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FREE VIBRATION ANALYSIS OF ORTHOTROPIC LAMINATED COMPOSITE PLATES USING FIRST ORDER SHEAR DEFORMATION THEORY

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Abstract:

In this paper free vibration analysis of orthotropic laminated composite plates using first order shear deformation theory. The existing first-order shear deformation theory contains five unknowns but present first order shear deformation theory contains only four unknowns and has many similarity with the classical plate theory such as equation of motion, boundary condition and stress resultant expressions. The equation of motion and boundary condition are derived from Hamilton's Principle for the calculation of frequency analysis of orthotropic laminated composite plates. Analytical closed form solution of simply supported anti-symmetric cross-ply and angle-ply laminated composite are obtained and results are compared with the exact three dimensional solution. Comparison studies shows that the present theory can achieve the same accuracy as of the existing first order shear deformation theory which has more number of unknowns.

Keywords

Classical laminated plate theory (CLPT), first-order shear deformation theory (FSDT), transverse shear, rotary inertia, strain energy, work done, kinetic energy, mass density, mass inertia.

1. Introduction

The increasing use of composite materials system in plates construction for which conventional method of analysis are inadequate. Among these systems laminated composites are widely used in the aerospace, automotive, marine and other structural applications because of advantageous features such as high ratio of stiffness and strength to weight and low maintenance cost. With the increase in application of engineering structures variety of laminated theories have been developed. The classical laminated plate theory (CLPT), is an extension of the Love–Kirchhoff hypothesis for isotropic plates and be applied if the laminate is thin and neglects the transverse shear deformation [1, 2, 3, 4, 5, 6] and rotary inertia effects and they discussed reasonable results for thin laminates. In order to overcome the limitations of CLPT, the shear deformation theories accounted for the effect of transverse shear deformation and rotary inertia have been recommended. The first-order shear deformation theory (FSDT) [7, 8, 9, 10, 11] also known as Reissner [12] and Mindlin [13] theory and accounts for the transverse shear effects but need some shear correction factors [14, 15, 16]. There are many studies of the bending of laminated composites plates have been carried out using FSDT [17, 18, 19]. The FSDT violates equilibrium conditions at the top and bottom surfaces of the plate, the shear correction factors are used to correct the unrealistic variation of the shear stress/strain through the thickness. The value of shear correction factor depends not only on the composite laminates and geometric parameters, but also on the loading and boundary conditions. For orthotropic materials a high ratio of in-plane or out-plane modulus of elasticity to transverse shear modulus, such that even for cases in which the cross-sectional thickness 'h' is very small compare with the smallest dimension, therefore the transverse shear deformation and rotary inertia effect are very significant.

In this paper is to develop the free vibration analysis of orthotropic type laminated composite plates by using FSDT method. A new FSDT method is compared with conventional FSDT method for different unknowns and obtained strong similarities. The split of transverse displacement into the bending and shear parts leads to a reduction in the number of unknowns and governing equations.

2. Theoretical formulation

2.1 Basic assumptions

Consider free vibration analysis of rectangular laminated plate of thickness *h* and edge dimension *a* and *b*. The plate is assumed to a Cartesian coordinate system x-y-z, where x, y plane is the middle plane of the plate and z axis is normal to the middle surface of the plate. For orthotropic laminated composite material the basic assumption are.

1. The displacement is small in comparison with the plate thickness 'h' and therefore, consider plain stress problems

2. The transverse displacement 'w' dividing three components extension w_a bending w_b and shear w_s., these components are functions of coordinates x and y only.

$$w(x,y) = w_a(x,y) + w_b(x,y) + w_s(x,y)$$

3. In comparison with in-plain stress σ_x and σ_y the transverse normal stress σ_z is negligible.

2.2 Kinematics

The displacement of the simple FSDT is given by

$$\begin{aligned} U(X,Y,Z) &= u_0(x,y) + z\psi_x \\ V(X,Y,Z) &= v_0(x,y) + z\psi_y \\ W(X,Y,Z) &= w_0(x,y) \end{aligned} \tag{1}$$

Where u_0, v_0 and w_0 are the unknown displacement functions of the corresponding point on the reference surface and ψ_x and ψ_y are the average rotation about y and x axes respectively of the normal to the mid-surface of the undeformed plate. The transverse displacement 'w' making further assumptions that the extension is very low as compare to bending and shear parts {i.e.,

$w_0(x,y) = w_b(x,y) + w_s(x,y)$ } and therefore $\psi_x = -\partial w_b/\partial x$ and $\psi_y = -\partial w_b/\partial y$, the displacement field of the simple FSDT can be rewritten as:

$$U(X,Y,Z)=u_0(x,y)-z\frac{\partial\psi_x}{\partial x}$$
$$V(X,Y,Z)=v_0(x,y)-z\frac{\partial\psi_y}{\partial y} \quad (2)$$
$$W(X,Y,Z)=w_b(x,y)+w_s(x,y)$$

The displacement of Eq. (2) contained only four unknowns. The idea of partitioning the transverse displacement into bending and shear components proposed by Huffington [20], Krishna Murty [21], Shimpi [22] and others.
The strain associated with the displacement in Eq. (2) is given by

$$\varepsilon_x=\frac{\partial U}{\partial X}=\frac{\partial u_0}{\partial x}-z\frac{\partial^2 w_b}{\partial x^2}$$
$$\varepsilon_y=\frac{\partial V}{\partial Y}=\frac{\partial v_0}{\partial y}-z\frac{\partial^2 w_b}{\partial y^2}$$
$$\gamma_{xy}=\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}=\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}-2z\frac{\partial^2 w_b}{\partial x\partial y} \quad (3)$$
$$\gamma_{yz}=\frac{\partial w_s}{\partial x}$$
$$\gamma_{xz}=\frac{\partial w_s}{\partial y}$$

2.3 Equation of motion

The Hamilton's principle is used to derive the equation of motion is given by

$$0=\int_0^T\left(\delta E+\delta W-\delta K\right)dt \quad (4)$$

Where δE, δW and δK are the variation of strain energy, work done and kinetic energy, respectively. The variation of strain energy can be expressed by

$$\delta E=\int_0^a\int_0^b\int_{-h/2}^{+h/2}\left(\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\sigma_{xy}\delta\gamma_{xy}+\sigma_{xz}\delta\gamma_{xz}+\sigma_{yz}\delta\gamma_{yz}\right)dxdydz$$

$$=\int_0^a\int_0^b[N_x\frac{\partial\delta u_0}{\partial x}-M_x\frac{\partial^2\delta w_b}{\partial x^2}+N_y\frac{\partial\delta v_0}{\partial y}-M_y\frac{\partial^2\delta w_b}{\partial y^2}+N_{xy}\left(\frac{\partial\delta u_0}{\partial y}+\frac{\partial\delta v_0}{\partial x}\right) \quad (5)$$
$$-2M_{xy}\frac{\partial^2\delta w_b}{\partial x\partial y}+Q_x\frac{\partial\delta w_s}{\partial x}+Q_y\frac{\partial\delta w_s}{\partial y}]dxdy$$

Where N, M and Q are the stress resultants is define by

$$
\begin{aligned}
N_x, N_y, N_{xy} &= \int_{-h/2}^{+h/2} \left(\sigma_x, \sigma_y, \sigma_{xy}\right) dz \\
M_x, M_y M_{xy} &= \int_{-h/2}^{+h/2} \left(\sigma_x, \sigma_y, \sigma_{xy}\right) z dz \\
Q_x, Q_y &= \int_{-h/2}^{+h/2} \left(\sigma_{xz}, \sigma_{yz}\right) dz
\end{aligned}
\tag{6}
$$

The variation of work done by external force is calculated by

$$
\delta W = -\int_0^a \int_0^b q \delta\left(w_b + w_s\right) dxdy \tag{7}
$$

Where q is the transverse external load. The variation of Kinetic energy is calculated by

$$
\begin{aligned}
\delta K &= \int_0^a \int_0^b \int_{-h/2}^{+h/2} \left(\dot{U}\delta\dot{U} + \dot{V}\delta\dot{V} + \dot{W}\delta\dot{W}\right) \rho dxdydz \\
&= \int_0^a \int_0^b \{ I_0 \left[\dot{u}_0 \delta\dot{u}_0 + \dot{v}_0 \delta\dot{v}_0 + \left(\dot{w}_b + \dot{w}_s\right) \delta\left(\dot{w}_b + \dot{w}_s\right)\right] \\
&- I_1 \left(\dot{u}_0 \frac{\partial \delta\dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial x} \delta\dot{u}_0 + \dot{v}_0 \frac{\partial \delta\dot{w}_b}{\partial y} + \frac{\partial \dot{w}_b}{\partial y} \delta\dot{v}_0\right) \\
&+ I_2 \left(\frac{\partial \dot{w}_b}{\partial x} \frac{\partial \delta w_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial y} \frac{\partial \delta w_b}{\partial y}\right) \} dxdy
\end{aligned}
\tag{8}
$$

Where dot- superscript indicates the differentiation with respect to time variable ‘t’, mass density is given by ρ and (I_0, I_1, I_2) are the mass inertias is given by

$$
I_0, I_0, I_0 = \int_{-h/2}^{+h/2} \left(1, z, z^2\right) \rho dz \tag{9}
$$

Substituting the expressions δE, δW and δK from Eqs. (5), (7) and (8) into Eq. (4) and integrating by parts and collecting the coefficient of δu_0, δv_0, δw_b and δw_s . The following equation of motion is given by

$$
\begin{aligned}
\delta u_0 &: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} \\
\delta v_0 &: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} \\
\delta w_b &: \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_2 \nabla^2 \ddot{w}_b \\
\partial w_s &: \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = I_0\left(\ddot{w}_b + \ddot{w}_s\right)
\end{aligned}
\tag{10}
$$

The natural boundary condition for Cartesian coordinate system are of the form:

$$\delta u_0 : N_x l_x + N_{xy} l_y$$
$$\delta v_0 : N_{xy} l_x + N_y l_y$$
$$\delta w_b : \left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - I_1 \ddot{u}_0 + I_2 \frac{\partial \ddot{w}_b}{\partial x} \right) l_x$$
$$+ \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - I_1 \ddot{v}_0 + I_2 \frac{\partial \ddot{w}_b}{\partial y} \right) l_y + \frac{\partial M_{ns}}{\partial s} \quad (11)$$

$$\delta w_s : Q_x l_x + Q_y l_y$$
$$\frac{\partial \delta w_b}{\partial n} : M_n$$
$$where$$
$$M_{ns} = \left(M_y - M_x \right) l_x l_y + M_{xy} \left(l_x^{\ 2} - l_y^{\ 2} \right)$$
$$M_n = M_x l_x^{\ 2} + M_y l_y^{\ 2} + 2 M_{xy} l_x l_y$$
$$\frac{\partial}{\partial n} = l_x \frac{\partial}{\partial x} + l_y \frac{\partial}{\partial y}$$
$$\frac{\partial}{\partial s} = l_x \frac{\partial}{\partial y} - l_y \frac{\partial}{\partial x}$$

Where l_x and l_y are the direction cosines of the unit normal to the mid plane boundary.

Now we consider different boundary condition in the explicit form:

i. Clamped edge

$u_0 = v_0 = w_b = w_s = \frac{\partial w_b}{\partial x} = 0$ at x = 0, a (12a)

$u_0 = v_0 = w_b = w_s = \frac{\partial w_b}{\partial y} = 0$ at y = 0, b

ii. Simply supported edge (cross-ply laminate)

$N_x = v_0 = w_b = w_s = M_x = 0$ at x = 0, a (12b)

$u_0 = N_y = w_b = w_s = M_y = 0$ at y = 0, b

iii. Simply supported edge (angle-ply laminate)

$u_0 = N_{xy} = w_b = w_s = M_x = 0$ at x = 0, a (12c)

$N_{xy} = v_0 = w_b = w_s = M_y = 0$ at y = 0, b

iv. Free edge

$$N_x = N_{xy} = \frac{\partial M_x}{\partial x} + 2\frac{\partial M_{xy}}{\partial y} - I_1\ddot{u}_0 + I_2\frac{\partial \ddot{w}_b}{\partial x} = Q_x = M_x = 0 \text{ at } x = 0, a \tag{12d}$$

$$N_{xy} = N_y = 2\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - I_1\ddot{v}_0 + I_2\frac{\partial \ddot{w}_b}{\partial y} = Q_y = M_y = 0 \text{ at } y = 0, b$$

2.4 Constitutive equations of orthotropic laminated plate

Consider a rectangular plate of thickness h composed of 'n' orthotropic laminated layers with the coordinate system x, y and z. as shown in the Fig. 1. Under the assumption that each layer have a plane of symmetry parallel to the x-y plane, the constitutive equations for a layer can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{13}$$

Where Q_{ij} are the engineering constant in the material axes of the layer given as

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$
$$Q_{66} = G_{12}, Q_{44} = G_{23}, Q_{55} = G_{13} \tag{14}$$

The laminate is made of several orthotropic layers with their material axes arbitrarily oriented with respect to the laminate coordinates. Each layer transformed to the laminate coordinates x, y and z. The stress-strain relation of the *k*th layer are given as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}^k = \begin{bmatrix} Q'_{11} & Q'_{12} & Q'_{16} & 0 & 0 \\ Q'_{12} & Q'_{22} & Q'_{26} & 0 & 0 \\ Q'_{16} & Q'_{26} & Q'_{66} & 0 & 0 \\ 0 & 0 & 0 & Q'_{44} & Q'_{45} \\ 0 & 0 & 0 & Q'_{45} & Q'_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{15}$$

Where Q'_{ij} are the constants of different transformed materials as:

$$Q'_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta$$
$$Q'_{12} = (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \tag{16}$$
$$Q'_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta$$
$$Q'_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta$$

$$Q'_{26} = (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta$$
$$Q'_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta)$$
$$Q'_{44} = Q_{44}\cos^2\theta + Q_{55}\sin^2\theta$$
$$Q'_{45} = (Q_{55} - Q_{44})\cos\theta\sin\theta$$
$$Q'_{55} = Q_{55}\cos^2\theta + Q_{44}\sin^2\theta$$

Where θ is the angle between global x-axis and local x-axis of each lamina.

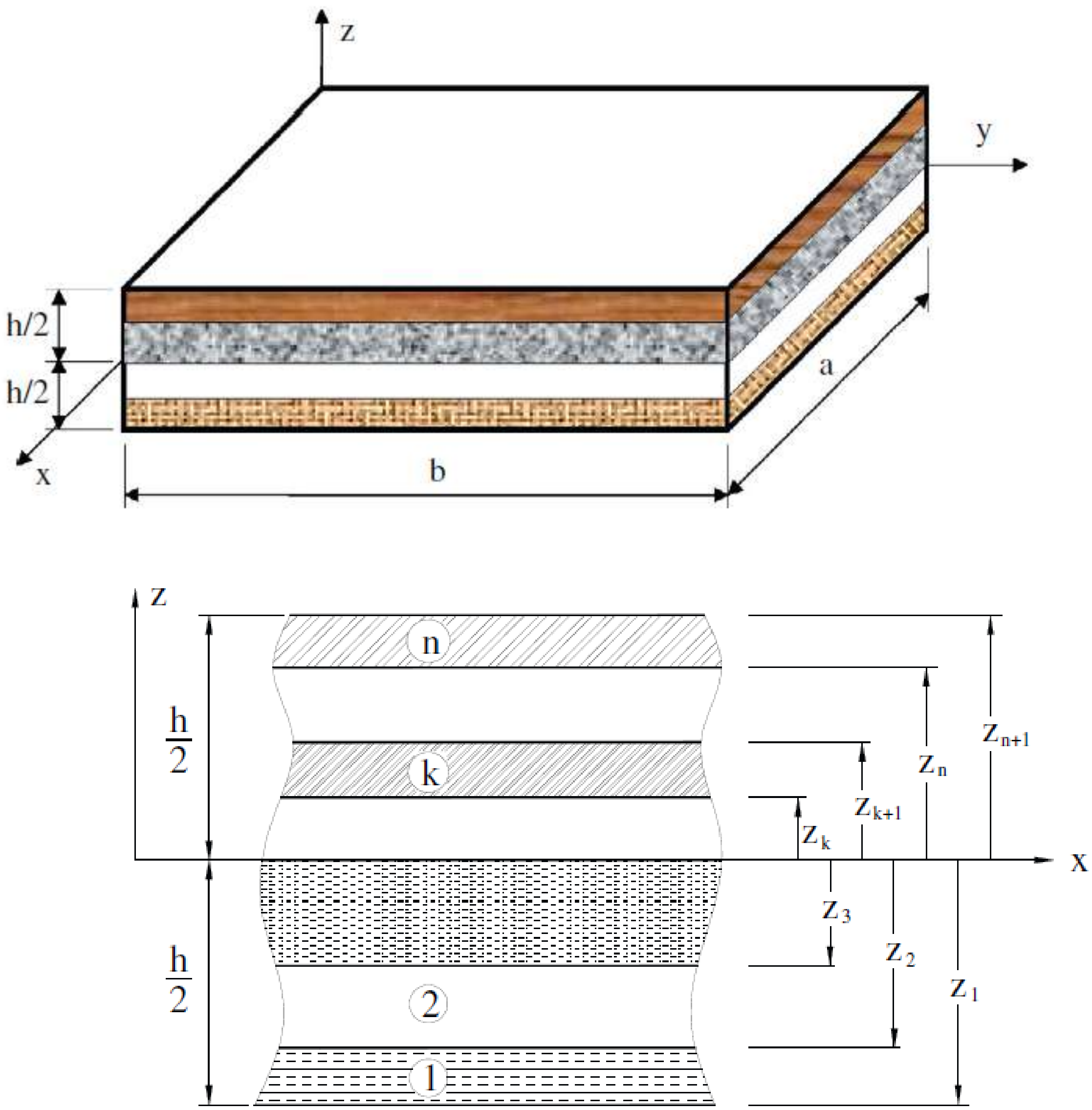

Fig.1. Coordinate system and layer numbers for a laminated plate

Put the value of Eq. (3) into Eq. (15) and also the subsequent results of Eq. (6), the stress resultants are obtained in terms of displacements (u_0, v_0, w_b, w_s) is given by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} \quad (17)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} \quad (18)$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = k \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial w_s}{\partial x} \\ \dfrac{\partial w_s}{\partial y} \end{Bmatrix} \quad (19)$$

Where k is defined as shear correction factor and (A_{ij}, B_{ij}, D_{ij}) are the coefficient of stiffness of plate and is defined by

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{+h/2} Q_{ij}\left(1, z, z^2\right) dz \quad (20)$$

3. Analytical solutions for antisymmetric cross-ply and angle- ply laminates

Consider a rectangular plate with all edges simply supported of length a and width b under transverse load q and based on Navier solution as

$$\begin{aligned} w_b(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{bmn} e^{i\omega t} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \\ w_s(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{smn} e^{i\omega t} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \end{aligned} \quad (21)$$

For antisymmetric cross-ply

$$u_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{u_0mn}e^{i\omega t}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
$$v_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{v_0mn}e^{i\omega t}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \quad (22)$$

For antisymmetric angle-ply

$$u_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{u_0mn}e^{i\omega t}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$
$$v_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{v_0mn}e^{i\omega t}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \quad (23)$$

Where $i=\sqrt{-1}$, (A_{u0mn}, B_{vomn}, C_{bmn}, C_{smn}) are coefficient and ω is the natural frequency of free vibration. The transverse load q(x, y) is expressed in terms of double Fourier sin series as

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \quad (24)$$

The Fourier coefficient Q_{mn} can be determined from the relationship

$$Q_{mn}=\frac{4}{ab}\int_0^a\int_0^b q(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy$$

$Q_{mn}=q_0$ for sinusoidal load (25)

$Q_{mn}=\dfrac{16q_0}{mn\pi}$ for uniform load (26)

By using Eq. (10) and the stress resultants Eqs. (17), (18), and (19) the analytical solution can be obtained for FSDT from governing differential equation for laminated plates is given by

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 \\ s_{12} & s_{22} & s_{23} & 0 \\ s_{13} & s_{23} & s_{33} & 0 \\ 0 & 0 & 0 & s_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & 0 \\ 0 & m_{22} & m_{23} & 0 \\ m_{13} & m_{23} & m_{33} & m_{34} \\ 0 & 0 & m_{34} & m_{44} \end{bmatrix}\right)\begin{Bmatrix} A_{u_0mn} \\ B_{v_0mn} \\ C_{bmn} \\ C_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ Q_{mn} \end{Bmatrix} \quad (27)$$

Where

$$s_{11} = A_{11}\left(\frac{m\pi}{a}\right)^2 + A_{66}\left(\frac{n\pi}{b}\right)^2, s_{12} = \left(A_{12} + A_{66}\right)\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)$$

$$s_{22} = A_{66}\left(\frac{m\pi}{a}\right)^2 + A_{22}\left(\frac{n\pi}{b}\right)^2, s_{33} = D_{11}\left(\frac{m\pi}{a}\right)^4 + 2\left(D_{12} + 2D_{66}\right)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^4 \quad (28)$$

$$s_{44} = k\left(A_{55}\left(\frac{m\pi}{a}\right)^2 + A_{44}\left(\frac{n\pi}{b}\right)^2\right)$$

$$\left.\begin{aligned} s_{13} &= -B_{11}\left(\frac{m\pi}{a}\right)^3 - \left(B_{12} + 2B_{66}\right)\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2 \\ s_{23} &= -B_{22}\left(\frac{n\pi}{b}\right)^3 - \left(B_{12} + 2B_{66}\right)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right) \end{aligned}\right\} \quad \text{for antisymmetric cross-ply} \quad (29)$$

$$\left.\begin{aligned} s_{13} &= -B_{26}\left(\frac{n\pi}{b}\right)^3 - 3B_{16}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right) \\ s_{23} &= -B_{16}\left(\frac{m\pi}{a}\right)^3 - 3B_{26}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2 \end{aligned}\right\} \quad \text{for antisymmetric angle-ply} \quad (30)$$

$$\begin{aligned} m_{11} &= m_{22} = m_{34} = m_{44} = I_0, m_{13} = -\left(\frac{m\pi}{a}\right)I_1 \\ m_{23} &= -\left(\frac{n\pi}{b}\right)I_1, m_{33} = I_0 + I_2\left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right\} \end{aligned} \quad (31)$$

For classical plate theory (CPT) the transverse shear displacement is zero (i.e. $w_s = 0$), therefore the analytical solution is

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}\right)\begin{Bmatrix} A_{u_0mn} \\ B_{v_0mn} \\ C_{bmn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \end{Bmatrix} \quad (32)$$

4. Numerical results

In this paper some results are discuss to verify the accuracy of the present theory with extending theory. The obtained results are compared with the exact solution with those predicted by other plate models. In this examples, a shear correction factor 5/6 has been used both present theory and FSDT. The lamina property for antisymmetric cross-ply and angle-ply is used:

$E_1/E_2 = 25$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu_{12} = 0.25$ is developed by (Reddy [23])

4.1 Vibration analysis

Table.1

Fundamental frequency of antisymmetric cross-ply $(0/90)_n$ square laminates under sinusoidal loads (Reddy [23])

a/h	Theory	n	
		1	3
2	CPT FSDT Present	6..2636 6.2092 6.2090	6.5617 6.5473 6.5574
10	CPT FSDT Present	6.9636 6.9373 6.6465	8.4617 8.4543 8.4234
20	CPT FSDT Present	7.6646 7.7071 7.7071	9.8327 9.9444 9.9452
40	CPT FSDT Present	8.5638 8.3343 8.8356	11.2627 11.5266 11.5267

Table.2

Fundamental frequency of antisymmetric angle-ply $(45/-45)_n$ square laminates under sinusoidal loads (Reddy [23])

a/h	Theory	n	
		1	4
2	CPT FSDT Present	4.5547 4.5593 4.9172	5.6452 5.5654 5.4664
10	CPT FSDT Present	7.1086 8.8972 8.9326	8.9463 11.4226 11.4934
20	CPT FSDT Present	7.1178 11.2975 11.2515	8.9652 16.2570 16.3393
40	CPT FSDT Present	8.2431 14.6015 14.5618	9.2324 20.2335 20.7612

5. Conclusions

A first order shear deformation theory was presented for bending analysis of laminated composite plates. The equation of motion discuss from Hamilton's principle which analytically solved for simply supported antisymmetric cross-ply and angle-ply laminated plates. In conventional FSTD the number of unknown is five is reduced by one of the present FSDT. The result of present FSDT and the conventional FSDT are almost same for the two cases. Therefore, it can be conclude that the present FSDT is not only accurate but also simple in analyzing the bending of laminated composite plates.

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