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ON SOSHEARENERGY OF TREES OF DIAMETER 4 - PART I

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Abstract:

Let G = (V, E) be a simple, non-trivial, finite, connected graph. A set D $\subset$ V is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. A dominating set D of G is called a minimal dominating set if no proper subset of D is a dominating set. Shear Energy of a graph with respect to the minimal dominating set in terms of idegree and odegree was introduced by B. D. Acharya et al [1]. There are many patterns in trees of diameter 4. In this paper, 4 patterns of trees of diameter 4 are considered and soShearEnergy are calculated for all possible minimal dominating set. SoShearEnergy curve for those graphs are plotted. Remaining patterns are discussed in the papers to come.

Key Words: idegree, odegree, oShearEnergy & soShearEnergy

1. Introduction:

Let G = (V, E) be a simple, finite, non trivial connected graph. A set D $\subset$ V is adjacent to some vertex in D. A dominating set D of G is called a minimal dominating set if no proper subset of D is a dominating set. In the year 2007, Shearenergy of a graph with respect to the minimal dominating set in terms of idegree and odegree was introduced by B.D.Acharya et al [1]. In the earlier paper soShearEnergy of many graph are been calculated.[2] [3] [4]. Let us consider some trees of diameter 4 and denote it by $T_{d=4}$. $T_{d=4}$ contains three internal vertices v_1, v_2, v_3 and the number of pendent vertices attached to these vertices are n_1, n_2 and n_3 respectively. Label a pendent vertex at v_1 and v_3 as u_1 and u_2 respectively. In this paper let us consider four types of trees of diameter 4. Remaing trees are considered in the papers to come.

The four types of trees which are considered are $T_{d=4}$ with

Type 1: $n_i \geq 2, i = 1,3 \; and \; n_2 \geq 1$

Type 2: $n_i \geq 2, i = 1,3 \; and \; n_2 = 0$

Type 3: $n_1 > 2, n_2 > 1 \; and \; n_3 = 1$

Type 4: $n_1 = 1, n_2 > 1 \; and \; n_3 \geq 1$

Basic definitions are given below

Definition 1.1: Let G be a graph and S be a subset of V(G). Let v $\in$ V-S, the idegree of v with respect to S is the number of neighbours of v in V-S and it is denoted by $id_S(v)$.

Definition 1.2: Let G be a graph and S be a subset of V(G). Let v $\in$ V-S, the odegree of v with respect to S is the number of neighbours of v in S and is denoted as $od_S(v)$.

Definition 1.3: Let G be graph and S be a subset of V(G). Let v $\in$ V-S, the oidegree of v with respect to S is $od_S(v) - id_S(v)$ if $od_S(v) > id_S(v)$ and it is denoted by $oid_S(v)$.

Definition 1.4: Let G be a graph and S be a subset of V(G). Let v $\in$ V-S, the iodegree of v with respect to S is $id_S(v) - od_S(v)$ if $id_S(v) > od_S(v)$ and it is denoted by $iod_S(v)$.

Definition 1.5: Let G be a graph and D be a dominating set, oShearEnergy of a graph with respect to D denoted by $os\varepsilon_D(G)$ is the summation of all oid if od > id or otherwise zero .

Definition 1.6: Let G be a graph and D be a minimal dominating set ,then energy curve is the curve obtained by joining the oShearEnergies with respect to D_{i-1} and D_i for $1 \leq i \leq n$, taking the number of vertices of D_i along the x axis and the oShearEnergy with respect to the D_i along the y axis.

Definition 1.7: Let G be a graph and D be a minimal dominating set, soShearEnergy of a graph with respect to D is $\sum_{i=0}^{|V-D|} os\varepsilon_{D_{i+1}}(G)$ where $D_{i+1} = D_i \cup V_{i+1}$, V_{i+1} is a singleton vertex with minimum oidegree of V-D_i and D_0 is a minimal dominating set where $0 \leq i \leq |V-D|$, it is denoted by $sos\varepsilon_D(G)$.

Definition 1.8: Let G be a graph and MDS(G) be the set of all minimal dominating set of G , then Hardihood^{+} of a graph G is max{ $sos\,\varepsilon_{MDS(G)}(G)$ } is denoted as HD^{+}(G).

Definition 1.9: Let G be a graph and MDS(G) be the set of all minimal dominating set of G , then Hardihood^{-} of a graph G is is min{ $sos\,\varepsilon_{MDS(G)}(G)$ } is denoted as HD^{-}(G). Let us denote tree of diameter 4 and of type t1,t2,t3,t4 by $T_{d=4,ti}$, i = 1,2,3,4.

Theorem 1.7:

Let $T_{d=n}$ be a tree of diameter n with $n_1, n_2, \ldots, n_{n-1}$ be the number of pendent vertices attached to $v_1, v_2, \ldots, v_{n-1}$. Let D be the minimal connected dominating set $\{v_1, v_2, \ldots, v_{n-1}\}$. Then $sos\,\varepsilon_{T_{d=n}}(D) = \frac{(os\varepsilon_{T_{d=n}}(D))(os\varepsilon_{T_{d=n}}(D)+1)}{2}$ where $os\varepsilon_{T_{d=n}}(D) = n_1 + n_2 + \ldots + n_{n-1}$.

Proof:

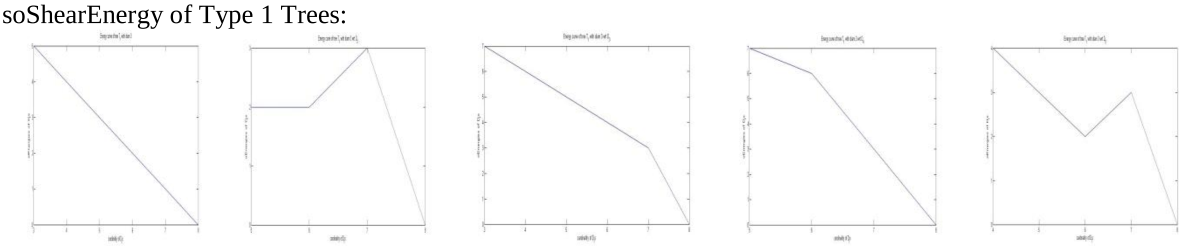
Let $T_{d=n}$ be a tree of diameter n with $n_1, n_2, \ldots, n_{n-1}$ be the number of pendent vertices attached to $v_1, v_2, \ldots, v_{n-1}$. Let D be the minimal connected dominating set and cardinality of D is n-1, then V-D contains all $n_1, n_2, \ldots, n_{n-1}$ pendent vertices, and the cardinality of the set V-D is $n_1 + n_2 + \ldots + n_{n-1}$. Since all the vertices in V-D are pendent vertices id=0 ,od=1 and oid=1 . Hence $os\varepsilon_{T_{d=n}}(D) = n_1 + n_2 + \ldots + n_{n-1}$

By a theorem and algorithm in [2], $sos\,\varepsilon_{T_{d=n}}(D) = \frac{(os\varepsilon_{T_{d=n}}(D))(os\varepsilon_{T_{d=n}}(D)+1)}{2}$ where

$os\varepsilon_{T_{d=n}}(D) = n_1 + n_2 + \ldots + n_{n-1}$.

Let us now consider the each type of trees one by one.

soShearEnergy of Type 1 Trees:



Lemma 2.1:

Let $T_{d=n,t1}$ be a tree of diameter 4 and of type 1 with given minimal dominating set ^{1}D which is connected, then $sos\,\varepsilon_{T_{d=4,t1}}(D) = \frac{(os\varepsilon_{T_{d=4,t1}}(^{1}D))(os\varepsilon_{T_{d=4,t1}}(^{1}D)+1)}{2}$ where $os\varepsilon_{T_{d=4,t1}}(^{1}D) = n_1 + n_2 + n_3$.

Proof:

Let $T_{d=n,t}$ be tree of diameter 4 and of type 1 with the given minimal dominating set ^{1}D which is connected dominating set. For the given tree, $|^{1}D| = 3$ and $|V - {}^{1}D| = \sum_{i=1}^{3} n_i$.

All the vertices in the set $V - {}^{1}D$ are pendent vertices, hence by theorem 1.9,

$sos\,\varepsilon_{T_{d=4,t1}}(D) = \frac{(os\varepsilon_{T_{d=4,t1}}(^{1}D))(os\varepsilon_{T_{d=4,t1}}(^{1}D)+1)}{2}$ where $os\varepsilon_{T_{d=4,t1}}(^{1}D) = n_1 + n_2 + n_3$.

Lemma 2.2:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with given minimal dominating set ^{2}D which is the compliment of the dominating set ^{1}D, then

- ✓ If $n_1 - 1 > n_2$ & n_3 then

$$sos\,\varepsilon_{T_{d=4,t1}}(^{2}D) = \begin{cases} 3n_1 + 2n_2 + n_3 - 4 & if\ n_2 - 2 > n_3 - 1 \\ 3n_1 + n_2 + 2n_3 - 4 & if\ n_3 - 1 > n_2 - 2 \\ 3n_1 + 2n_2 + n_3 - 1 & if\ n_2 - 2 = n_3 - 1 \end{cases}$$

- ✓ If $n_2 - 1 > n_1$ & n_3 then

$$sos\,\varepsilon_{T_{d=4,t1}}(^{2}D) = \begin{cases} n_1 + 3n_2 + 2n_3 - 2 & if\ n_3 - 1 > n_1 - 1 \\ 2n_1 + 3n_2 + n_3 - 2 & if\ n_1 - 1 > n_3 - 1 \\ n_1 + 3n_2 + 2n_3 - 3 & if\ n_1 - 1 = n_3 - 1 \end{cases}$$

✓ If $n_3 - 1 > n_1$ & n_2 then

$$sos\varepsilon_{T_{d=4,t1}}(^2D) = \begin{cases} n_1 + 2n_2 + 3n_3 - 2 & if\ n_2 - 2 > n_1 - 1 \\ 2n_1 + n_2 + 3n_3 - 2 & if\ n_1 - 1 > n_2 - 2 \\ 2n_1 + n_2 + 3n_3 - 3 & if\ n_2 - 2 = n_1 - 1 \end{cases}$$

✓ If $n_1 - 1 = n_2 - 1 = n_3 - 1$ then

$$sos\varepsilon_{T_{d=4,t1}}(^2D) = 2n_1 + n_2 + 3n_3 - 1$$

Proof:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with the given minimal dominating set ^2D which is the set of all pendent vertices, with $\left|^2D\right| = n_1 + n_2 + n_3 + 2$ and $\left|V - ^2D\right| = 3$.

$id(v_i) = 1$, $od(v_i) = n_i$ and $oid(v_i) = n_i - 1$ for $v_i \in V - ^2D$ such that $d(v_i) = n_i + 1, i = 1,3$.

$id(v) = 2, od(v) = n_2$ & $oid(v) = n_2 - 2$ for $v \in V - ^2D$ such that $d(v) = n_2 + 2$.

Therefore $os \in_{T_{d=4,t1}}(^2D) = n_1 + n_2 + n_3 - 4$.

Case (i): Let us consider $n_1 - 1 > n_2 - 2$ & $n_3 - 1$, then the vertex to be shifted is the vertex of degree $n_3 + 1$, then there is change in vertex of degree $n_2 + 2$, then $id(v) = 1, od(v) = n_2 + 1$ & $oid(v) = n_2$ for $v \in V - ^2D\ and\ d(v) = n_2 + 2$.

Hence $os \in_{T_{d=4,t1}}(^2D_1) = n_1 + n_2 - 1$. If $n_1 > n_2$, then the vertex to be shifted is vertex of degree $n_2 + 2$ then $os\varepsilon_{T_{d=4,t1}}(^2D_2) = n_1 + 1$

Therefore $sos\varepsilon_{T_{d=4,t1}}(^2D) = \begin{cases} 3n_1 + 2n_2 + n_3 - 4 & if\ n_2 - 2 > n_3 - 1 \\ 3n_1 + n_2 + 2n_3 - 4 & if\ n_3 - 1 > n_2 - 2 \end{cases}$

Subcase: Let us consider $n_1 - 1 > n_2 - 2 = n_3 - 1$. Since the vertex v_2 have higher idegree, v_2 is shifted to the dominating set. Then there is change in idegree, odegree and oidegrees of vertices v_1 and v_3. For vertices v_1 and v_3 idegree is 0, odegrees of v_1 and v_3 are $n_1 + 1, n_3 + 1$ respectively. Then $os\varepsilon_{T_{d=4,t1}}(^2D_2) = n_1 + n_3 + 2$. Shifting vertex v_3, $os\varepsilon_{T_{d=4,t1}}(^2D_3) = n_1 + 1$.

Then $sos\varepsilon_{T_{d=4,t1}}(^2D) = 3n_1 + n_2 + 2n_3 - 1$.

Case (ii): Let us consider $n_2 - 2 > n_3 - 1$ & $n_1 - 1$, then by the above argument,

$$sos\varepsilon_{T_{d=4,t1}}(^2D) = \begin{cases} n_1 + 3n_2 + 2n_3 - 2\ if\ n_2 - 2 > n_3 \\ n_1 + 2n_2 + 3n_3 - 2\ otherwise \end{cases}$$

Subcase (i): Let us consider $n_2 - 2 > n_3 - 1 = n_1 - 1$. Since $n_3 - 1 = n_1 - 1$, they have the same idegree, any one of the vertices v_1 or v_3 can be shifted to the dominating set. With out loss of generality, let us choose the vertex v_1 and shift vertex v_1 to the dominating set. Then $id(v_2) = 1, od(v_2) = n_2 + 1$ and $oid(v_2) = n_2$. Then $os\varepsilon_{T_{d=4,t1}}(^2D_2) = n_2 + n_3 + 1$. Then shifting vertex v_3 to the dominating set we get $os\varepsilon_{T_{d=4,t1}}(^2D_3) = n_2 + 2$. Therefore $sos\varepsilon_{T_{d=4,t1}}(^2D) = n_1 + 3n_2 + n_3 - 3$.

Case (iii): Let us consider $n_3 - 1 > n_2 - 2 > n_1 - 1$, then the vertex to be shifted is the vertex of degree $n_1 + 1$, then as in the above case there is change in the vertex of degree $n_2 + 2$ and $os\varepsilon_{T_{d=4,t1}}(^2D_2) = n_2 + n_3 - 1$. Therefore $sos\varepsilon_{T_{d=4,t1}}(^2D) = \begin{cases} n_1 + 2n_2 + 3n_3 - 2\ if\ n_3 - 1 > n_2 \\ n_1 + 2n_2 + 3n_3 - 2\ otherwise \end{cases}$

Subcase (i): Let us consider. By similar argument in Subcase of (i), we get the result $sos\varepsilon_{T_{d=4,t1}}(^2D) = 2n_1 + n_2 + 3n_3 - 1.$

Case (iv): Let us consider $n_1 - 1 = n_2 - 2 = n_3 - 1 = x$. As the three vertices v_1, v_2, v_3 are in a path, $id(v_i) = 1, i = 1,3, od(v_i) = n_i, i = 1,3$ and $oid(v_i) = n_i - 1.$

For the vertex $v_2, id(v_2) = 2, od(v_2) = n_2$ and $oid(v_2) = n_2.$ As all the three oids are equal choose vertex v_2 with greater id. Then $id(v_i) = 0, i = 1,3; od(v_i) = n_i + 1, i = 1,3$ and $oid(v_i) = n_i + 1.$ Hence $os\varepsilon_{T_{d=4,t1}}(^2D_2) = n_1 + n_3 + 2.$

Without loss of generality choose vertex v_1 and shift to the dominating set, then $os\varepsilon_{T_{d=4,t1}}(^2D_3) = n_3 + 1.$

Hence $sos\varepsilon_{T_{d=4,t1}}(^2D) = 2n_1 + n_2 + 3n_3 - 1.$

Lemma 2.3:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with given minimal dominating sets ^3D. If ^3D = A U B where A is set of vertices of degree $n_1 + 2$ and $n_3 + 2$ and B is the n_2 pendent vertices , then $sos\varepsilon_{T_{d=4,t1}}(D) = (n_1 + n_3 + 1)(n_2 + 2)\frac{(n_1 + n_3)}{2}$.

Proof:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with the given minimal dominating set ^3D . The dominating set ^3D contains vertices of degree n_1+2, $n_3 +2$ and n_2 pendent vertices. Then $\left|^3D\right| = n_2 + 2$. The set V-D contains $n_1 + n_3$ pendent vertices and a vertex of degree $n_2 + 2$, then $\left|V - {}^3D\right| = n_1 + n_3 + 1$.

All the vertices in the set $V - {}^3D$ are not adjacent to each other have idegree zero. For the n1 + n_3 pendent vertices odegrees are 1, oidegrees is also 1. For the remaining one vertex odegree is $n_2 + 2$, oidegree is $n_2 + 2$. Therefore $os\varepsilon_{T_{d=4,t1}}(^3D_1) = n_1 + n_3 + n_2 + 2.$

Then by theorem in [2], we get $sos\varepsilon_{T_{d=4,t1}}(D) = (n_1 + n_3 + 1)(n_2 + 2)\frac{(n_1 + n_3)}{2}$.

Lemma 2.4:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with given minimal dominating set $^4D = (^3D)^c$, then $sos\varepsilon_{T_{d=4,t1}}(^4D) = \begin{cases} s-(s-1)+\ldots+(s-n_2)+n_3 \ if \ n_1 < n_3 \\ s-(s-1)+\ldots+(s-n2)+n_1 \ if \ n_3 > n_1 \end{cases}$ where s = $os\varepsilon_{T_{d=4}}(^4D_1)$.

Proof:

Let $T_{d=4,t1}$ be tree of diameter 4 and of type 1 with the given minimal dominating set $^4D = (^3D)^c$. Then by above case $\left|^4D\right| = n_1 + n_3 + 1$ and $\left|V - {}^4D\right| = n_2 + 2.$ idegree of all the vertices in the set $V - {}^4D$ are zero and odegree of both the internal vertices are n_1 and n_3 and of the pendent vertices are one . Therefore $os\varepsilon_{T_{d=4,t1}}(^4D_1) = n_1 + n_2 + n_3$

By algorithm and by simplification, we can write

$sos\varepsilon_{T_{d=4,t1}}(^4D) = s - (s-1) + \ldots + (s - n_2) + n_3$ if $n_1 < n_3$.

If $n_1 > n_3$, $sos\varepsilon_{T_{d=4,t1}}(^4D) = s - (s-1) + \ldots + (s - n_2) + n_1$.

Lemma 2.5:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with given minimal dominating set ^5D. If $^5D = AUB$ where A is set of $n_2 + n_3$ pendent vertices , B is vertex of degree $n_1 + 2$ and C is the pendent vertex at distance 3 from vertex of degree $n_1 + 2$, then

$$sos\varepsilon_{T_{d=4,t1}}(^5D) = \begin{cases} s-(s-1)+\ldots+(s-n_1)+n_3 \ if \ n_2 < n_3 \\ s-(s-1)+\ldots+(s-n_1)+n_2 \ if \ n_3 > n_2 \end{cases}$$

Where $s = n_1 + n_2 + n_3 - 1$

Proof:

Let $T_{d=4,t1}$ be tree of diameter 4 and of type 1 with the given minimal dominating set ${}^5D$. The dominating set ${}^5D$ contains the vertex v_1 and the pendent vertices of v_2 and v_3. Therefore $|{}^5D| = 1 + n_2 + n_3$ and $|V - {}^5D| = n_1 + 2$. The idegree and odegree of all the n_1 pendent vertices are 0 and 1, therefore iodegree is one. For the remaining two vertices, idegree is 1 and odegree is n_2 + 1 and n_3. Therefore the oidegree of these two vertices are n_2 and n_3. Therefore $os\varepsilon_{T_{d=4,t1}}({}^5D_1) = n_1 + n_2 + n_3 - 1$. On simplifcation, we can write $sos\varepsilon_{T_{d=4,t1}}({}^5D) = s + (s-1) + \ldots + (s - n_1) + n_3$ if $n_2 < n_3 - 1$ $sos\varepsilon_{T_{d=4,t1}}({}^5D) = s + (s-1) + \ldots + (s - n_1) + n_2$ if $n_2 > n_3 - 1$ where $s = n_1 + n_2 + n_3 - 1.$

Lemma 2.6:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with given minimal dominating set ${}^6D$. If ${}^6D = AUB$ where A is the singleton set v_3 and B is the $n_1 + n_3$ pendent vertices, then

$$sos\varepsilon_{T_{d=4,t1}}({}^6D) = \begin{cases} s - (s-1) + \ldots + (s - n_3) + n_1 \ if \ n_2 < n_1 \\ s - (s-1) + \ldots + (s - n_3) + n_2 \ if \ n_3 > n_2 \end{cases} \text{ where } s = n_1 + n_2 + n_3 - 1.$$

Proof:

Let $T_{d=4,t1}$ be tree of diameter 4 and of type 1 with the given minimal dominating set ${}^6D$. The dominating set ${}^6D$ contains v_3 and the $n_1 + n_3$ pendent vertices. Replacing all n_1 with n_3 in the above case we get ,

$$sos\varepsilon_{T_{d=4,t1}}({}^6D) = \begin{cases} s - (s-1) + \ldots + (s - n_3) + n_1 \ if \ n_2 < n_1 \\ s - (s-1) + \ldots + (s - n_3) + n_2 \ if \ n_3 > n_2 \end{cases}$$

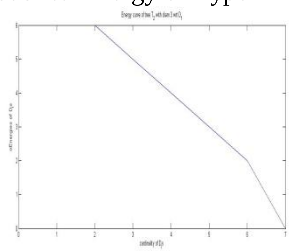
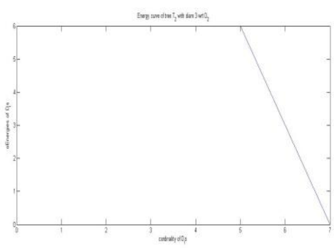
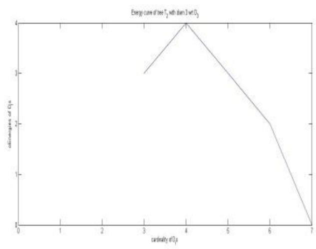
From the above result we can conclude the following result with $n_1 = 2, n_2 = 1, n_3 = 2.$

Theorem 2.7:

Let $T_{d=4,t1}$ be a tree of diameter 4 and of type 1 with given minimal dominating set ${}^iD, i = 1,2,3,4,5,6$, then

1. $HD^+(T_{d=4,t1}) = sos\varepsilon_{T_{d=4,t1}}({}^3D)$
2. $HD^-(T_{d=4,t1}) = sos\varepsilon_{T_{d=4,t1}}({}^2D)$

3. soShearEnergy of Type 2 Trees:

Lemma 3.1:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2 with the given minimal dominating set ${}^1D$ where ${}^1D = \{v : d(v) \geq 3\}$ then, $sos\varepsilon_{T_{d=4,t2}}({}^1D) = \frac{n_1 + n_3}{2}(5 + n_1 + n_3).$

Proof:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2. Let ${}^1D$ be the minimal dominating set with vertices of degree greater than 3, then $|{}^1D| = 2$ and $|V - {}^1D| = 1 + n_1 + n_3$ .Therefore the $os\varepsilon_{T_{d=4,t2}}({}^1D_1) = 2 + n_1 + n_3.$

By theorem in [2] and by simplification we get $sos\varepsilon_{T_{d=4,t2}}({}^1D) = \frac{n_1 + n_3}{2}(5 + n_1 + n_3).$

Lemma 3.2:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2 with the given minimal dominating set ${}^2D = ({}^1D)^C$ then,

$$sos\varepsilon_{T_{d=4,t2}}({}^1D)=\begin{cases}2n_1+n_3+3\ if\ n_1>n_3\\ n_1+2n_3+3\ otherwise\end{cases}$$

Proof:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2 with the given minimal dominating set ${}^2D=({}^1D)^C$, then by the above case , $\left|{}^2D\right|=n_1+n_3+1$ and $\left|V-{}^2D\right|=2$. The idegree and odegree of both the vertices are zero, odegree of vertex of degree n_1+1 is n_1+1 and for the vertex of degree n_3+1 is n_3+1. Hence oidegree of these two vertices are n_1+1 and n_3+1 . Hence $os\varepsilon_{T_{d=4,t2}}({}^2D_1)=n_1+n_3+2.$ Therefore by theorem in [2],

$$sos\varepsilon_{T_{d=4,t2}}({}^1D)=\begin{cases}2n_1+n_3+3\ if\ n_1>n_3\\ n_1+2n_3+3\ otherwise\end{cases}.$$

Lemma 3.3:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2 with the given minimal dominating set ${}^3D=AUB$ where A is a vertex of degree n_1, B is the set of all n_3 pendent vertices then, $sos\varepsilon_{T_{d=4,t2}}({}^3D)=n_1(\frac{2n_3+n_1+1}{2})+(n_3-1)+2.$

Proof:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2. Let ${}^3D$ be the minimal dominating set with n_3 pendent vertices , one vertex of degree n_1. Then $\left|{}^3D\right|=n_3+1$ and $\left|V-{}^3D\right|=n_1+3.$

The idegree of n_1 pendent vertices are zero, odegree is one, oidegree is 1. The idegree and odegree of vertex v_2 is 1, hence oidegree is 0. Remaing vertex of degree n_1+2 have idegree 1 and odegree n_3 and oidegree is n_3-1. Therefore the $os\varepsilon_{T_{d=4,t2}}({}^3D_1)=(n_1+n_3).$ By the algorithm , the vertex to be shifted to the D set is vertex of degree 1, n_1 pendent vertices are shifted one by one. At the n_1 th stage $os\varepsilon_{T_{d=4,t2}}({}^3D_{n_1})=n_3-1.$ At the n_1+1^{th} stage vertex v_3 is shifted to the D set, then $os\varepsilon_{T_{d=4,t2}}({}^3D_{n_1+1})=2.$

By definition, $sos\varepsilon_{T_{d=4,t2}}({}^3D\ )=(n_1+n_3)+n_3+(n_1-1)+...+n_3+(n_3-1)+2.$

$sos\varepsilon_{T_{d=4,t2}}({}^3D)=n_1(\frac{2n_3+n_1+1}{2})+(n_3-1)+2.$

Lemma 3.4:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2 with the given minimal dominating set ${}^4D=AUB$ where A is a vertex of degree n_3, B is the set of all n_1 pendent vertices be the given minimal dominating set then $sos\varepsilon_{T_{d=4,t2}}({}^4D)=n_3(\frac{2n_1+n_3+1}{2})+(n_1-1)+2.$

Proof:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2 with n_1, n_3 be the pendent vertices attached to the vertex v_1 and v_3. Let ${}^4D$ is the minimal dominating set with n_1 pendent vertices, one vertex of degree $n_3+2.$ Then $\left|{}^4D\right|=n_1+1$ and $\left|V-{}^4D\right|=n_3+3$. By replacing n_1 with n_3 in the above theorem we get the result

$$sos\varepsilon_{T_{d=4,t2}}({}^4D)=n_3(\frac{2n_1+n_3+1}{2})+(n_1-1)+2.$$

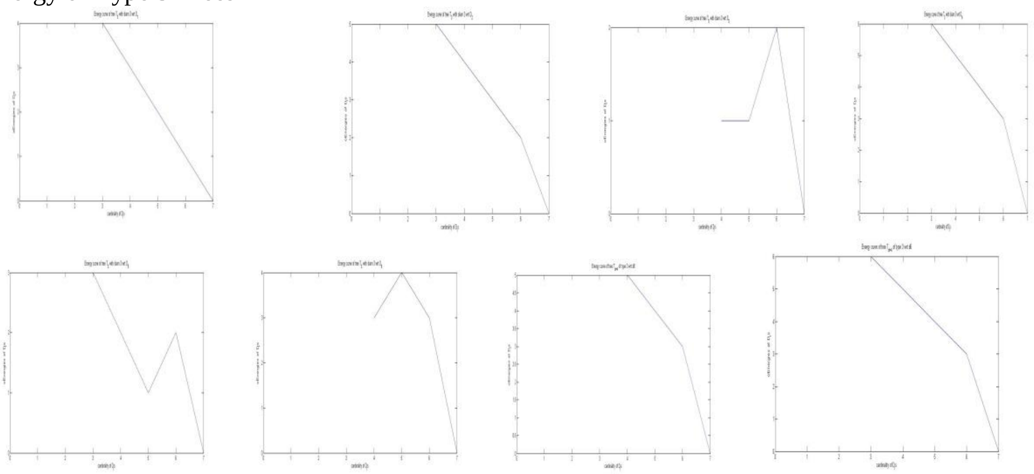
Theorem 3.5:

Let $T_{d=4,t2}$ be a tree of diameter 4 and of type 2 with the given minimal dominating set ${}^1D,\ {}^2D,\ {}^3D$, ${}^4D$ with $n_1=2, n_2=0, n_3=2,\ then$

1. $HD^+(T_{d=4,t2})=sos\varepsilon_{T_{d=4,t2}}({}^4D).$

2. HD'($T_{d=4,t2}$) = $sos\varepsilon_{T_{d=4,t2}}({}^2D)$.

Proof: From Lemma 3.1, Lemma 3.2, Lemma 3.3, Lemma 3.4, the results holds good.**4.** soShearEnergy of Type 3 Trees



Lemma 4.1:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^1D$ which is the connected dominating set then, $sos\varepsilon_{T_{d=4,t3}}({}^4D) = (\frac{os\varepsilon_{T_{d=4,t3}}({}^1D_1)(os\varepsilon_{T_{d=4,t3}}({}^1D_1)+1)}{2}$ where $os\varepsilon_{T_{d=4,t3}} = n_1 + n_2$.

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^1D$ which is a connected dominating set . Since the given tree is of diameter 4, $\left|{}^1D\right| = 3$ and $\left|V - {}^1D\right| = n_1 + n_2$.

By theorem 1.9, $sos\varepsilon_{T_{d=4,t3}}({}^4D) = (\frac{os\varepsilon_{T_{d=4,t3}}({}^1D_1)(os\varepsilon_{T_{d=4,t3}}({}^1D_1)+1)}{2}$ where $os\varepsilon_{T_{d=4,t3}} = n_1 + n_2$.

Lemma 4.2:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^2D$ is the compliment of ${}^1D$ and $n_1 - 1, n_2 - 2 > 1,$ then,

$$sos\varepsilon_{T_{d=4,t3}} = \begin{cases} 3n_1 + n_2 + 1 \; if \; n_1 - 1 > n_2 - 2 \\ n_1 + 2n_2 \; otherwise \end{cases}$$

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^2D$ is the set of all vertices of degree one, then $\left|{}^2D\right| = n_1 + n_2$ and $\left|V - {}^2D\right| = 3$. It consists of the vertices v_1, v_2, v_3 of degree $n_1 + 1, n_2 + 2, 2$. Therefore idegree of the vertex v_1 is 1, of the vertex v_2 is 2 , of the vertex v_3 is 1. The odegree of these vertices are $n_1, n_2, 1$. Therefore oidegree of these vertices are $n_1 + 1, n_2 + 2, 2$ and 0 respectively. Therefore $os\varepsilon_{T_{d=4,t3}}({}^2D_1) = n_1 + n_2 - 3.$

Case (i): Let us consider $n_1 - 1 > n_2 - 2 > 1$, then vertex to be shifted is vertex of degree $n_2 + 2.$ $os\varepsilon_{T_{d=4,t3}}({}^2D_2) = n_1 + 3.$ Vertex of degree 2 is shifted and $os\varepsilon_{T_{d=4,t3}}({}^2D_3) = n_1 + 1.$

Hence $sos\varepsilon_{T_{d=4,t3}}({}^2D) = 3n_1 + n_2 + 1.$

Case (ii): Let us consider $n_2 - 2 > n_1 - 1 > 1$ then similar to the above case , $sos\varepsilon_{T_{d=4,t3}}({}^2D) = n_1 + 2n_2.$

Hence $sos\varepsilon_{T_{d=4,t3}} = \begin{cases} 3n_1 + n_2 + 1 \; if \; n_1 - 1 > n_2 - 2 \\ n_1 + 2n_2 \; otherwise \end{cases}$

Lemma 4.3:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^3D = A \cup B$ where A is the set of vertices v_1, v_2 and B is singleton set u_2

Then $sos\varepsilon_{T_{d=4,t3}}({}^3D)=\dfrac{os\varepsilon_{T_{d=4,t3}}({}^3D_1)(os\varepsilon_{T_{d=4,t3}}({}^3D_1)-2)}{2}$. where $os\varepsilon_{T_{d=4,t3}}({}^3D_1)=n_1+n_2+2.$

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^3D$. ${}^3D$ is the set of vertices of degree greater than 3 and the pendent vertex n_2. Then $|{}^3D|=3$ and $|V-{}^3D|=n_1+n_2+1$. As n_1+n_2 vertices in the set are pendent vertices their iodegree is 0 and odegree is 1. For the remaing one vertex, from the construction it is celar that, it is a vertex of degree 2. As the end vertex is also in the set D idegree is 0 and odegree is 2. Therefore $os\varepsilon_{T_{d=4,t3}}({}^3D_1)=n_1+n_2+2.$

Then, by theorem in [2] and by simplification, $sos\varepsilon_{T_{d=4,t3}}({}^3D)=\dfrac{os\varepsilon_{T_{d=4,t3}}({}^3D_1)(os\varepsilon_{T_{d=4,t3}}({}^3D_1)-2)}{2}$.

Lemma 4.4:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^4D=({}^3D)^c$, then $sos\varepsilon_{T_{d=4,t3}}=\begin{cases}3n_1+2n_2 & if\ n_1>n_2-1\\ 3n_1+n_2+3 & if\ n_1>n_2=0\\ 2n_1+3n_2 & if\ n_2>n_1-1\\ n_1+3n_2+2 & if\ n_2>n_1=1\end{cases}$

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^4D=({}^3D)^c$ .The set contains vertices of degree n_1+1, n_2+2. Hence $|V-{}^4D|=3.$ The vertices of degree n_1+1 and n_2+2 are adjacent to each other, hence their idegrees are 1 and odegree are n_1 and n_2+1 respectively, hence oidegree are $n_1-1\,\&\,n_2$. For the vertex u_2 idegree is 0, odegree is 1 and oidegree is 1.

Hence $os\varepsilon_{T_{d=4,t3}}({}^4D_1)=n_1-1+n_2+1=n_1+n_2$.

If $n_1>n_2>1$, then $os\varepsilon_{T_{d=4,t3}}({}^4D_2)=\begin{cases}n_1+n_2-1 & if\ n_1>n_2>1\\ n_1+2 & if\ n_1>n_2=1\end{cases}$.

$os\varepsilon_{T_{d=4,t3}}({}^4D_3)=\begin{cases}n_1+1 & if\ n_1>n_2>1\\ n_2 & if\ n_1>n_2=1\end{cases}$.

Hence $sos\varepsilon_{T_{d=4,t3}}({}^4D\ )=\begin{cases}3n_1+2n_2 & if\ n_1>n_2>1\\ 3n_1+n_2+3 & if\ n_1>n_2=1\end{cases}$.

If $n_2>n_1>1$, then $os\varepsilon_{T_{d=4,t3}}({}^4D_2)=\begin{cases}n_1+n_2-1 & if\ n_2>n_1>1\\ n_2+2 & if\ n_2>n_1=1\end{cases}$.

$os\varepsilon_{T_{d=4,t3}}({}^4D_3)=n_2+1$.

Hence $sos\varepsilon_{T_{d=4,t3}}({}^4D)=\begin{cases}2n_1+3n_2 & if\ n_2>n_1-1\\ n_1+3n_2+2 & if\ n_2>n_1=1\end{cases}$

Combining the two cases we get

$$sos\varepsilon_{T_{d=4,t3}}({}^4D)=\begin{cases}3n_1+2n_2 & if\ n_1>n_2-1\\ 3n_1+n_2+3 & if\ n_1>n_2=0\\ 2n_1+3n_2 & if\ n_2>n_1-1\\ n_1+3n_2+2 & if\ n_2>n_1=1\end{cases}$$

Lemma 4.5:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^5D=A\cup B$ where A is the set of vertices v_1,v_3 and B is set of n_2 pendent vertices , then $sos\varepsilon_{T_{d=4,t3}}({}^5D)=os\varepsilon_{T_{d=4,t3}}({}^5D_1)+(os\varepsilon_{T_{d=4,t3}}({}^5D_1)-1)+\ldots+(n_2+2)$.

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^5D$, which contain vertices of degrees $n_1+1,2$ & n_2 pendent vertices. Then $|{}^5D| = n_2+2$. The set $V-{}^5D$ contains vertex of degree n_2+2, n_1 pendent vertices, a vertex of degree 1, u_1. Then $|V-{}^5D| = n_1+2$. There are n_1+1 pendent vertices, hence oidegree of n_1+1 vertices are 1. For the vertex of degree n_2+2, idegree 0, odegree is n_2+2, oidegree is n_2+2. Hence $os\varepsilon_{T_{d=4,t3}}({}^5D_1) = n_1+n_2+3$.

By theorem in [2], $sos\varepsilon_{T_{d=4,t3}}({}^5D) = os\varepsilon_{T_{d=4,t3}}({}^5D_1) + (os\varepsilon_{T_{d=4,t3}}({}^5D_1)-1) + \ldots + (n_2+2)$.

Lemma 4.6:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^6D = ({}^5D)^c$, then $sos\varepsilon_{T_{d=4,t3}}({}^6D) = (n_1+n_2+2)+(n_1+n_2+1)+\ldots+(n_2+2)+n_2$.

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^6D = ({}^5D)^c$. By the above case $|{}^6D| = n_1+2$ and $|V-{}^6D| = n_2+2$. All the vertices in $V-{}^6D$ are independent vertices and therefore idegrees of all the n_2+2 vertices are 0. odegrees of the n_2 pendent vertices and 1 and hence oidegree is aslo 1. For v_1 odegree and oidegree are n_1+1. The degree of vertex v_3 is 2. Hence oidegree is also 2. Therefore $os\varepsilon_{T_{d=4,t3}}({}^6D_1) = n_1+n_2+2$. By algorithm in [2], the value decreses one by one for n_1 number of times. Hence $os\varepsilon_{T_{d=4,t3}}({}^6D_{n_1}) = n_2+2, os\varepsilon_{T_{d=4,t3}}({}^6D_{n_1+1}) = n_2$.

Hence $sos\varepsilon_{T_{d=4,t3}}({}^6D) = (n_1+n_2+2)+(n_1+n_2+1)+\ldots+(n_2+2)+n_2$.

Lemma 4.7:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^7D = A\cup B\cup C$ where A is the singleton set v_1 and B is the vertex u_2 and C is the n_2 pendent vertices, then

$$sos\varepsilon_{T_{d=4,t3}}({}^7D) = \begin{cases} (n_1+n_2)+(n_1+n_2-1)+\ldots+(n_2+1)+n_2+2 & if \quad n_2>2 \\ (n_1+1)+(n_1+2)+(n_1+1)+\ldots+3+2 & if \ n_2=1 \end{cases}$$

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set ${}^7D$ which contains vertices v_1, u_2 and n_2 pendent vertices. Then $|{}^7D| = n_2+2$ and $|V-{}^7D| = n_1+2$. It contains vertex of degree n_2+2, v_2, n_1 pendent vertices and a vertex of degree 2. For the n_1 pendent vertices, oidegree is 1. For the vertex of degree 2, idegree is 1, odegree 1 and oidegree 0. For the vertex of degree n_2+2, idegree is 1, odegree is n_2+1,oidegree is n2 . Hence $os\varepsilon_{T_{d=4,t3}}({}^7D_1) = n_1+n_2$. If $n_2>2$, then $os\varepsilon_{T_{d=4,t3}}$ reduces one by one for n_1+1 number of steps. Then $os\varepsilon_{T_{d=4,t3}}({}^7D_{n_1}) = n_2$. The vertex of degree n_2+2 is shifted to the D set then $os\varepsilon_{T_{d=4,t3}}({}^7D_{n_1+3}) = 2$.

Hence $sos\varepsilon_{T_{d=4,t3}}({}^7D) = (n_1+n_2+1)+(n_1+n_2)+\ldots.+(n_2+1)+n_2+2$.

If $n_2=1$, then the vertex to be shifted is vertex of degree n_2+2, then the oidegree of vertex of degree 2 is changed to 2. Then $os\varepsilon_{T_{d=4,t3}}({}^7D_2) = n_1+2$. Then the value reduces one by one for n_1 number of times. Then $os\varepsilon_{T_{d=4,t3}}({}^7D_{n1+2}) = 2$. Therefore $sos\varepsilon_{T_{d=4,t3}}({}^7D) = (n_1+1)+(n_1+2)+(n_1+1)+\ldots+3+2$

Combining both the cases,

$$sos\varepsilon_{T_{d=4,t3}}({}^7D) = \begin{cases} (n_1+n_2)+(n_1+n_2-1)+\ldots+(n_2+1)+n_2+2 & if \quad n_2>2 \\ (n_1+1)+(n_1+2)+(n_1+1)+\ldots+3+2 & if \ n_2=1 \end{cases}$$

Lemma 4.8:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set $^{8}D = (^{7}D)^{c}$ then, $sos\varepsilon_{T_{d=4,t3}}(^{8}D) = (n_1+2+n_2)+(n_1+1+n_2)+\ldots+(n_1+1).$

Proof:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating set $^{8}D = (^{7}D)^{c}$. By above case $\left|^{8}D\right| = n_1+2$ and $\left|V - {}^{8}D\right| = n_2+2$. All the vertices are independent in $V - {}^{8}D$ and they have idegree 0. Hence they have their degrees as their oidegree. There are n_2+1 pendent vertices of degree 1, a vertex of degree n_1+1. Therefore $os\varepsilon_{T_{d=4,t3}}(^{8}D_1) = n_{1.}+2+n_2.$

Hence by theorem in [2], $sos\varepsilon_{T_{d=4,t3}}(^{8}D) = (n_1+2+n_2)+(n_1+1+n_2)+\ldots+(n_1+1).$

Theorem 4.9:

Let $T_{d=4,t3}$ be a tree of diameter 4 and of type 3 with the given minimal dominating sets $^{i}D = 1,2,\ldots 8$ with $n_1 = 2, n_2 = 1, n_3 = 2$ then

(i) HD+($T_{d=4,t3}$)= $sos\varepsilon_{T_{d=4,t3}}(^{5}D)$

(ii) HD($T_{d=4,t3}$) = $sos\varepsilon_{T_{d=4,t3}}(^{2}D)$

Proof:

By the Lemma 4.1, Lemma 4.2, Lemma 4.3, Lemma 4.4, Lemma 4.5, Lemma 4.6, Lemma 4.7 and Lemma 4.8, the results hold good.

Remark 4.10: Trees of type 3 and type 4 are isomorphic to each other. Hence replacing n_1 by n_3 in the $sos\varepsilon_{T_{d=4,t3}}(D)$ we get the result.

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