



A MATHEMATICAL MODEL FOR THE EFFECT OF CORTICOSTERONE USING FUZZY EXPONENTIAL DISTRIBUTION

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Abstract:

A fuzzy mathematical model was developed and used this model to calculate the expected mean and variance of Corticosterone level in the given time interval after lights onsets Releasing Hormone treatment. Formulae of fuzzy Exponential distribution and its α -cut sets were presented. Using fuzzy Exponential distribution, we showed that if the Lower α -cut of Mean and variance are increases when different alpha values and upper α -cut of Mean and variance are increases when different alpha values with respect to the time intervals.

Key Words: Fuzzy Exponential Distribution, Corticosterone, Fuzzy Mean & Fuzzy Variance

1. Introduction:

The Exponential distribution is widely used in statistical models for life data. Among all statistical techniques it may be in use for engineering analysis with smaller sample sizes than any other method. The Exponential distribution plays a central role in reliability modeling since it is the only continuous distribution with a constant hazard function. The exponential distribution is perhaps the extensively realistic statistical distribution in several fields. One of the reasons for its prominence is that the exponential distribution has constant hazard rate function. The exponentiated exponential (EE) distribution was introduced by Gupta et. al. [1]. Also generalized exponential distribution development was discussed by Gupta and Kundu [3].

Placing animals in a new environment generally resulted in the same kind of behavior irrespective of the moment of the light-dark cycle. All the animals reared on their hind paws and moved around actively in their new cage for $\approx 10-15$ min At ZT2 intact animals resumed their sleeping position, while at ZT14 the activity turned back to normal basal levels. Only ADX animals defecated several times after their move to a new cage. SCN- lesioned animals, in contrast remained active during the whole 60 min of during the experiments. Our only consider Plasma Corticosterone values after intact rats were placed in a new cage at the time 2 h after lights onset. [2], [3], [4], [5], [6], [7], [8].

2. Notations:

| | | |
|-------------------------|----|--|
| λ | – | Scale parameter |
| $\bar{\lambda}[\alpha]$ | – | Alpha cut of scale value |
| μ | -- | Mean of Exponential distribution |
| σ^2 | -- | Variance of Exponential distribution |
| $\bar{\mu}$ | -- | Fuzzy Mean of Exponential distribution |
| $\bar{\sigma}^2$ | -- | Fuzzy Variance of Exponential distribution |
| t | – | Test termination time |

3. Mathematical Model Using Fuzzy Exponential Distribution:

The continuous probability measure of a crisp event the distribution of which is based by a density $f(x)$, can be represented by and integral

$$f(A) = \int_A f(x)dx = \int_v \chi_{A(x)} f(x)dx$$

from which, replacing the characteristic function, characteristic function

$$\chi_{A(x)} = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

By the membership function, immediately the probability of a fuzzy event A is obtained

$$f(A) = \int_v \chi_{A(x)} f(x)dx$$

The analogous construction for continuous probabilities is obvious. The concept proved workable in many applications, although the usual interpretation of probability as the measure of chance for the event that the next realization will fall into the crisp set A rises considerable difficulties in comprehension: the position of a crisp

singleton, realization within the fuzzy set A would be possible, according to the principle of inclusion for the fuzzy sets, only within the core A_1 of A. Here an interpretation of $f(A)$ may help as the expected value of the membership function $\mu(x)$

$$f(A) = E_p \mu_A(x) [9], [10], [11].$$

The Exponential distribution is widely used in statistical method for life data. Among all statistical techniques it may be in use for engineering analysis with smaller sample sizes than any other method. A continuous random variable T with Exponential distribution $ED(\lambda)$ where, $\lambda > 0$ is scale parameter has the probability density function

$$f(t) = \lambda^{-1} e^{-\left(\frac{t}{\lambda}\right)}, t > 0, \lambda \geq 0$$

In this experiment let us assume that $f(E)$ is not known precisely and it needs to be estimated or obtained from expert opinion. So the f values are uncertain and we substitute $\bar{\lambda}$ for λ . Now $\bar{f}(t)$ is the fuzzy probability of t then the α -cuts of the fuzzy Exponential Distribution

$$\bar{f}(x)[\alpha] = \left\{ \frac{1}{\bar{\lambda}} e^{-\left(\frac{t}{\bar{\lambda}}\right)} / \bar{\lambda} \varepsilon \bar{\lambda} [\alpha] \right\}$$

The distribution function for Fuzzy Exponential distribution is

$$F(t) = \int_0^t \frac{1}{\bar{\lambda}} e^{-\left(\frac{t}{\bar{\lambda}}\right)} dt = \left[\frac{1}{\bar{\lambda}} e^{-\left(\frac{t}{\bar{\lambda}}\right)} \right]_0^t = 1 - e^{-\left(\frac{t}{\bar{\lambda}}\right)}$$

$$E(t) = \int_0^t x \frac{1}{\bar{\lambda}} e^{-\left(\frac{x}{\bar{\lambda}}\right)} dx = \left[\frac{t \frac{1}{\bar{\lambda}} e^{-\left(\frac{t}{\bar{\lambda}}\right)}}{-\frac{1}{\bar{\lambda}}}\right]_0^t - \left[\frac{\frac{1}{\bar{\lambda}} e^{-\left(\frac{t}{\bar{\lambda}}\right)}}{-\frac{1}{\bar{\lambda}^2}}\right]_0^t = \bar{\lambda}$$

$\bar{\mu} = \bar{\lambda}$

$$E(t^2) = \int_0^t x^2 \frac{1}{\bar{\lambda}} e^{-\left(\frac{x}{\bar{\lambda}}\right)} dx = \left[\frac{x^2 \frac{1}{\bar{\lambda}} e^{-\left(\frac{x}{\bar{\lambda}}\right)}}{-\frac{1}{\bar{\lambda}}}\right]_0^t - \left[\frac{2x \frac{1}{\bar{\lambda}} e^{-\left(\frac{x}{\bar{\lambda}}\right)}}{\frac{1}{\bar{\lambda}^2}}\right]_0^t + \left[\frac{2 \frac{1}{\bar{\lambda}} e^{-\left(\frac{x}{\bar{\lambda}}\right)}}{-\frac{1}{\bar{\lambda}^3}}\right]_0^t = 2\bar{\lambda}^2$$

$$\bar{\sigma}^2 = E(t^2) - [E(t)]^2 = 2\bar{\lambda}^2 - (\bar{\lambda})^2 = \bar{\lambda}^2$$

The scale parameter gives the flexibility of Exponential distribution by changing the value of scale parameter. However sometimes we face situations when the parameter is imprecise. Therefore we consider the Exponential distribution with fuzzy parameters by replacing the scale parameter λ into the fuzzy number $\bar{\lambda}$.

For $\alpha \in [0,1]$, the alpha cuts of Mean of Fuzzy Exponential distribution is $\bar{\mu}[\alpha] = [\bar{\mu}_1[\alpha], \bar{\mu}_2[\alpha]]$

Where

$$\bar{\mu}_1[\alpha] = \text{Inf} \{ \bar{\lambda} t / \bar{\lambda} \in \bar{\lambda}[\alpha], t > 0 \}$$

$$\bar{\mu}_2[\alpha] = \text{Sup} \{ \bar{\lambda} t / \bar{\lambda} \in \bar{\lambda}[\alpha], t > 0 \}$$

For $\alpha \in [0,1]$, the alpha cuts of Variance of Fuzzy Threshold Shock Model with three parameter weibull distribution is $\bar{\sigma}^2[\alpha] = [\bar{\sigma}_1^2[\alpha], \bar{\sigma}_2^2[\alpha]]$

Where

$$\bar{\sigma}_1^2[\alpha] = \text{Inf} \{ \bar{\lambda}^2 t / \bar{\lambda} \in \bar{\lambda}[\alpha], t > 0 \}$$

$$\bar{\sigma}_2^2[\alpha] = \text{Sup} \{ \bar{\lambda}^2 t / \bar{\lambda} \in \bar{\lambda}[\alpha], t > 0 \}$$

4. Application:

Let us take an experiment conducted by [1] in the Plasma Corticosterone values after intact rats were placed in a new cage at circadian time. 2 hours after light onset features and elevated Corticosterone levels. We define these patients as corticosterone by their features and by their response to the corticosterone test, which may be a useful tool in diagnosing Corticosterone. Placing the animal in new environment Plasma Corticosterone values after intact rats were placed in a new cage at the time 2 hours after lights onset was determined

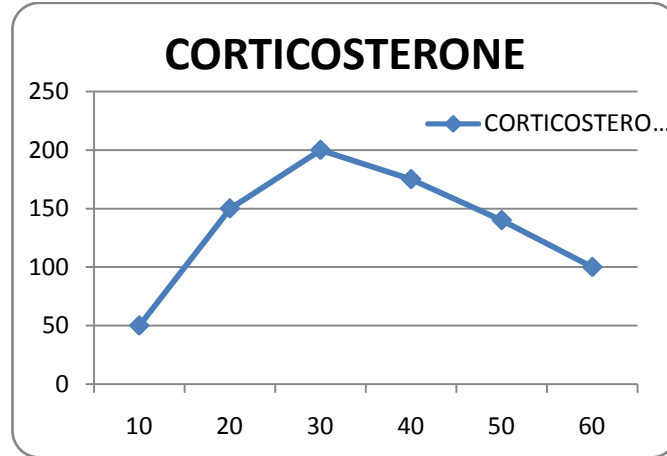


Figure 4.1: Plasma Corticosterone values

In some situations the value of the scale parameter of the Exponential distribution are not known precisely. Therefore we consider triangular numbers for the scale parameter. The triangular fuzzy number of the scale parameter is $\bar{\lambda} = [0.007, 0.0074, 0.008]$

The alpha cut of scale parameter is $\bar{\lambda}[\alpha] = [0.007 + 0.0004\alpha, 0.008 - 0.0006\alpha]$

Table 4.1: Lower α -cut of Mean of Fuzzy Exponential distribution

| α | T=60 | T=50 | T=40 | T=30 | T=20 | T=10 |
|----------|-------|-------|-------|-------|-------|-------|
| 0 | 0.024 | 0.02 | 0.016 | 0.012 | 0.008 | 0.004 |
| 0.1 | 0.066 | 0.055 | 0.044 | 0.033 | 0.022 | 0.011 |
| 0.2 | 0.102 | 0.085 | 0.068 | 0.051 | 0.034 | 0.017 |
| 0.3 | 0.144 | 0.12 | 0.096 | 0.072 | 0.048 | 0.024 |
| 0.4 | 0.18 | 0.15 | 0.12 | 0.09 | 0.06 | 0.03 |
| 0.5 | 0.222 | 0.185 | 0.148 | 0.111 | 0.074 | 0.037 |
| 0.6 | 0.264 | 0.22 | 0.176 | 0.132 | 0.088 | 0.044 |
| 0.7 | 0.3 | 0.25 | 0.2 | 0.15 | 0.1 | 0.05 |
| 0.8 | 0.342 | 0.285 | 0.228 | 0.171 | 0.114 | 0.057 |
| 0.9 | 0.378 | 0.315 | 0.252 | 0.189 | 0.126 | 0.063 |
| 1 | 0.42 | 0.35 | 0.28 | 0.21 | 0.14 | 0.07 |

Table 4.2: Upper α -cut of Mean of Fuzzy Exponential distribution

| A | T=60 | T=50 | T=40 | T=30 | T=20 | T=10 |
|-----|-------|-------|-------|-------|-------|-------|
| 0 | 0.48 | 0.4 | 0.32 | 0.24 | 0.16 | 0.08 |
| 0.1 | 0.438 | 0.365 | 0.292 | 0.219 | 0.146 | 0.073 |
| 0.2 | 0.39 | 0.325 | 0.26 | 0.195 | 0.13 | 0.065 |
| 0.3 | 0.348 | 0.29 | 0.232 | 0.174 | 0.116 | 0.058 |
| 0.4 | 0.3 | 0.25 | 0.2 | 0.15 | 0.1 | 0.05 |
| 0.5 | 0.258 | 0.215 | 0.172 | 0.129 | 0.086 | 0.043 |
| 0.6 | 0.216 | 0.18 | 0.144 | 0.108 | 0.072 | 0.036 |
| 0.7 | 0.168 | 0.14 | 0.112 | 0.084 | 0.056 | 0.028 |
| 0.8 | 0.126 | 0.105 | 0.084 | 0.063 | 0.042 | 0.021 |
| 0.9 | 0.078 | 0.065 | 0.052 | 0.039 | 0.026 | 0.013 |
| 1 | 0.036 | 0.03 | 0.024 | 0.018 | 0.012 | 0.006 |

Table 4.3: Lower α -cut of Variance of Fuzzy Exponential distribution

| A | T=60 | T=50 | T=40 | T=30 | T=20 | T=10 |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| 0 | 0.0000096 | 0.000008 | 0.0000064 | 0.0000048 | 0.0000032 | 0.0000016 |
| 0.1 | 0.0000726 | 0.0000605 | 0.0000484 | 0.0000363 | 0.0000242 | 0.0000121 |
| 0.2 | 0.0001734 | 0.0001445 | 0.0001156 | 0.0000867 | 0.0000578 | 0.0000289 |
| 0.3 | 0.0003456 | 0.000288 | 0.0002304 | 0.0001728 | 0.0001152 | 0.0000576 |

| | | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.4 | 0.00054 | 0.00045 | 0.00036 | 0.00027 | 0.00018 | 0.00009 |
| 0.5 | 0.0008214 | 0.0006845 | 0.0005476 | 0.0004107 | 0.0002738 | 0.0001369 |
| 0.6 | 0.0011616 | 0.000968 | 0.0007744 | 0.0005808 | 0.0003872 | 0.0001936 |
| 0.7 | 0.0015 | 0.00125 | 0.001 | 0.00075 | 0.0005 | 0.00025 |
| 0.8 | 0.0019494 | 0.0016245 | 0.0012996 | 0.0009747 | 0.0006498 | 0.0003249 |
| 0.9 | 0.0023814 | 0.0019845 | 0.0015876 | 0.0011907 | 0.0007938 | 0.0003969 |
| 1 | 0.00294 | 0.00245 | 0.00196 | 0.00147 | 0.00098 | 0.00049 |

Table 4.4: Upper α -cut of Variance of Fuzzy Exponential distribution

| A | T=60 | T=50 | T=40 | T=30 | T=20 | T=10 |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| 0 | 0.00384 | 0.0032 | 0.00256 | 0.00192 | 0.00128 | 0.00064 |
| 0.1 | 0.0031974 | 0.0026645 | 0.0021316 | 0.0015987 | 0.0010658 | 0.0005329 |
| 0.2 | 0.002535 | 0.0021125 | 0.00169 | 0.0012675 | 0.000845 | 0.0004225 |
| 0.3 | 0.0020184 | 0.001682 | 0.0013456 | 0.0010092 | 0.0006728 | 0.0003364 |
| 0.4 | 0.0015 | 0.00125 | 0.001 | 0.00075 | 0.0005 | 0.00025 |
| 0.5 | 0.0011094 | 0.0009245 | 0.0007396 | 0.0005547 | 0.0003698 | 0.0001849 |
| 0.6 | 0.0007776 | 0.000648 | 0.0005184 | 0.0003888 | 0.0002592 | 0.0001296 |
| 0.7 | 0.0004704 | 0.000392 | 0.0003136 | 0.0002352 | 0.0001568 | 0.0000784 |
| 0.8 | 0.0002646 | 0.0002205 | 0.0001764 | 0.0001323 | 0.0000882 | 0.0000441 |
| 0.9 | 0.0001014 | 0.0000845 | 0.0000676 | 0.0000507 | 0.0000338 | 0.0000169 |
| 1 | 0.0000216 | 0.000018 | 0.0000144 | 0.0000108 | 0.0000072 | 0.0000036 |

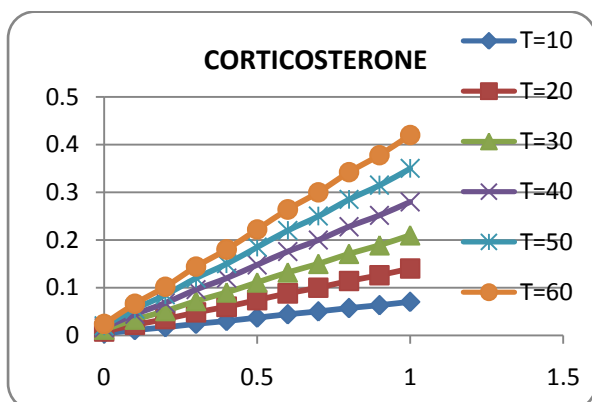


Figure 4.2: Lower α -cut of Mean of Fuzzy Exponential distribution

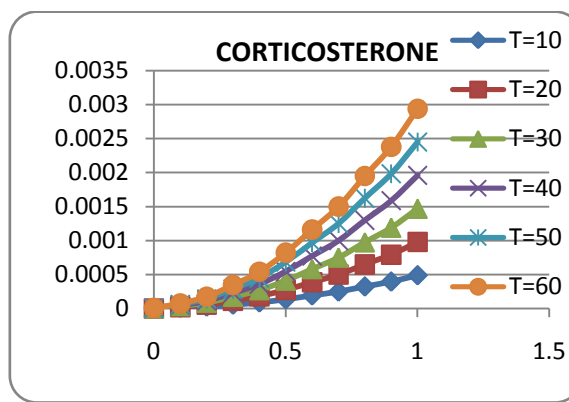


Figure 4.4: Lower α -cut of Variance of Fuzzy Exponential distribution

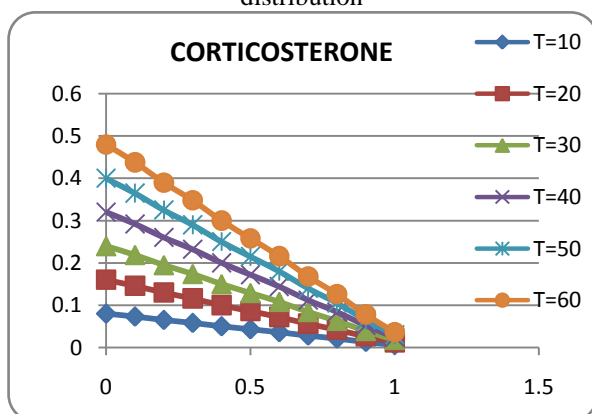


Figure 4.3: Upper α -cut of Mean of Fuzzy Exponential distribution

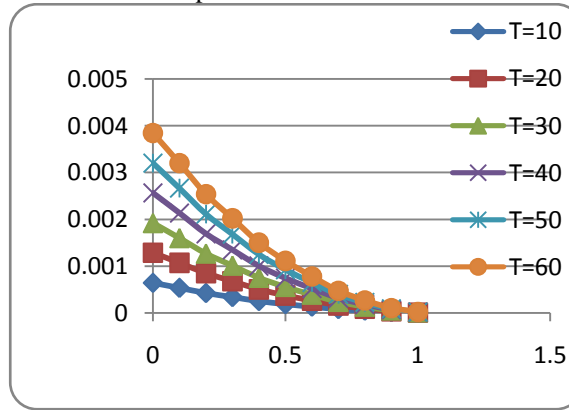


Figure 4.5: Upper α -cut of Variance of Fuzzy Exponential distribution

Lower Mean and Variance of the Fuzzy Exponential distribution for the effect of Secretion of corticosterone with $t=10, 20, 30, 40, 50, 60$. it is clear that the α -cut for the fuzzy Exponential distribution Mean values Variance values are increases when t increases. Upper Mean and Variance of the Fuzzy Exponential distribution for the effect of Secretion of corticosterone with $t=10, 20, 30, 40, 50, 60$. it is clear that the α -cut for the fuzzy Exponential distribution Mean values and Variance values increases when t increases.

5. Conclusion:

In this paper, some results are discussed .Fuzzy Exponential distribution for the effect of Secretion of Corticosterone with different t values. Using Exponential distribution, it is clear that the α -cut for the fuzzy

Exponential Distribution Lower Mean values and Variance values are increases when t increases and Upper mean values and variance values are decreases when t increases . This shows that if the test termination time increases, the Lower Fuzzy Exponential distribution for the effect of Secretion of Corticosterone increases. Upper Fuzzy Exponential distribution for the effect of Secretion of Corticosterone decreases.

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