Adaptive Active Noise Control for Uncertain Secondary Pathes

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Abstract

A new direct adaptive algorithm for feedforward active noise control (ANC) is proposed in a general case when the primary and secondary path responses are all uncertain. In order to reduce the actual canceling error, the two artificial errors are introduced and are forced into zero by adjusting three adaptive FIR filters in an on-line manner. The distinctive feature of the method is that the adjusted parameters do not have to converge to the true FIR parameters but some constants, then the reduction of the two errors can attain the canceling at the objective point. The effectiveness of the proposed algorithm is validated in experiments using an air duct.

1. Introduction

Active noise control (ANC) is a way of suppressing unwanted noises generated by the primary sound sources at objective positions by producing artificial secondary control sounds. Together with the development of high speed DSP, the ANC recently has found more and more applications in improving industrial and living environment [1]. In the aspect of control methodology, the adaptive feedforward control is a powerful tool in the ANC, since the path dynamics cannot be precisely obtained and may be uncertainly changing.

A variety of filtered-x adaptive algorithms have been proposed to attain improved canceling performance in the ANC [3][4][5]. We also presented a modified filtered-x adaptive algorithms using the extended error signal and proved the robust stability [6]. Almost all filtered-x approaches assume that the secondary path model is known a priori. Therefore, if the secondary path responses are unknown or uncertainly changeable, identification of the secondary path model is required and the filtered-x algorithms should be updated on a basis of the identified model, or the feedforward controller should also be redesigned in a real-time manner according to the identified path models [7]. However, the former approach is not stability-guaranteed and the latter scheme suffers from computational burden.

The aim of this paper is to propose a new direct adaptive control approach in which two modified errors are introduced and are forced into zero by adjusting three kinds of adaptive filters. The proposed method does not require explicit identification of the primary and secondary path dynamics, so the computational burden is improved compared to the indirect adaptive approach presented by the authors [7]. Finally its effectiveness is validated in exper-

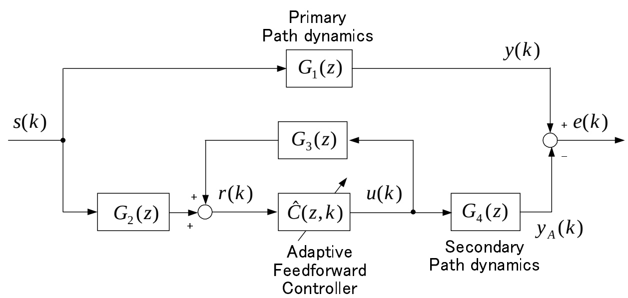

Figure 1: Schematic diagram of adaptive feedforward active noise control system

The schematic diagram of a single channel active noise control system is shown in Fig.1. The sound from the primary noise source is denoted by $s(k)$ which is detected as $r(k)$ by the reference microphone. The detected signal $r(k)$ is used as the input to the adaptive feedforward controller $\hat{C}(z,k)$. The output of the controller $u(k)$ is given to control the loudspeaker which emitts the secondary artificial sound so that the noise at the objective point may be cancelled. The canceling error at the point is denoted by $e(k)$. All of the signals are characterized by four path dynamics, in which $G_1(z)$ and $G_2(z)$ represent the primary path dynamics, and $G_3(z)$ and $G_4(z)$ the secondary path dynamics, respectively, as given in Fig.1. Since all of the path dynamics may contain model uncertainty and parameter changeability, the adaptive control approaches are essentially important to deal with the problems. It follows from Fig.1 that

$$e(k) = G_1(z)s(k) - G_4(z)u(k) \tag{1}$$

$$u(k) = C(z)r(k) \tag{2}$$

$$r(k) = G_2(z)s(k) + G_3(k)v(k) \tag{3}$$

where $C(z)$ be an FIR type of feedforward controller. Let $\hat{C}(z.k)$ be an adaptive version of $C(z)$, then the secondary control sound is generated by

$$u(k) = \hat{C}(z,k)r(k) = \hat{\boldsymbol{\theta}}^T(k)\boldsymbol{\varphi}(k) \tag{4}$$

where $\hat{\boldsymbol{\theta}}(k) = [\hat{\theta}_1(k),\ \cdots,\ \hat{\theta}_m(k)]^T$ and $\boldsymbol{\varphi}(k) = [r(k-1),\ \cdots,\ r(k-m)]^T$. Then the adaptive parameters to be adjusted and the canceling error $e(k)$ can be expressed from (1) $\sim$ (4) as

$$
\begin{aligned}
e(k) &= \bar{G}_1(z)r(k) - \bar{G}_4(z)u(k) && (5)\\
&= \bar{G}_4(z)\left(\frac{\bar{G}_1(z)}{\bar{G}_4(z)} - \hat{C}(z,k)\right)r(k) && (6)\\
&= \bar{G}_4(z)[(\boldsymbol{\theta}^* - \hat{\boldsymbol{\theta}}(k))^T\varphi(k)] && (7)
\end{aligned}
$$

where $\bar{G}_1(z) = G_1(z)/G_2(z)$ and $\bar{G}_4(z) = G_4(z) + G_1(z)G_3(z)\ /G_2(z)$. Thus it should be noted that the error system (7) for adaptation depends on all of the path dynamics.

2.2 Conventional Algorithms

1) Case with known secondary path dynamics

If the secondary path dynamics $G_3(z)$ is known, the feedback effect by $G_3(z)$ is cancelled by subtracting $G_3(z)u(k)$ from $r(k)$ to obtain $G_3(z) = 0$. As a result, it follows that $\bar{G}_4(z) = G_4(z)$, then we can realize the filtered-x algorithm if $G_4(z)$ is also known, as

a) Filtered-x algorithms [3][5]

$$
\begin{aligned}
&\hat{\boldsymbol{\theta}}(k+1) = \hat{\boldsymbol{\theta}}(k) + \mu\boldsymbol{\psi}(k)e(k) && (8)\\
&\boldsymbol{\psi}(k) = G_4(z)\boldsymbol{\varphi}(k) && (9)
\end{aligned}
$$

where $\mu > 0$ is a step size, and the normalized form is also used which is obtained by deviding the correction term in (8) by $\rho + \boldsymbol{\psi}(k)^T\boldsymbol{\psi}(k)$, $(\rho > 0)$.

b) Extended Error Based Filtered-x algorithm [6]

$$
\begin{aligned}
&\hat{\boldsymbol{\theta}}(k+1) = \hat{\boldsymbol{\theta}}(k) + \gamma(k)\boldsymbol{\psi}(k)\varepsilon(k) && (10)\\
&\boldsymbol{\psi}(k) = G_4(z)\boldsymbol{\varphi}(k) && (11)\\
&\varepsilon(k) = \frac{\eta(k)}{1+\gamma(k)\boldsymbol{\psi}^T(k)\boldsymbol{\psi}(k)} && (12)\\
&\eta(k) = e(k) + G_4(z)u(k) - \hat{\boldsymbol{\theta}}^T(k)\boldsymbol{\psi}(k) && (13)
\end{aligned}
$$

where the extended error $\varepsilon(k)$ defined by (12) and (13) is used instead of $e(k)$. The convergence and stability of the algorithm has been investigated by the authors [6].

2) Case with uncertain secondary path dynamics

A model of path dynamics includes uncertainties and actual dynamics will change due to temperature variations as in exhaust gas duct. When all the path dynamics are uncertain or changeable, the following on-line identification-based approach is essentially needed [7].

First, to identify the path dynamics $\bar{G}_1(z)$ and $\bar{G}_4(z)$ in (5) by using the accessible signals $e(k)$, $u(k)$ and $r(k)$, we adopt the identification models $\hat{H}_1(z)$ and $\hat{H}_4(z)$ as

$$
\hat{e}(k) = \hat{H}_1(z)r(k) + \hat{H}_2(z)u(k) \qquad (14)
$$

The FIR coefficients in $\hat{H}_1(z)$ and $\hat{H}_2(z)$ can be updated by the LMS adaptive algorithm [7]. The dither signal modulated by the canceling error $e(k)$ is needed in the explicit identification for achieving the PE condition.

Next, we compute the FIR adaptive feedforward controller $\hat{C}(z.k)$ as $\hat{C}(z.k) = \hat{H}_1(z)/\hat{H}_4(z)$. This polynomial divisions can be executed by FFT and IFFT to obtain the stable $\hat{C}(z.k)$ in real-time manner.

Thus, the various types of filtered-x adaptive algorithms need the reliable models for the secondary path dynamics. The above indirect adaptive approach is one of the solutions, but the computational burden of FFT and IFFT at every instant is rather high for real-time adaptation.

3. New Direct Fully Adaptive Algorithm

The conventional and extended-error based filtered-x algorithms require prior knowledge on the secondary path dynamics $G_3(z)$ and $G_4(z)$. The aim of this paper is to propose a new direct adaptive algorithm which does not need explicit identification of the uncertain secondary path dynamics.

The basic structure of the proposed adaptive feedforward control algorithm is illustrated in Fig.2. It follows from the block diagram, $e(k)$, $e'(k)$ and $e''(k)$ are computed as follows:

$$
\begin{aligned}
e(k) &= \overline{G}_1(z)r(k) - \overline{G}_4(z)u(k) && (15)\\
e'(k) &= e(k) + \hat{K}(z,k)u(k) - \hat{D}(z,k)r(k) && (16)\\
e''(k) &= \hat{D}(z,k)r(k) - \hat{C}(z,k)x(k) && (17)
\end{aligned}
$$

where $\overline{G}_1(z)$ and $\overline{G}_4(z)$ are defined in Section 2.1, and $e'(k)$ and $e''(k)$ are newly introduced as artificial errors for execution of the proposed direct adaptive algorithm. The control input $u(k)$ and the auxiliary input $x(k)$ are also defined as

$$
\begin{aligned}
u(k) &= \hat{C}(z,k)r(k) && (18)\\
x(k) &= \hat{K}(z,k)r(k) && (19)
\end{aligned}
$$

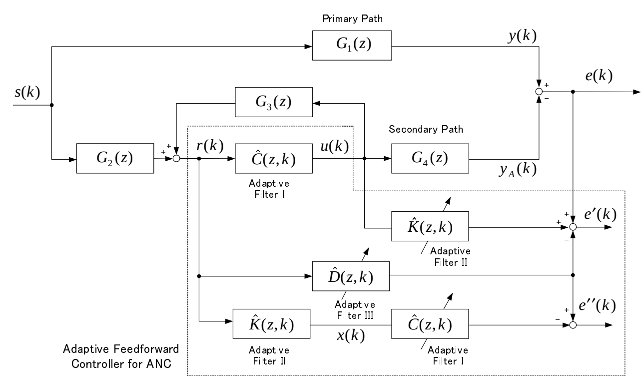

Figure 2: Direct adaptive algorithm for active noise control

It seems that $\hat{D}(z,k)$ and $\hat{K}(z,k)$ are the identified models for $\overline{G}_1(z)$ and $\overline{G}_4(z)$ respectively and the adaptive controller $\hat{C}(z,k)$ is adjusted according to the identified models of the secondary path dynamics. However, the advantage of the proposed algorithm does not require the convergence of the adjusted parameters to their true values, but the convergence of their parameters to ar constants such that the errors $e'(k)$ and $e''(k)$ can co: verge to zero. Therefore, any probing signal is not neede unlike the indirect adaptive algorithm.

Then it follows from Fig.2 that

$$e'(k) + e''(k) = e(k) + \hat{K}(z,k)\hat{C}(z,k)r(k) - \hat{C}(z,k)\hat{K}(z,k)r(k) \quad (2$$

Thus if $e'(k)$ and $e''(k) \to 0$ for $k \to \infty$ are both satisfie and the FIR parameters of $\hat{C}(z,k)$ and $\hat{K}(z,k)$ conver to any constants, then the second and third terms in tl right hand side of (20) can be cancelled, then by tl relation $e'(k)+e''(k) = e(k)$, thus it can be attained th: $e(k) \to 0$.

In the indirect adaptive algorithm, since accuracy identification affects the canceling performance, dith signals are required to attain the PE property of the r gressor signals, which sometimes degrades the cancelir error $e(k)$. The problem can also be overcome by tl proposed direct adaptive approach.

In the following, we give the direct adaptive algorith to update the parameters of three FIR controllers:

$$\hat{C}(z,k) = c_1(k)z^{-1} + \cdots + c_{L_C}(k)z^{-L_C} \quad (21$$
$$\hat{K}(z,k) = k_1(k)z^{-1} + \cdots + k_{L_K}(k)z^{-L_K} \quad (21$$
$$\hat{D}(z,k) = d_1(k)z^{-1} + \cdots + d_{L_D}(k)z^{-L_D} \quad (21$$

where $\hat{\boldsymbol{\theta}}_c(k)$, $\hat{\boldsymbol{\theta}}_k(k)$, $\hat{\boldsymbol{\theta}}_d(k)$, $\boldsymbol{\varphi}(k)$, $\boldsymbol{\zeta}(k)$ and $\boldsymbol{\xi}(k)$ are defined respectively as:

$$\hat{\boldsymbol{\theta}}_C(k) = [c_1(k), c_2(k), \ldots, c_{L_C}(k)]^T \quad (22a)$$
$$\hat{\boldsymbol{\theta}}_K(k) = [k_1(k), k_2(k), \ldots, k_{L_K}(k)]^T \quad (22b)$$
$$\hat{\boldsymbol{\theta}}_D(k) = [d_1(k), d_2(k), \ldots, d_{L_D}(k)]^T \quad (22c)$$
$$\boldsymbol{\varphi}(k) = [x(k-1), x(k-2), \ldots, x(k-L_C)]^T \quad (22d)$$
$$\boldsymbol{\zeta}(k) = [u(k-1), u(k-2), \ldots, u(k-L_K)]^T \quad (22e)$$
$$\boldsymbol{\xi}(k) = [r(k-1), r(k-2), \ldots, r(k-L_D)]^T \quad (22f)$$

By using the notations, $e'(k)$ can be modified as

$$\begin{aligned} e'(k) &= e(k) + \hat{K}(z,k)\hat{C}(z,k)r(k) - \hat{D}(z,k)r(k) \\ &= [\overline{G_1}(z) - \hat{D}(z,k)]r(k) \\ &\quad +[\hat{K}(z,k) - \overline{G_4}(z)]\hat{C}(z,k)r(k) \\ &= [\boldsymbol{\theta}_{D*} - \hat{\boldsymbol{\theta}}_D(k)]^T\boldsymbol{\xi}(k) + [\hat{\boldsymbol{\theta}}_K(k) - \boldsymbol{\theta}_{K*}]^T\boldsymbol{\zeta}(k) \quad (23) \end{aligned}$$

where $\boldsymbol{\theta}_{D*}$ and $\boldsymbol{\theta}_{K*}$ are the true parameter vector corresponding to $\overline{G}_1(z)$ and $\overline{G}_4(z)$. Similarly, the error $e''(k)$ can also be rewritten by

$$\begin{aligned} e''(k) &= \hat{D}(z,k)r(k) - \hat{C}(z,k)\hat{K}(z,k)r(k) \\ &= \left[\hat{D}(z,k)\hat{K}(z,k)^{-1} - \hat{C}(z,k)\right]K(z,k)r(k) \\ &= [\boldsymbol{\theta}_{C*} - \hat{\boldsymbol{\theta}}_C(k)]^T\boldsymbol{\varphi}(k) \quad (24) \end{aligned}$$

where $\boldsymbol{\theta}_{C*}$ is the FIR coefficient vector approximating $\hat{C}(z,k)\hat{K}(z,k)^{-1}$, and $\hat{D}(z,k)$ and $\hat{K}(z,k)$ are actually time-varying system, then $\boldsymbol{\theta}_{C*}$ becomes also time-varying. However, by treating the parameter vector as

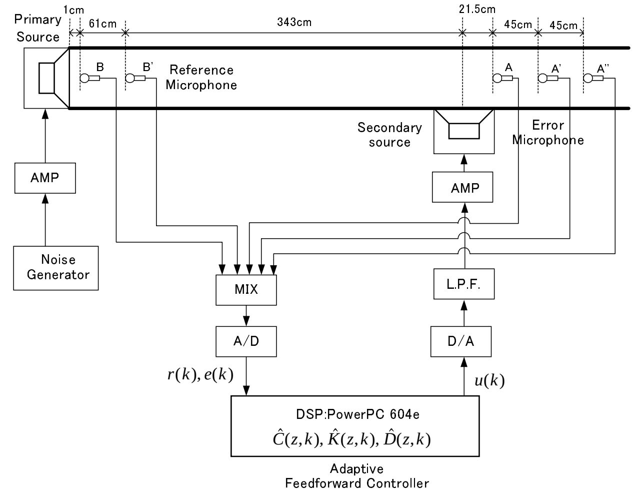

Figure 3: Experimental setup using air duct

time-invariant, we can write down the adaptive algorithm for updating the three controller parameter vectors as:

$$\hat{\boldsymbol{\theta}}_D(k+1) = \hat{\boldsymbol{\theta}}_D(k) + \gamma_D\boldsymbol{\xi}(k)\varepsilon(k) \quad (25)$$
$$\hat{\boldsymbol{\theta}}_K(k+1) = \hat{\boldsymbol{\theta}}_K(k) - \gamma_K\boldsymbol{\zeta}(k)\varepsilon(k) \quad (26)$$
$$\hat{\boldsymbol{\theta}}_C(k+1) = \hat{\boldsymbol{\theta}}_C(k) + \gamma_C\boldsymbol{\varphi}(k)\varepsilon'(k) \quad (27)$$
$$\varepsilon(k) = \frac{e'(k)}{1 + \gamma_D\boldsymbol{\xi}^T(k)\boldsymbol{\xi}(k) + \gamma_K\boldsymbol{\zeta}^T(k)\boldsymbol{\zeta}(k)} \quad (28)$$
$$\varepsilon'(k) = \frac{e''(k)}{1 + \gamma_C\boldsymbol{\varphi}^T(k)\boldsymbol{\varphi}(k)} \quad (29)$$

Local convergence can be investigated by the aid of the averaging approach [8] in which parameter convergence profiles can be calculated.

4. Experimental Results in Air Duct

Fig.3 depicts an experimental setup for a noise suppression system in an air duct. The primary source noise $s(k)$ is generated from a loudspeaker by passing white noises through a lowpass filter with a passband of 400 Hz. The sampling frequency is chosen 1kHz. The source noise is detected by the reference microphones placed at B and B', while the error microphones are placed at A, A' and A". By switching the two reference microphones by the mixer, we can simulate unknown changes of the path dynamics $G_2(z)$ and $G_3(z)$. Similarly by switching the three error microphones, we can also simulate changes of the path dynamics $G_1(z)$ and $G_4(z)$. Under the various changes of all path dynamics, we compared three adaptive algorithms: (a) Conventional filtered-x algorithm using a nominal secondary path model as prior knowledge, (b) the extended error based filtered-x algorithm using the same nominal secondary path model as above, and (c) the proposed direct adaptive algorithm. The degree of all of the polynomials $\hat{C}(z)$, $\hat{K}(z)$ and $\hat{D}(z)$ is 80, and the same degree of $\hat{C}(z)$ is used in (a) and (b). All of the step size are chosen as $\gamma = \gamma_C = \gamma_K = \gamma_D = 10$.

Experiment 1: Let the error microphone be fixed at the location A, and the secondary path model was identified

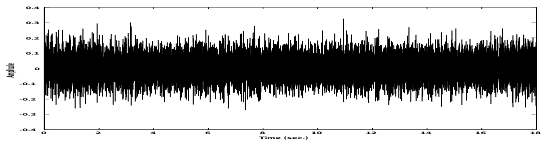

Figure 4: $e(k)$ in case without control in Experiment 1

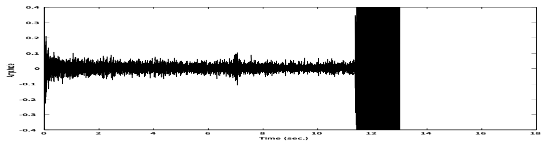

Figure 5: $e(k)$ obtained by conventional filtered-x algorithm in Experiment 1

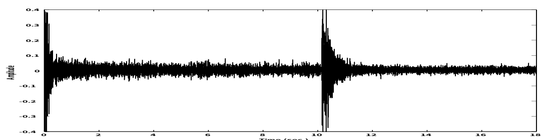

Figure 6: $e(k)$ obtained by extended error based filtered-x algorithm in Experiment 1

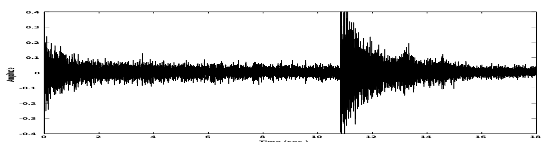

Figure 7: $e(k)$ obtained by the direct fully adaptive algorithm in Experiment 1

before the experiment, and the obtained model was used in the algorithms (a) and (b) as the nominal model. At about 10 seconds after the start of control, the position of the reference microphone was switched from B' to B, which simulates uncertain changes in $G_2(z)$ and $G_3(z)$. the Figs.4 to 7 show the time profiles of the error $e(k)$ in cases with no control and $e(k)$ in comparison with the three algorithms (a), (b) and (c). Since there is no changes in $G_4(z)$ and very small changes in $G_3(z)$, the algorithms (b) and (c) could give very nice and stable performance (see Figs.6 and 7, respectively), but the algorithm (a) diverged because it is not stability-guaranteed even if $G_4(z)$ is known (see Fig.5).

Experiment 2: The reference microphone was fixed at the position B, and the location of the error microphone was changed as A"$\to$ A $\to$ A" $\to$ A', that simulates uncertain changes of $G_3(z)$ and $G_4(z)$. The step size of the all algorithm was chosen 2. The error $e(k)$ is summarized in Figs.8 to 11. The algorithms (a) and (b) cannot be robust to the changes in $G_3(z)$ and $G_4(z)$ (see Figs.9 and 10 respectively), while the proposed direct adaptive algorithm (c) could give very stable performance to any changes in $G_4(z)$ as shown in Fig.11.

5. Conclusion

We have presented the direct adaptive algorithm for updating the feedforward active noise controller, which will be applicable when the primary and secondary path

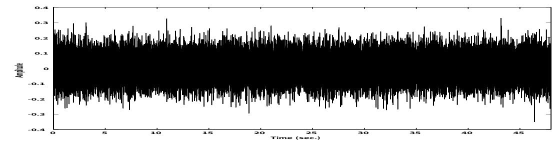

Figure 8: $e(k)$ in case without control in Experiment 2

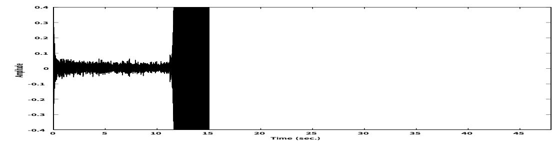

Figure 9: $e(k)$ obtained by conventional filtered-x algorithm in Experiment 2

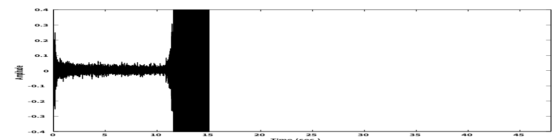

Figure 10: $e(k)$ obtained by extended error based filtered-x LMS algorithm in Experiment 2

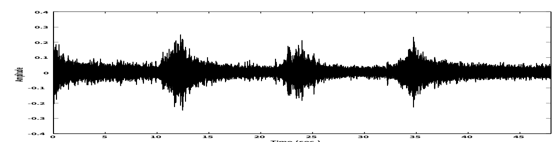

Figure 11: $e(k)$ obtained by the direct fully adaptive algorithm in Experiment 2

dynamics are uncertain or unknown. The proposed algorithm involves three groups of adaptive filters updated so that the two artificial errors may be eliminated and then the actual canceling error can be cancelled.

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