Email: memaths@yahoo.com
[2] Department of Mathematics, Barani Institute of Management Sciences(BIMS), Rawalpindi, Pakistan. Email: sh.asim.hussain@gmail.com

Abstract. The aim of this paper is to propose a new type of graph called neutrosophic soft graphs. We have established a link between graphs and neutrosophic soft sets. Basic operations of neutrosophic soft graphs such as union, intersection and complement are defined here. The concept of strong neutrosophic soft graphs is also discussed in this paper.

Keywords: Soft Sets, Graphs, Neutrosophic soft sets, Neutrosophic soft graphs. Strong neutrosophic soft graphs

1 Introduction

Graph theory is a nice tool to depict information in a very nice way. Usually graphs are represented pictorially, algebraically in the form of relations or by matrices. Their representation depends on application for which a graph is being employed. Graph theory has its origins in a 1736 paper by the celebrated mathematician Leonhard Euler [13] known as the father of graph theory, when he settled a famous unsolved problem known as Ko¨nigsburg Bridge problem. Subject of graph theory may be considered a part of combinatorial mathematics. The theory has greatly contributed to our understanding of programming, communication theory, switching circuits, architecture, operational research, civil engineering anthropology, economics linguistic and psychology. From the standpoint of applications it is safe to say that graph theory has become the most important part of combinatorial mathematics. A graph is also used to create a relationship between a given set of elements. Each element can be represented by a vertex and the relationship between them can be represented by an edge.

L.A. Zadeh [26] introduced the notion of fuzzy subset of a set in 1965 which is an extension of classical set theory. His work proved to be a mathematical tool for explaining the concept of uncertainty in real life problems. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. In 1975 Azriel Rosenfeld [20] considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs which have many applications in modeling, Environmental science, Social science, Geography and Linguistics etc. which deals with problems in these areas that can be better studied using the concept of fuzzy graph structures. Many researchers contributed a lot and gave some more generalized forms of fuzzy graphs which have been studied in [8] and [10]. These contributions show a new dimension of graph theory.

Molodstov introduced the theory of soft sets [18] which is generally used to deal with uncertainty and vagueness. He introduced the concept as a mathematical tool free from difficulties and presented the fundamental results of the new theory and successfully applied it to several directions. During recent past soft set theory has gained popularity among researchers, scholars practitioners and academicians. The theory of neutrosophic set is introduced by Smarandache [21] which is useful for dealing real life problems having imprecise, indeterminacy and inconsistent data. The theory is generalization of classical sets and fuzzy sets and is applied in decision making problems, control theory, medicines, topology and in many more real life problems. Maji [17] first time proposed the definition of neutrosophic soft sets and discussed many operations such as union, intersection and complement etc of such sets. Some new theories and ideas about neutrosophic sets can be studied in [6], [7] and [12]. In the present paper neutrosophic soft sets are employed to study graphs and give rise to a new class of graphs called neutrosophic soft graphs. We have discussed different operations defined on neutrosophic soft graphs using examples to make the concept easier. The concept of strong neutrosophic soft graphs and the complement of strong neutrosophic soft graphs is also discussed. Neutrosophic soft graphs are pictorial representation in which each vertex and each edge is an element of neutrosophic soft sets. This paper has been arranged as the following;

In section 2, some basic concepts about graphs and neutrosophic soft sets are presented which will be employed in later sections. In section 3, concept of neutrosophic soft graphs is given and some of their fundamental properties have been studied. In section 4, the concept of strong neutrosophic soft graphs and its complement is studied. Conclusion are also given at the end of section 4.

2 PRELIMINARIES

In this section, we have given some definitions about graphs and neutrosophic soft sets. These will be helpful in later sections.

2.1 Definition [25]: A graph G^* consists of set of finite objects $V=\{v_1,v_2,v_3,......v_n\}$ called vertices (also called points or nodes) and other set $E=\{e_1,e_2,e_3,......e_n\}$ whose elements are called edges (also called lines or arcs). Usually a graph is denoted as $G^*=(V,E)$. Let G^* be a graph and $\{u,v\}$ an edge of G^*. Since $\{u,v\}$ is 2-element set, we may write $\{v,u\}$ instead of $\{u,v\}$. It is often more convenient to represent this edge by uv or vu. If $e=uv$ is an edges of a graph G^*, then we say that u and v are adjacent in G^* and that e joins u and v. A vertex which is not adjacent to any other node is called isolated vertex.

2.2 Definition [25]: An edge of a graph that joins a node to itself is called loop or self loop.

2.3 Definition [25]: In a multigraph no loops are allowed but more than one edge can join two vertices, these edges are called multiple edges or parallel edges and a graph is called multigraph.

2.4 Definition [25]: *A graph which has neither loops nor multiple edges is called a simple graph.*

2.5 Definition [25]: A sub graph H^* of G^* is a graph having all of its vertices and edges in G^*. If H^* is a sub graph of G^*, then G^* is a super graph of H^*.

2.6 Definition [25]: Let $G_1^*=(V_1,E_1)$ and $G_2^*=(V_2,E_2)$ be two graphs. A function $f:V_1\to V_2$ is called isomorphism if

i) f is one to one and onto.

ii) for all $a,b\in V_1,\{a,b\}\in E_1$ if and only if $\{f(a),f(b)\}\in E_2$ when such a function exists, G_1^* and G_2^* are called isomorphic graphs and is written as $G_1^*\cong G_2^*$.

In other words, two graph G_1^* and G_2^* are said to be isomorphic to each other if there is a one to one correspondence between their vertices and between edges such that incidence relationship is preserved.

2.7 Definition [25]: The union of two simple graphs $G_1^*=(V_1,E_1)$ and $G_2^*=(V_2,E_2)$ is the simple graph with the vertex set $V_1\cup V_2$ and edge set $E_1\cup E_2$. The union of G_1^* and G_2^* is denoted by $G^*=G_1^*\cup G_2^*=(V_1\cup V_2,E_1\cup E_2)$.

2.8 Definition [25]: The join of two simple graphs $G_1^*=(V_1,E_1)$ and $G_2^*=(V_2,E_2)$ is the simple graph with the vertex set $V_1\cup V_2$ and edge set $E_1\cup E_2\cup E'$ where E' is the set of all edges joining the nodes of V_1 and V_2 assume that $V_1\cap V_2\neq\phi$. The join of G_1^* and G_2^* is denoted by $G^*=G_1^*+G_2^*=(V_1\cup V_2,E_1\cup E_2\cup E')$.

2.9 Definition [18]: Let U be an initial universe and E be the set of all possible parameters under consideration with respect to U. The power set of U is denoted by $P(U)$ and A is a subset of E. Usually parameters are attributes, characteristics, or properties of objects in U.
A pair (F,A) is called a soft set over U, where F is a mapping $F:A\to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e\in A,F(e)$ may be considered as the set of $e-$ approximate elements of the soft set (F,A).

2.10 Definition [21]: A neutrosophic set A on the universe of discourse X is defined as $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle,x\in X\}$, where $T,I,F:X\to]^-0,1^+[$ and $^-0\le T_A(x)+I_A(x)+F_A(x)\le 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0,1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

2.11 Definition [17]: Let $N(U)$ be the set of all neutrosophic sets on universal set U,E be the set of parameters that describes the elements of U and $A\subseteq E$. A pair (F,A) is called a neutrosophic soft set NSS over U, where F is a mapping given by $F:A\to N(U)$. A neutrosophic soft set is a mapping from parameters to $N(U)$. It is a parameterized family of neutrosophic subsets of U. For $e\in A$, $F(e)$ may be considered as the set of e-approximate elements of the neutrosophic soft set (F,A). The neutrosophic soft set (F,A) is parameterized family $\{F(e_i),i=1,2,3,e\in A\}$.

2.12 Definition [17]: Let $E_1,E_2\in E$ and $(F,E_1),(G,E_2)$ be two neutrosophic soft sets over U then (F,E_1) is said to be a neutrosophic soft subset of (G,E_2) if

(1) $E_1 \subseteq E_2$

(2) $\begin{cases} T_{F(e)}(x) \le T_{G(e)}(x), I_{F(e)}(x) \le I_{G(e)}(x), \\ F_{F(e)}(x) \ge F_{G(e)}(x) \end{cases}$

for all $e \in E_1, x \in U$.

In this case, we write $(F,E_1) \subseteq (G,E_2)$.

2.13 Definition [17]: Two neutrosophic soft sets (F,E_1) and (G,E_2) are said to be neutrosophic soft equal if (F,E_1) is a neutrosophic soft subset of (G,E_2) and (G,E_2) is a neutrosophic soft subset of (F,E_1) In this case, we write $(F,E_1) = (G,E_2)$.

2.14 Definition [14]: Let U be an initial universe, E be the set of parameters, and $A \subseteq E$.

(a) (H,A) is called a relative whole neutrosophic soft set *(*with respect to the parameter set A *), denoted by* ϕ_A *, if* $T_{H(e)}(x) = 1, I_{H(e)}(x) = 1, F_{H(e)}(x) = 0,$ for all $e \in A,$ $x \in U$.

(b) (G,A) is called a relative null neutrosophic soft set (with respect to the parameter set A), denoted by ϕ_A , if $T_{H(e)}(x) = 0, I_{H(e)}(x) = 0, F_{H(e)}(x) = 1,$ for all $e \in A,$ $x \in U$.

The relative whole neutrosophic soft set with respect to the set of parameters E is called the absolute neutrosophic soft set over U and simply denoted by U_E . In a similar way, the relative null neutrosophic soft set with respect to E is called the null neutrosophic soft set over U and is denoted by ϕ_E .

2.15 Definition [17]: The complement of a NSS (G,A) is denoted by $(G,A)^c$ and is defined by $(G,A)^c = (G^c, \neg A)$ where $G^c : \neg A \to N(U)$ is a mapping given by $G^c(\neg e)$ = neutrosophic soft complement with $T_{G^c(\neg e)} = F_{G(e)}, I_{G^c(\neg e)} = I_{G(e)}, F_{G^c(\neg e)} = T_{G(e)}$.

2.16 Definition [14](1): Extended union of two NSS (H,A) and (G,B) over the common universe U is denoted by $(H,A) \cup_E (G,B)$ and is define as $(H,A) \cup_E (G,B) = (K,C),$ where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows

$$T_{k(e)}(x) = \begin{cases} T_{H(e)}(x) & \text{if } e \in A - B, \\ T_{G(e)}(x) & \text{if } e \in B - A, \\ \max\{T_{H(e)}(x), T_{G(e)}(x)\} & \text{if } e \in A \cap B \end{cases}$$

$$I_{k(e)}(x) = \begin{cases} I_{H(e)}(x) & \text{if } e \in A - B, \\ I_{G(e)}(x) & \text{if } e \in B - A, \\ \max\{I_{H(e)}(x), I_{G(e)}(x)\} & \text{if } e \in A \cap B \end{cases}$$

$$F_{k(e)}(x) = \begin{cases} F_{H(e)}(x) & \text{if } e \in A - B, \\ F_{G(e)}(x) & \text{if } e \in B - A, \\ \min\{F_{H(e)}(x), F_{G(e)}(x)\} & \text{if } e \in A \cap B \end{cases}$$

2.17 Definition [14]: The restricted union of two NSS (H,A) and (G,B) over the common universe U is denoted by $(H,A) \cup_R (G,B)$ and is define as $(H,A) \cup_R (G,B) = (K,C),$ where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows

$$T_{K(e)}(x) = \max\{T_{H(e)}(x), T_{G(e)}(x)\} \text{ if } e \in A \cap B,$$

$$I_{K(e)}(x) = \max\{I_{H(e)}(x), I_{G(e)}(x)\} \text{ if } e \in A \cap B,$$

$$F_{K(e)}(x) = \min\{F_{H(e)}(x), F_{G(e)}(x)\} \text{ if } e \in A \cap B.$$

2.18 Definition [14]: Extended intersection of two NSS (H,A) and (G,B) over the common universe U is denoted by $(H,A) \cap_E (G,B)$ and is define as $(H,A) \cap_E (G,B) = (K,C),$ where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows

$$T_{k(e)}(x) = \begin{cases} T_{H(e)}(x) & \text{if } e \in A - B, \\ T_{G(e)}(x) & \text{if } e \in B - A, \\ \min\{T_{H(e)}(x), T_{G(e)}(x)\} & \text{if } e \in A \cap B \end{cases}$$

$$I_{k(e)}(x) = \begin{cases} I_{H(e)}(x) & \text{if } e \in A - B, \\ I_{G(e)}(x) & \text{if } e \in B - A, \\ \min\{I_{H(e)}(x), I_{G(e)}(x)\} & \text{if } e \in A \cap B \end{cases}$$

$$F_{k(e)}(x)=\begin{cases} F_{H(e)}(x) & \text{if } e\in A-B, \\ F_{G(e)}(x) & \text{if } e\in B-A, \\ \max\left\{F_{H(e)}(x),F_{G(e)}(x)\right\} & \text{if } e\in A\cap B \end{cases}$$

2.19 Definition [14]: The restricted intersection of two NSS (H,A) and (G,B) over the common universe U is denoted by $(H,A)\cap_R(G,B)$ and is define as $(H,A)\cap_R(G,B)=(K,C)$, where $C=A\cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows

$$T_{K(e)}(x)=\min\left\{T_{H(e)}(x),T_{G(e)}(x)\right\} \text{ if } e\in A\cap B,$$

$$I_{K(e)}(x)=\min\left\{I_{H(e)}(x),I_{G(e)}(x)\right\} \text{ if } e\in A\cap B,$$

$$F_{K(e)}(x)=\max\left\{F_{H(e)}(x),F_{G(e)}(x)\right\} \text{ if } e\in A\cap B.$$

3 Neutrosophic soft graphs

3.1 Definition Let $G^*=(V,E)$ be a simple graph and A be the set of parameters. Let $N(V)$ be the set of all neutrosophic sets in V. By a neutrosophic soft graph NSG, we mean a 4-tuple $G=(G^*,A,f,g)$ where $f:A\to N(V), g:A\to N(V\times V)$ defined as $f(e)=f_e=\left\{\langle x,T_{fe}(x),I_{fe}(x),F_{fe}(x)\rangle, x\in V\right\}$ and $g(e)=g_e=\left\{\langle (x,y),T_{fe}(x,y),I_{fe}(x,y),F_{fe}(x,y)\rangle,(x,y)\in V\times V\right\}$ are neutrosophic sets over V and $V\times V$ respectively, such that

$$T_{ge}(x,y)\le \min\left\{T_{fe}(x),T_{fe}(y)\right\},$$

$$I_{ge}(x,y)\le \min\left\{I_{fe}(x),I_{fe}(y)\right\},$$

$$F_{ge}(x,y)\ge \max\left\{F_{fe}(x),F_{fe}(y)\right\}.$$

for all $(x,y)\in V\times V$ and $e\in A$. We can also denote a NSG by $G=(G^*,A,f,g)=\{N(e):e\in A\}$ which is a parameterized family of graphs $N(e)$ we call them Neutrosophic graphs.

3.2 Example

Let $G^*=(V,E)$ be a simple graph with $V=\{x_1,x_2,x_3\}, A=\{e_1,e_2,e_3\}$ be a set of parameters. A NSG is given in Table 1 below and $T_{ge}(x_i,x_j)=0, I_{ge}(x_i,x_j)=0$ and $F_{ge}(x_i,x_j)=1,$ for all $(x_i,x_j)\in V\times V\setminus\{(x_1,x_2),(x_2,x_3),(x_3,x_1)\}$ and for all $e\in A$.

Table 1

f	x_1	x_2	x_3
e_1	(0.4,0.5,0.6)	(0.4,0.5,0.7)	(0,0,1)
e_2	(0.3,0.4,0.5)	(0.1,0.3,0.4)	(0.1,0.3,0.6)
e_3	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
g	(x_1,x_2)	(x_2,x_3)	(x_1,x_3)
e_1	(0.2,0.3,0.8)	(0,0,1)	(0,0,1)
e_2	(0.1,0.3,0.6)	(0,0,1)	(0.1,0.3,0.8)
e_3	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)

$N(e_1)$ Corresponding to e_1

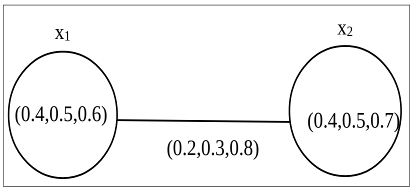

figure 1

$N(e_2)$ Corresponding to e_2

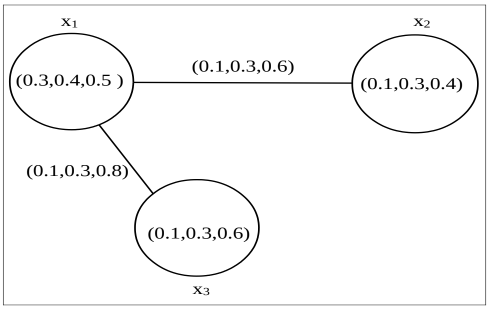

figure 2

$N(e_3)$ Corresponding to e_3

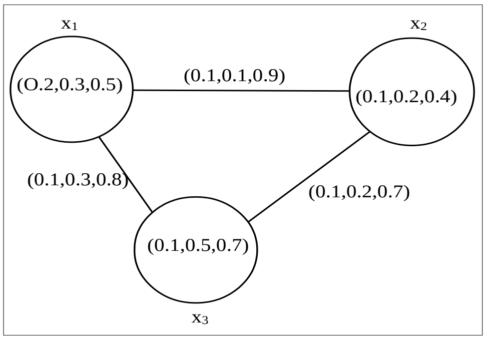

figure 3

3.3 Definition A neutrosophic soft graph $G=\left(G^*,A^1,f^1,g^1\right)$ is called a neutrosophic soft subgraph of $G=\left(G^*,A,f,g\right)$ if

(i) $A^1\subseteq A$

(ii) $f_e^1\subseteq f$, that is,

$T_{f_e^1}(x)\le T_{fe}(x), I_{f_e^1}(x)\le I_{fe}(x), F_{f_e^1}(x)\ge F_{fe}(x).$

(iii) $g_e^1\subseteq g$, that is,

$T_{g_e^1}(x,y)\le T_{ge}(x,y), I_{g_e^1}(x,y)\le I_{ge}(x,y), F_{g_e^1}(x,y)\ge F_{ge}(x,y).$

for all $e\in A^1$.

3.4 Example

Let $G^*=(V,E)$ be a simple graph with $V=\{x_1,x_2,x_3\}$ and set of parameters $A=\{e_1,e_2\}$. A neutrosophic soft subgraph of example 3.2 is given in Table 2 below and $T_{ge}(x_i,x_j)=0, I_{ge}(x_i,x_j)=0$ and $F_{ge}(x_i,x_j)=1,$ for all $(x_i,x_j)\in V\times V\setminus\{(x_1,x_2),(x_2,x_3),(x_3,x_1)\}$ and for all $e\in A.$

Table 2.

f^1	x_1	x_2	x_3
e_1	(0.3,0.2,0.5)	(0.3,0.2,0.6)	(0,0,1)
e_2	(0.1,0.1,0.5)	(0.1,0.2,0.4)	(0.1,0.2,0.6)
g^1	(x_1,x_2)	(x_2,x_3)	(x_1,x_3)
e_1	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
e_2	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.8)

$N(e_1)$ Corresponding to e_1

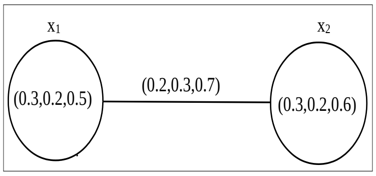

figure 4

$N(e_2)$ Corresponding to e_2

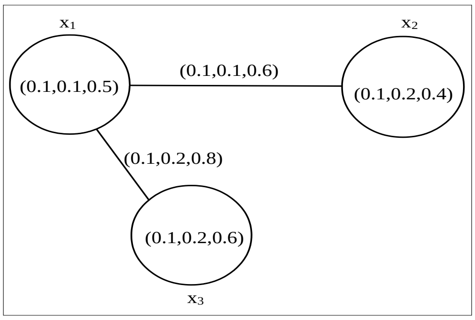

figure 5

3.5 Definition A neutrosophic soft subgraph $G=\left(G^*,A^1,f^1,g^1\right)$ is said to be spanning neutrosophic soft subgraph of $G=\left(G^*,A,f,g\right)$ if $f_e^1(x)=f(x)$, for all $x\in V, e\in A^1.$

(Here two neutrosophic soft graphs have the same neutrosophic soft vertex set, But have opposite edge sets.

3.6 Definition The union of two neutrosophic soft graphs $G_1=\left(G_1^*,A_1,f^1,g^1\right)$ and $G_2=\left(G_2^*,A_2,f^2,g^2\right)$ is denoted by $G=\left(G^*,A,f,g\right)$, with $A=A_1\cup A_2$ where the truth-membership, indeterminacy-membership and falsity-membership of union are as follows

$$T_{fe}(x)=\begin{cases} T_{f_e^1}(x)\begin{cases}\text{if } e\in A_1-A_2 \text{ and } x\in V_1-V_2 \text{ or}\\ \text{if } e\in A_1-A_2 \text{ and } x\in V_1\cap V_2 \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } x\in V_1-V_2.\end{cases}\\ T_{f_e^2}(x)\begin{cases}\text{if } e\in A_2-A_1 \text{ and } x\in V_2-V_1 \text{ or}\\ \text{if } e\in A_2-A_1 \text{ and } x\in V_1\cap V_2 \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } x\in V_2-V_1.\end{cases}\\ \max\left\{T_{f_e^1}(x),T_{f_e^2}(x)\right\}\{\text{if } e\in A_1\cap A_2 \text{ and } x\in V_1\cap V_2\}\\ 0,\ \text{otherwise}\end{cases}$$

$$I_{fe}(x)=\begin{cases} I_{f_e^1}(x)\begin{cases}\text{if } e\in A_1-A_2 \text{ and } x\in V_1-V_2 \text{ or}\\ \text{if } e\in A_1-A_2 \text{ and } x\in V_1\cap V_2 \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } x\in V_1-V_2.\end{cases}\\ I_{f_e^2}(x)\begin{cases}\text{if } e\in A_2-A_1 \text{ and } x\in V_2-V_1 \text{ or}\\ \text{if } e\in A_2-A_1 \text{ and } x\in V_1\cap V_2 \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } x\in V_2-V_1.\end{cases}\\ \max\left\{I_{f_e^1}(x),I_{f_e^2}(x)\right\}\left\{\begin{matrix}\text{if } e\in A_1\cap A_2 \text{ and}\\ x\in V_1\cap V_2\end{matrix}\right\}\\ 0,\ \text{otherwise}\end{cases}$$

$$F_{fe}(x)=\begin{cases} F_{f_e^1}(x)\begin{cases}\text{if } e\in A_1-A_2 \text{ and } x\in V_1-V_2 \text{ or}\\ \text{if } e\in A_1-A_2 \text{ and } x\in V_1\cap V_2 \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } x\in V_1-V_2.\end{cases}\\ F_{f_e^2}(x)\begin{cases}\text{if } e\in A_2-A_1 \text{ and } x\in V_2-V_1 \text{ or}\\ \text{if } e\in A_2-A_1 \text{ and } x\in V_1\cap V_2 \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } x\in V_2-V_1.\end{cases}\\ \min\left\{F_{f_e^1}(x),F_{f_e^2}(x)\right\}\left\{\begin{matrix}\text{if } e\in A_1\cap A_2 \text{ and}\\ x\in V_1\cap V_2\end{matrix}\right\}\\ 0,\ \text{otherwise}\end{cases}$$

Also

$$T_{ge}(x,y)=\begin{cases} T_{g_e^1}(x,y)\begin{cases}\text{if } e\in A_1-A_2 \text{ and } (x,y)\in(V_1\times V_1)\text{-}(V_2\times V_2) \text{ or}\\ \text{if } e\in A_1-A_2 \text{ and } (x,y)\in(V_1\times V_1)\cap(V_2\times V_2) \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } (x,y)\in(V_1\times V_1)\text{-}(V_2\times V_2).\end{cases}\\ T_{g_e^2}(x,y)\begin{cases}\text{if } e\in A_2-A_1 \text{ and } (x,y)\in(V_2\times V_2)\text{-}(V_1\times V_1) \text{ or}\\ \text{if } e\in A_2-A_1 \text{ and } (x,y)\in(V_1\times V_1)\cap(V_2\times V_2)\text{or}\\ \text{if } e\in A_1\cap A_2 \text{ and } (x,y)\in(V_2\times V_2)\text{-}(V_1\times V_1).\end{cases}\\ \max\left\{T_{g_e^1}(x,y),T_{g_e^2}(x,y)\right\}\left\{\begin{matrix}\text{if } e\in A_1\cap A_2 \text{ and}\\ (x,y)\in(V_1\times V_1)\cap(V_2\times V_2)\end{matrix}\right\}\\ 0,\ \text{otherwise}\end{cases}$$

$$I_{ge}(x,y)=\begin{cases} I_{g_e^1}(x,y)\begin{cases}\text{if } e\in A_1-A_2 \text{ and } (x,y)\in(V_1\times V_1)\text{-}(V_2\times V_2) \text{ or}\\ \text{if } e\in A_1-A_2 \text{ and } (x,y)\in(V_1\times V_1)\cap(V_2\times V_2) \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } (x,y)\in(V_1\times V_1)\text{-}(V_2\times V_2).\end{cases}\\ I_{g_e^2}(x,y)\begin{cases}\text{if } e\in A_2-A_1 \text{ and } (x,y)\in(V_2\times V_2)\text{-}(V_1\times V_1) \text{ or}\\ \text{if } e\in A_2-A_1 \text{ and } (x,y)\in(V_1\times V_1)\cap(V_2\times V_2)\text{or}\\ \text{if } e\in A_1\cap A_2 \text{ and } (x,y)\in(V_2\times V_2)\text{-}(V_1\times V_1).\end{cases}\\ \max\left\{I_{g_e^1}(x,y),I_{g_e^2}(x,y)\right\}\left\{\begin{matrix}\text{if } e\in A_1\cap A_2 \text{ and}\\ (x,y)\in(V_1\times V_1)\cap(V_2\times V_2)\end{matrix}\right\}\\ 0,\ \text{otherwise}\end{cases}$$

$$F_{ge}(x,y)=\begin{cases} F_{g_e^1}(x,y)\begin{cases}\text{if } e\in A_1-A_2 \text{ and } (x,y)\in(V_1\times V_1)\text{-}(V_2\times V_2) \text{ or}\\ \text{if } e\in A_1-A_2 \text{ and } (x,y)\in(V_1\times V_1)\cap(V_2\times V_2) \text{ or}\\ \text{if } e\in A_1\cap A_2 \text{ and } (x,y)\in(V_1\times V_1)\text{-}(V_2\times V_2).\end{cases}\\ F_{g_e^2}(x,y)\begin{cases}\text{if } e\in A_2-A_1 \text{ and } (x,y)\in(V_2\times V_2)\text{-}(V_1\times V_1) \text{ or}\\ \text{if } e\in A_2-A_1 \text{ and } (x,y)\in(V_1\times V_1)\cap(V_2\times V_2)\text{or}\\ \text{if } e\in A_1\cap A_2 \text{ and } (x,y)\in(V_2\times V_2)\text{-}(V_1\times V_1).\end{cases}\\ \min\left\{F_{g_e^1}(x,y),F_{g_e^2}(x,y)\right\}\left\{\begin{matrix}\text{if } e\in A_1\cap A_2 \text{ and}\\ (x,y)\in(V_1\times V_1)\cap(V_2\times V_2)\end{matrix}\right\}\\ 0,\ \text{otherwise}\end{cases}$$

3.7 Example

Let $G_1^*=(V_1,E_1)$ be a simple graph with $V_1=\{x_1,x_2,x_3\}$ and set of parameters $A_1=\{e_1,e_2,e_3\}$. Let $G_2^*=(V_2,E_2)$ be a simple graph with $V_2=\{x_2,x_3,x_5\}$ and set of parameters $A_2=\{e_2,e_4\}$. A NSG $G_1=(G_1^*,A_1,f^1,g^1)$ is given in Table 3 below and $T_{ge}(x_i,x_j)=0, I_{ge}(x_i,x_j)=0$ and $F_{ge}(x_i,x_j)=1,$ for all $(x_i,x_j)\in V_1\times V_1\setminus\{(x_1,x_4),(x_3,x_4),(x_1,x_3)\}$ and for all $e\in A_1$.

Table 3

f^1	x_1	x_3	x_4
e_1	(0.1, 0.2, 0.3)	(0.2, 0.3, 0.4)	(0.2, 0.5, 0.7)
e_2	(0.1, 0.3, 0.7)	(0.4, 0.6, 0.7)	(0.1, 0.2, 0.3)
e_3	(0.5, 0.6, 0.7)	(0.6, 0.8, 0.9)	(0.3, 0.4, 0.6)

g^1	(x_1,x_4)	(x_3,x_4)	(x_1,x_3)
e_1	(0.1, 0.2, 0.7)	(0.1, 0.3, 0.8)	(0.1, 0.2, 0.5)
e_2	(0.1, 0.2, 0.7)	(0.1, 0.1, 0.9)	(0.1, 0.2, 0.8)
e_3	(0.1, 0.3, 0.8)	(0.2, 0.3, 0.9)	(0, 0, 1)

$N(e_1)$ Corresponding to e_1

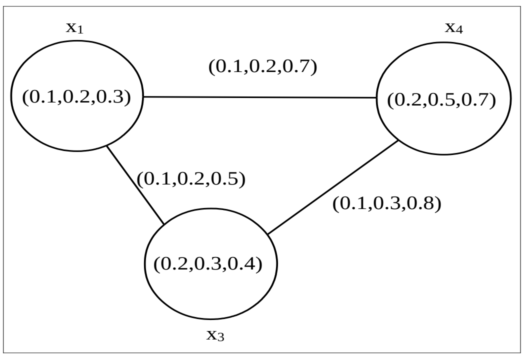

figure 6

$N(e_2)$ Corresponding to e_2

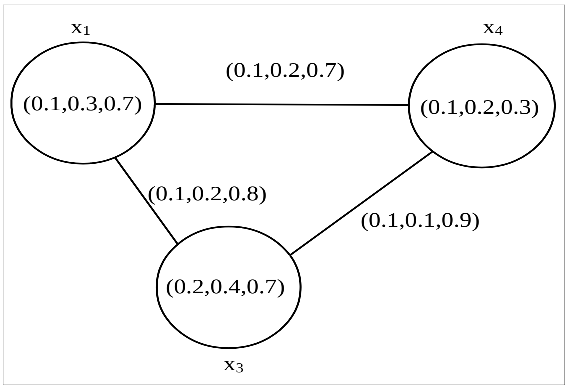

figure 7

$N(e_3)$ Corresponding to e_3

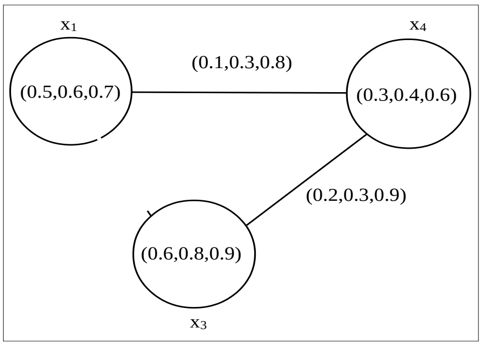

figure 8

A NSG $G_2 = \left(G_2^*, A_2, f^2, g^2\right)$ is given in Table 4 below and $T_{ge}\left(x_i, x_j\right) = 0, I_{ge}\left(x_i, x_j\right) = 0$ and $F_{ge}\left(x_i, x_j\right) = 1,$ for all $\left(x_i, x_j\right) \in V_2 \times V_2 \setminus \left\{\left(x_2, x_3\right), \left(x_3, x_5\right), \left(x_2, x_5\right)\right\}$ and for all $e \in A_2$.

Table 4

f^2	x_2	x_3	x_5
e_1 e_2	(0.1,0.2,0.4) (0.3,0.6,0.8)	(0.2,0.3,0.4) (0.5,0.7,0.9)	(0.4,0.6,0.7) (0.3,0.4,0.5)
g^2	$\left(x_2, x_3\right)$	$\left(x_3, x_5\right)$	$\left(x_2, x_5\right)$
e_1 e_2	(0.1,0.2,0.8) (0.1,0.1,0.9)	(0.2,0.3,0.9) (0.2,0.2,0.9)	(0,0,1) (0.2,0.3,0.8)

$N(e_2)$ Corresponding to e_2

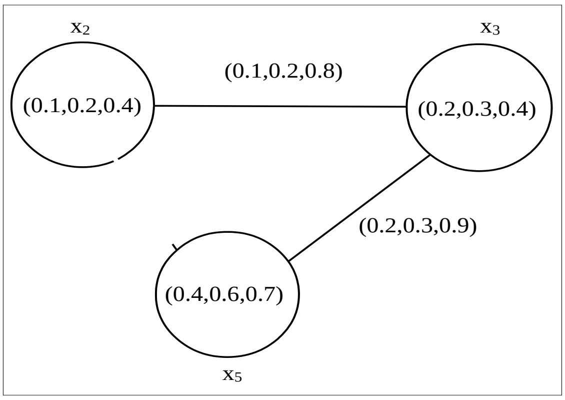

figure 9

$N(e_4)$ Corresponding to e_4

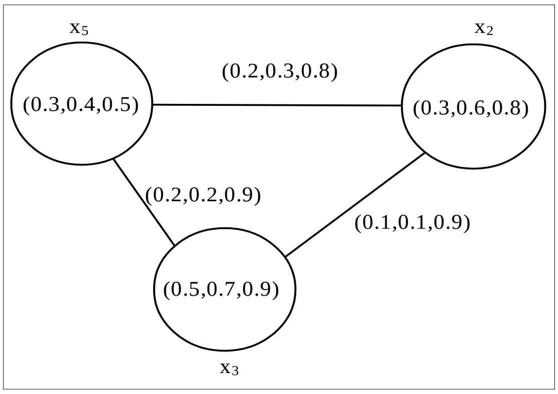

figure 10

The union $G = \left(G^*, A, f, g\right)$ is given in Table 5 below and $T_{ge}\left(x_i, x_j\right) = 0, I_{ge}\left(x_i, x_j\right) = 0$ and $F_{ge}\left(x_i, x_j\right) = 1,$ for all $\left(x_i, x_j\right) \in V \times V \setminus \left\{\left(x_1, x_4\right), \left(x_3, x_4\right), \left(x_1, x_3\right), \left(x_2, x_3\right), \left(x_3, x_5\right), \left(x_2, x_5\right)\right\}$ and for all $e \in A$.

Table 5

f	x_1	x_2	x_3	x_4	x_5
e_1	$(0.1,0.2,0.3)$	$(0,0,1)$	$(0.2,0.5,0.7)$	$(0.2,0.3,0.4)$	$(0,0,1)$
e_2	$(0.1,0.3,0.7)$	$(0.1,0.2,0.3)$	$(0.2,0.4,0.4)$	$(0,.1,0,2,0,3)$	$(0.4,0.6,0.7)$
e_3	$(0.5,0.6,0.7)$	$(0, 0, 1)$	$(0.6,0.8,0.9)$	$(0.3,0.4,0.6)$	$(0,0,1)$
e_4	$(0,0,1)$	$(0.3, 0.6, 0.8)$	$(0.5,0.7,0.9)$	$(0,0,1)$	$(0.3,0.4,0.5)$

g	(x_1,x_4)	(x_3,x_4)	(x_1,x_3)	(x_2,x_3)	(x_3,x_5)	(x_2,x_5)
e_1	$(0.1,0.2,0.7)$	$(0.1,0.3,0.8)$	$(0.1,0.2,0.8)$	$(0,0,1)$	$(0,0,1)$	$(0,0,1)$
e_2	$(0.1,0.2,0.7)$	$(0.1,0.1,0.9)$	$(0.1,0.2,0.8)$	$(0,.1,0,2,0,8)$	$(0.2,0.3,0.9)$	$(0,0,1)$
e_3	$(0.1,0.3,0.8)$	$(0.2,0.3,0.9)$	$(0,0,1)$	$(0,0,1)$	$(0,0,1)$	$(0,0,1)$
e_4	$(0,0,1)$	$(0,0,1)$	$(0,0,1)$	$(0.1,0.1,0.9)$	$(0.2,0.2,0.9)$	$(0.2,0.3,0.8)$

$N(e_1)$ Corresponding to e_1

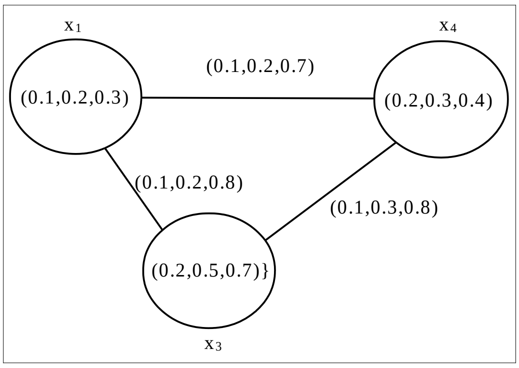

figure 11

$N(e_2)$ Corresponding to e_2

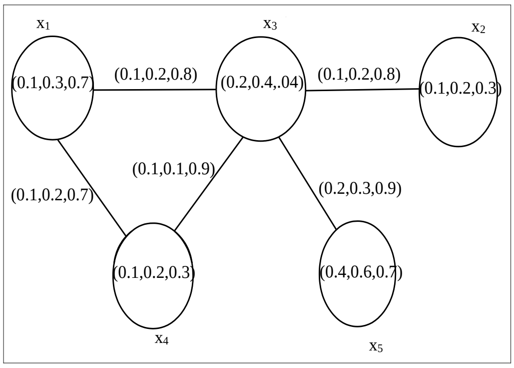

figure 12

$N(e_3)$ Corresponding to e_3

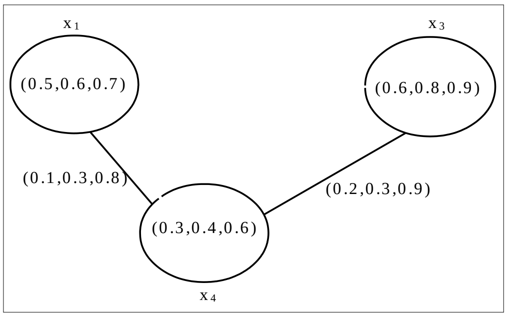

figure 13

$N(e_4)$ Corresponding to e_4

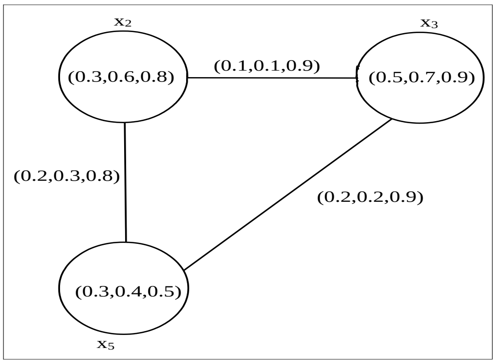

figure 14

3.8 Proposition

The union $G^{*}=(V,A,f,$ of two neutrosophic soft graph $G_1=\left(G^{*},A_1,f^{1},g^{1}\right)$ and $G_2=\left(G^{*},A_2,f^{2},g^{2}\right)$ is a neutrosophic soft graph.

Proof

Case i) If $e \in A_1 - A_2$ and $(x, y) \in (V_1 \times V_1) - (V_2 \times V_2)$, then

$T_{ge}(x, y) = T_{g_e^1}(x, y) \le \min\{T_{1_{fe}}(x), T_{1_{fe}}(y)\}$

$= \min\{T_{fe}(x), T_{fe}(y)\}$

so $T_{ge}(x, y) \le \min\{T_{fe}(x), T_{fe}(y)\}$

Also $I_{ge}(x, y) = I_{g_e^1}(x, y) \le \min\{I_{1_{fe}}(x), I_{1_{fe}}(y)\}$

$= \min\{I_{fe}(x), I_{fe}(y)\}$

so $I_{ge}(x, y) \le \min\{I_{fe}(x), I_{fe}(y)\}$

Now $F_{ge}(x, y) = F_{g_e^1}(x, y) \ge \max\{F_{1_{fe}}(x), F_{1_{fe}}(y)\}$

$= \max\{F_{fe}(x), F_{fe}(y)\}$

Similarly If $\{e \in A_1 - A_2$ and $(x, y) \in (V_1 \times V_1) \cap (V_2 \times V_2)\}$, or If $\{e \in A_1 \cap A_2$ and $(x, y) \in (V_1 \times V_1) - (V_2 \times V_2)\}$, we can show the same as done above.

Case ii) If $e \in A_1 \cap A_2$ and $(x, y) \in (V_1 \times V_1) \cap (V_2 \times V_2)$, then

$T_{ge}(x, y) = \max\{T_{1_{ge}}(x, y), T_{2_{ge}}(x, y)\}$

$\le \max\{\min\{T_{1_{fe}}(x), T_{1_{fe}}(y)\}, \min\{T_{2_{fe}}(x), T_{2_{fe}}(y)\}$

$\le \min\{\max\{T_{1_{fe}}(x), T_{2_{fe}}(x)\}, \max\{T_{1_{fe}}(y), T_{2_{fe}}(y)\}\}$

$= \min\{T_{fe}(x), T_{fe}(y)\}$

Also $I_{ge}(x, y) = \max\{I_{g_e^1}(x, y), I_{g_e^2}(x, y)\}$

$\le \max\{\min\{I_{1_{fe}}(x), I_{1_{fe}}(y)\}, \min\{I_{2_{fe}}(x) I_{2_{fe}}(y)\}\}$

$\le \min\{\max\{I_{1_{fe}}(x), I_{2_{fe}}(x)\}, \max\{I_{1_{fe}}(y), I_{2_{fe}}(y)\}\}$

$= \min\{I_{fe}(x), I_{fe}(y)\}$

Now $F_{ge}(x, y) = \min\{F_{g_e^1}(x, y), F_{g_e^1}(x, y)\}$

$\ge \min\{\max\{F_{1_{fe}}(x), F_{1_{fe}}(y)\}, \max\{F_{2_{fe}}(x), F_{2_{fe}}(y)\}\}$

$\ge \max\{\min\{F_{1_{fe}}(x), F_{2_{fe}}(x)\}, \min\{F_{1_{fe}}(y), F_{2_{fe}}(y)\}\}$

$= \max\{F_{fe}(x), F_{fe}(y)\}$

Hence the union $G = G_1 \cup G_2$ is a neutrosophic soft graph.

3.9 Definition The intersection of two neutrosophic soft graphs $G_1 = (G_1^*, A_1, f^1, g^1)$ and $G_2 = (G_2^*, A_2, f^2, g^2)$ is denoted by $G = (G^*, A, f, g)$ where $A = A_1 \cap A_2, V = V_1 \cap V_2$ and the truth-membership, indeterminacy-membership and falsity-membership of intersection are as follows

$$T_{f_e}(x) = \begin{cases} T_{f_e}^1(x) & \text{if } e \in A_1 - A_2 \\ T_{f_e}^2(x) & \text{if } e \in A_2 - A_1 \\ \min\{T_{f_e}^1(x), T_{f_e}^2(x)\} & \text{if } e \in A_1 \cap A_2 \end{cases},$$

$$I_{f_e}(x) = \begin{cases} I_{f_e}^1(x) & \text{if } e \in A_1 - A_2 \\ I_{f_e}^2(x) & \text{..if } e \in A_2 - A_1 \\ \min\{I_{f_e}^1(x), I_{f_e}^2(x)\}. & \text{if } e \in A_1 \cap A_2 \end{cases}$$

$$F_{f_e}(x) = \begin{cases} F_{f_e}^1(x) & \text{if } e \in A_1 - A_2 \\ F_{f_e}^2(x) & \text{if } e \in A_2 - A_1 \\ \max\{F_{f_e}^1(x), F_{f_e}^2(x)\} & \text{if } e \in A_1 \cap A_2 \end{cases}$$

$$T_{g_e}(x, y) = \begin{cases} T_{g_e^1}(x, y) & \text{if } e \in A_1 - A_2 \\ T_{g_e^2}(x, y) & \text{if } e \in A_2 - A_1 \\ \min\{T_{g_e^1}(x, y), T_{g_e^2}(x, y)\} & \text{if } e \in A_1 \cap A_2 \end{cases}$$

$$I_{g_e}(x, y) = \begin{cases} I_{g_e^1}(x, y) & \text{if } e \in A_1 - A_2 \\ I_{g_e^2}(x, y) & \text{if } e \in A_2 - A_1 \\ \min\{I_{g_e^1}(x, y), I_{g_e^2}(x, y)\} & \text{if } e \in A_1 \cap A_2 \end{cases},$$

$$F_{g_e}(x, y) = \begin{cases} F_{g_e^1}(x, y) & \text{if } e \in A_1 - A_2 \\ F_{g_e^2}(x, y) & \text{if } e \in A_2 - A_1 \\ \max\{F_{g_e^1}(x, y), F_{g_e^2}(x, y)\} & \text{if } e \in A_1 \cap A_2 \end{cases} \quad \textbf{3.10}$$

3.10 Example

Let $G_1^* = (V_1, E_1)$ be a simple graph with with $V_1 = \{x_1, x_2, x_3\}$ and set of parameters $A_1 = \{e_1, e_2\}$. A NSG $G_1 = (V_1, A_1, f^1, g^1)$ is given in Table 6 below and $T_{ge}(x_i, x_j) = 0, I_{ge}(x_i, x_j) = 0$ and $F_{ge}(x_i, x_j) = 1,$ for all $(x_i, x_j) \in V_1 \times V_1 \setminus \{(x_1, x_5), (x_1, x_2), (x_2, x_5)\}$ and for all $e \in A_1$.

Table 6

f^1	x_1	x_2	x_5
e_1	(0.1,0.2,0.3)	(0.2,0.4,0.5)	(0.1,0.5,0.7)
e_2	(0.2,0.3,0.7)	(0.4,0.6,0.7)	(0.3,0.4,0.6)
g^1	(x_1, x_5)	(x_2, x_5)	(x_1, x_2)
e_1	(0.1,0.1,0.8)	(0.1,0.3,0.8)	(0.1,0.1,0.6)
e_2	(0.2,0.3,0.7)	(0.3,0.4,0.8)	(0.2,0.3,0.7)

$N(e_1)$ Corresponding to e_1

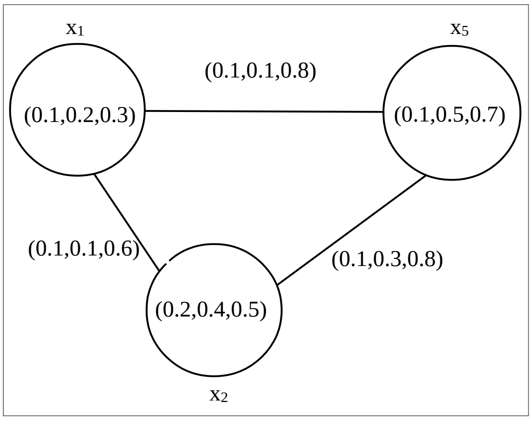

figure 15

$N(e_2)$ Corresponding to e_2

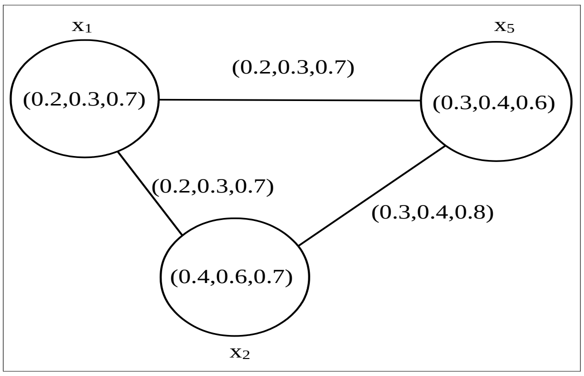

figure 16

Let $G_2^*=(V_2,E_2)$ be a simple graph with $V_2=\{x_1,x_2,x_3\}$ and set of parameters $A_2=\{e_2,e_3\}$ $A_2=\{e_2,e_3\}$. A NSG $G_2=(V_2,A_2,f^2,g^2)$ is given in Table 7 below and $T_{ge}(x_i,x_j)=0, I_{ge}(x_i,x_j)=0$ and $F_{ge}(x_i,x_j)=1,$ for all $(x_i,x_j)\in V_2\times V_2\setminus\{(x_2,x_3),(x_3,x_5),(x_2,x_5)\}$ and for all $e\in A_2$.

Table 7.

f^2	x_2	x_3	x_5
e_2	(0.3,0.5,0.6)	(0.2,0.4,0.6)	(0.4,0.5,0.9)
e_3	(0.2,0.4,0.5)	(0.1,0.2,0.6)	(0.1,0.5,0.7)
g^2	(x_2,x_3)	(x_3,x_5)	(x_2,x_5)
e_2	(0.1,0.3,0.7)	(0.2,0.4,0.9)	(0.2,0.4,0.9)
e_3	(0.1,0.2,0.8)	(0.1,0.2,0.9)	(0.1,0.4,0.8)

$N(e_2)$ corresponding to e_2

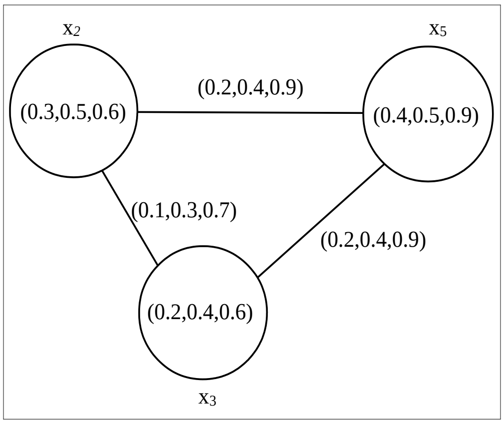

figure 17

$N(e_3)$ Corresponding to e_3

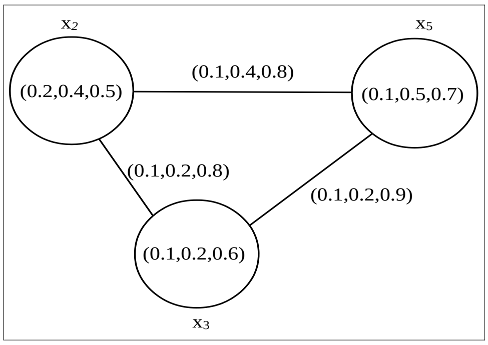

figure 18

Let $V=V_1\cap V_2=\{x_2,x_5\}, A=A_1\cup A_2=\{e_1,e_2,e_3\}$

The intersection of two neutrosophic soft graphs $G_1=(G_1^*,A_1,f^1,g^1)$ and $G_2=(G_2^*,A_2,f^2,g^2)$ is given in Table 8.

Table 8.

f	x_2	x_5	g	(x_2,x_5)
e_1	(0.2,0.4,0.5)	(0.1,0.5,0.7)	e_1	(0.1,0.3,0.8)
e_2	(0.3,0.5,0.7)	(0.3,0.4,0.9)	e_2	(0.2,0.4,0.9)
e_3	(0.2,0.4,0.5)	(0.1,0.5,0.7)	e_3	(0.1,0.4,0.8)

$N(e_1)$ corresponding to e_1

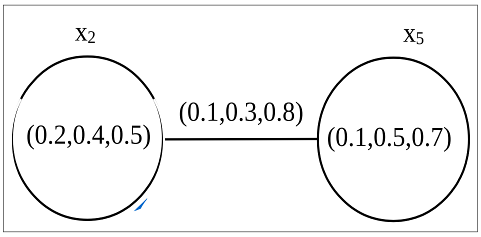

figure 19

$N(e_2)$ corresponding to e_2

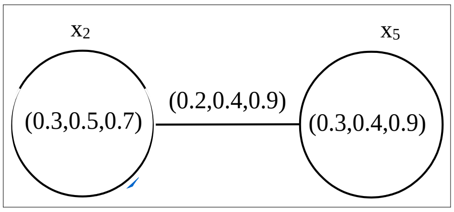

figure 20

$N(e_3)$ Corresponding to e_3

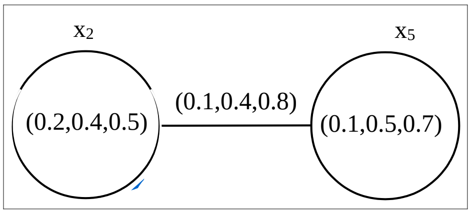

figure 21

3.11 Proposition

The intersection $G=(G^*,A,f,g)$ of two neutrosophic soft graphs $G_1=(G^*,A_1,f^1,g^1)$ and $G_2=(G^*,A_2,f^2,g^2)$ is a neutrosophic soft graph where , $A=A_1\cup A_2$ and $V=V_1\cap V_2$.

Proof

Case i) If $e\in A_1-A_2$ *then* $T_{ge}(x,y)=T_{g_e^1}(x,y)$

$\le \min\{T_{f_e^1}(x),T_{f_e^1}(y)\}=\min\{T_{fe}(x),T_{fe}(y)\}$

so $T_{ge}(x,y)\le \min\{T_{fe}(x),T_{fe}(y)\}$

Also $I_{ge}(x,y)=I_{g_e^1}(x,y)\le \min\{I_{f_e^1}(x),I_{f_e^1}(y)\}$

$=\min\{I_{fe}(x),I_{fe}(y)\}$

so $I_{ge}(x,y)\le \min\{I_{fe}(x),I_{fe}(y)\}$

Now $F_{ge}(x,y)=F_{g_e^1}(x,y)\ge \max\{F_{f_e^1}(x),F_{f_e^1}(y)\}$

$=\max\{F_{fe}(x),F_{fe}(y)\}$

Similarly If $e\in A_2-A_1$ we can show the same as done above.

Case ii) If $e\in A_1\cap A_2$ then $T_{ge}(x,y)=\min\{T_{g_e^1}(x,y),T_{g_e^2}(x,y)\}$

$\le \min\{\min\{T_{f_e^1}(x),T_{f_e^1}(y)\},\min\{T_{f_e^2}(x),T_{f_e^2}(y)\}\}$

$\le \min\{\min\{T_{f_e^1}(x),T_{f_e^2}(x)\},\min\{T_{f_e^1}(y),T_{f_e^2}(y)\}\}$

$=\min\{T_{fe}(x),T_{fe}(y)\}$

Also $I_{ge}(x,y)=\min\{I_{g_e^1}(x,y),I_{g_e^2}(x,y)\}$

$\le \min\{\min\{I_{f_e^1}(x),I_{f_e^1}(y)\},\min\{I_{f_e^2}(x),I_{f_e^2}(y)\}\}$

$\le \min\{\min\{I_{f_e^1}(x),I_{f_e^2}(x)\},\min\{I_{f_e^1}(y)I_{f_e^2}(y)\}\}$

$=\min\{I_{fe}(x),I_{fe}(y)\}$

Now $F_{ge}(x,y)=\max\{F_{g_e^1}(x,y),F_{g_e^1}(x,y)\}$

$\ge \max\{\max\{F_{f_e^1}(x),F_{f_e^1}(y)\},\max\{F_{f_e^2}(x),F_{f_e^2}(y)\}\}$

$\ge \max\{\max\{F_{f_e^1}(x),F_{f_e^2}(x)\},\max\{F_{f_e^1}(y),F_{f_e^2}(y)\}\}$

$=\max\{F_{fe}(x),F_{fe}(y)\}$

Hence the intersection $G=G_1\cap G_2$ is a neutrosophic soft graph.

4 Strong Neutrosophic Soft Graph

4.1 Definition A neutrosophic soft graph $G=(G^*,A,f,g)$, is called strong if $g_e(x,y)=f_e(x)\cap f_e(y)$, for all $x,y\in V, e\in A$. That is if

$$T_{ge}(x,y)=\min\{T_{fe}(x),T_{fe}(y)\},$$
$$I_{ge}(x,y)=\min\{I_{fe}(x),I_{fe}(y)\},$$
$$F_{ge}(x,y)=\max\{F_{fe}(x),F_{fe}(y)\}.$$

for all $(x,y)\in E$.

4.2 Example

Let $V=\{x_1,x_2,x_3\}, A=\{e_1,e_2\}$. A strong NSG $G=(G^*,A,f,g)$ is given in Table 9 below and $T_{ge}(x_i,x_j)=0, I_{ge}(x_i,x_j)=0$ and $F_{ge}(x_i,x_j)=1$, for all $(x_i,x_j)\in V\times V\setminus\{(x_1,x_2),(x_2,x_3),(x_1,x_3)\}$ and for all $e\in A$.

Table 9.

f	x_1	x_2	x_5
e_1	(0.1,0.2,0.4)	(0.2,0.3,0.5)	(0.3,0.4,0.7)
e_2	(0.3,0.6,0.8)	(0.4,0.5,0.9)	(0.3,0.4,0.5)
g	(x_1,x_2)	(x_2,x_3)	(x_1,x_3)
e_1	(0.1,0.2,0.5)	(0.2,0.3,0.7)	(0,0,1)
e_2	(0.3,0.5,0.9)	(0.3,0.4,0.9)	(0.3,0.4,0.8)

$N(e_1)$ Corresponding to e_1

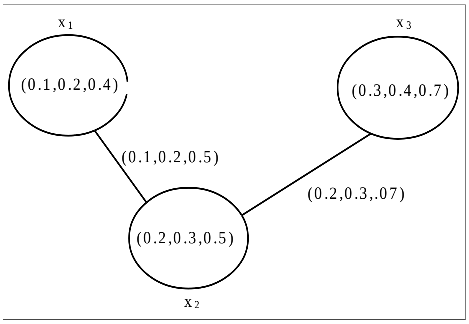

figure 22

$N(e_2)$ Corresponding to e_2

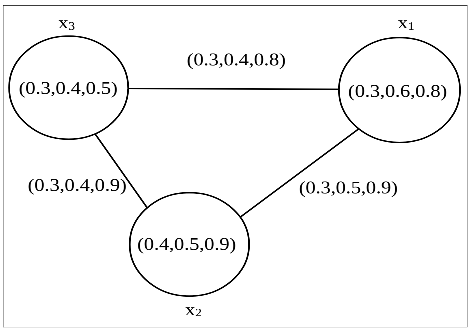

figure 23

4.3 Definition Let $G=(G^*,A,f,g)$ be a strong neutrosophic soft graph that is $g_e(x,y)=f_e(x)\cap f_e(y)$, for all for all $x,y\in V, e\in A$. The complement $\bar{G}=(\bar{G}^*,\bar{A},\bar{f},\bar{g})$ of strong neutrosophic soft graph $G=(G^*,A,f,g)$ is neutrosophic soft graph where

(i) $\bar{A}=A$

(ii) $T_{fe}(x)=\overline{T_{fe}}(x), I_{fe}(x)=\overline{I_{fe}}(x), F_{fe}(x)=\overline{F_{fe}}(x)$ for all x $\in V$

$$(iii)\ \overline{T_{fe}}(x,y)=\begin{cases}\min\{T_{fe}(x),T_{fe}(y)\} \text{ if } T_{ge}(x,y)=0\\ 0 \quad \text{otherwise}\end{cases}$$

$$\bar{I}_{ge}(x,y)=\begin{cases}\min\{I_{fe}(x),I_{fe}(y)\} \text{ if } I_{ge}(x,y)=0\\ 0 \quad \text{otherwise}\end{cases}$$

$$\bar{F}_{ge}(x,y)=\begin{cases}\max\{F_{fe}(x),F_{fe}(y)\} \text{ if } F_{ge}(x,y)=0\\ 0 \quad \text{otherwise}\end{cases}$$

4.4 Example

For the strong neutrosophic soft graph in previous example, the complements are given below for e_1 and e_2.
Corresponding to e_1, the complement of

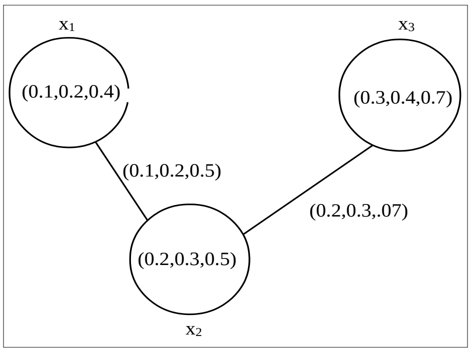

figure 24

is given by

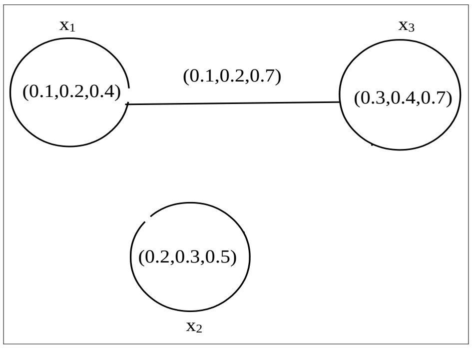

figure 25

Corresponding to e_2 ,the complement of

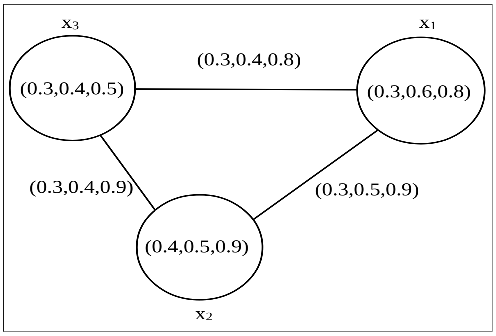

figure 26

is given by

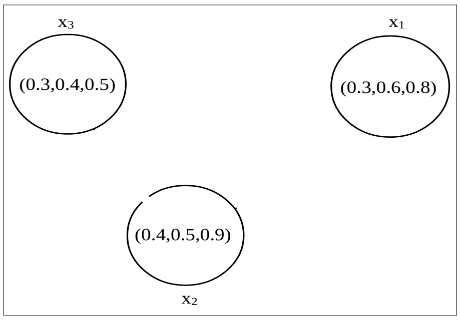

figure 27

Conclusion: Neutrosophic soft set theory is an approach to deal with uncertainty having enough parameters so that it is free from those difficulties which are associated with other contemporary theories dealing with study of uncertainty. A graph is a convenient way of representing information involving relationship between objects. In this paper we have combined both the theories and introduced and discussed neutrosophic soft graphs which are representatives of neutrosophic soft sets.

References

[1] M. Akram, Bipolar fuzzy graphs. information Sciences 181 (2011) 5548--5564 .

[2] M. Akram, W. A. Dudek, Interval-valued fuzzy graphs. Computers and Mathematics with Applications 61(2011)289--299.

[3] H. Aktas, Naim Cagman, Soft sets and soft groups, Information Sciences 177 (2007) 2726--2735.

[4] M. I. Ali, F. Feng, X. Y. Liu, W. K. Min, M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57 (2009) 1547--1553.1

[5] M. I. Ali, M. Shabir, M. Nazz, Algebraic structures of soft sets associated with new operations, Computers and Mathematics with Applications 61(9)(2011) 2647-2654.

[6] M. Ali, and F. Smarandache, Complex Nuetrosophic Set, Neural Computing and Applications, Vol. 27, no. 01. DOI: 10.1007/s00521-015-2154-y.

[7] M. Ali, I. Deli, F. Smarandache, The Theory of Neutrosophic Cubic Sets and Their Applications in Pattern Recognition, Journal of Intelligent and Fuzzy Systems, (In press), DOI:10.3233/IFS-151906

[8] P. Bhattacharya, Some remarks on fuzzy graphs. Pattern Recognition Letters 6 (1987) 297--302.

[9] K. R. Bhutani, On automorphism of fuzzy graphs. Pattern Recognition Letters 9 (1989) 159--162.

[10] K. R. Bhutani, A. Rosenfeld, Strong arcs in fuzzy graphs. Information Sciences 152 (2003) 319--322.

[11] K. R. Bhutani, A. Rosenfeld, Fuzzy end nodes in fuzzy graphs. Information Sciences 152 (2003)323--326.

[12] I. Deli, M. Ali, and F. Smarandache, Bipolar Neutrosophic Sets And Their Application Based On Multi-Criteria Decision Making Problems. (Proceeding of the 2015 International Conference on Advanced Mechatronic Systems, Beijing, China, August 22-24, 2015. IEEE Xplore, DOI: 10.1109/ICAMechS.2015.7287068

[13] L. Euler, Solutio problematis ad geometriam situs pertinentis, Commentarii Academiae Scientiarum Imperialis Petropolitanae 8 (1736) 128--140

[14] A. Hussain, M. Shabir, Algebraic Structures Of Neutrosophic Soft Sets, Neutrosophic Sets and Systems, 7,(2015) 53-61.

[15] P.K. Maji, R. Biswas, R. Roy, An application of soft sets in decision making problems, Computers and Mathematics with Applications 44 (2002) 1077--1083.

[16] P. K. Maji, R. Biswas, R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555--562.

[17] P.K. Maji, Neutrosophic soft set, Annals of Fuzzy Mathematics and Informatics, 5(1) (2013) 157-168.

[18] D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications 37 (1999) 19--31.

[19] K. H. Rosen, Discrete Mathematics and its Applications, McGraw-Hill, Washington, 1995.

[20] A.Rosenfeld,Fuzzy Graphs, Fuzzy Sets and their Applications to Cognitive and Decision Process, M.Eds. Academic Press, New York, 77-95,1975.

[21] F. Smarandache, Neutrosophic Set, a generalisation of the intuitionistic fuzzy sets, Inter. J.Pure Appl. Math. 24 (2005) 287{297.

[22] F.Smarandache, Neutrosophy. *Neutrosophic Probability, Set, and Logic*, Amer. Res. Press, Rehoboth, USA, 105 p., 1998 ; **http://fs.gallup.unm.edu/eBook-neutrosophics4.pdf** (4th edition).

[23] F. Smarandache, Neutrosophic Graphs, in his book Symbolic Neutrosophic Theory, EuropaNova,

Brussels, 2015.
[24] G. S. Singh, Graph theory, PHI Learning limited, New Delhi-110001 (2010).
[25] C. Vasudev, Graph theory with applications, New age international publishers (2006).
[26] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338--353.

Received: Feb. 20, 2016. Accepted: Mar. 30, 2016