Physical-Layer Network Coding with Non-Binary Channel Codes

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Abstract

We apply non-binary coding to the two-way relay channel and evaluate two decoding schemes for the multiple-access phase at the relay: joint decoding for both users and separate decoding for a linear combination of both packets. We evaluate several efficient modulation schemes which combine favorably with non-binary coding and find that joint decoding can offer significant performance benefits.

I. Introduction

Physical-layer or wireless network coding combines the principles of packet combining [1] and multi-user detection. In this paper, we consider the symmetrical two-way relay channel, in which two users exchange information via a relay. Soon after the invention of network coding, it has been recognized that additional gains are possible by considering the received information on signal level [2]. The application of channel coding and the joint consideration of network and channel coding brought further improvements [3], [4]. In the following, we extend these schemes to include non-binary channel coding with joint decoding and decoding for linear combinations [5]–[7].

II. System Model

A. The Two-Way Relay Channel

We consider the two-way relay channel in which the relay makes use of physical-layer network coding. In this scenario, two users A and B exchange packets solely via a relay in two phases:

1) In the multiple-access phase, both users transmit their information to the relay, which decodes the superposed signal.
2) In the broadcast phase, the relay transmits a combination of both users' packets, from which each user can recover the packet from the other source exploiting information about its own packet.

Physical-layer network coding exploits the aspect that the relay needs to recover only a combination of the two packets and not both packets individually. In the following, we will focus on a symmetric channel in which both users experience the same average SNR and we will consider only the multiple-access phase since this is the bottleneck in this scenario. The block diagram for the multiple-access phase is shown in Fig. 1. The information packets $\mathbf{u}_\mathsf{a}$, $\mathbf{u}_\mathsf{b}$ are encoded by a channel encoder, represented by its generator matrix $\mathbf{G}$, and the resulting codewords $\mathbf{c}_\mathsf{a}$, $\mathbf{c}_\mathsf{b}$ are mapped to the QAM symbol sequences $\mathbf{x}_\mathsf{a}$, $\mathbf{x}_\mathsf{b}$, which are transmitted over a flat channel.

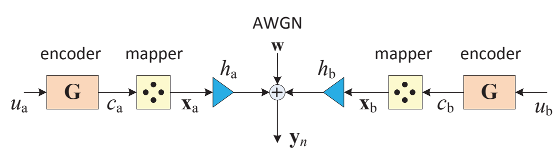

Figure 1. Two-user multiple-access channel

The received signal at the receiver is given by the complex-valued sequence

$$\mathbf{y}_n = h_{\mathsf{a},n}\mathbf{x}_{\mathsf{a},n} + h_{\mathsf{b},n}\mathbf{x}_{\mathsf{b},n} + \mathbf{w}_n, \quad \mathbf{y}_n \in \mathbb{C}^{1\times T}, \mathbf{w}_n \sim \mathcal{CN}\left(0, \mathbf{I}_T\right), n = 1, 2, \ldots, N_\mathsf{q} \tag{1}$$

The index n is the discrete time index, while T denotes the number of channel uses per codeword symbol.

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B. Channel Coding

We apply a non-binary channel code, which is defined over a Galois field $\mathbb{F}_q = GF(q)$ with q being a power of two. The most prominent examples for this class of codes are non-binary LDPC and turbo codes but also Reed-Solomon codes. For binary LDPC and for convolutional codes, this scheme has been treated in [8]–[10]. The packets carrying the user information are given as vectors $\mathbf{u}_\mathsf{a} = [u_{\mathsf{a},1}, \ldots, u_{\mathsf{a},K}]$, $\mathbf{u}_\mathsf{b} = [u_{\mathsf{b},1}, \ldots, u_{\mathsf{b},K}]$ of K symbols in $\mathbb{F}_q$, which corresponds to $K_{\mathsf{bin}} = K \cdot \log_2 q$ bits. These packets are encoded to the codewords

$$\begin{aligned} \mathbf{c}_\mathsf{a} &= \mathbf{u}_\mathsf{a}\mathbf{G} = [c_{\mathsf{a},1}, c_{\mathsf{a},2}, \ldots, c_{\mathsf{a},N}] \in \mathbb{F}_q^{1\times N} \\ \mathbf{c}_\mathsf{b} &= \mathbf{u}_\mathsf{b}\mathbf{G} = [c_{\mathsf{b},1}, c_{\mathsf{b},2}, \ldots, c_{\mathsf{b},N}] \in \mathbb{F}_q^{1\times N} \end{aligned} \tag{2}$$

using the same channel code, which is here defined by its generator matrix $\mathbf{G} \in \mathbb{F}_q^{K\times N}$. Arithmetic for message and codeword symbols is defined in the Galois field $\mathbb{F}_q$. We associate each Galois field element to an integer number out of $\mathbb{Z}_q = \{0, 1, \ldots, q-1\}$, which we denote by $[c_{k,n}] \in \mathbb{Z}_q$[1]. This allows to define a combined codeword

$$\mathbf{d} \triangleq [d_1, d_2, \ldots, d_N] \in \mathbb{Z}_{q^2}^{1\times N}, \text{ with } d_n \triangleq [c_{\mathsf{a},n}] \cdot q + [c_{\mathsf{b},n}] \tag{3}$$

In the other direction, we define the mappings μ_a, μ_b from integer $d \in \mathbb{Z}_{q^2}$ to two integers in $\mathbb{Z}_q$: $\mu_\mathsf{a}(d), \mu_\mathsf{b}(d) \in \mathbb{Z}_q$ such that $d = \mu_\mathsf{a}(d) \cdot q + \mu_\mathsf{b}(d) \in \mathbb{Z}_{q^2}$.

C. Modulations

Non-binary coding with $q > 2$ facilitates mapping and in particular demapping for QAM constellations with $M \leq q$ constellation points. We consider constellations which map *one* codeword symbol $c_{\mathsf{a},n} \in \mathbb{F}_q$ to $T \in \mathbb{N}$ channel uses. This mapping preserves the memorylessness of the channel which is a property which is assumed by most decoding algorithms. In other words, the QAM constellation is defined in T complex dimensions as $\chi : \mathbb{Z}_q \to \mathbb{X} \subset \mathbb{C}^{1\times T}$, where the constellation $\mathbb{X}$ has always cardinality $|\mathbb{X}| = q$. The QAM symbols are hence written as

$$\mathbf{x}_{\mathsf{a},n} = \chi\left([c_{\mathsf{a},n}]\right), \ \mathbf{x}_{\mathsf{b},n} = \chi\left([c_{\mathsf{b},n}]\right) \tag{4}$$

In the following, we apply a channel code with $q = 16$, i.e. each codeword symbol corresponds to 4 bits. The "natural" modulation for this code is 16-QAM since then one codeword symbol corresponds to one QAM constellation symbol and one channel use and demapping, to be defined in the next Section, is particularly simple. Since each codeword symbol carries $\log_2 q = 4$ bits and the code rate is K/N, the rate per user is given by $R = \frac{K \log_2 q}{NT}$. Table I shows the selected modulations for $T \in \{1, 2, 3, 4\}$. For $T = 3$, none of the usual QAM modulations can be applied but it is not difficult to carve a constellation out of a sphere in 6 real dimensions [11].

Table I
MODULATIONS: A CODEWORD SYMBOL OUT OF $\mathbb{F}_{16}$ IS MAPPED TO T CHANNEL USES.

T	1	2	3	4
modulation	16-QAM	QPSK	Subset of E_6 sphere packing	BPSK

III. DECODING METHODS

A. Joint Decoding (JD) Based on Posterior Probabilities for Joint Symbols

For optimum decoding, we have to consider the codeword symbols from both users jointly. This follows from the fact that the received symbol $\mathbf{y}_n$ depends on both $c_{\mathsf{a},n}$ and $c_{\mathsf{b},n}$ as follows from (1), (4). We therefore consider the *a posteriori probabilities* (APP) for all values of the combined codeword symbols d_n, as defined in (3).

$$p_n(b) \triangleq P\left[d_n = b \mid \mathbf{y}_n\right] \propto \exp\left(-\left\|\mathbf{y}_n - h_{\mathsf{a},n}\chi\left(\mu_\mathsf{a}(b)\right) - h_{\mathsf{b},n}\chi\left(\mu_\mathsf{b}(b)\right)\right\|^2\right), \text{ for } b \in \mathbb{Z}_{q^2}$$

These APPs are fed into a joint decoder, which is described for a q-ary LDPC code in [6]. The decoder tries to find the joint codeword $\mathbf{d}$ or equivalently both messages $\mathbf{u}_\mathsf{a}$, $\mathbf{u}_\mathsf{b}$. At this point, we can take advantage of the non-binary code and consider not only the sum of both packets, i.e. $\mathbf{u}_\mathsf{a} + \mathbf{u}_\mathsf{b}$, which is equivalent to a bitwise XOR operation, but all linear combinations $\alpha\mathbf{u}_\mathsf{a} + \beta\mathbf{u}_\mathsf{b}$. The linearity of the code implies $(\alpha\mathbf{u}_\mathsf{a} + \beta\mathbf{u}_\mathsf{b})\,\mathbf{G} = \alpha\mathbf{c}_\mathsf{a} + \beta\mathbf{c}_\mathsf{b}$, i.e. a linear combination of the messages corresponds to the same linear combination of codewords.

For joint decoding, we may consider both messages or any linear combination of both messages, i.e. after decoding, the relay can *seek* for a correct linear combination [5]. For the simulations, we simply assume that perfect error detection is possible

[1] This means that $u_{\mathsf{a},1} + u_{\mathsf{b},1}$ refers to addition in the finite field $\mathbb{F}_q$ while $[u_{\mathsf{a},1}] + [u_{\mathsf{b},1}]$ means the usual addition of integers.

while in a real implementation this can be realized at low complexity with an CRC code in addition to the inherent error detection capabilities of LDPC codes. Once the relay has found a valid linear combination, it retransmits this combined packet along with the coefficients α, β in the broadcast phase. Each user, with the knowledge of his own packet and the coefficients, can then recover the other packet.

B. Separate Decoding (SD)

Another approach with less complexity and which allows to apply the standard single-user decoder reverses the order of decoding and seeking for linear combinations. For this, we exploit the property of linear codes in $\mathbb{F}_q$ that every linear combination $\alpha\mathbf{c}_\mathsf{a} + \beta\mathbf{c}_\mathsf{b}$ is also a codeword for all $\alpha, \beta \in \mathbb{F}_q$. Therefore, we can define the APPs for the linear combination $\alpha c_{\mathsf{a},n} + \beta c_{\mathsf{b},n}$,

$$p_n(b;\alpha,\beta) \triangleq P\left[\alpha c_{\mathsf{a},n} + \beta c_{\mathsf{b},n} = b \mid \mathbf{y}_n\right] \propto p\left(\mathbf{y}_n \mid \alpha c_{\mathsf{a},n} + \beta c_{\mathsf{b},n} = b\right) = \sum_{d\in\mathcal{D}^b_{\alpha\beta}} p_n(d) \tag{5}$$

with the index set $\mathcal{D}^b_{\alpha\beta} \triangleq \{[c_\mathsf{a}]\cdot q + [c_\mathsf{b}] : c_\mathsf{a}, c_\mathsf{b} \in \mathbb{F}_q, \alpha c_\mathsf{a} + \beta c_\mathsf{b} = b\}$ for all $\alpha, \beta \in \mathbb{F}_q$ and $b \in \mathbb{Z}_q$. For each pair α, β, an APP vector with q entries is defined by (5), which is fed to the decoder. This decoder is the usual soft-input decoder for the channel code defined in $\mathbb{F}_q$, thus this approach is also directly applicable to Reed-Solomon codes.

IV. Simulation Results

A. Fast Fading

For fast fading, we assume that the channel is constant during one (multi-dimensional) QAM symbol, i.e. for T channel uses and i.i.d. Rayleigh distributed, i.e. $h_{\mathsf{a},n}, h_{\mathsf{b},n} \sim \mathcal{CN}\left(0, \sqrt{\mathrm{SNR}}\right)$; this channel is also known as perfectly interleaved fading channel. Fig. 2 shows the simulated packet error rates (PER) for all modulation and decoding options. As was to be expected, joint decoding performs best. It is however remarkable that for all modulations, the error rate for decoding both packets is the same as for decoding any combination. In other words, for fast fading, the observation that the relay only requires a combination of the packets does not result in any gain. On the other hand, the gain of joint decoding w.r.t. separate decoding for a linear combination is significant. For separate decoding to work, it is important to either allow to search for all linear combinations or to determine the appropriate values of α and β since setting $\alpha = \beta = 1$ does not work.

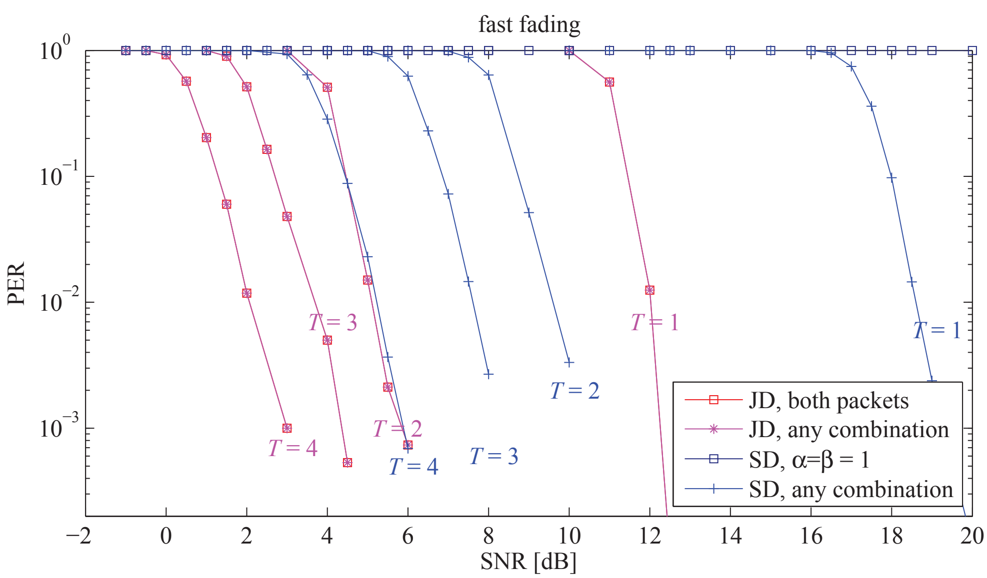

Figure 2. Packet error rates for modulations with $T = 1, 2, 3, 4$ over a fast fading symmetrical channel.

B. Block Fading

With block fading, the channel coefficients are constant during an entire packet, i.e. $h_{\mathsf{a},n} = h_\mathsf{a} \sim \mathcal{CN}\left(0, \sqrt{\mathrm{SNR}}\right)$, $h_{\mathsf{b},n} = h_\mathsf{b} \sim \mathcal{CN}\left(0, \sqrt{\mathrm{SNR}}\right)$ and are chosen indepedently for the next packet. For this case, there is no time diversity within a packet and therefore the slope of the error curves in Fig. 3 exhibits diversity order one. For the eight-dimensional modulation with

$T = 4$ (BPSK), JD performs identically to SD while for the modulation with the same order as the codeword symbols, i.e. 16-QAM, JD again shows a noticeable performance gain over SD.

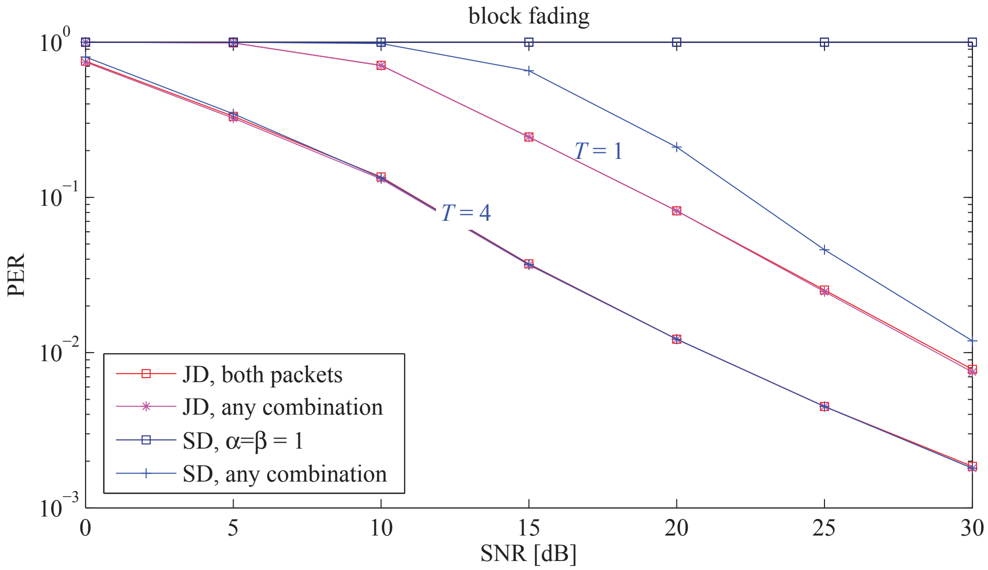

Figure 3. PER for block Rayleigh fading and 16-QAM ($T = 1$) and BPSK ($T = 4$) modulations.

C. AWGN Channel

For completeness and for reference, we have also conducted simulations for the AWGN channel with $h_\mathsf{a} = h_\mathsf{b} = \sqrt{\text{SNR}}$, although it has to be noted that this case is not very realistic since it assumes identical phases for both users. As is the case with binary coding, the relay can in no case recover both messages but it is possible to find a linear combination of both messages. For this channel, there is a moderate difference between joint and separate decoding.

V. Conclusions

We have applied non-binary coding schemes to the two-way relay channel and have exploited principles from physical-layer network coding and multi-user detection. It was found that joint decoding provides signifcant performance benefits for the fast fading channel and for higher-order modulations in the block fading channel while for the AWGN channel the gains are moderate. We have shown how non-binary channel coding combines well with the principles of network coding.

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