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# SUPPLEMENTARY MATERIAL: DEGREES OF FREEDOM FOR PIECEWISE LIPSCHITZ ESTIMATORS

FREDERIK RIIS MIKKELSEN AND NIELS RICHARD HANSEN

## 1. Computational Time and Number of Selected Predictors

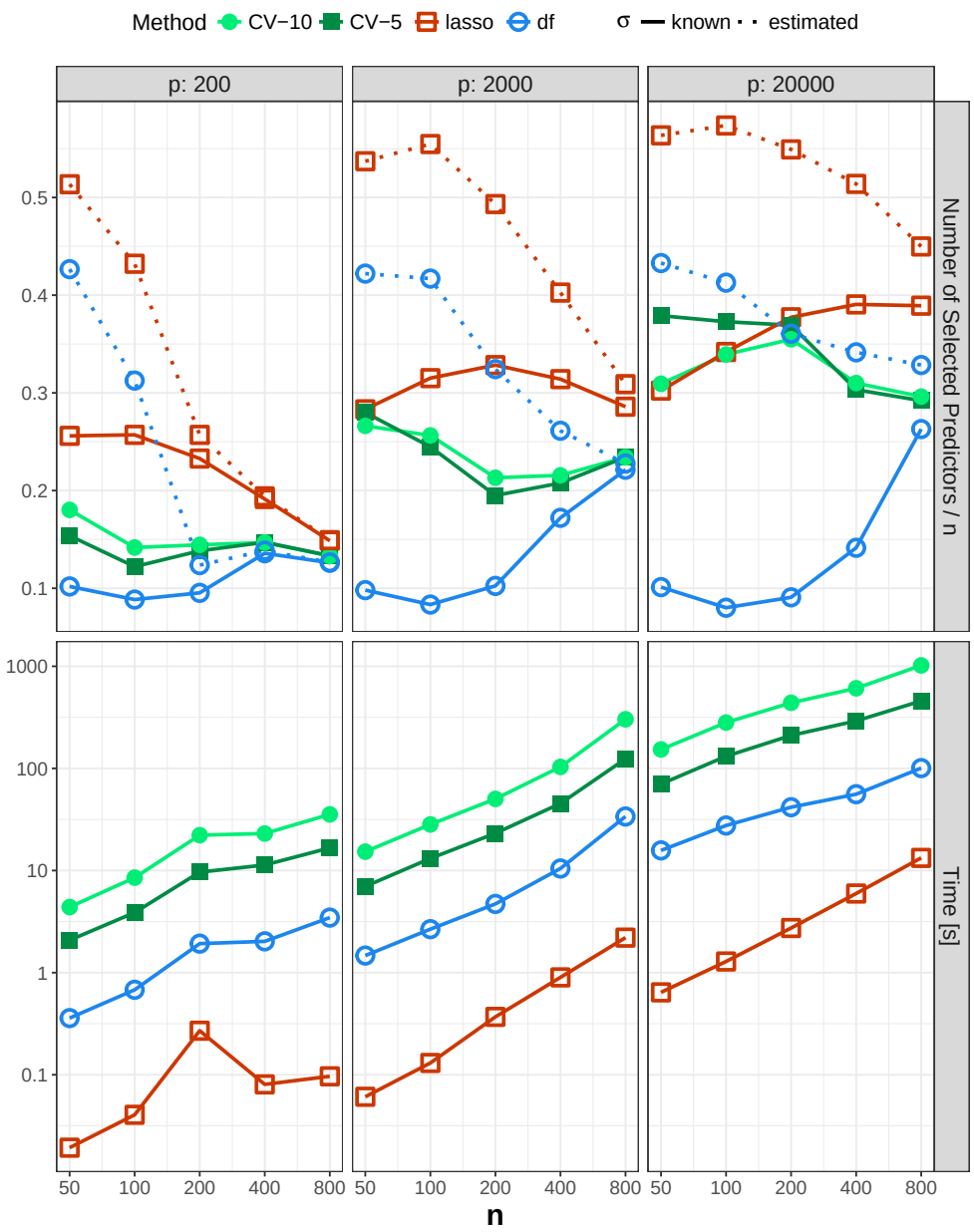
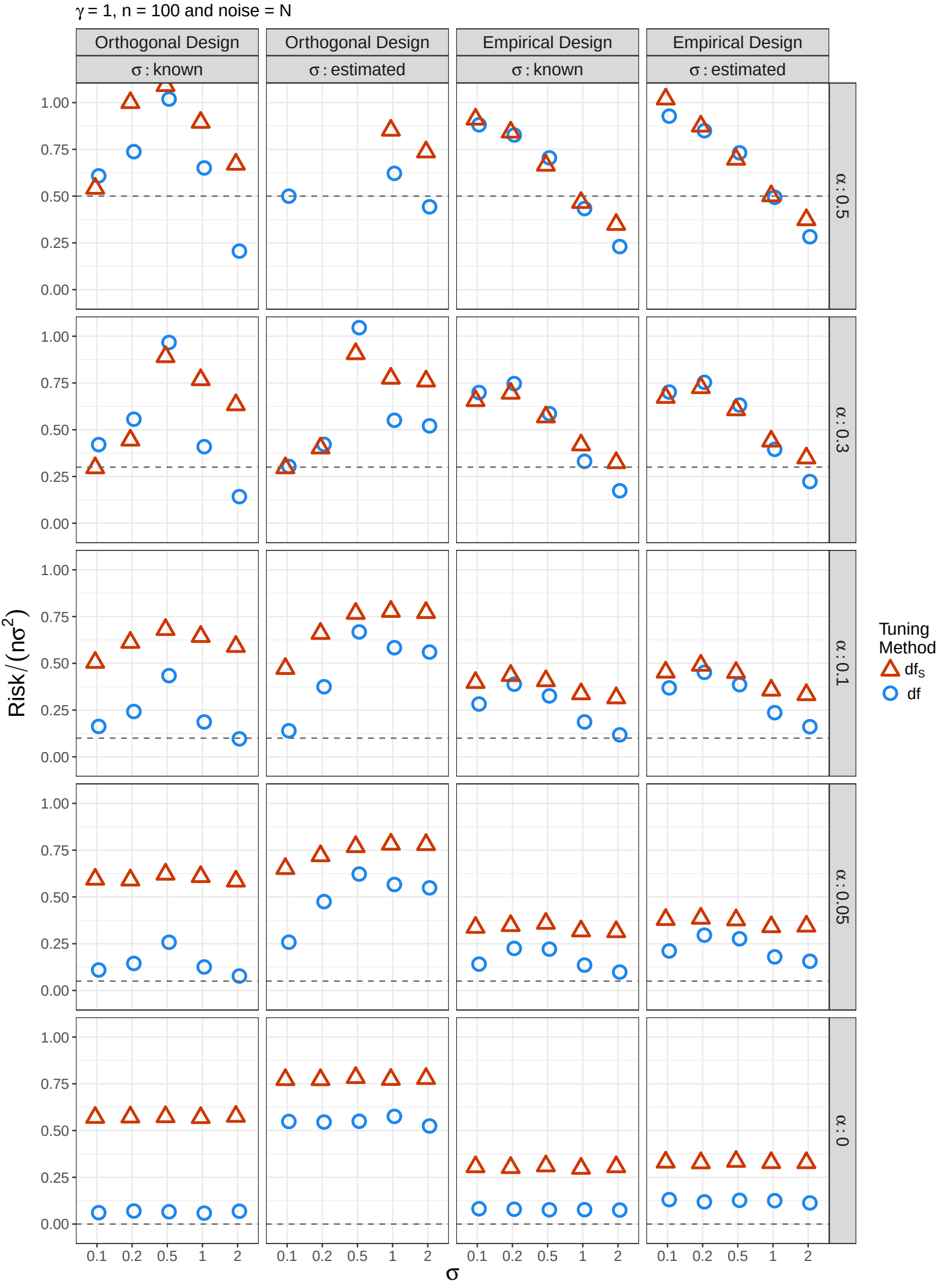
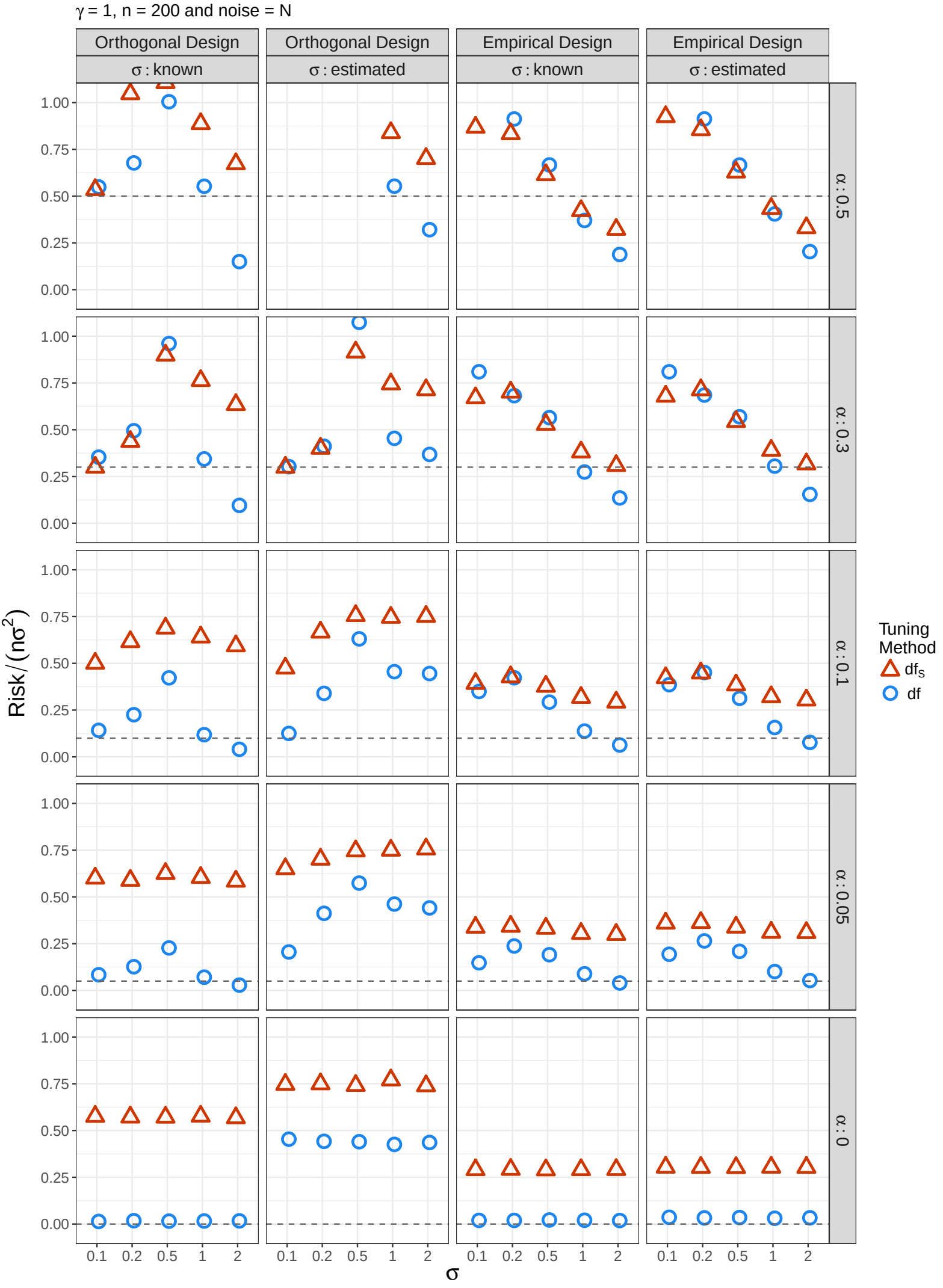



FIGURE 1. The number of selected predictors divided by $n$ (top), along with computational time of evaluating the estimator and tuning the $\lambda$-parameter using the different methods (bottom). The design parameters were: $\sigma = 0.5$, $\gamma = 1$, $\alpha = 0.1$, and the design type was (S) with a constant correlation of $\rho = 0.1$ (see Section 4)

## 2. Risk estimates

Plots of the risk estimates relative to $n\sigma^2$ for the estimators $\hat{\mu}^{\hat{\lambda}_{\mathrm{df}_{\mathbb{S}}}}_{\mathrm{OLS.l}}$ and $\hat{\mu}^{\hat{\lambda}_{\mathrm{df}}}_{\mathrm{OLS.l}}$. The dashed lines are the relative risks for the oracle-OLS estimator.


γ = 1, n = 100 and noise = N
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
df_S
df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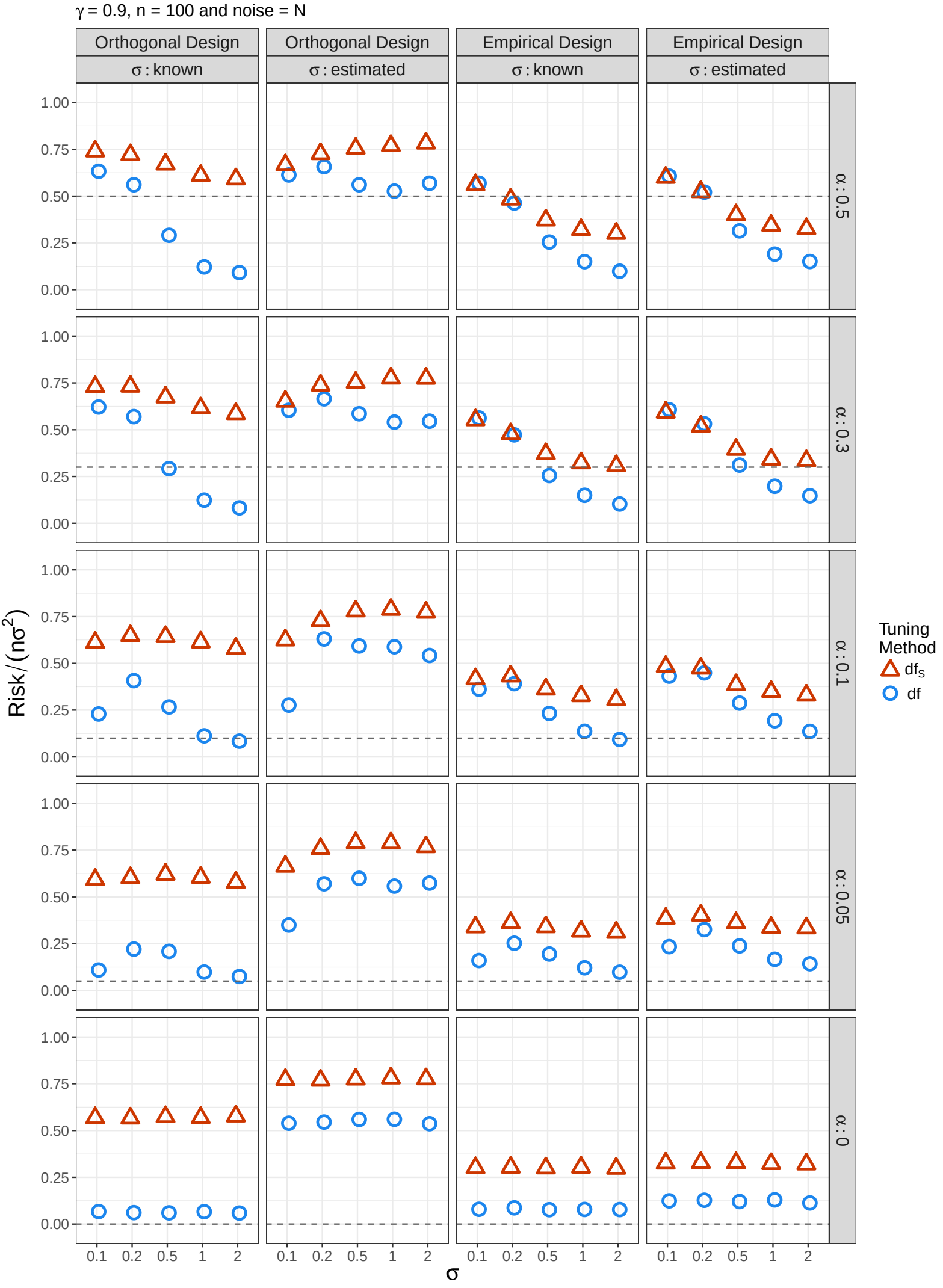
γ = 0.9, n = 100 and noise = N
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
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α : 0.5
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Tuning Method
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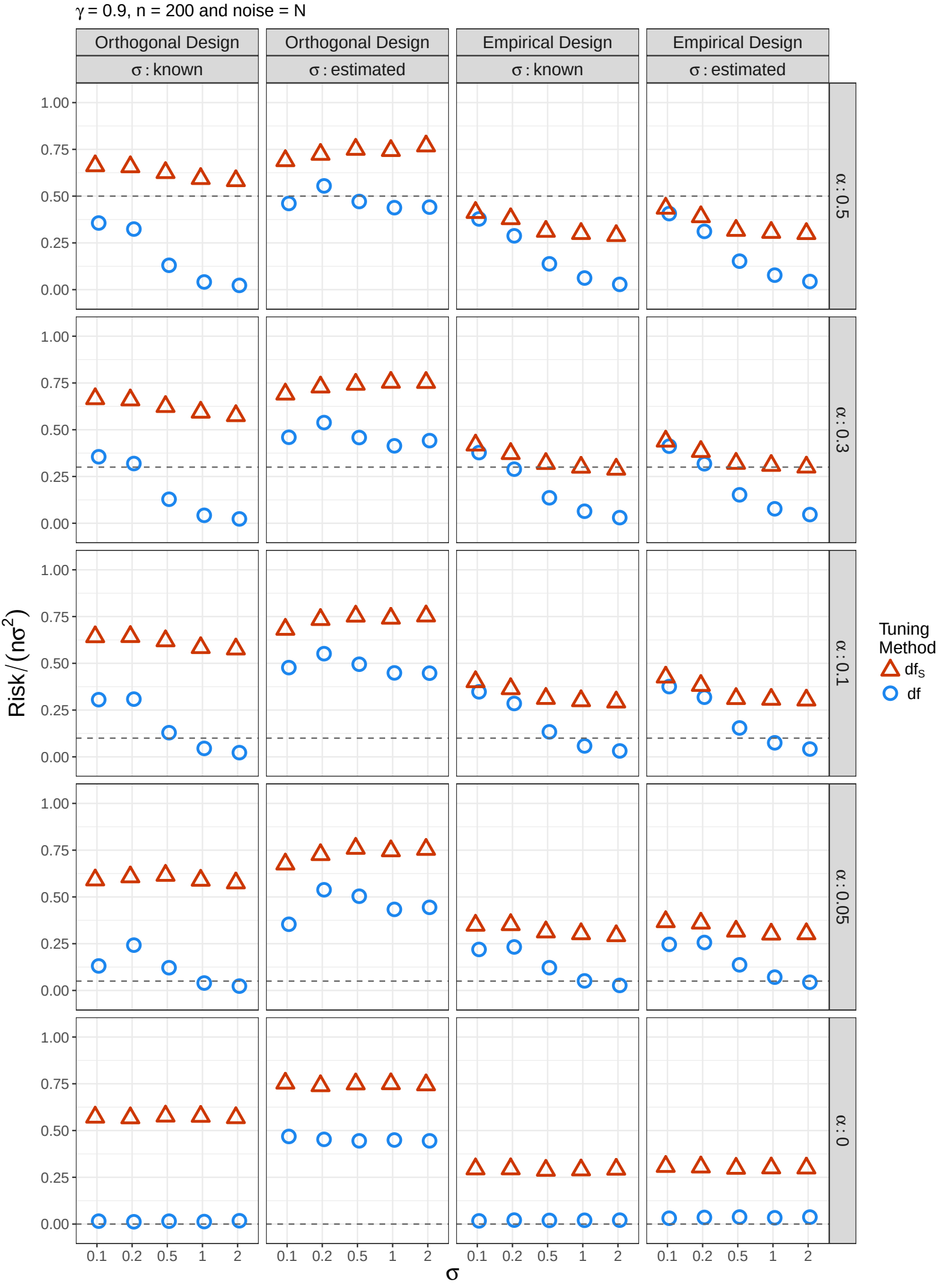
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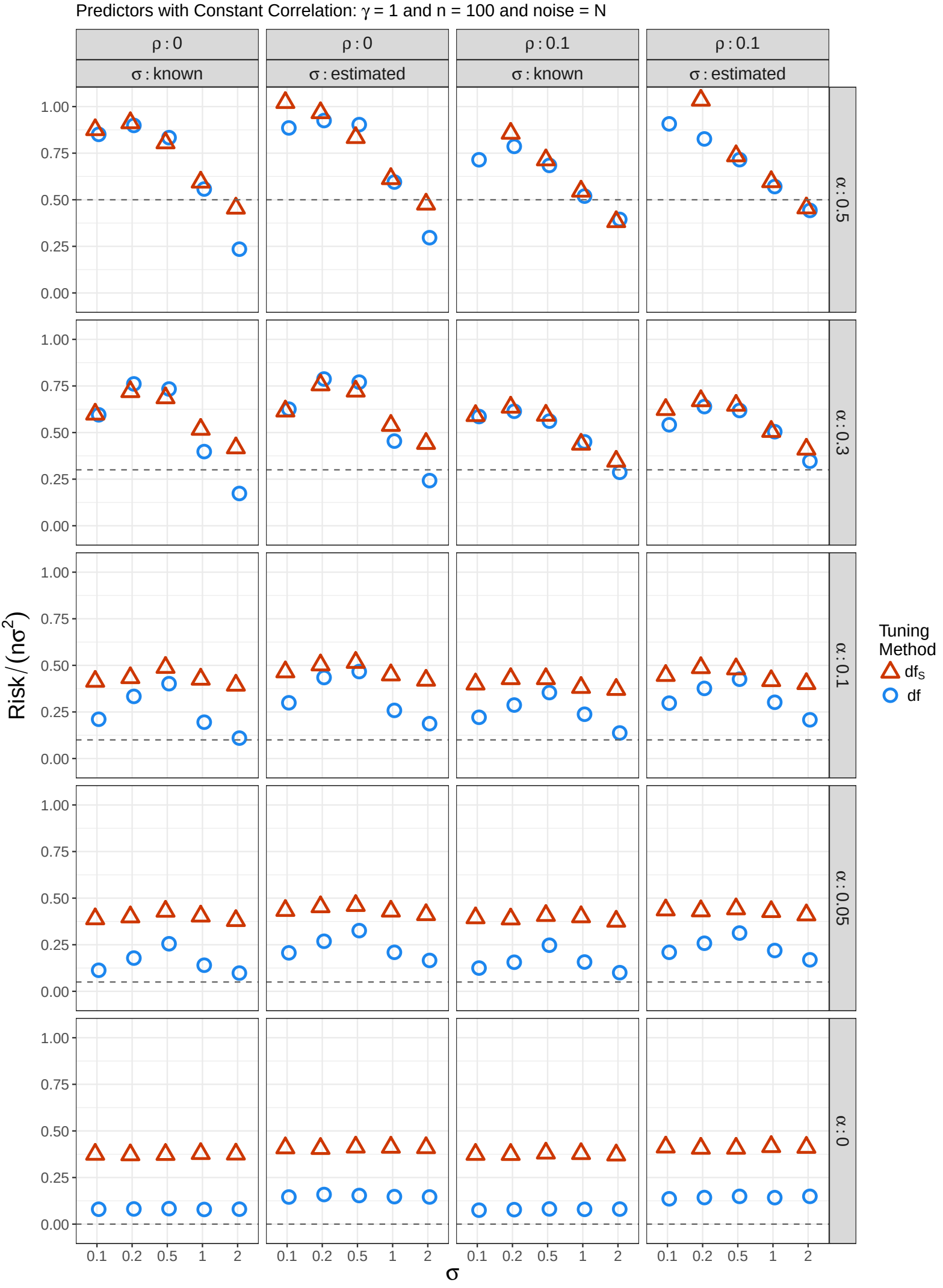
Predictors with Constant Correlation: γ = 1 and n = 100 and noise = N
ρ : 0
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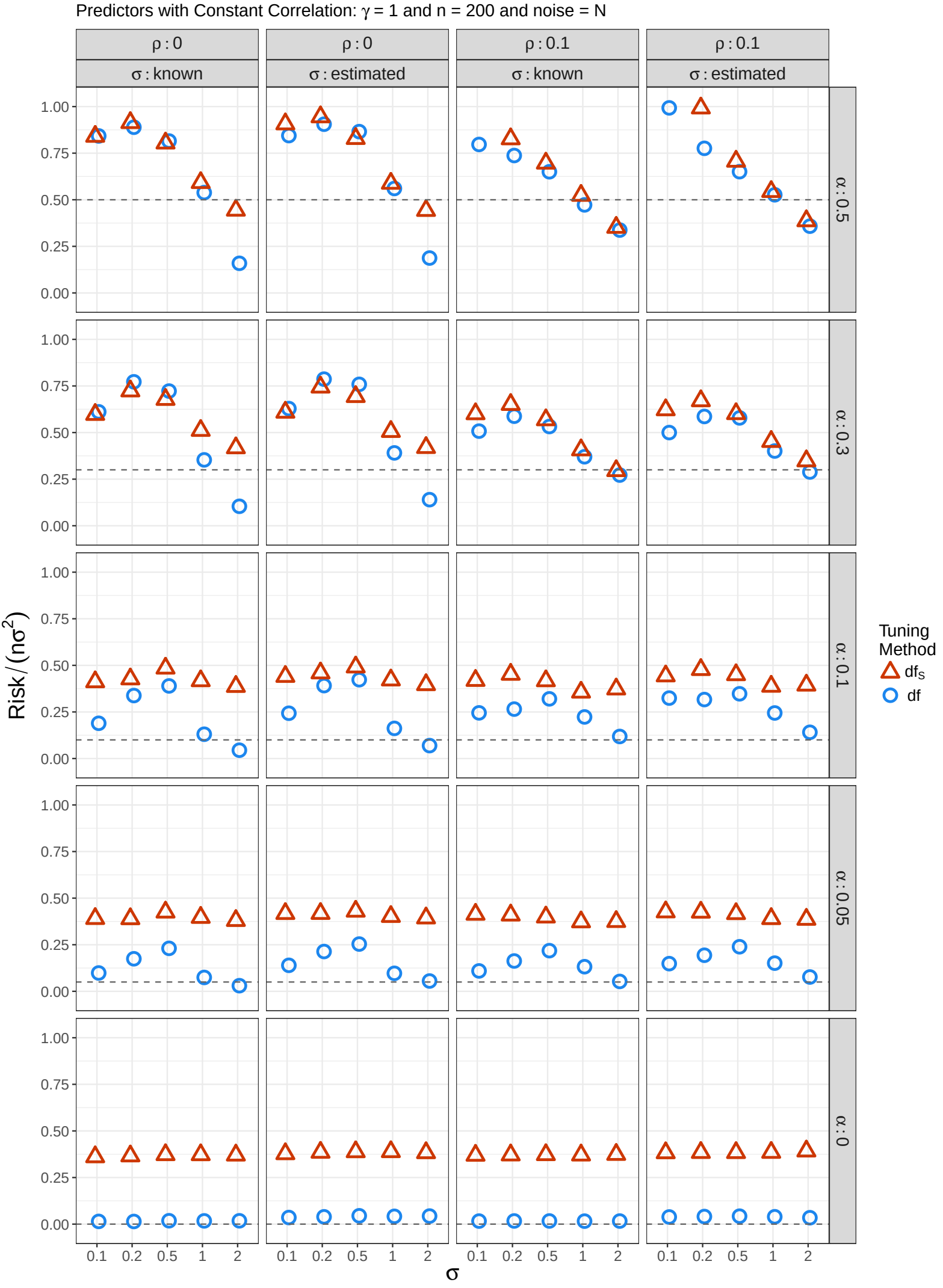
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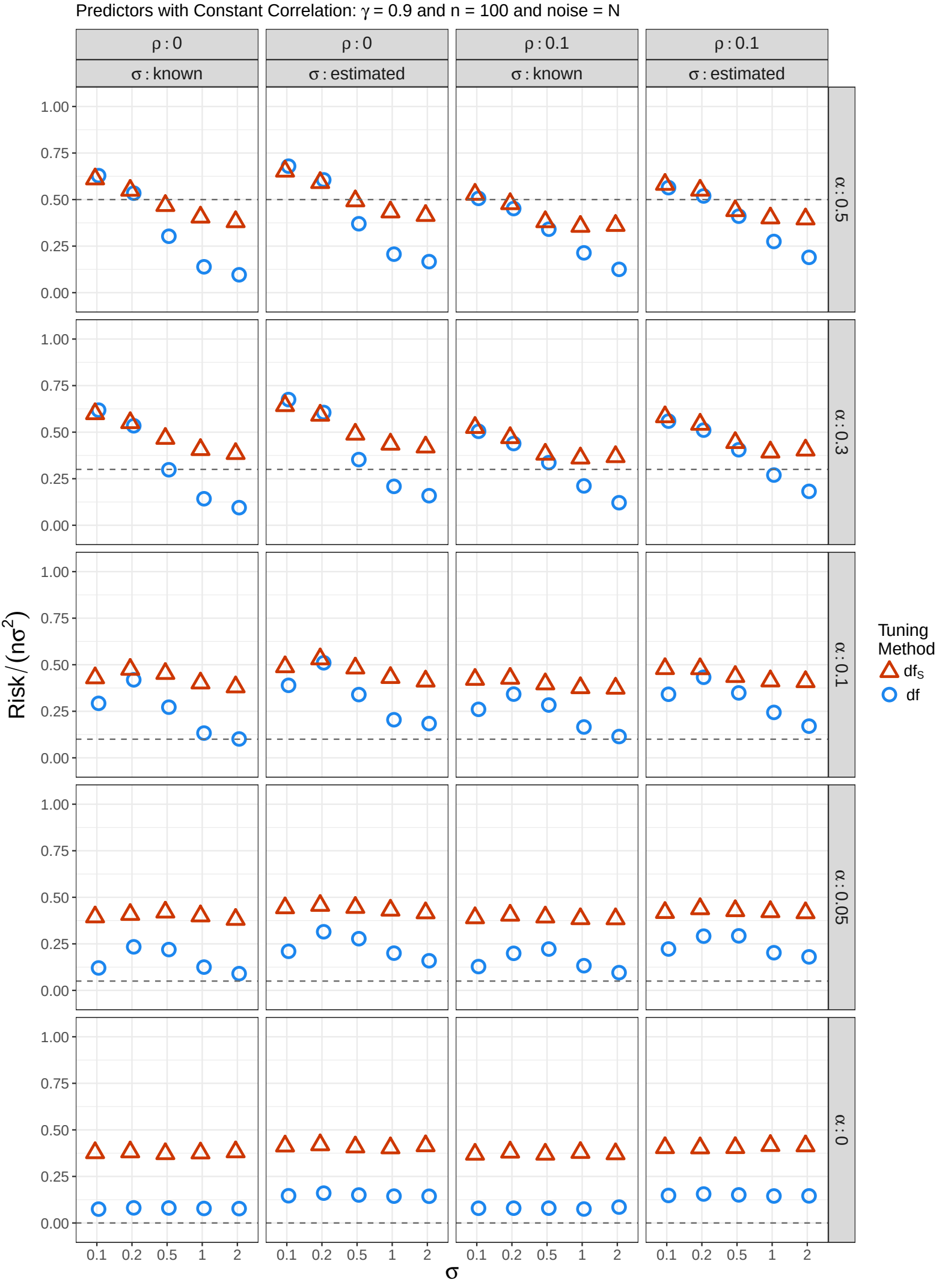
df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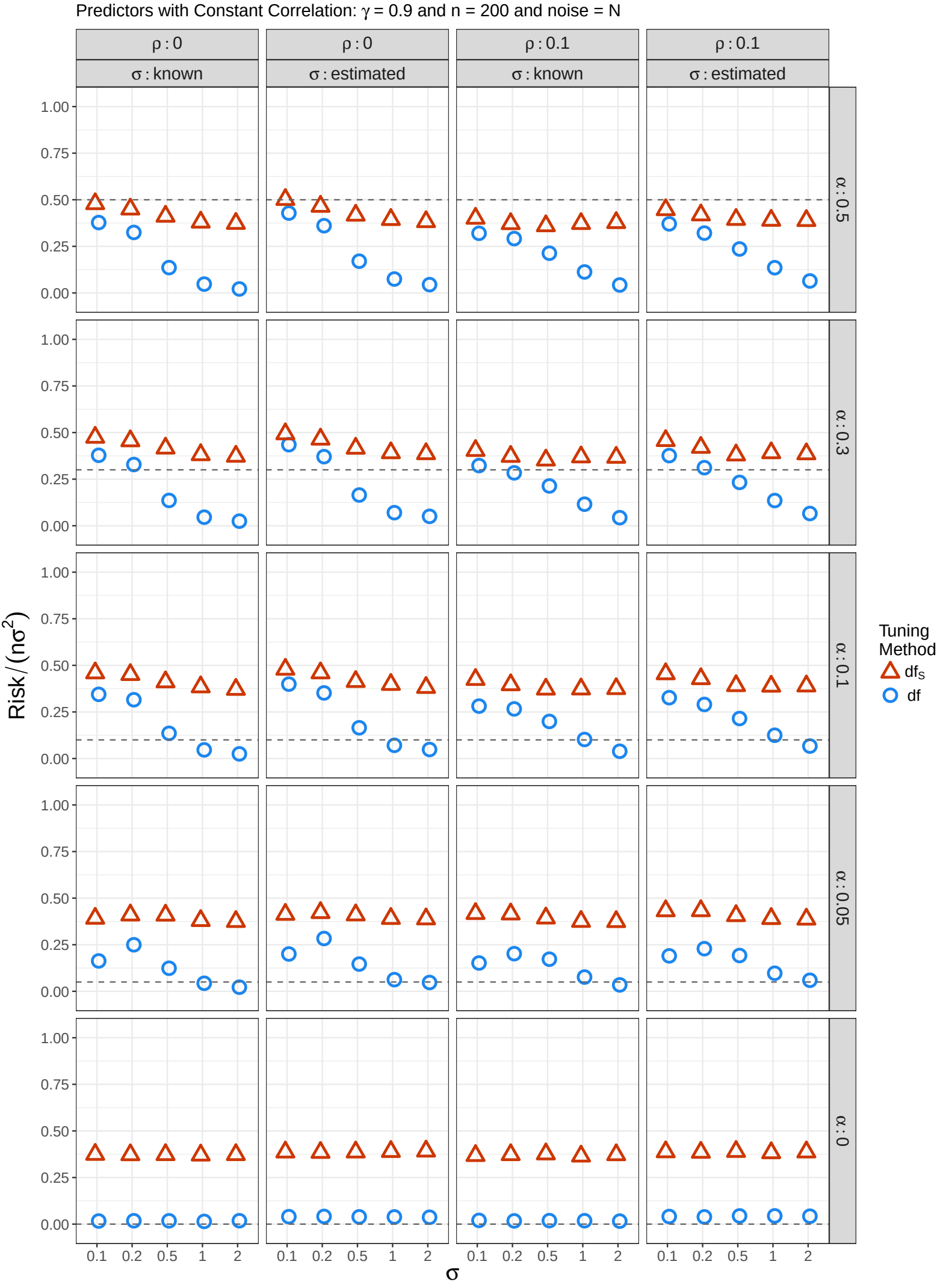
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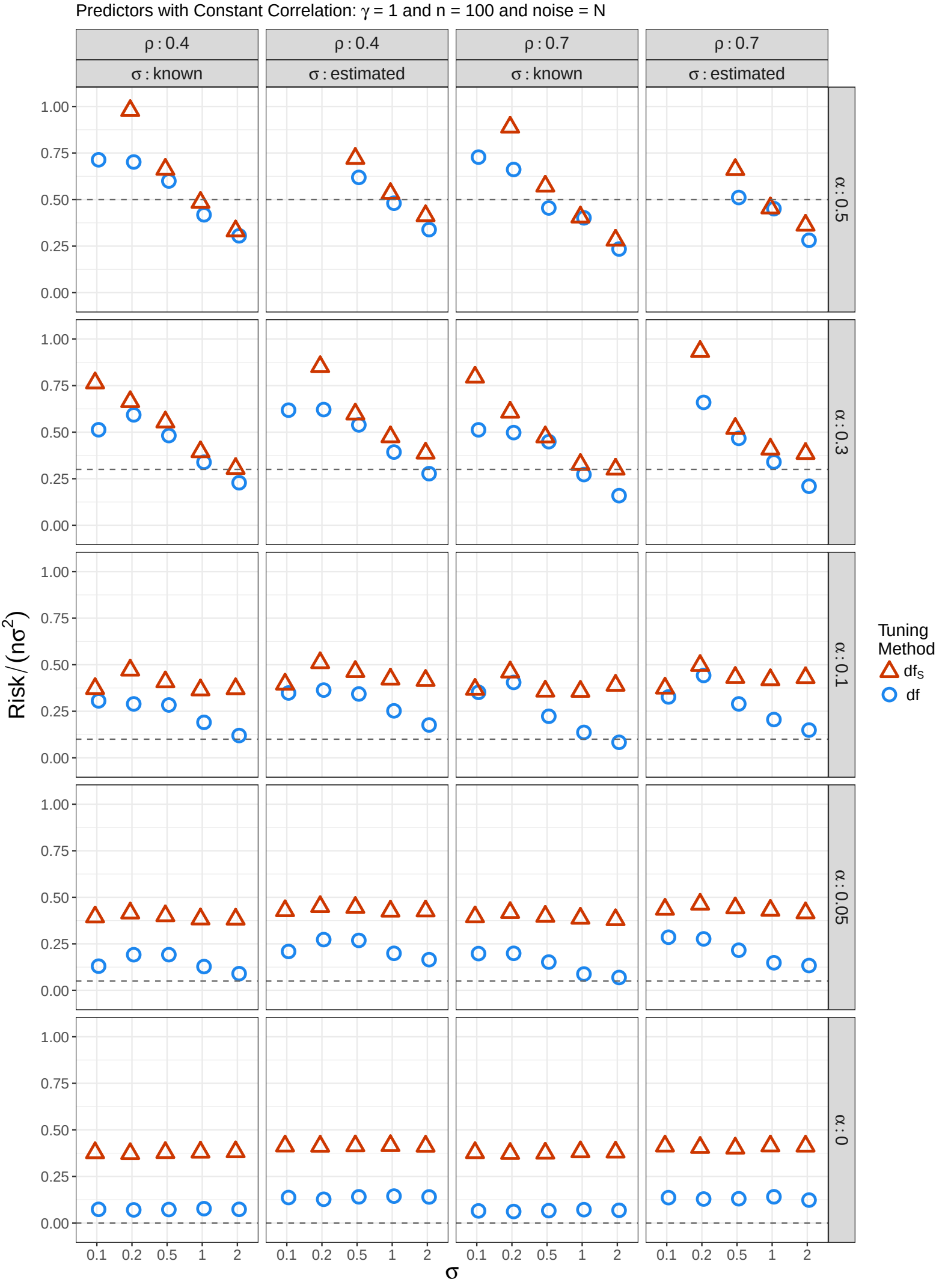
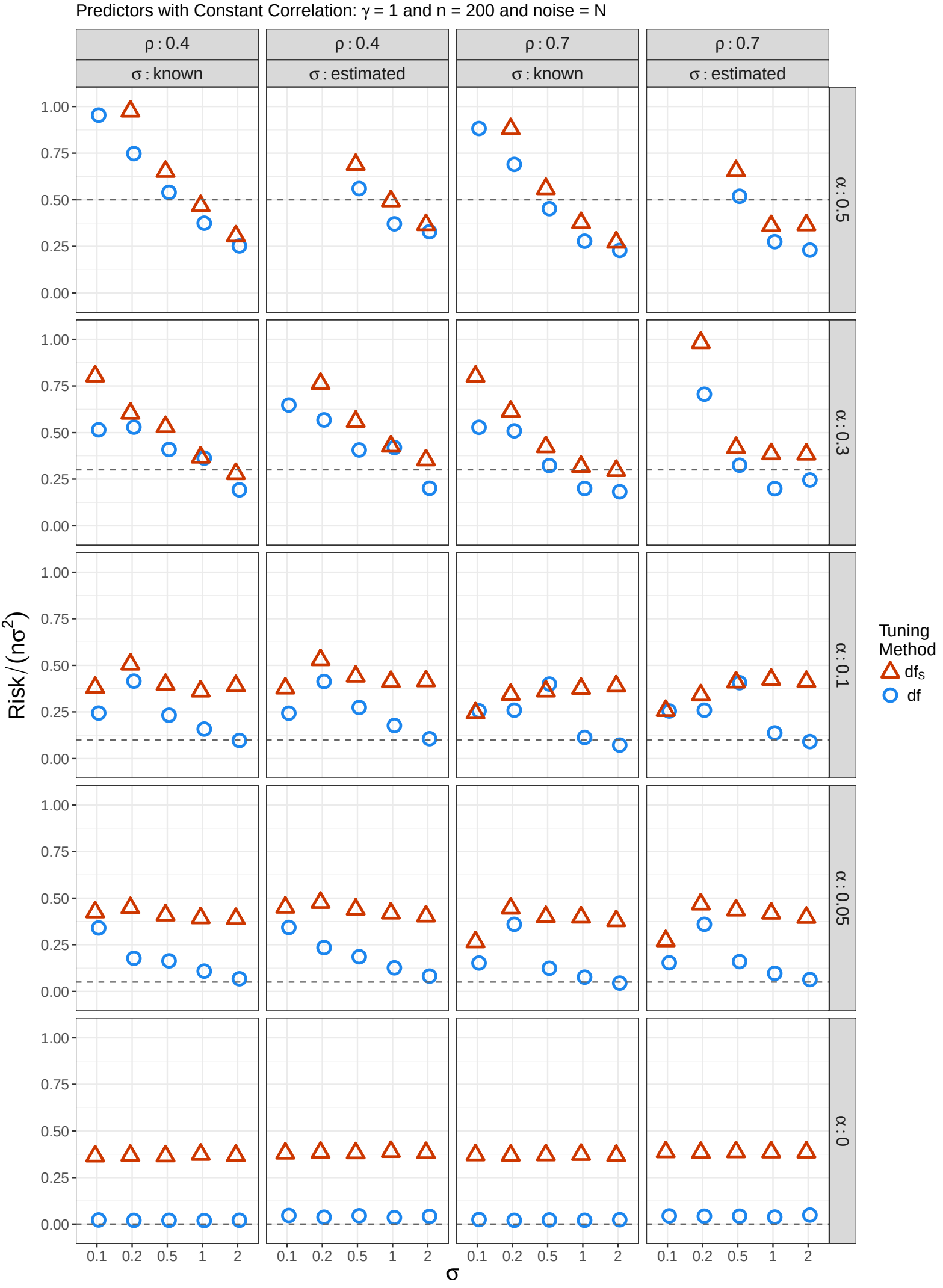
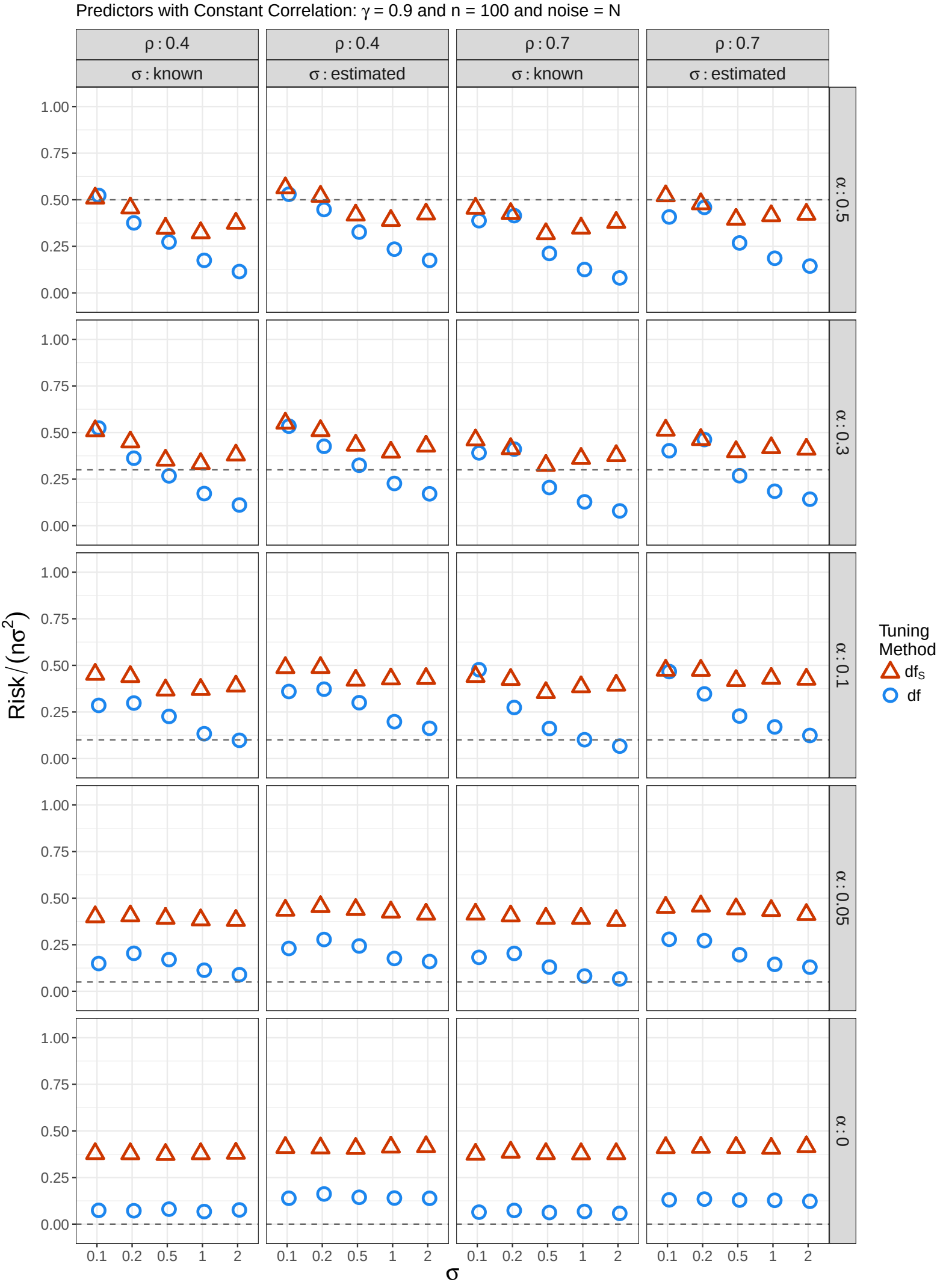

Predictors with Constant Correlation: γ = 1 and n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
dfS
df


Predictors with Constant Correlation: γ = 1 and n = 200 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk / (nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
df_S
df


Predictors with Constant Correlation: γ = 0.9 and n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
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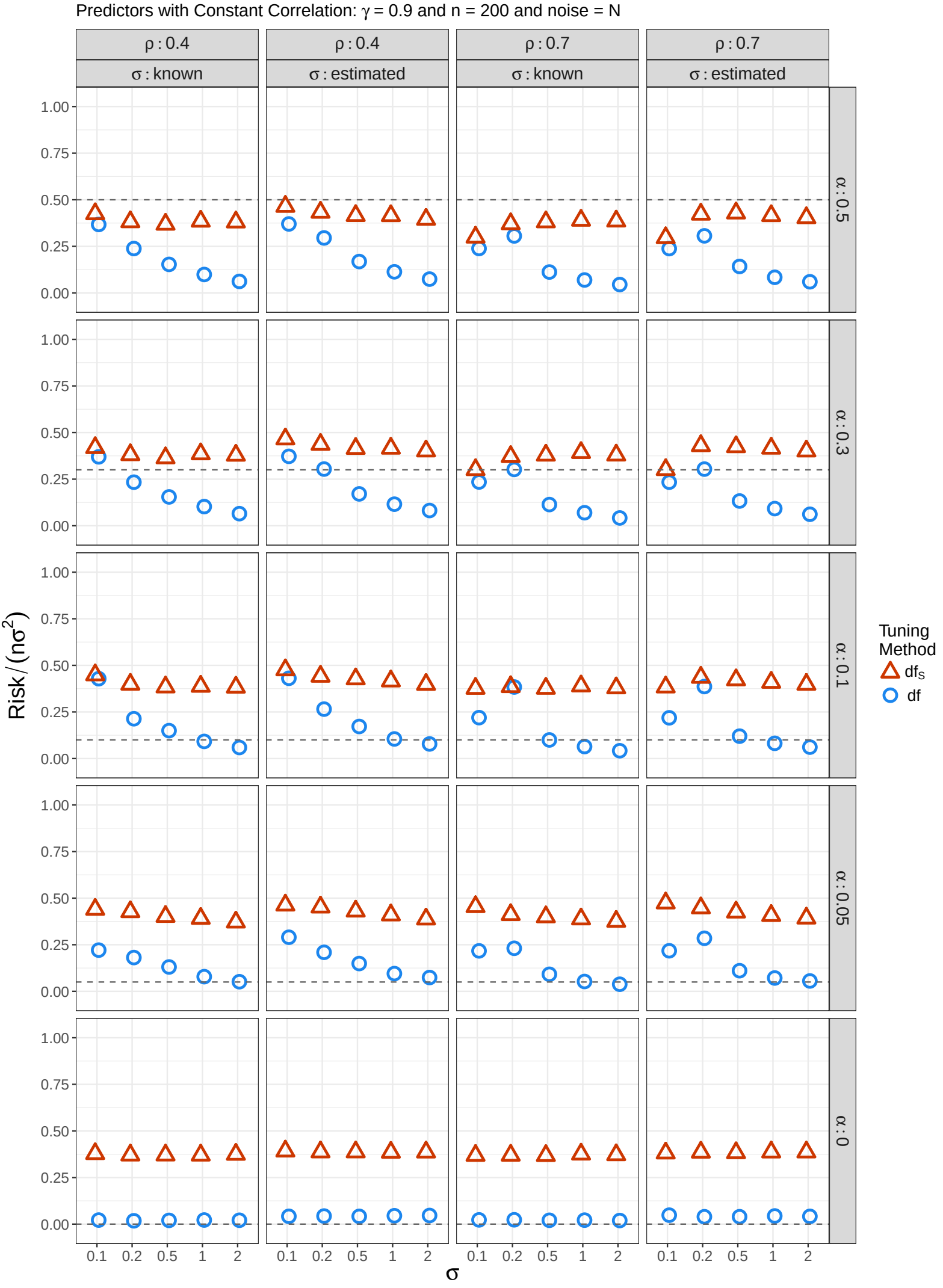
dfS
df


Predictors with Constant Correlation: γ = 0.9 and n = 200 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
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Risk/(nσ²)
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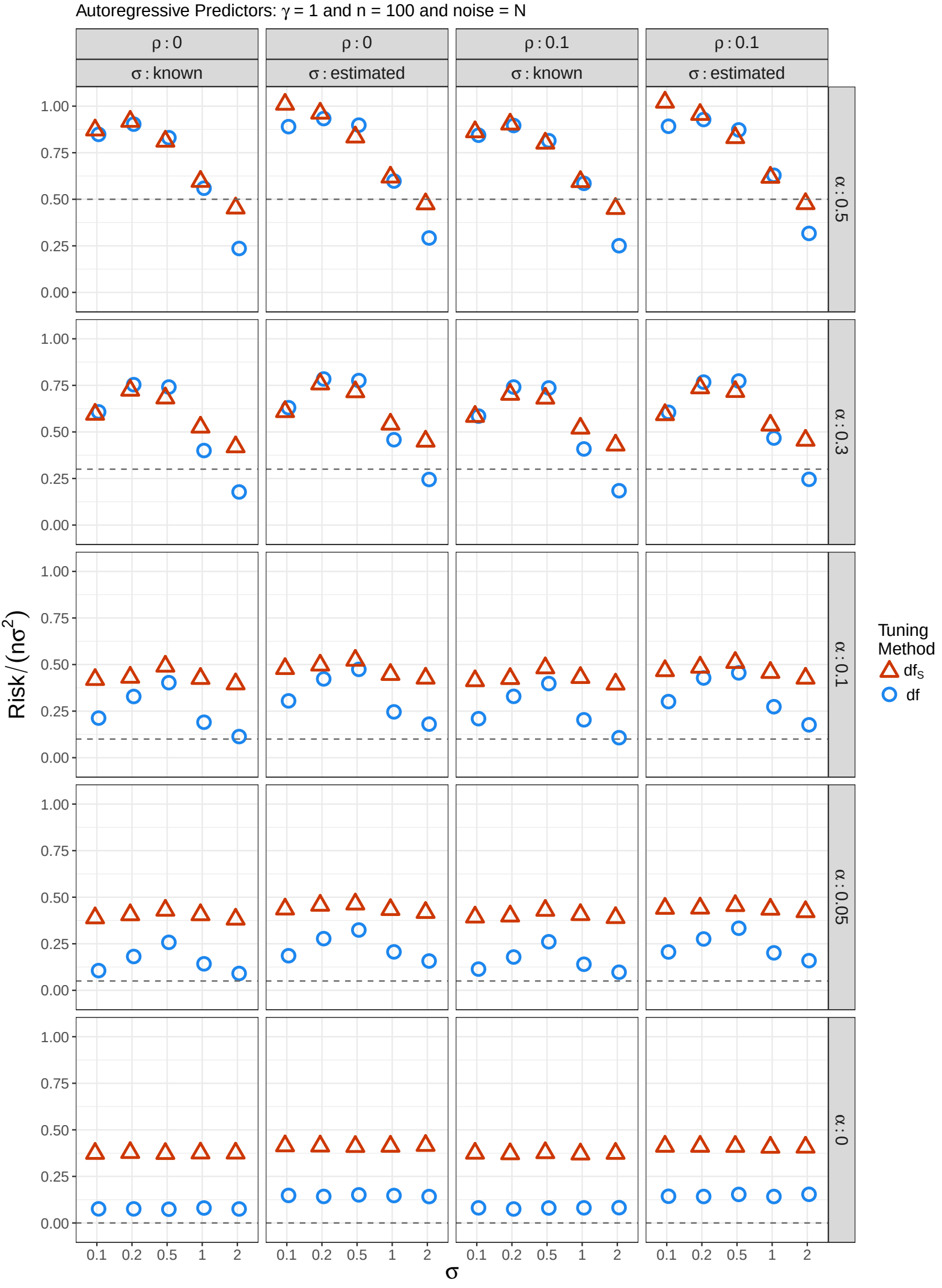
Tuning Method
df_S
df


Autoregressive Predictors: γ = 1 and n = 100 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
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0.25
0.00
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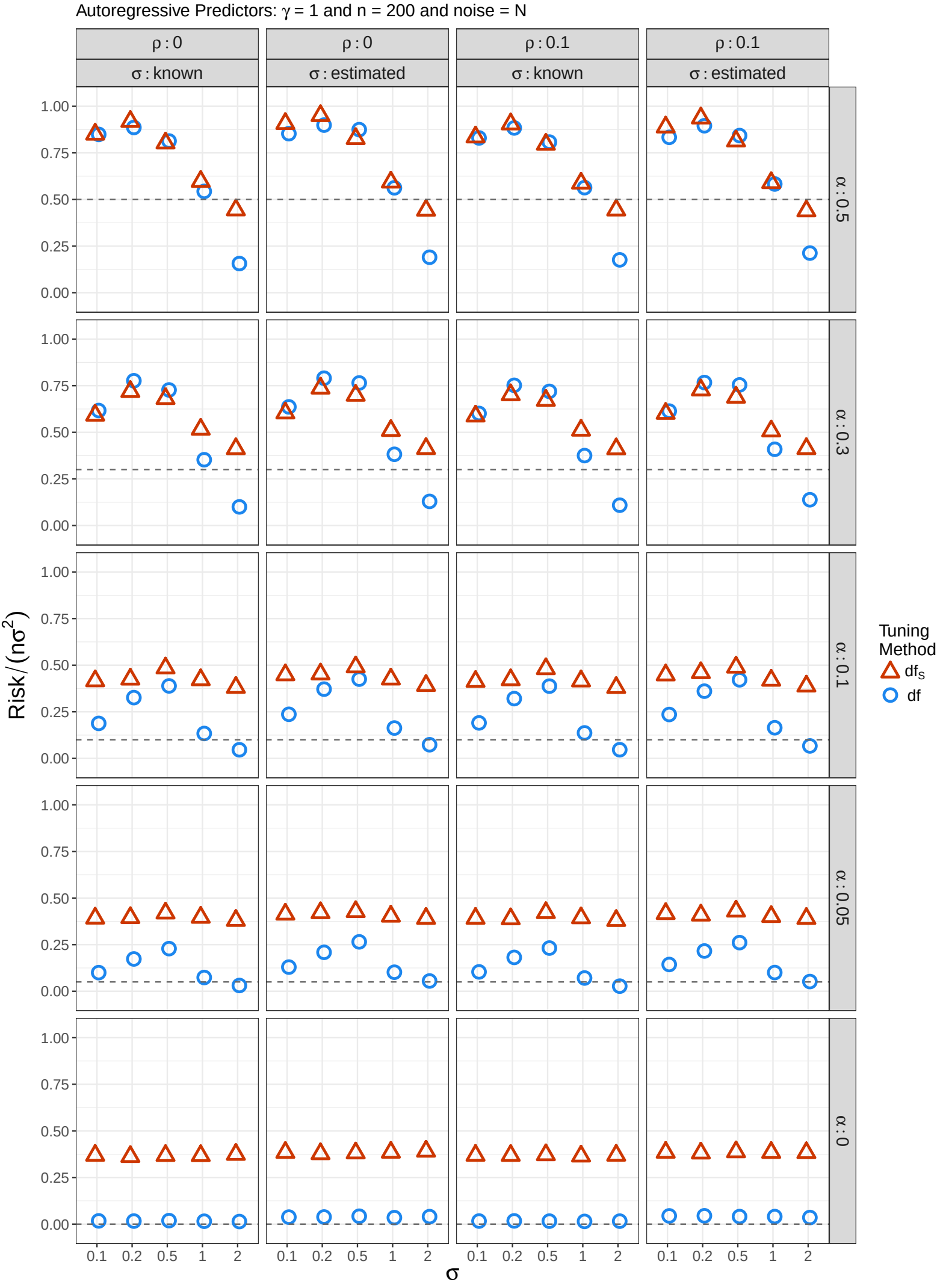
Tuning Method
dfS
df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ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
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Tuning Method
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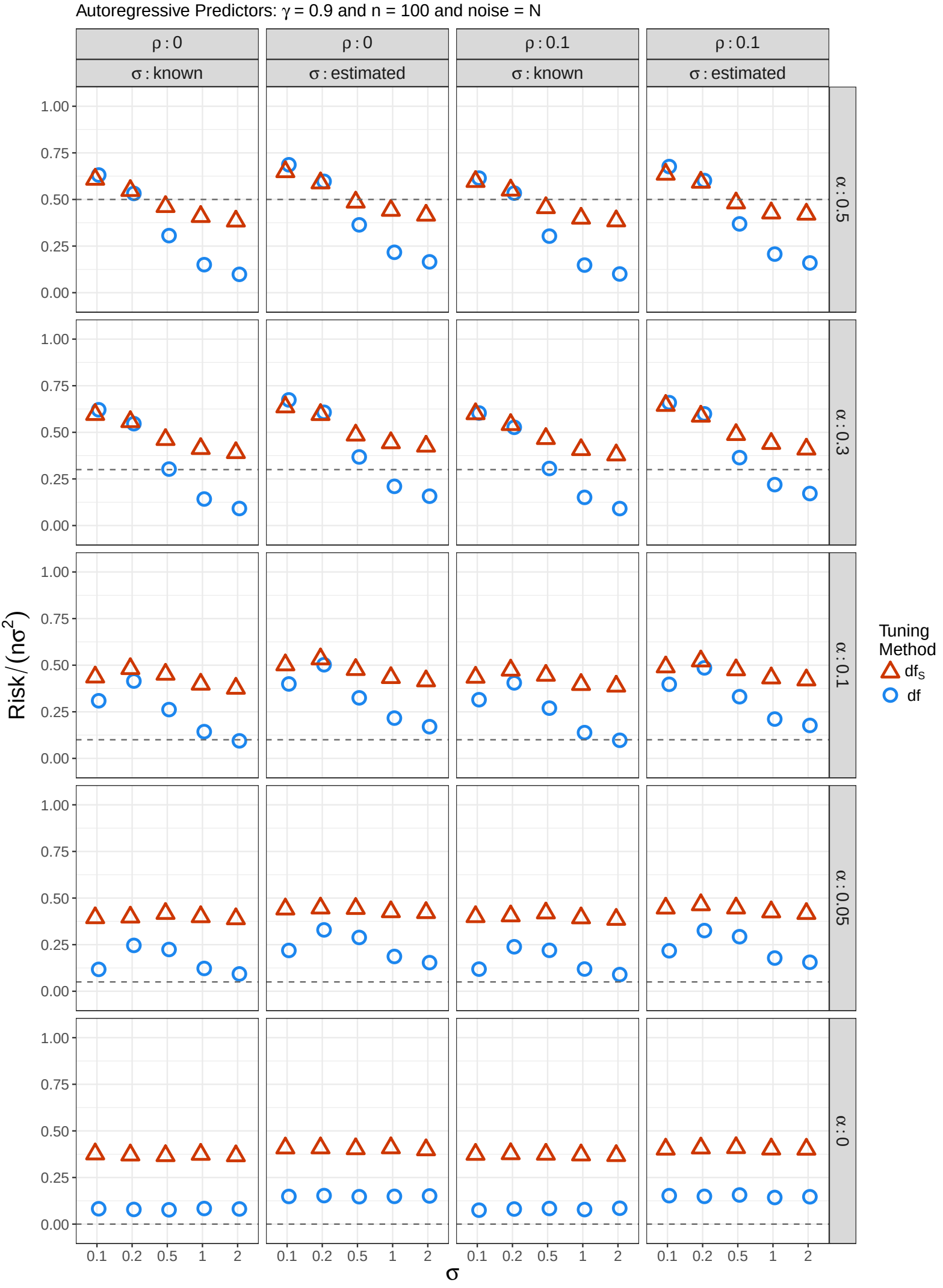
df


Autoregressive Predictors: γ = 0.9 and n = 100 and noise = N
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ρ : 0
ρ : 0.1
ρ : 0.1
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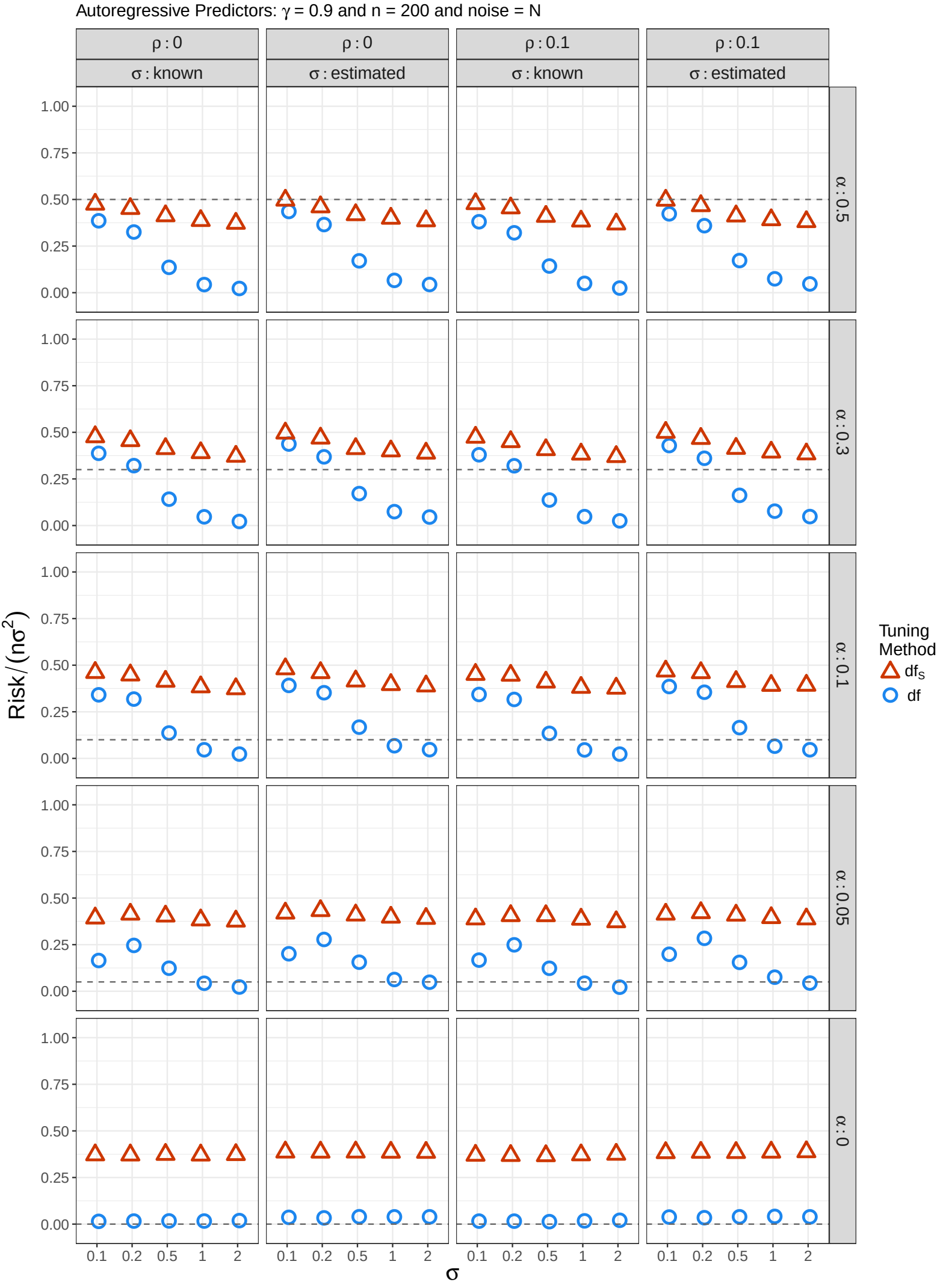
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Tuning Method
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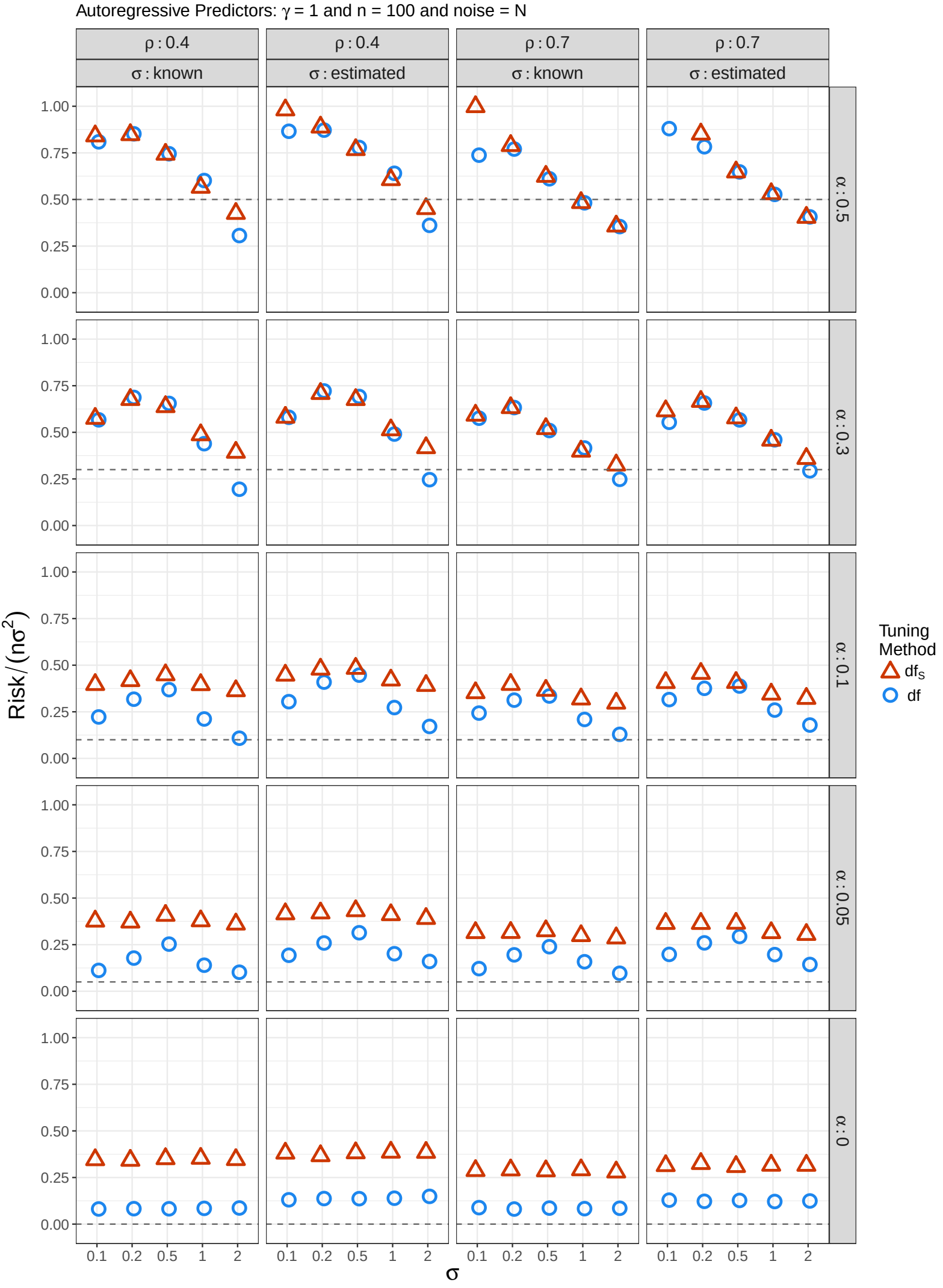
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ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
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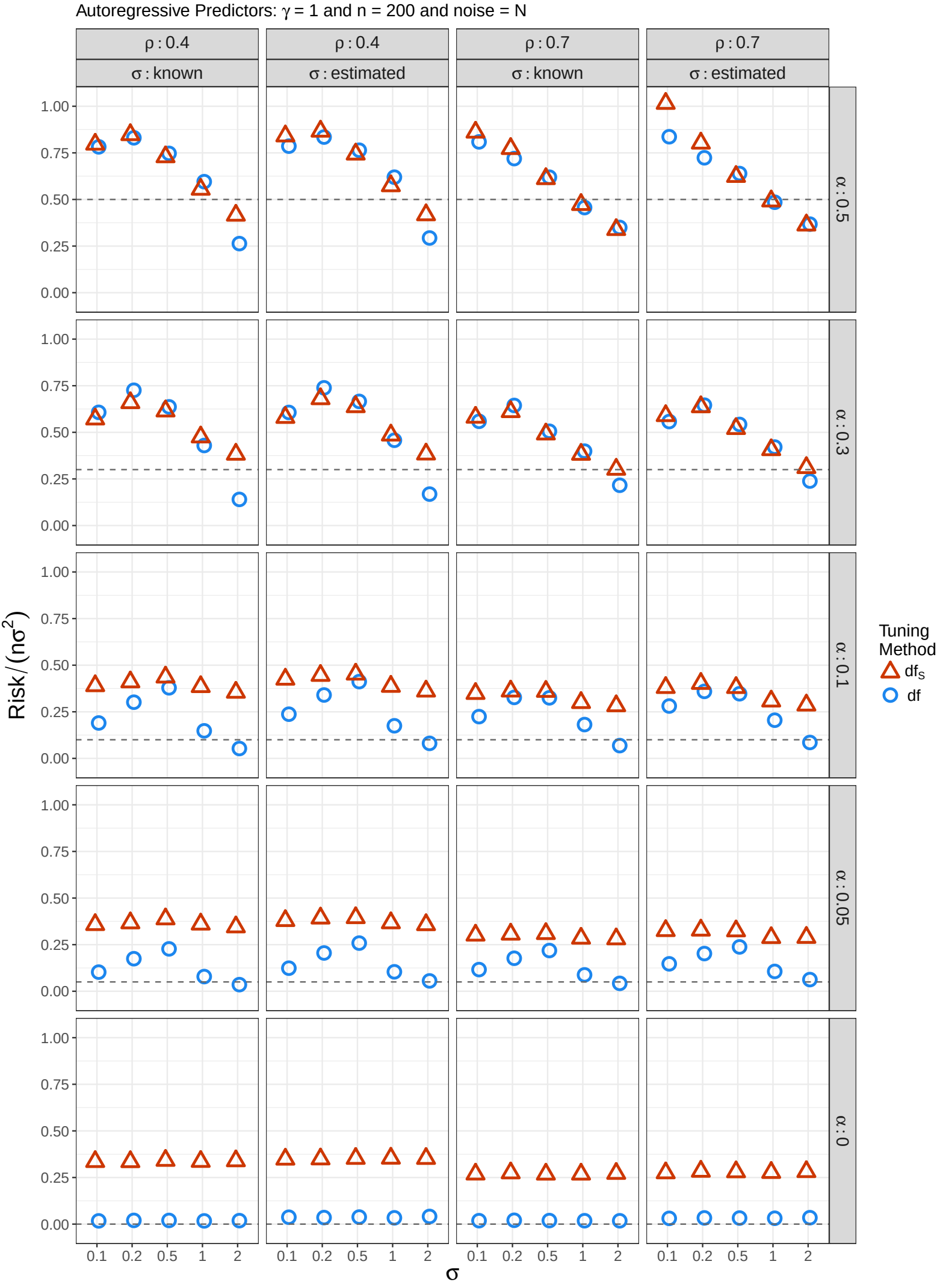
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
df_S
df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ρ : 0.4
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Tuning Method
df_S
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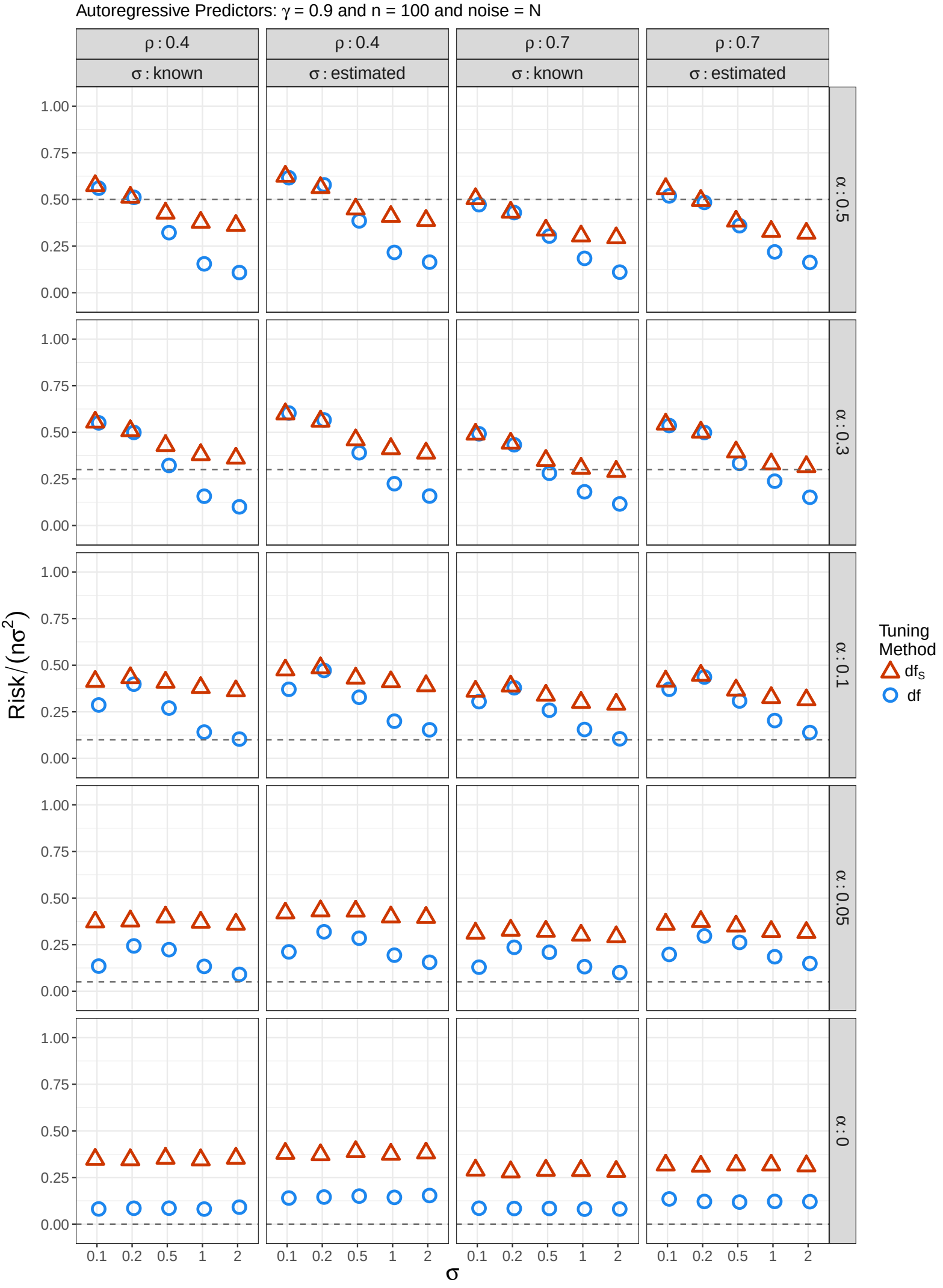
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0.25
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0.2
0.5
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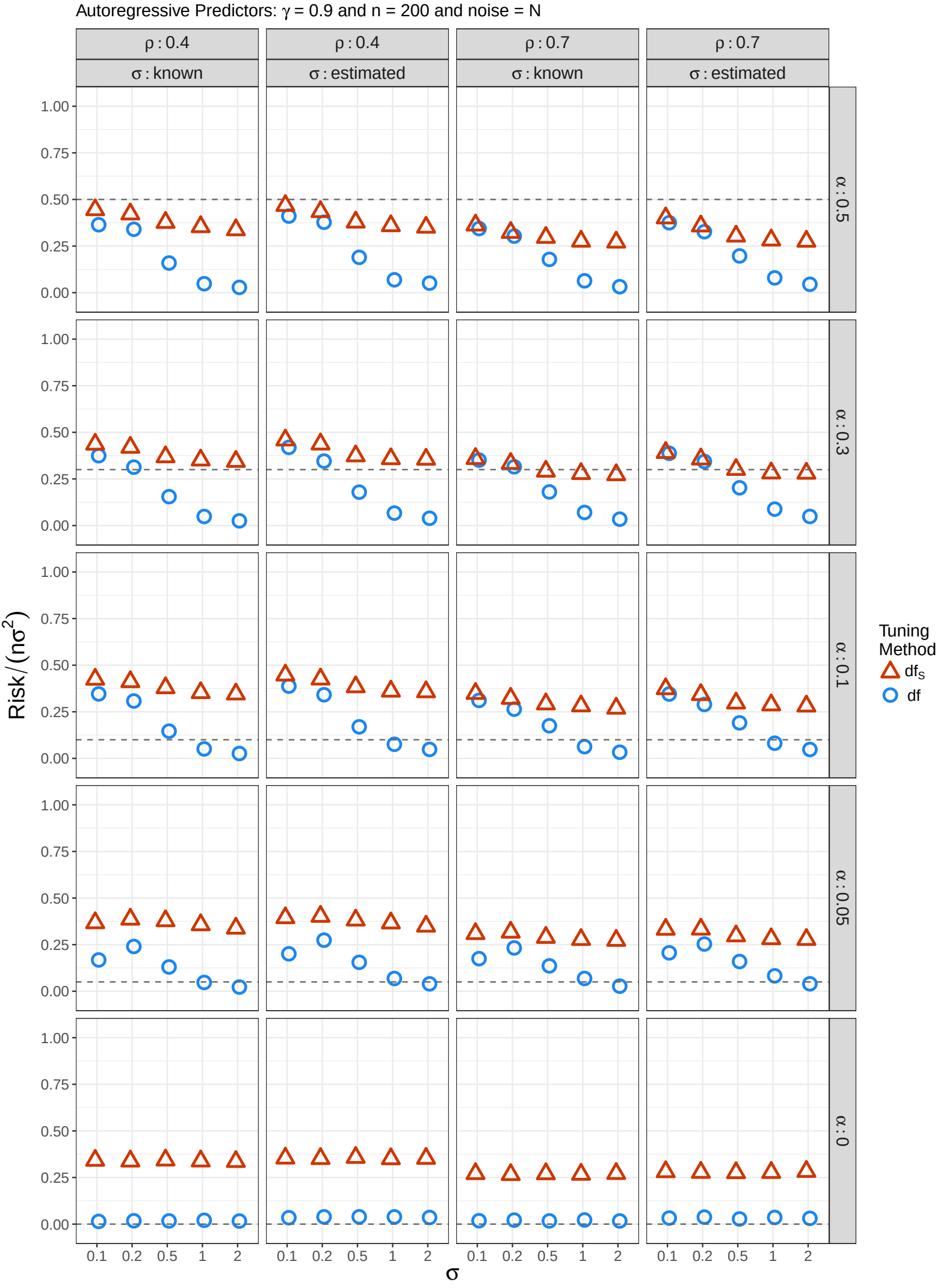
2
σ
Tuning Method
df_S
df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α : 0
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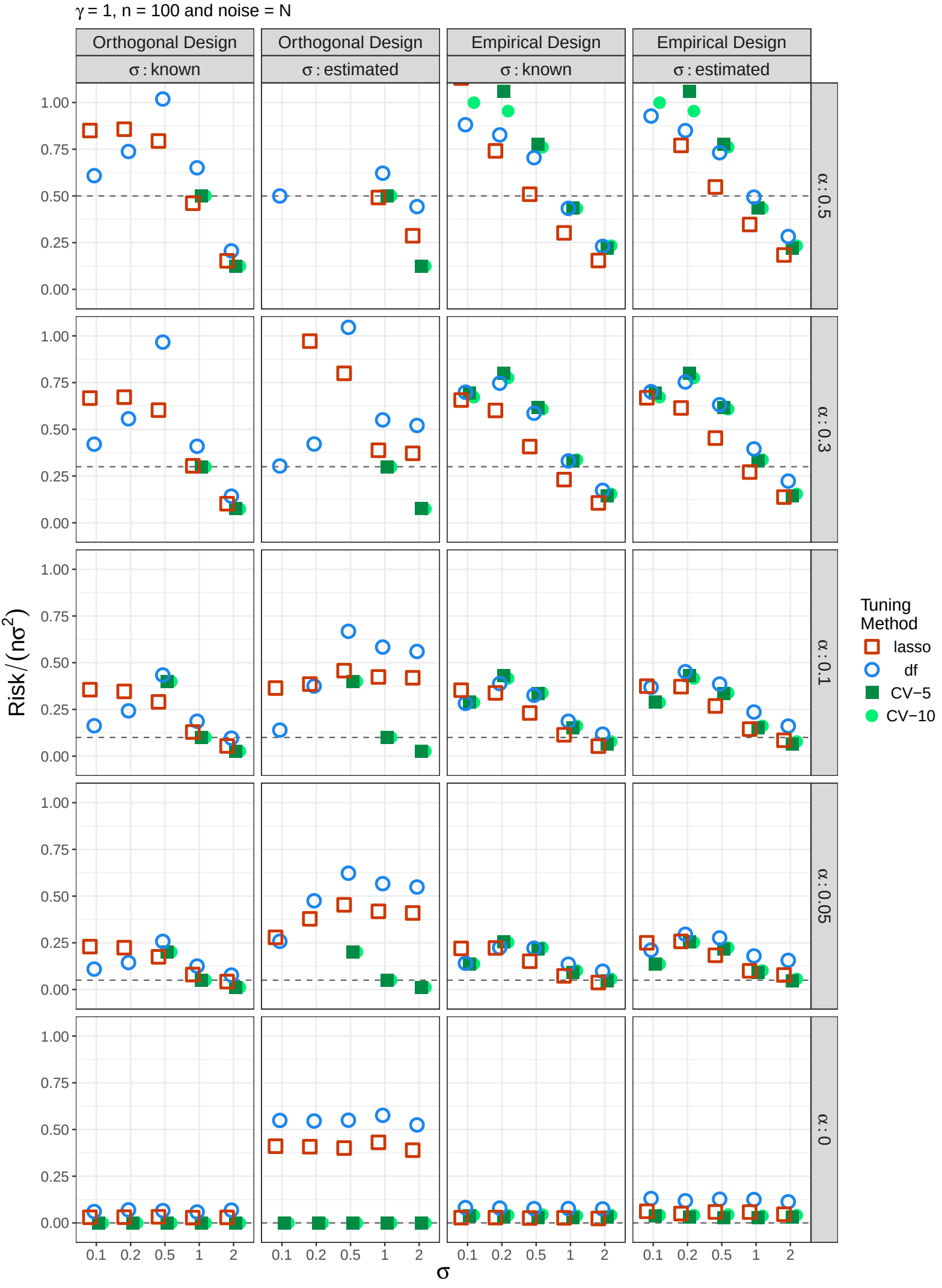
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
df_S
df

Plots of the risk estimates relative to $n\sigma^2$ for the estimators $\hat{\mu}_{\text{OLS.l}}^{\hat{\lambda}_{\text{df}}}$, $\hat{\mu}_{\text{OLS.l}}^{\hat{\lambda}_{\text{CV}-5}}$, $\hat{\mu}_{\text{OLS.l}}^{\hat{\lambda}_{\text{CV}-10}}$ and $\hat{\mu}_{\text{lasso}}^{\hat{\lambda}_{\text{df}_S}}$. The dashed lines are the relative risks for the oracle-OLS estimator.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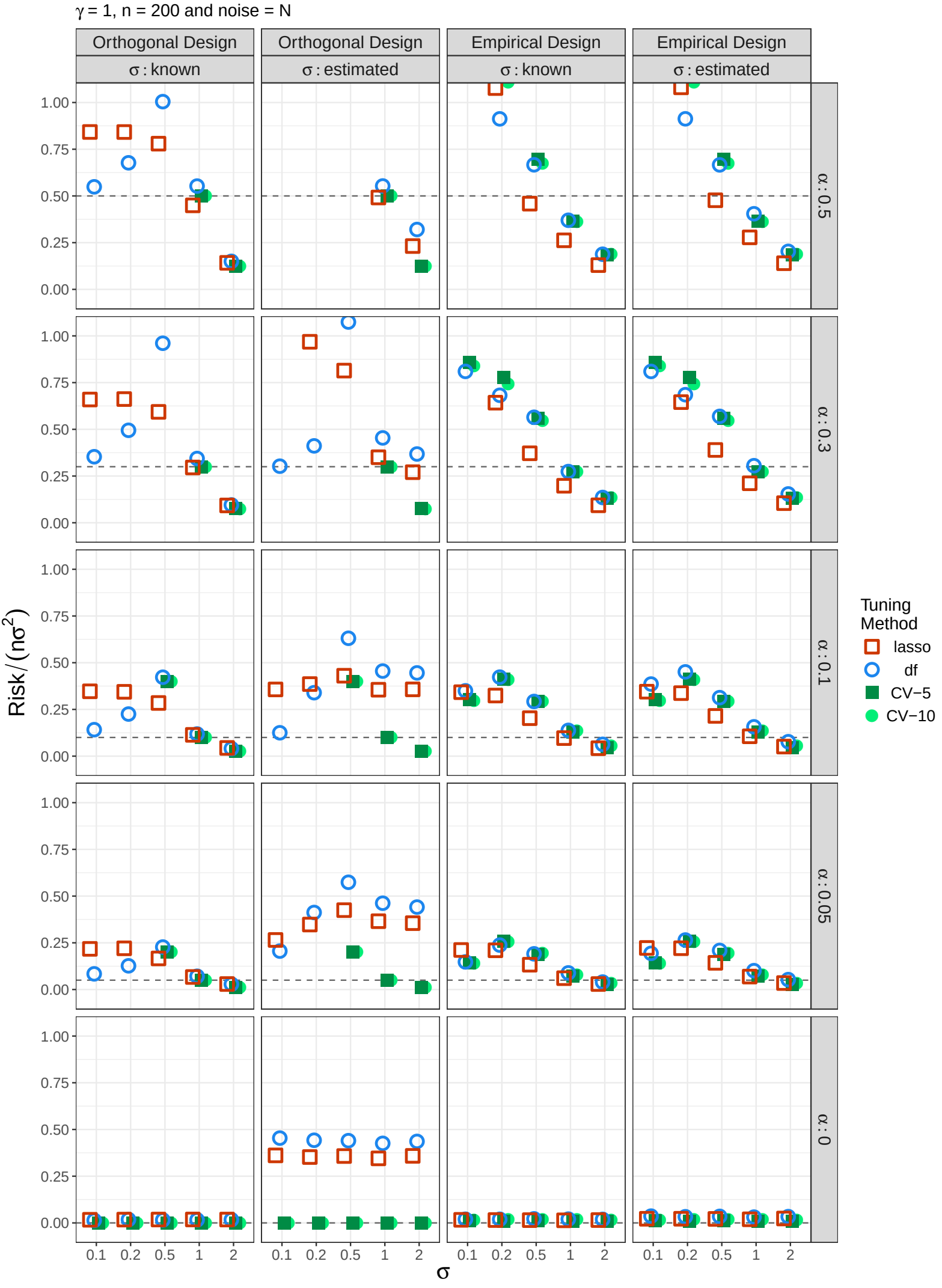
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10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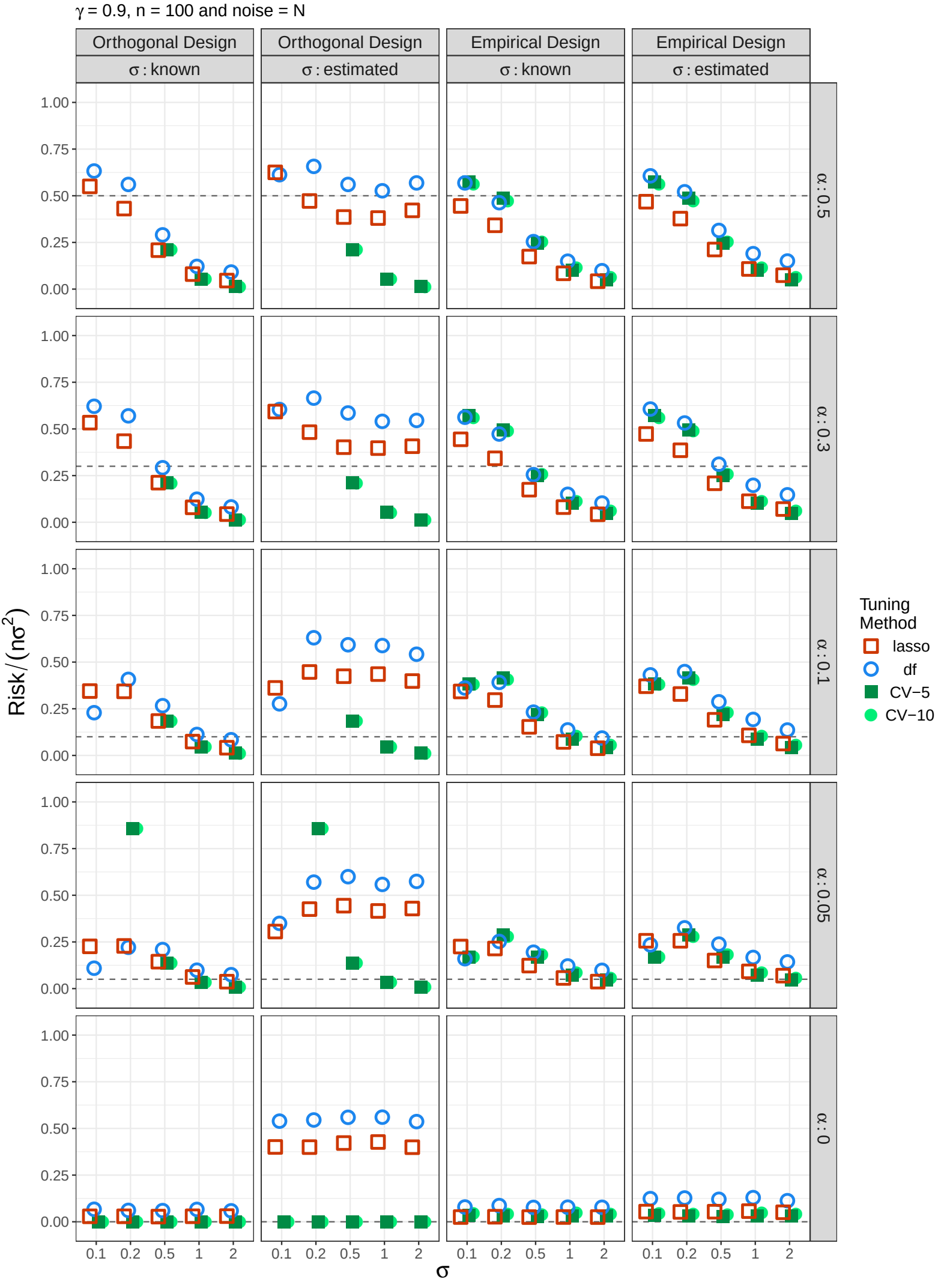
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10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α : 0.05
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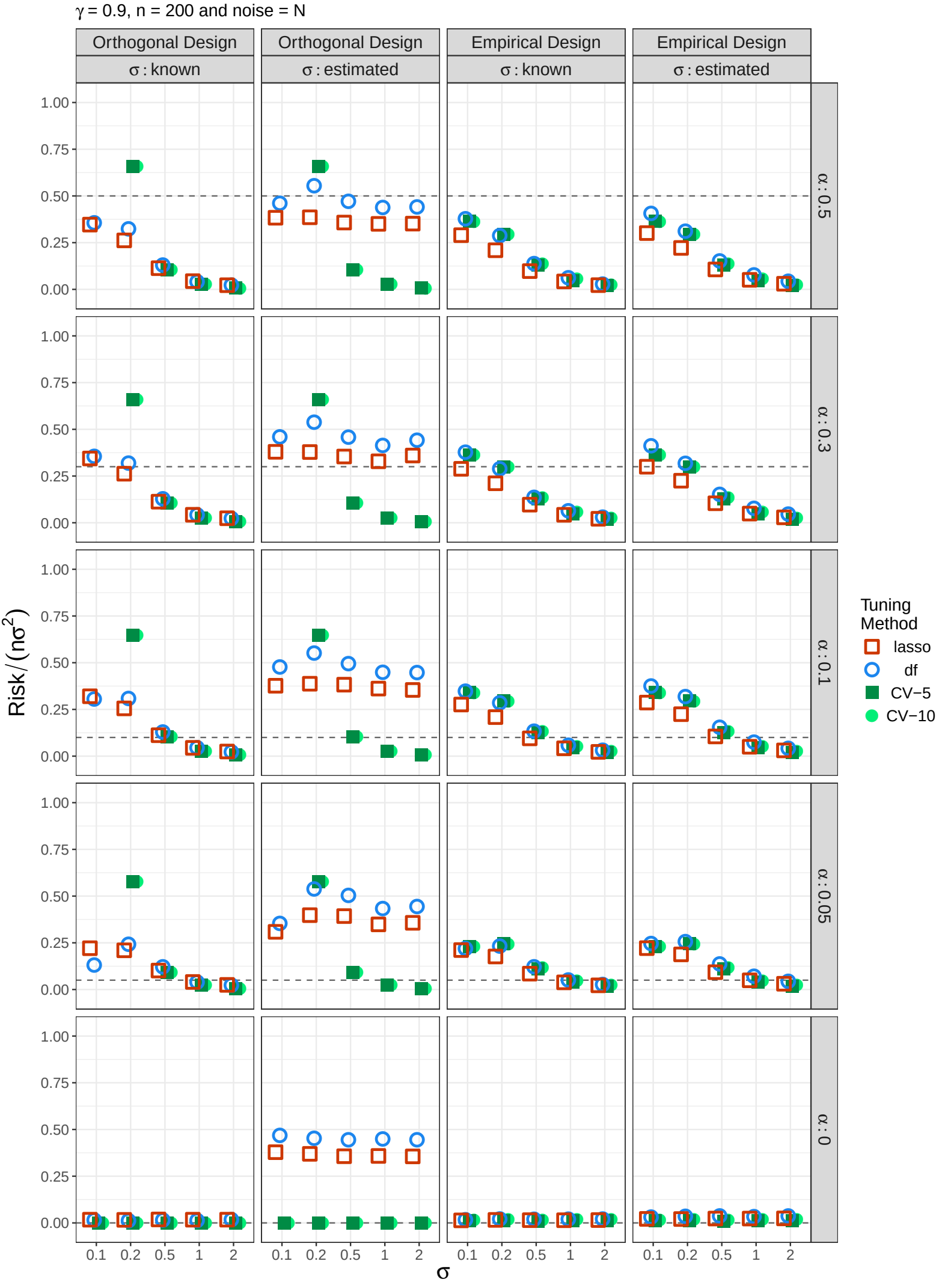
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10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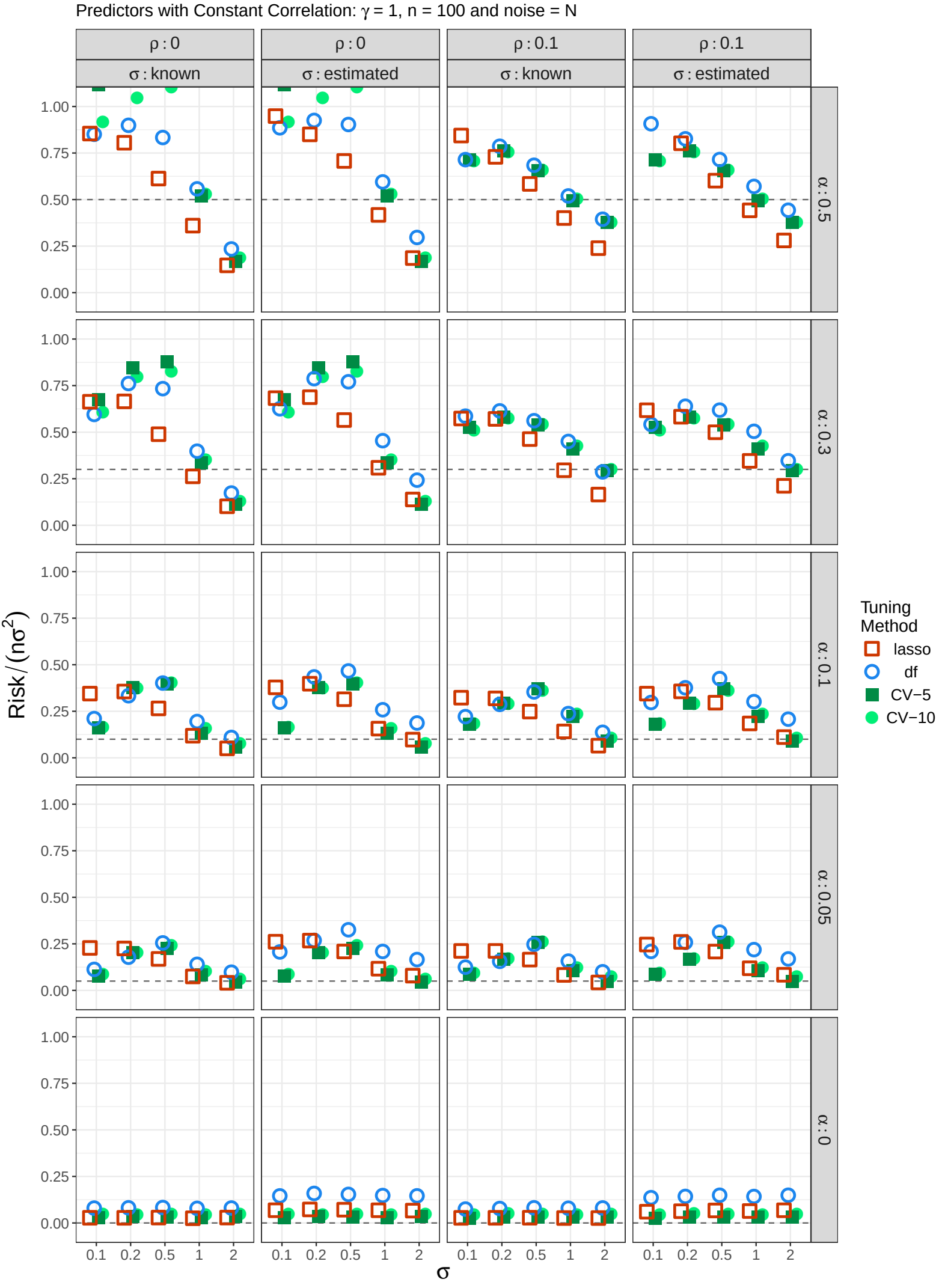
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
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σ : estimated
α : 0.5
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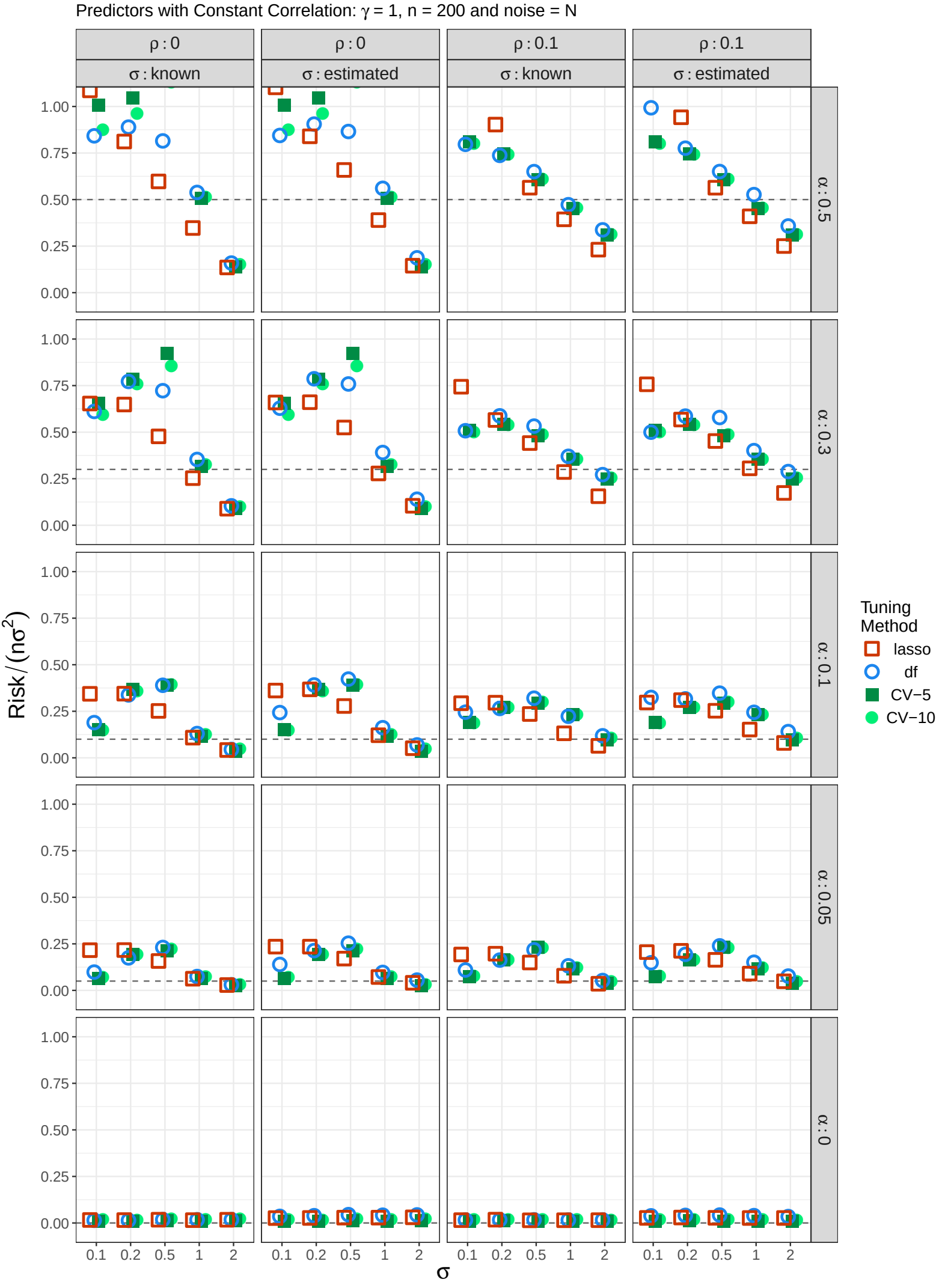
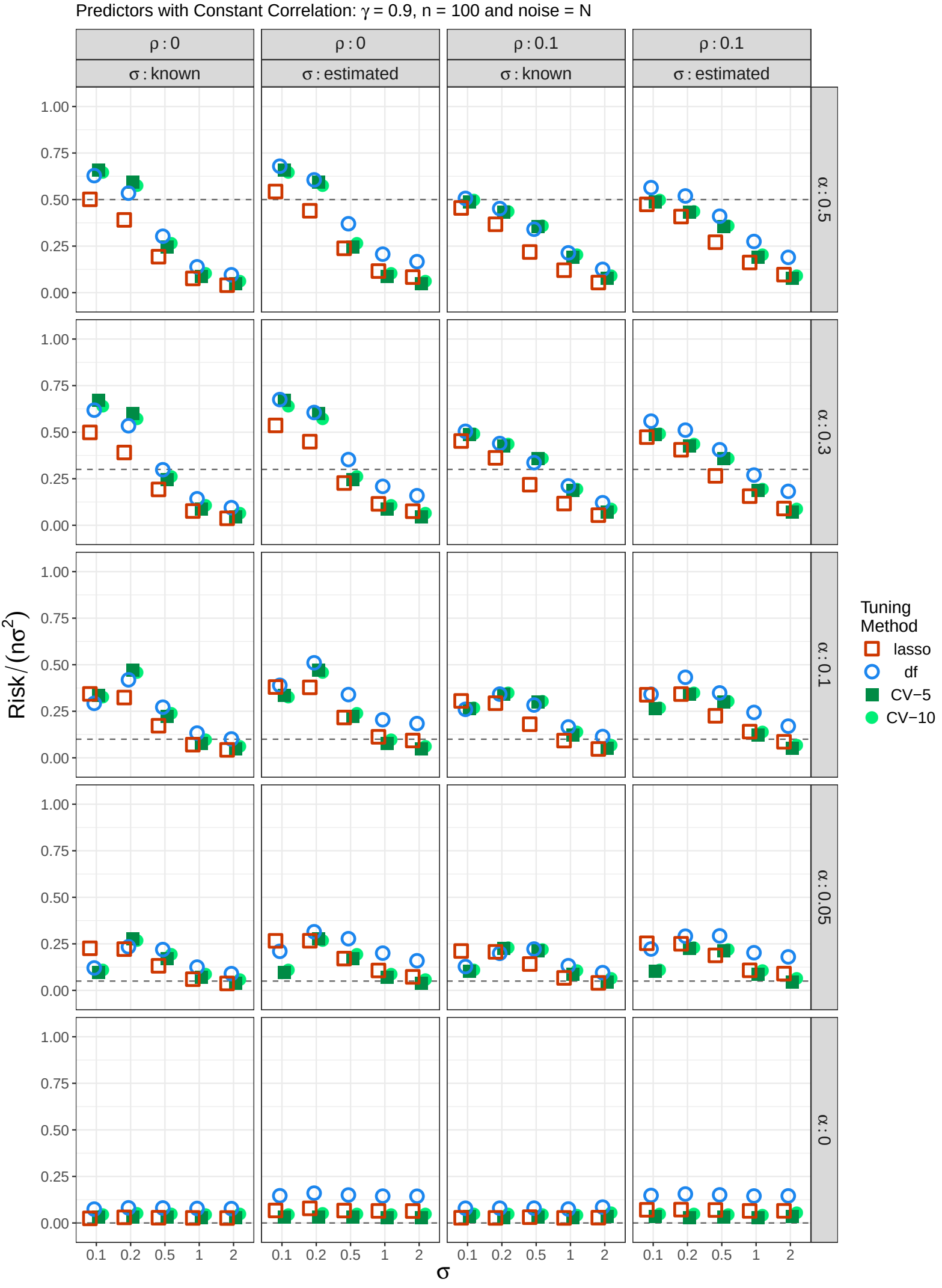
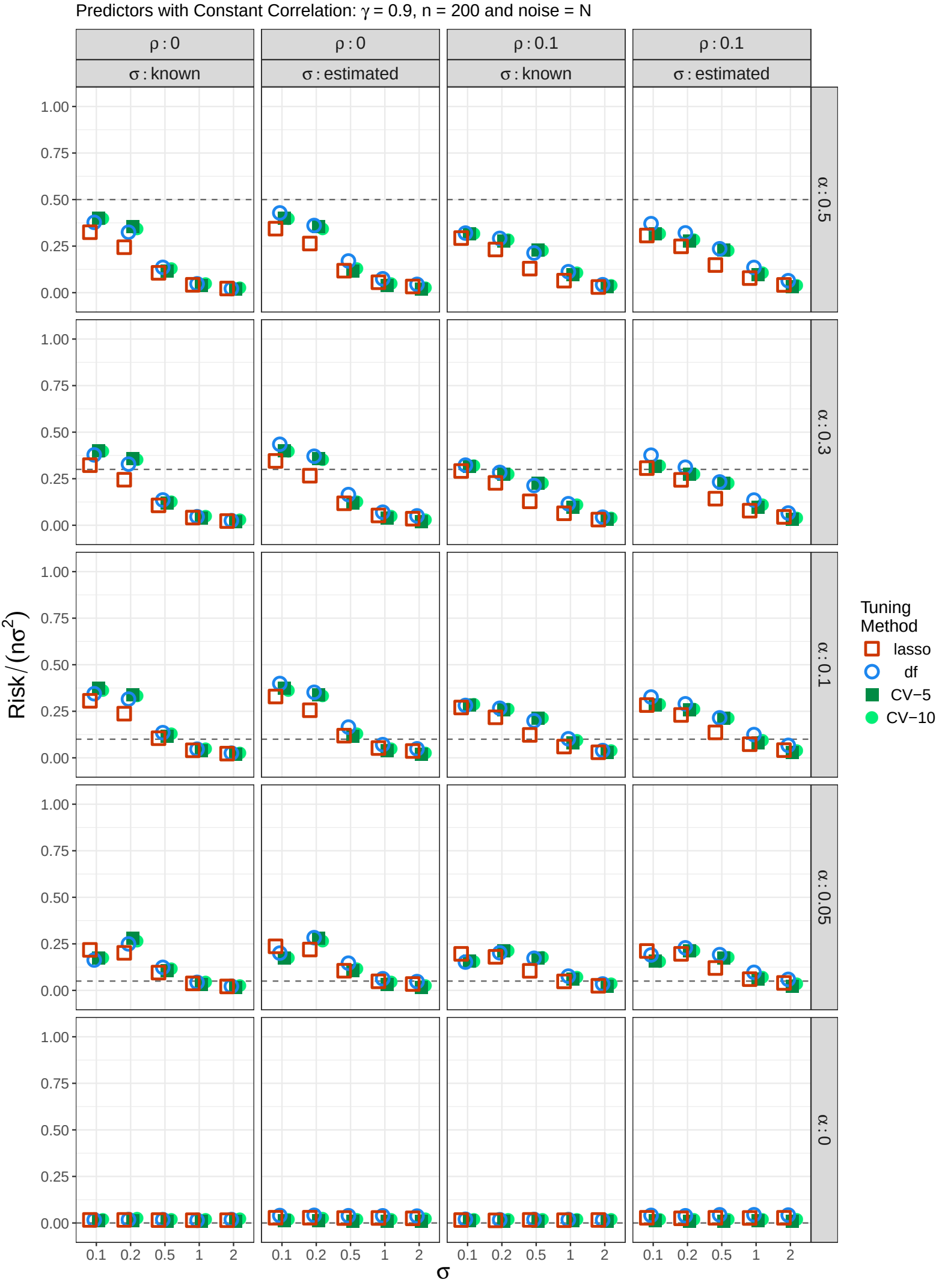
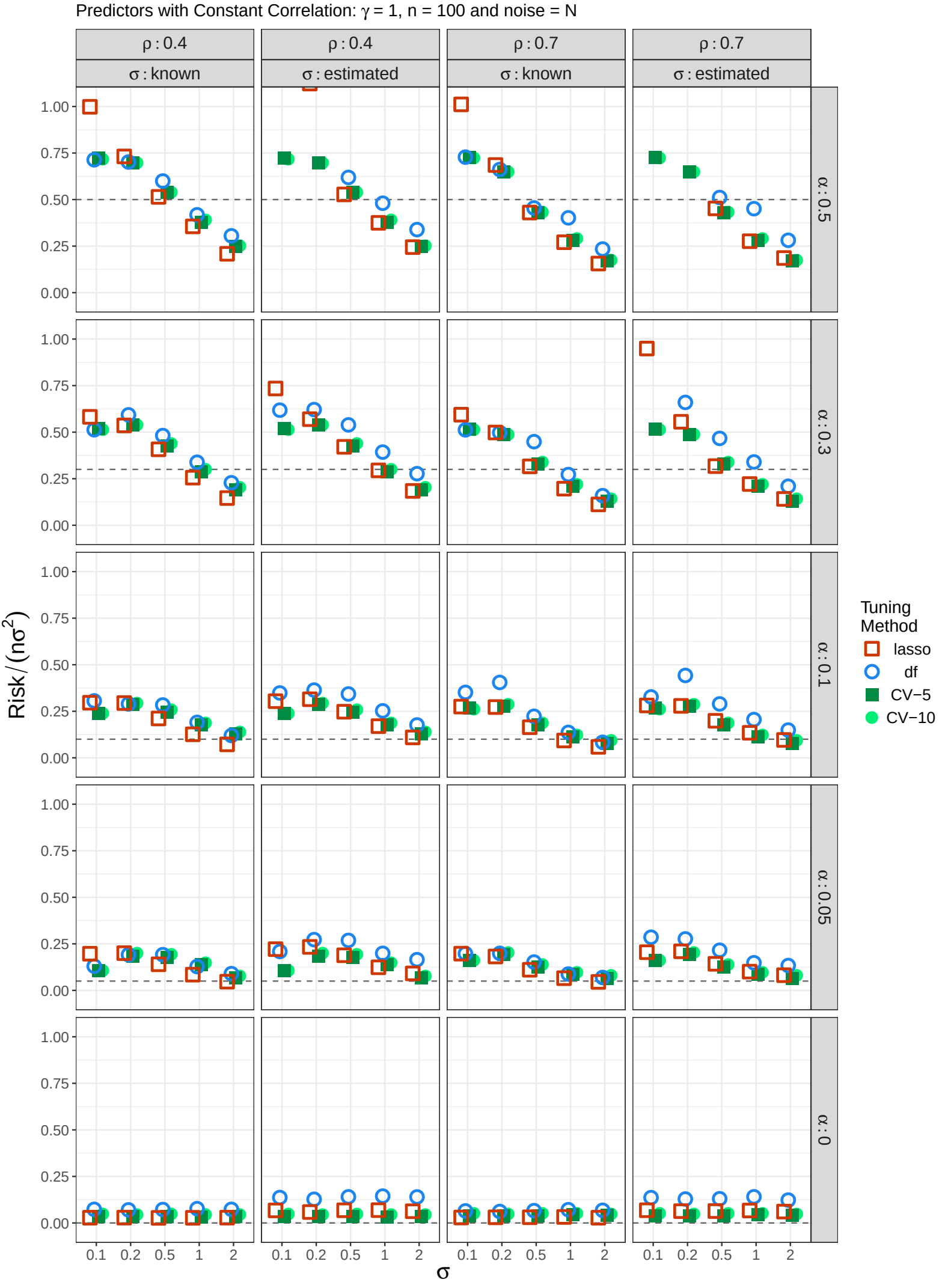
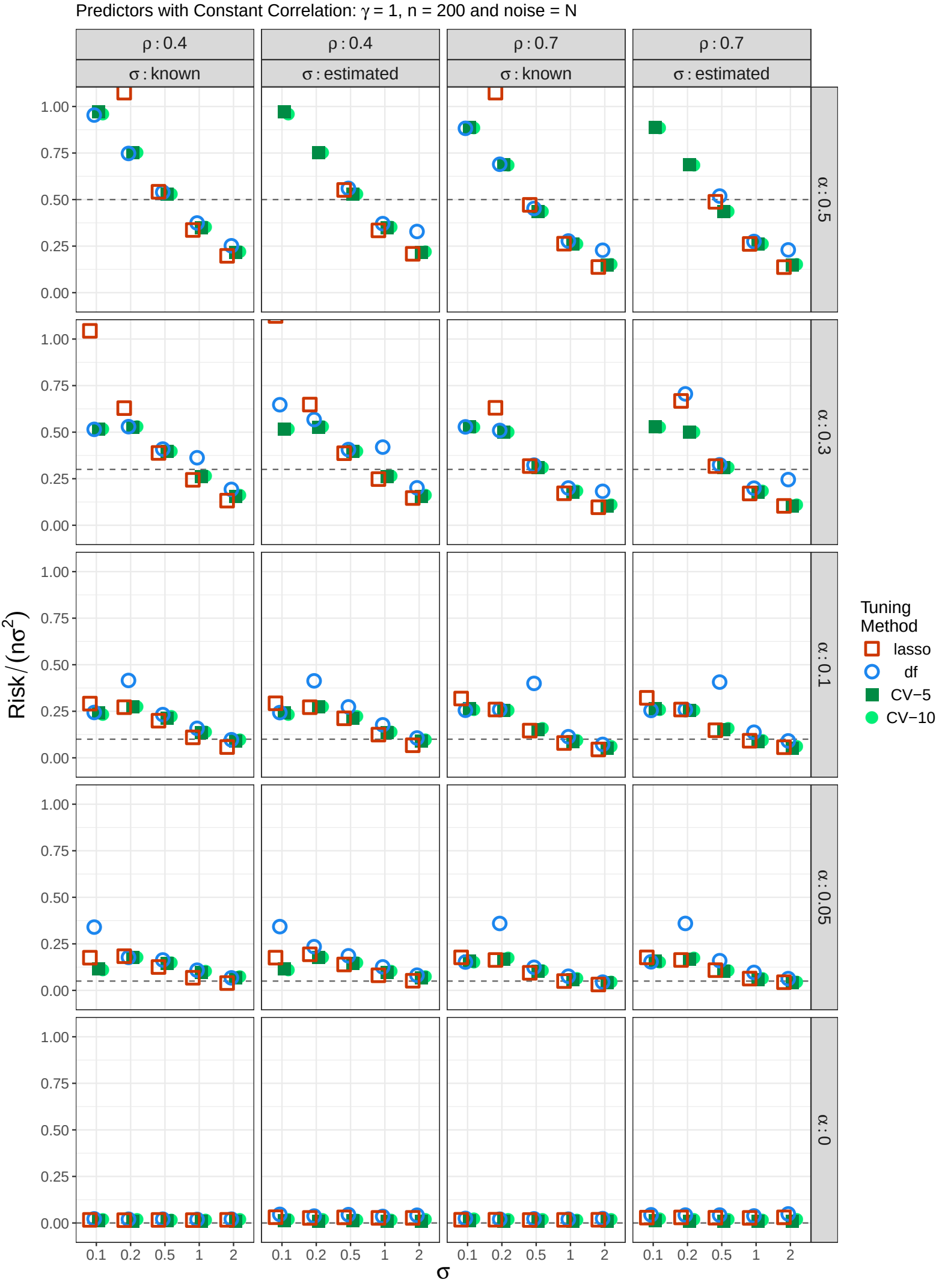
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

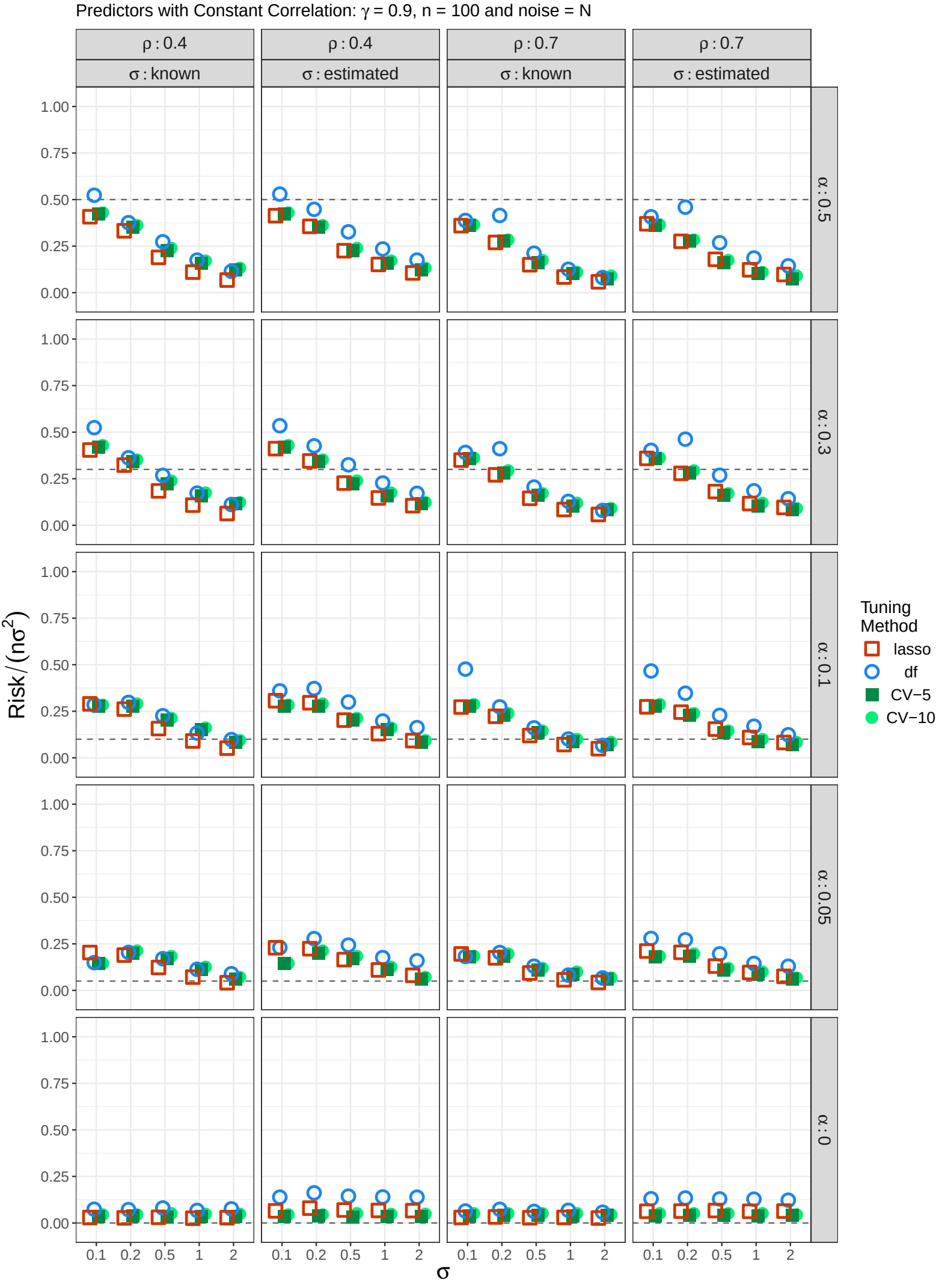

Predictors with Constant Correlation: γ = 1, n = 200 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

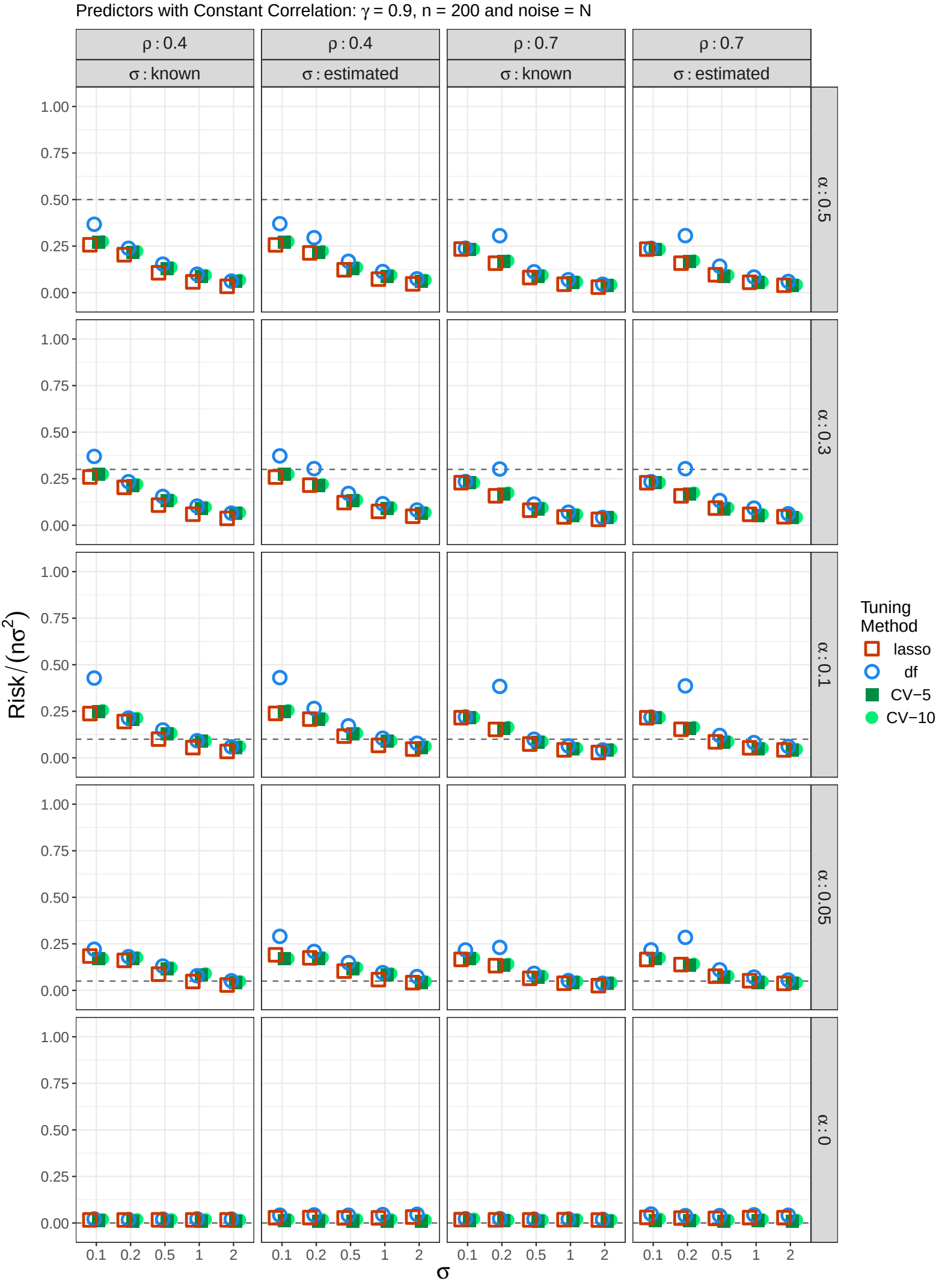

Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

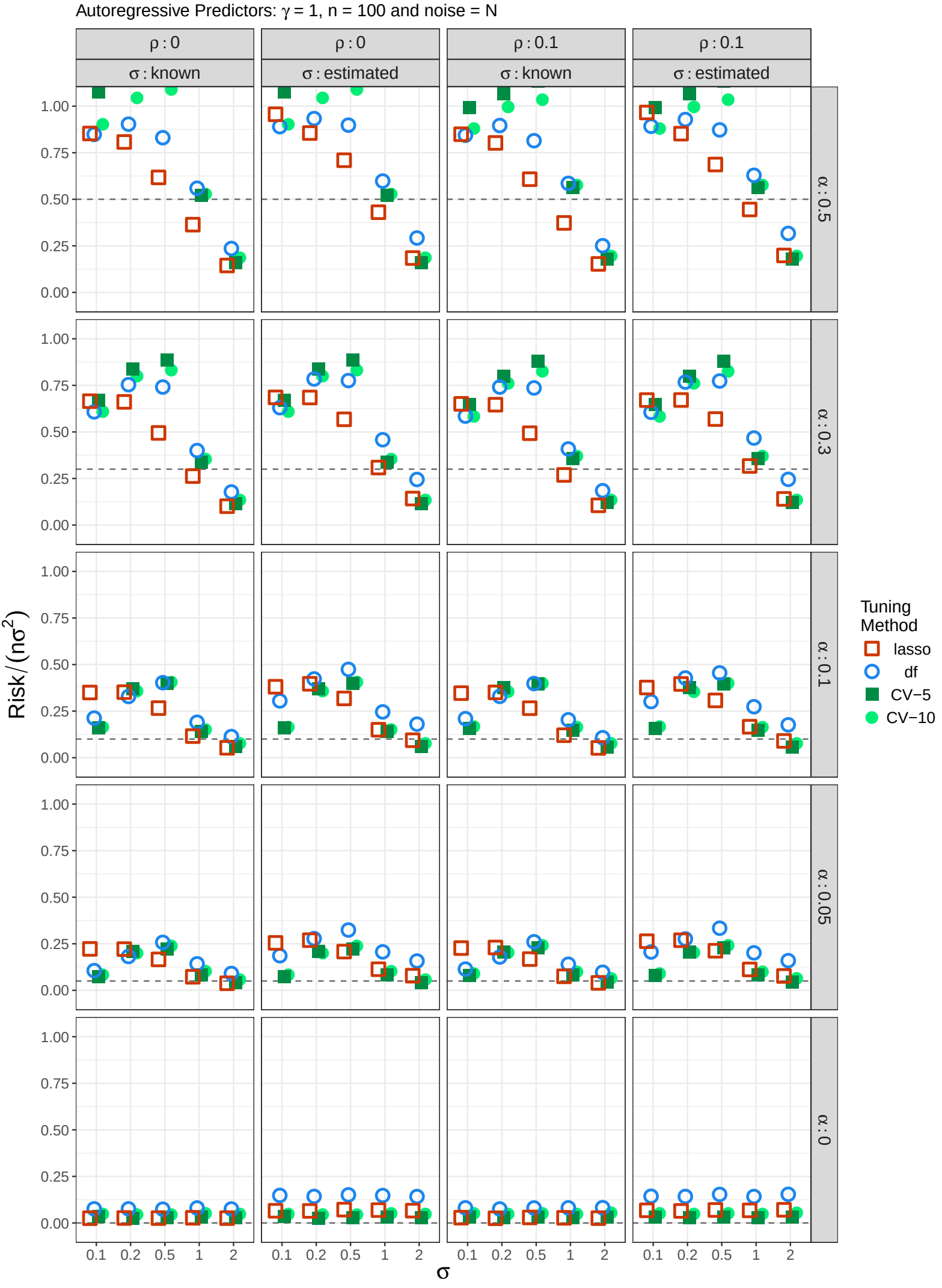

Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

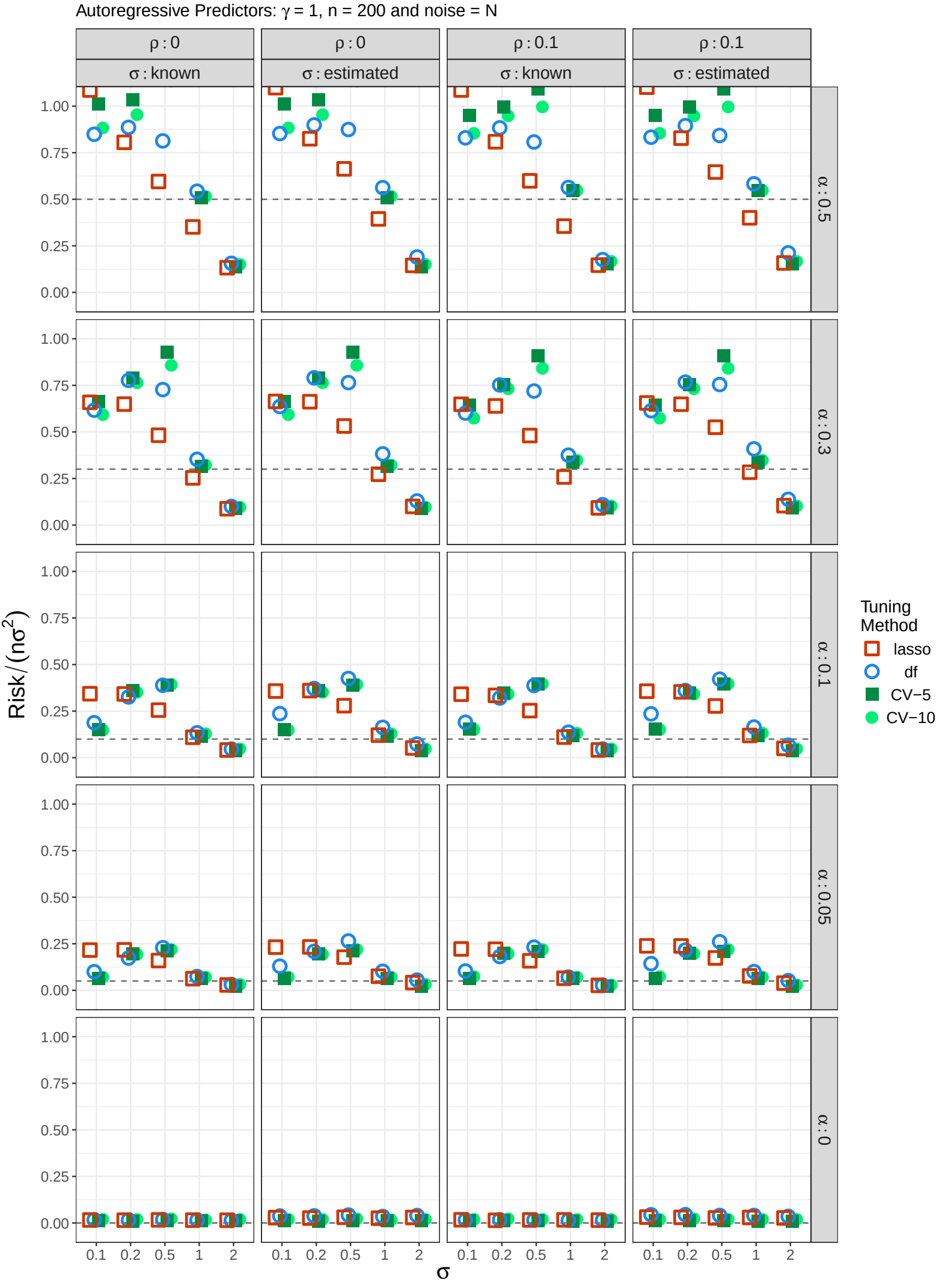

Predictors with Constant Correlation: γ = 1, n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

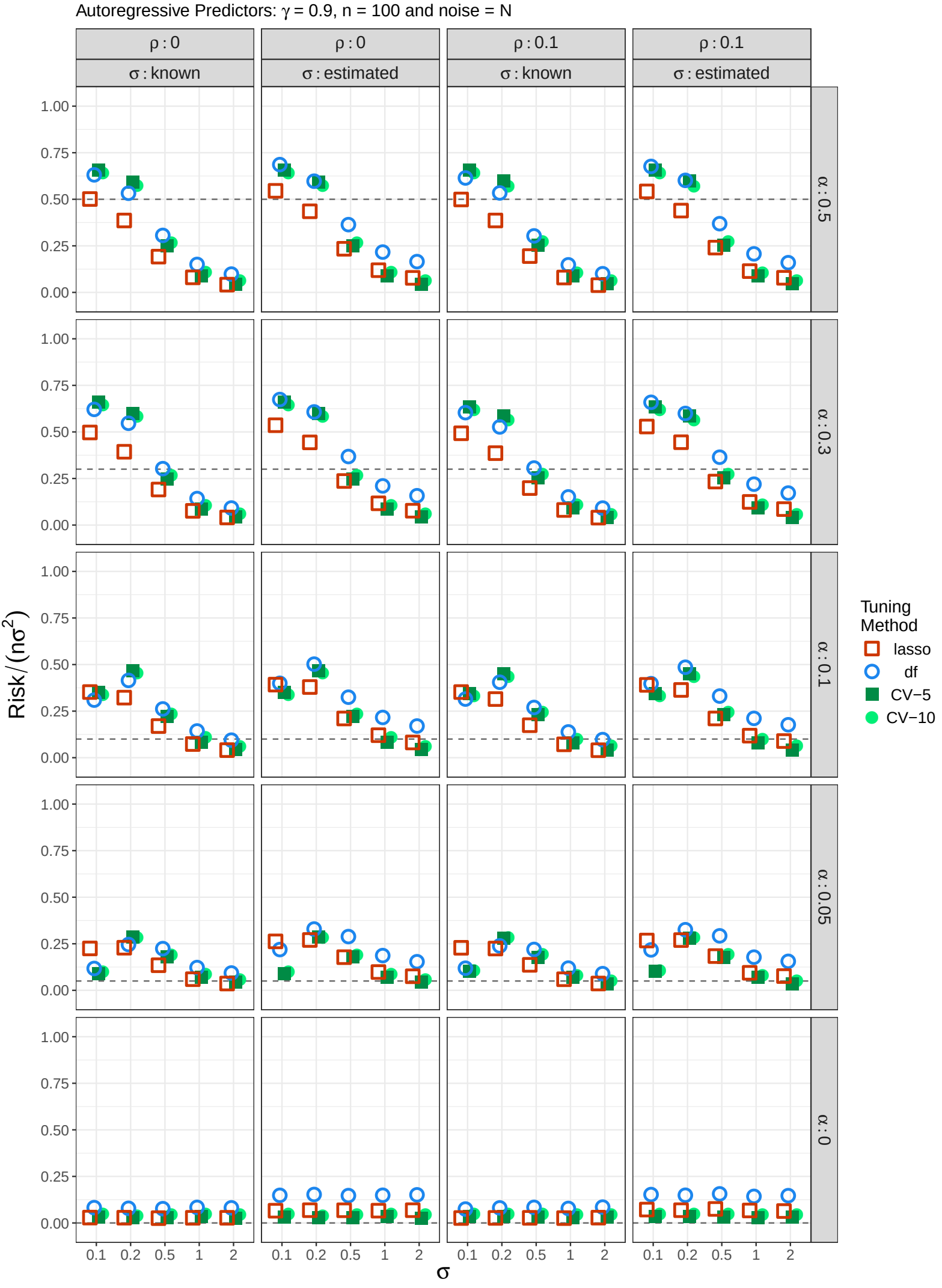

Predictors with Constant Correlation: γ = 1, n = 200 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

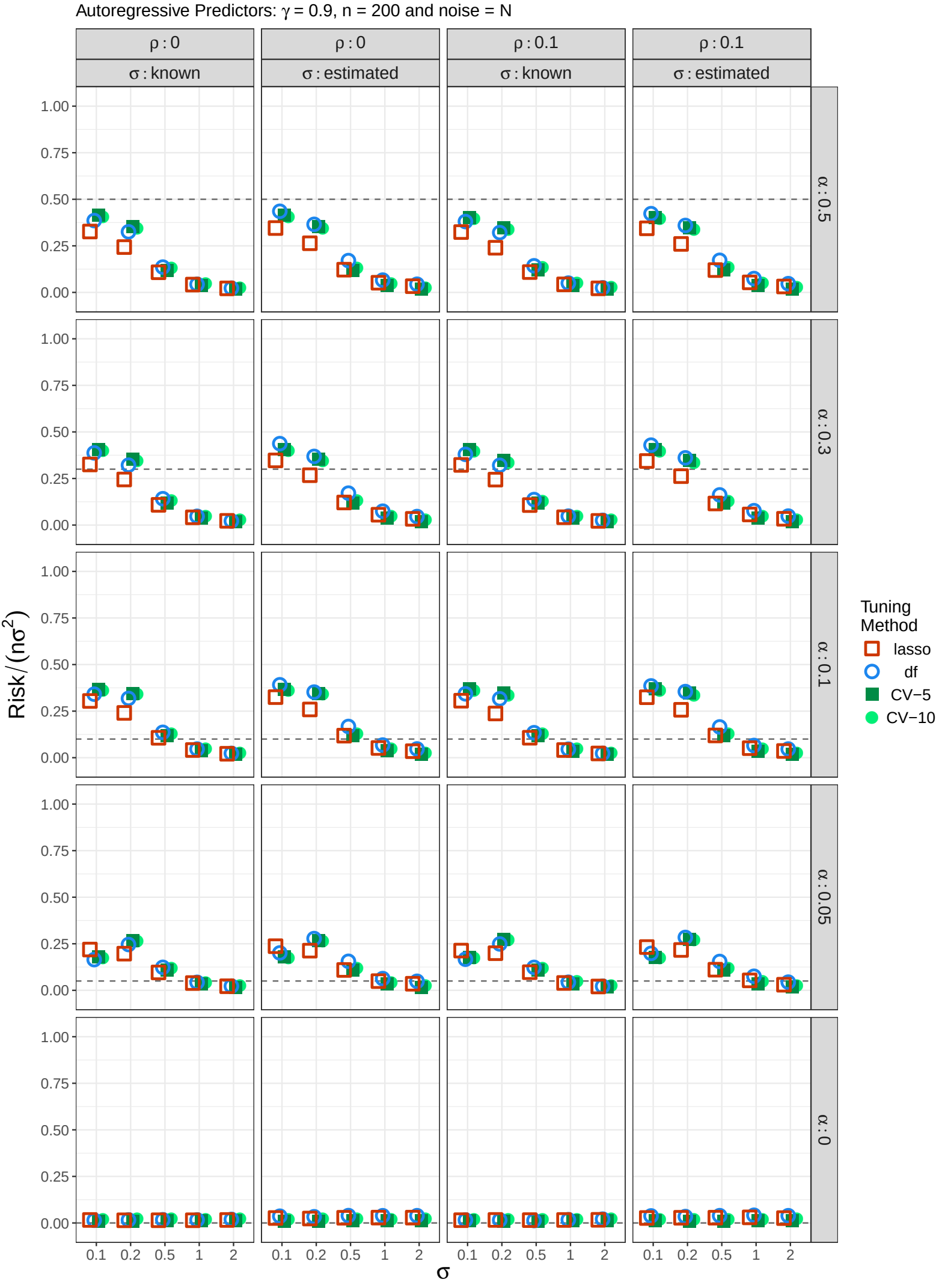

Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = N
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

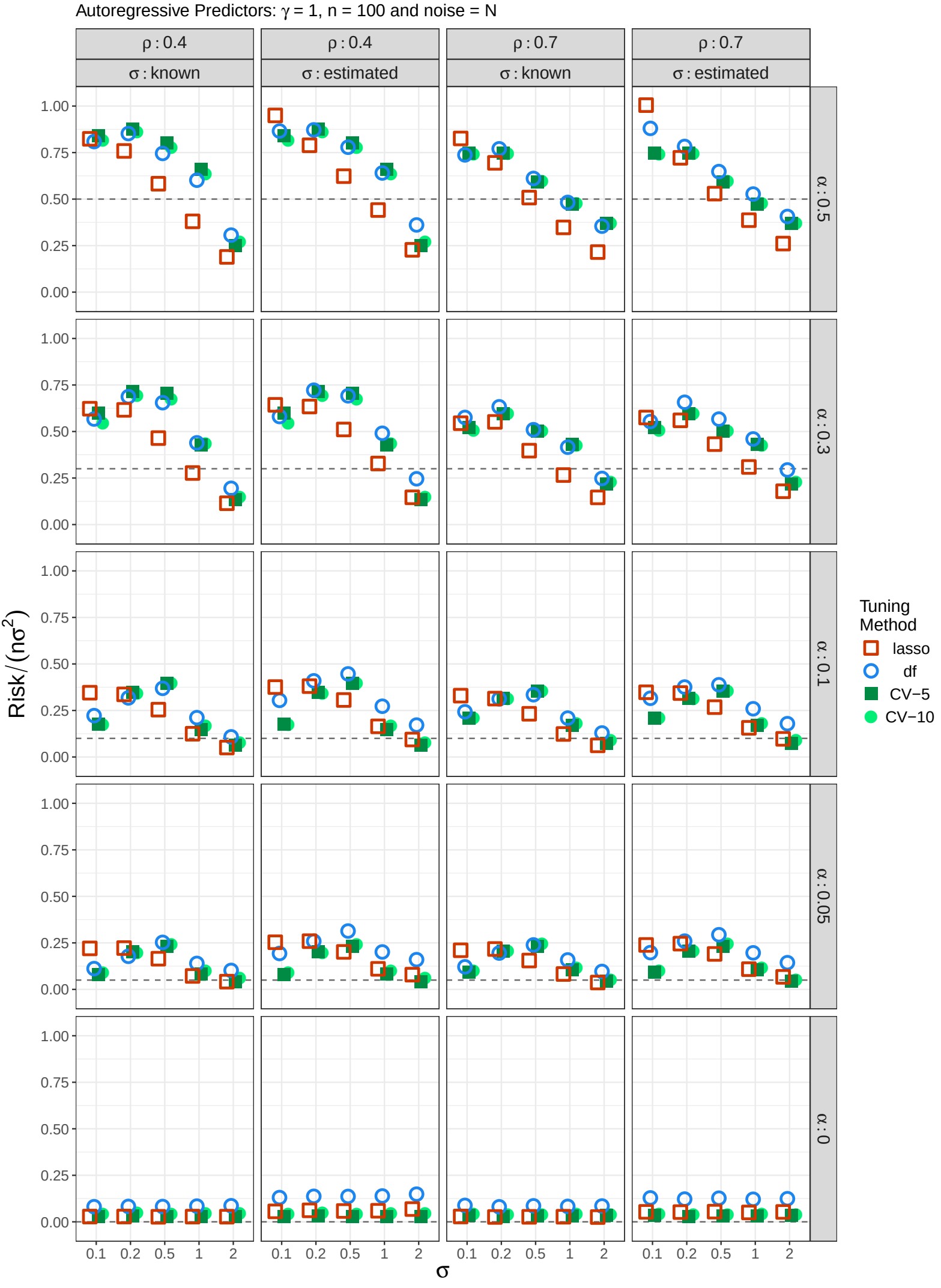

Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = N
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

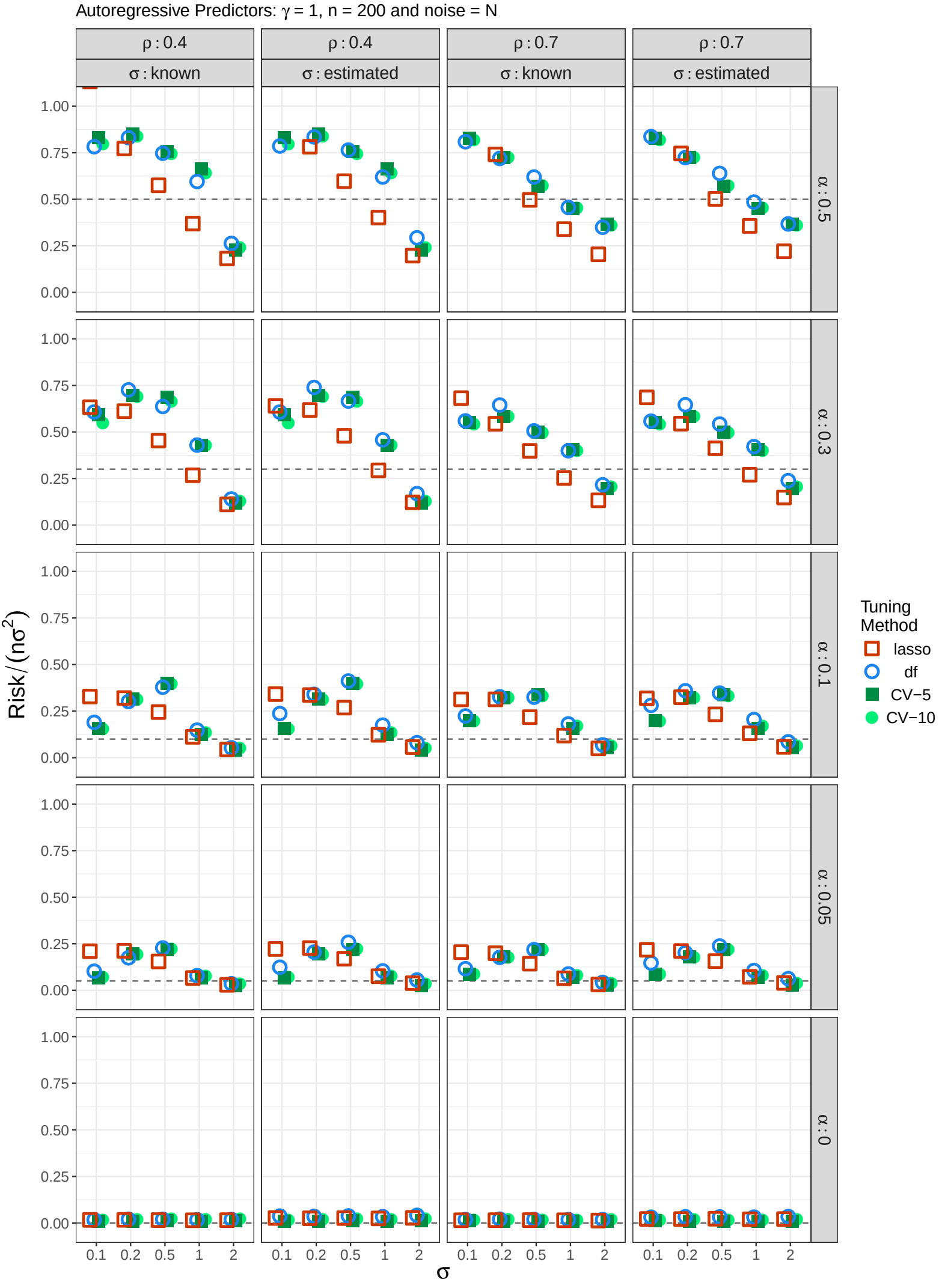

Autoregressive Predictors: γ = 1, n = 100 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

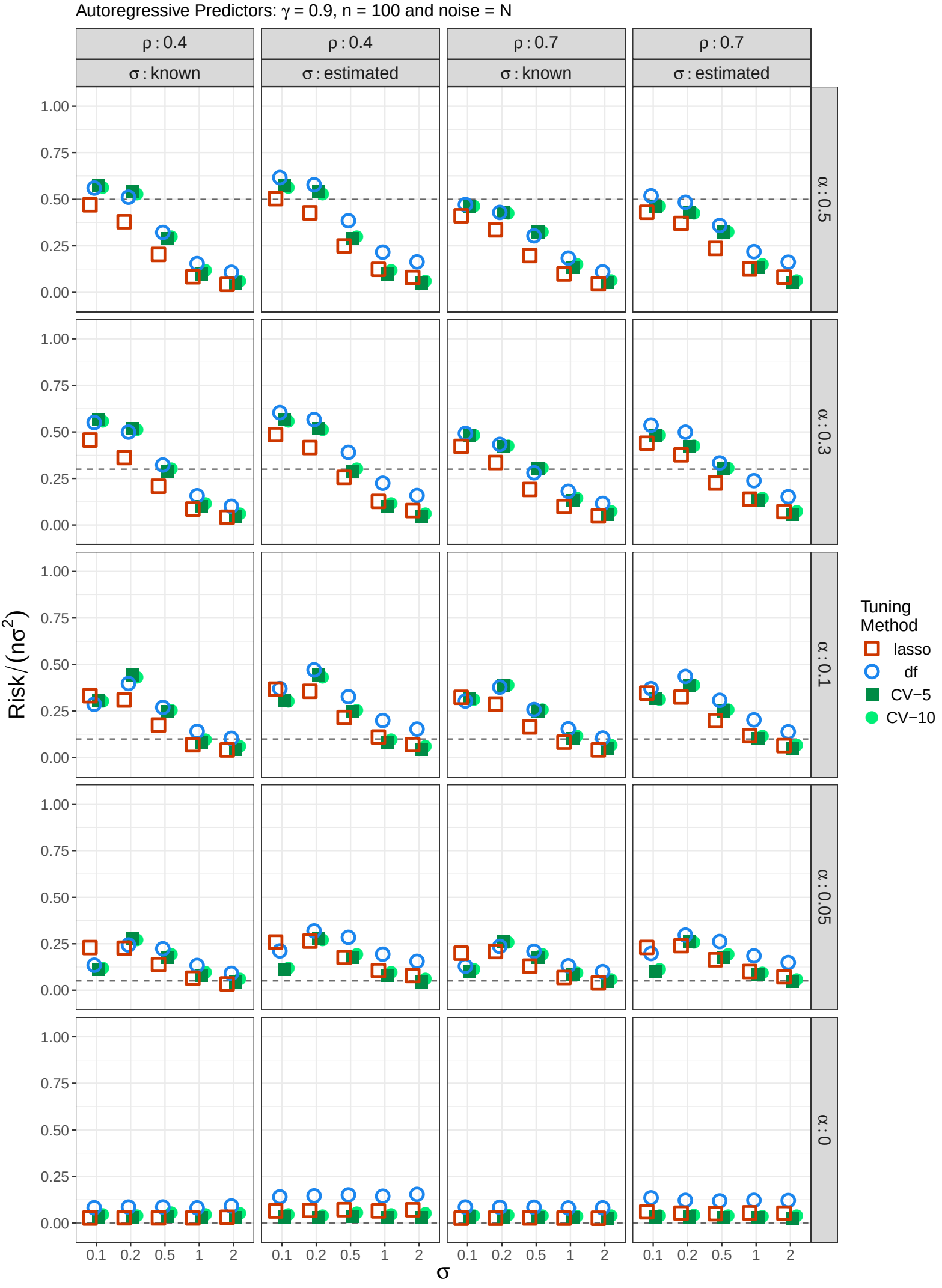

Autoregressive Predictors: γ = 1, n = 200 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

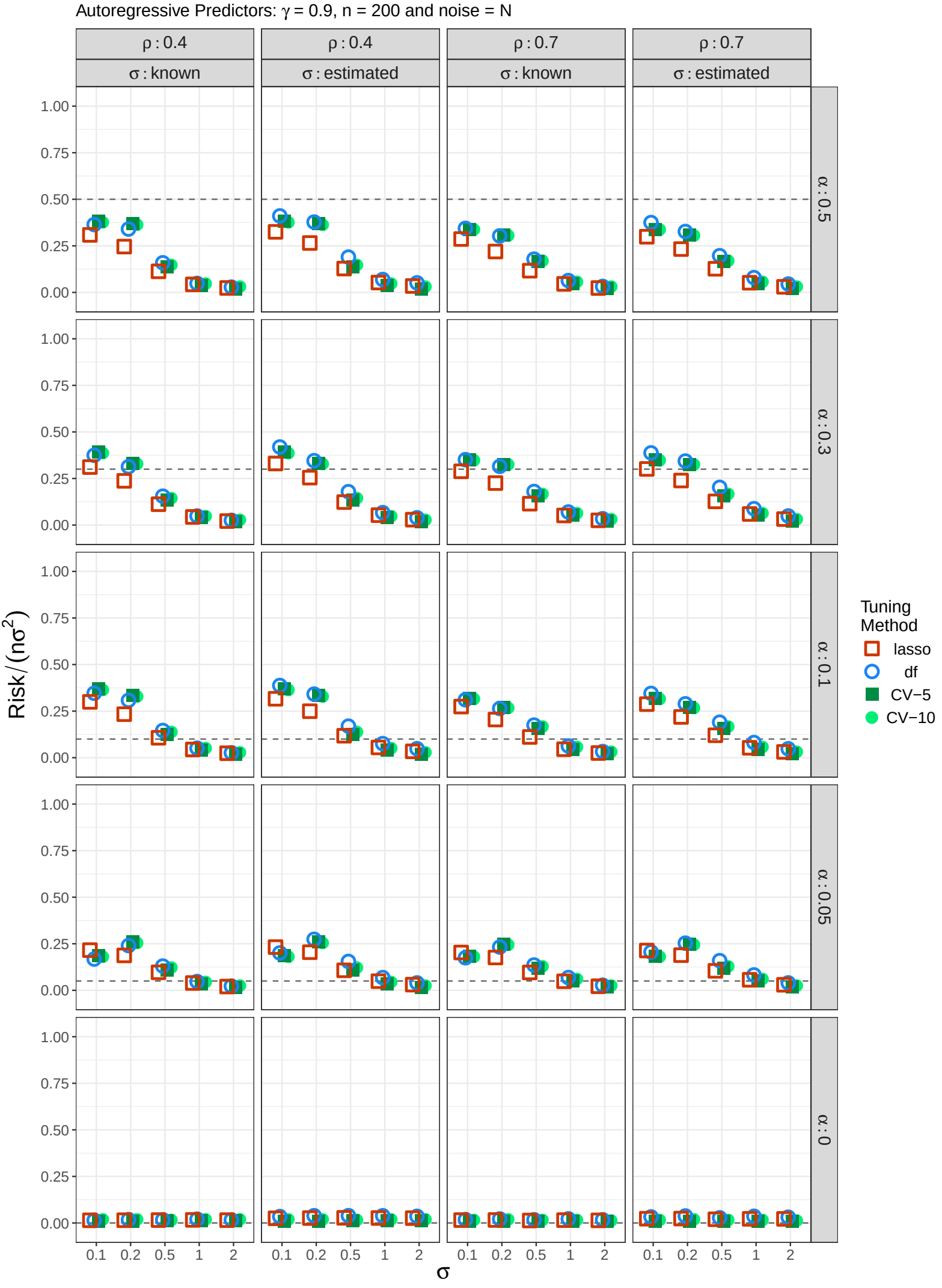

Autoregressive Predictors: γ = 0.9, n = 100 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

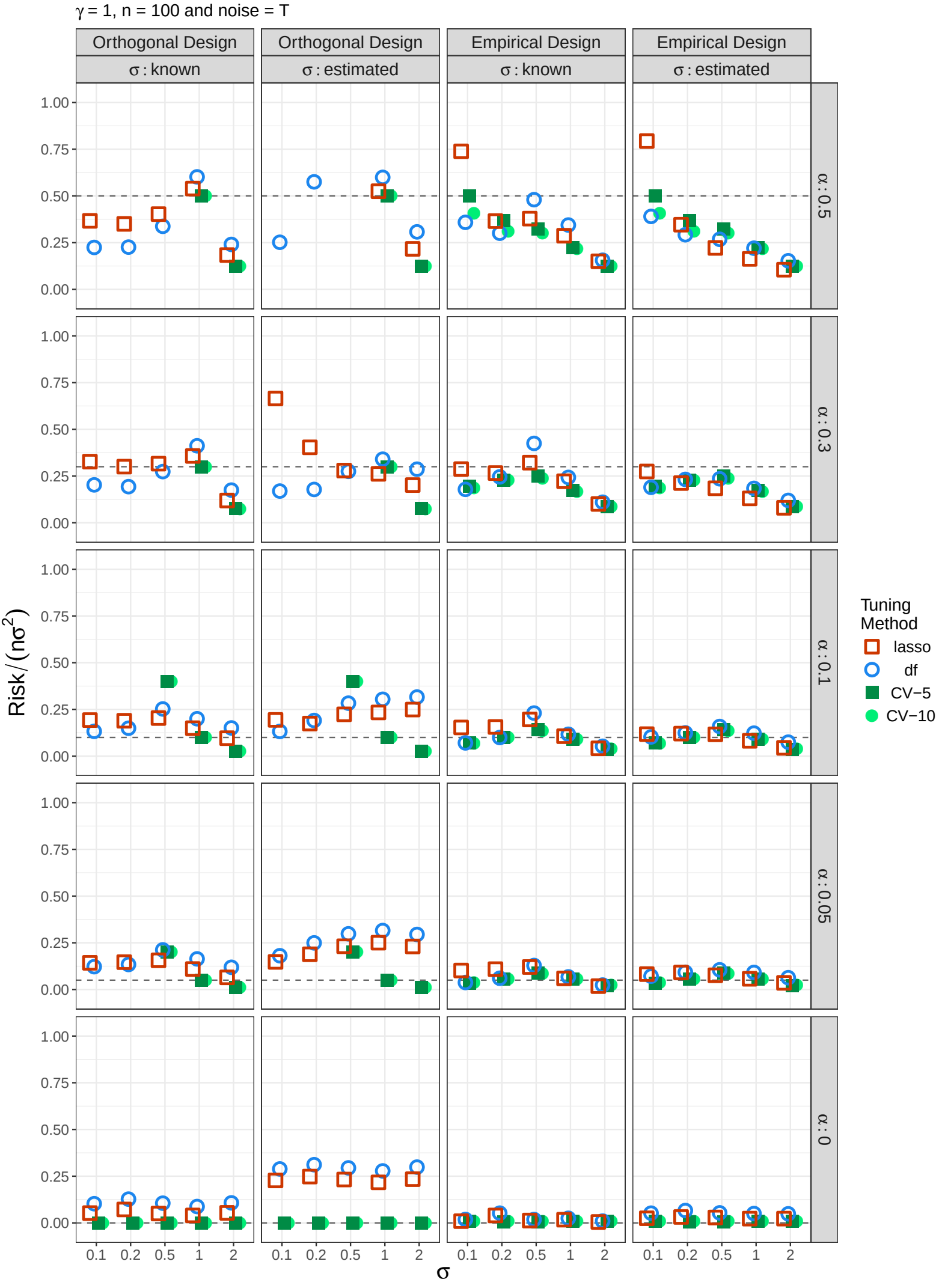

Autoregressive Predictors: γ = 0.9, n = 200 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

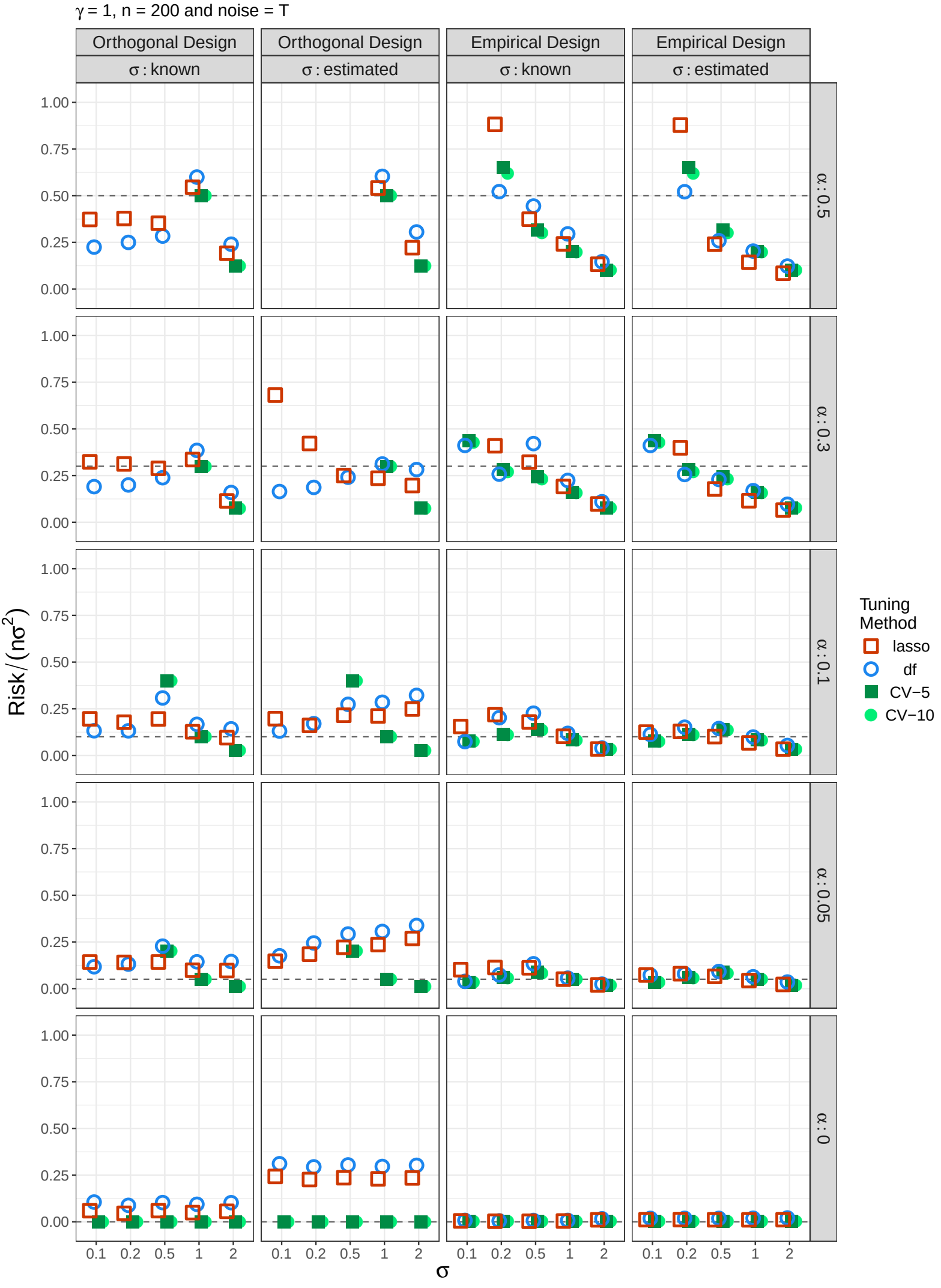

Autoregressive Predictors: γ = 1, n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

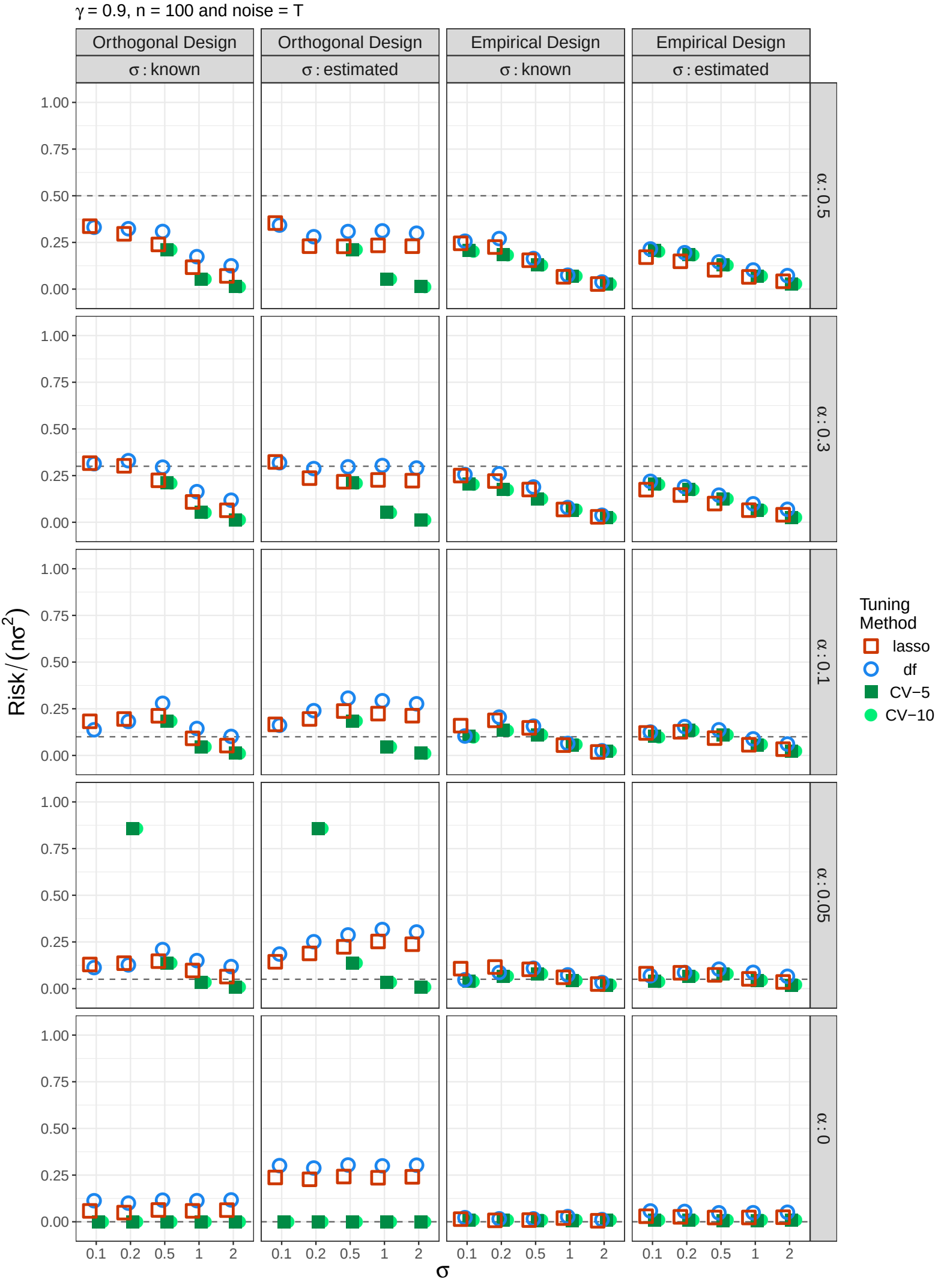

Autoregressive Predictors: γ = 1, n = 200 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

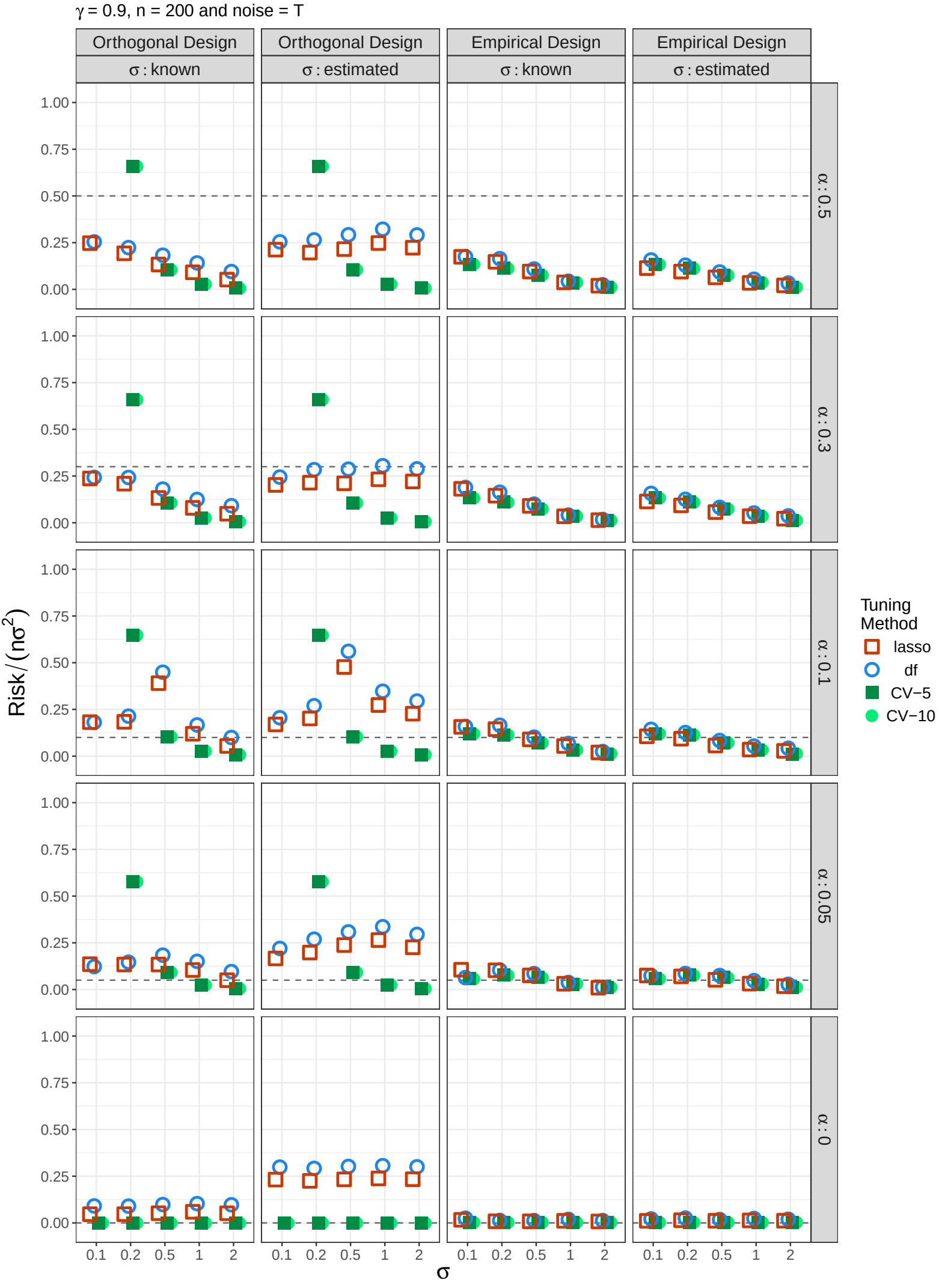

Autoregressive Predictors: γ = 0.9, n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = N
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
1.00
0.75
0.50
0.25
0.00
Risk/(nσ²)
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


γ = 1, n = 100 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


γ = 1, n = 200 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


γ = 0.9, n = 100 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


γ = 0.9, n = 200 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
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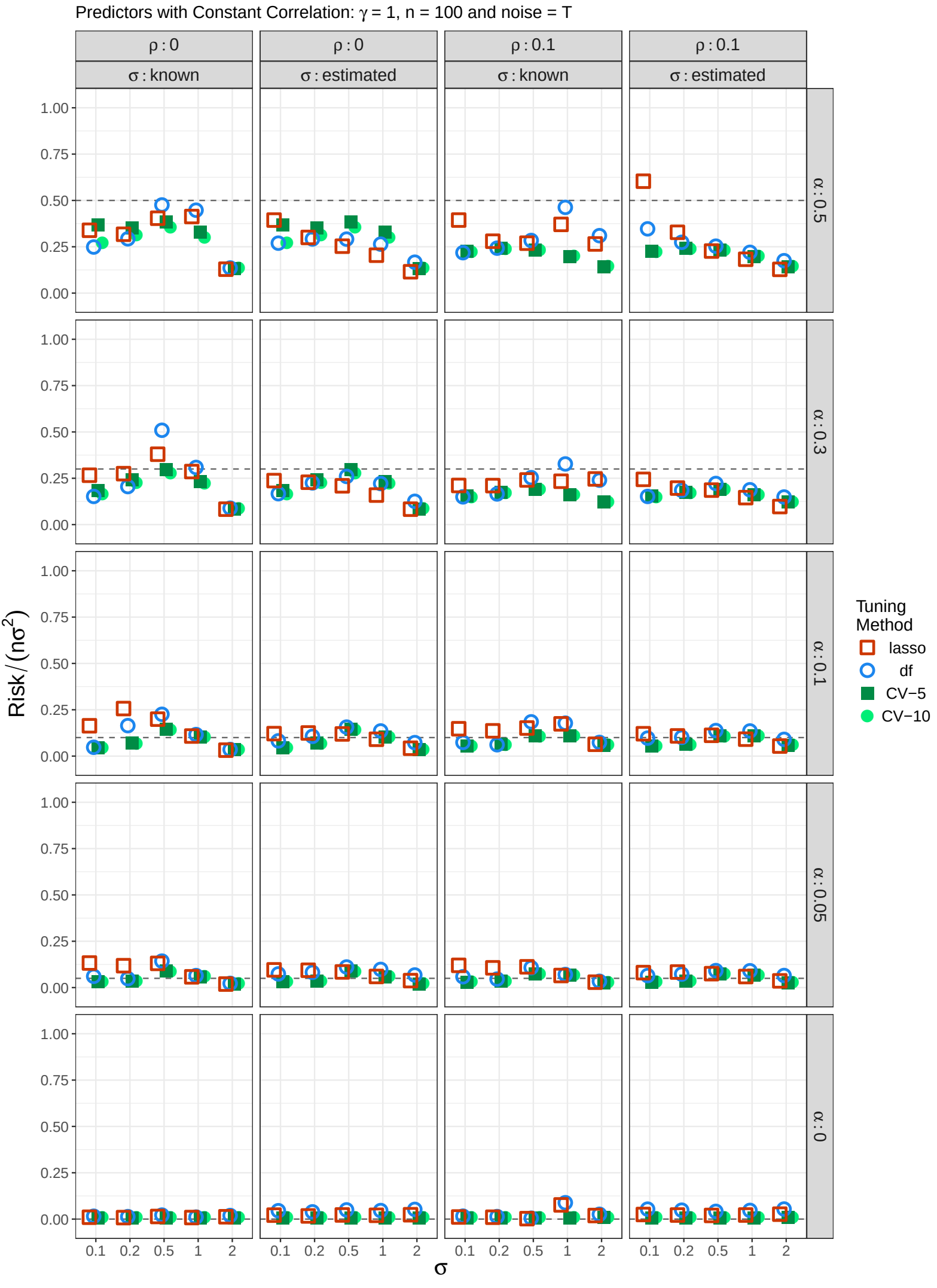
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = T
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
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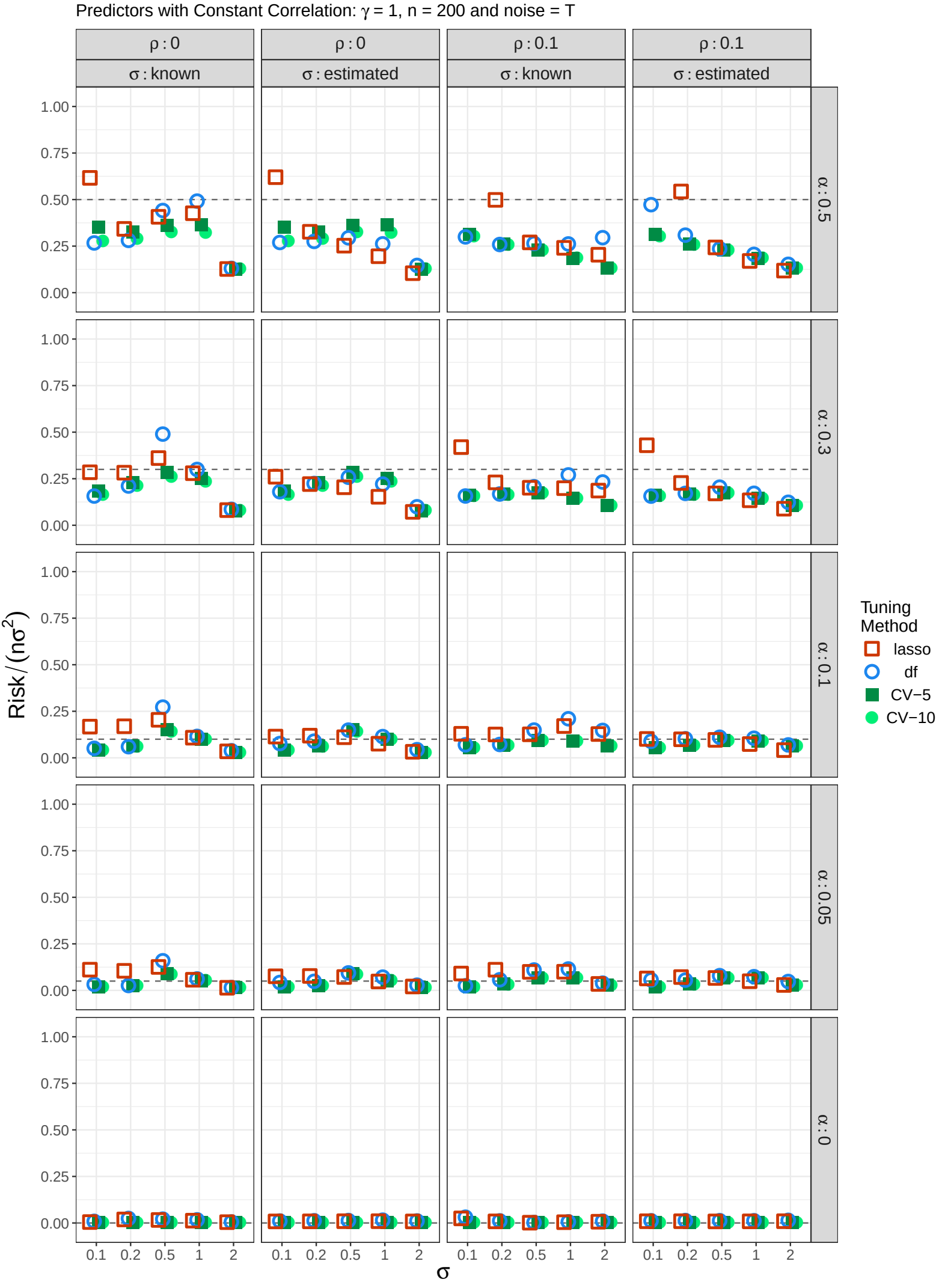
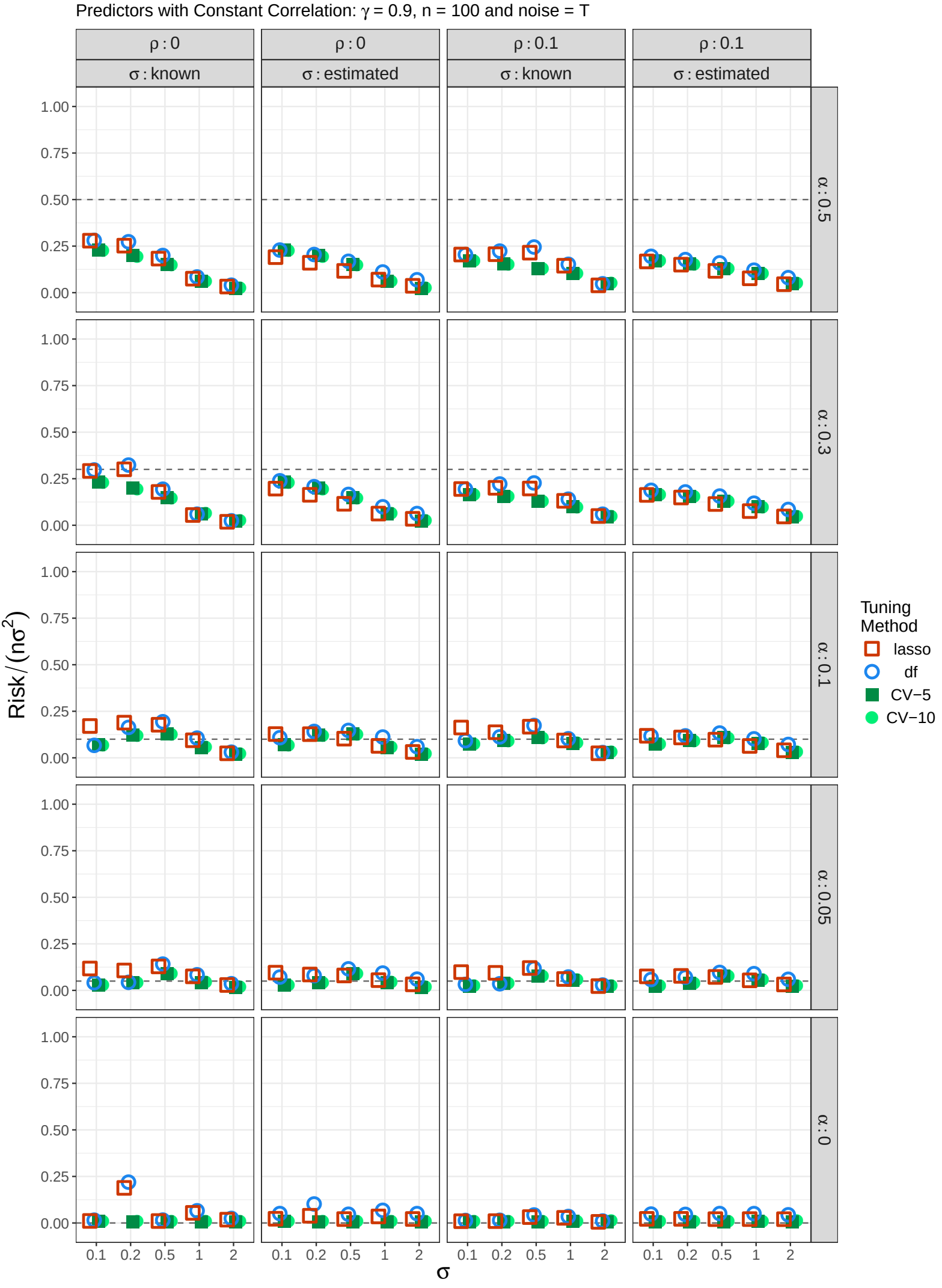
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = T
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = T
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
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0.2
0.5
1
2
σ
Tuning Method
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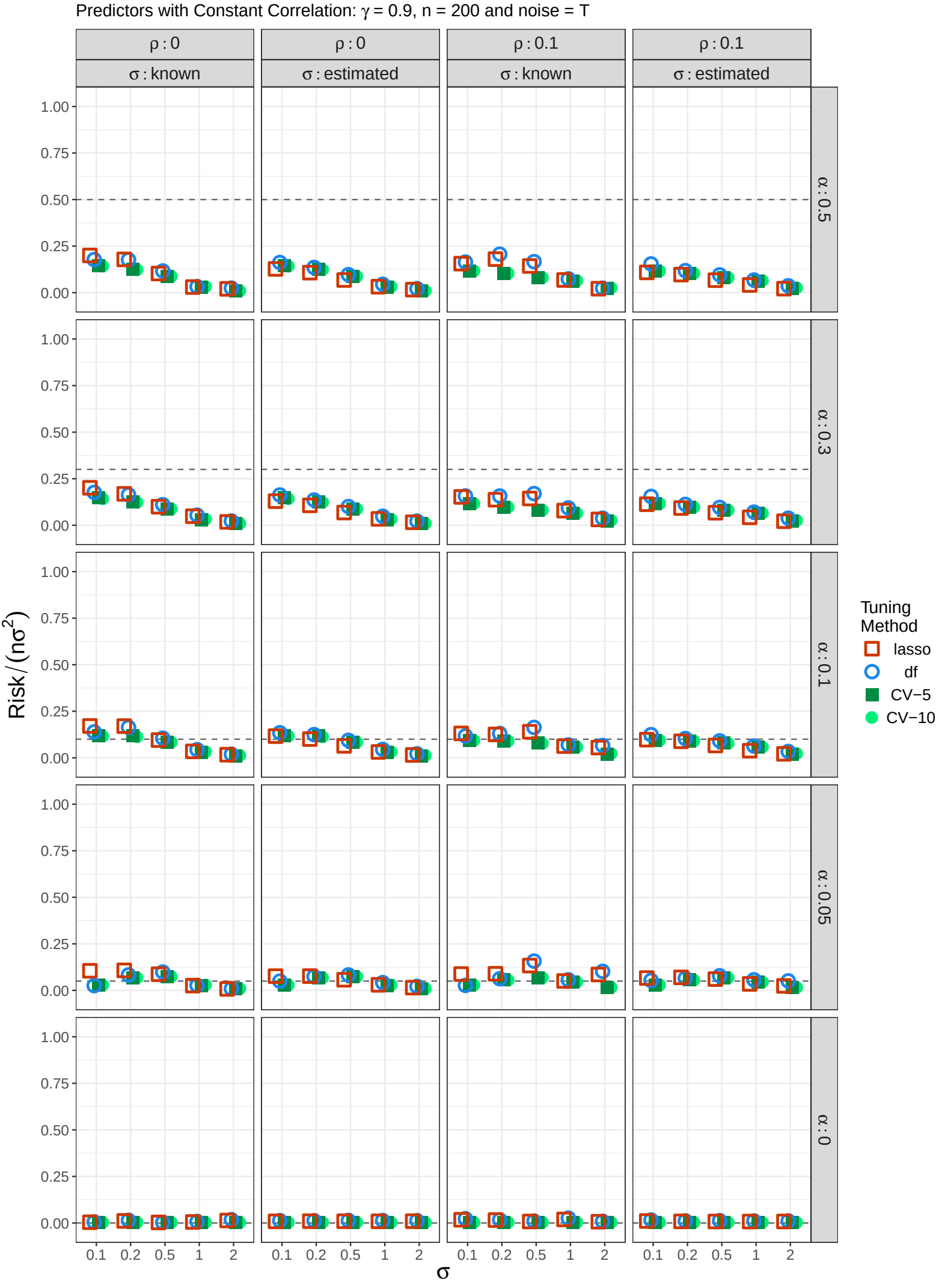
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = T
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
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α : 0
Risk/(nσ²)
1.00
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Tuning Method
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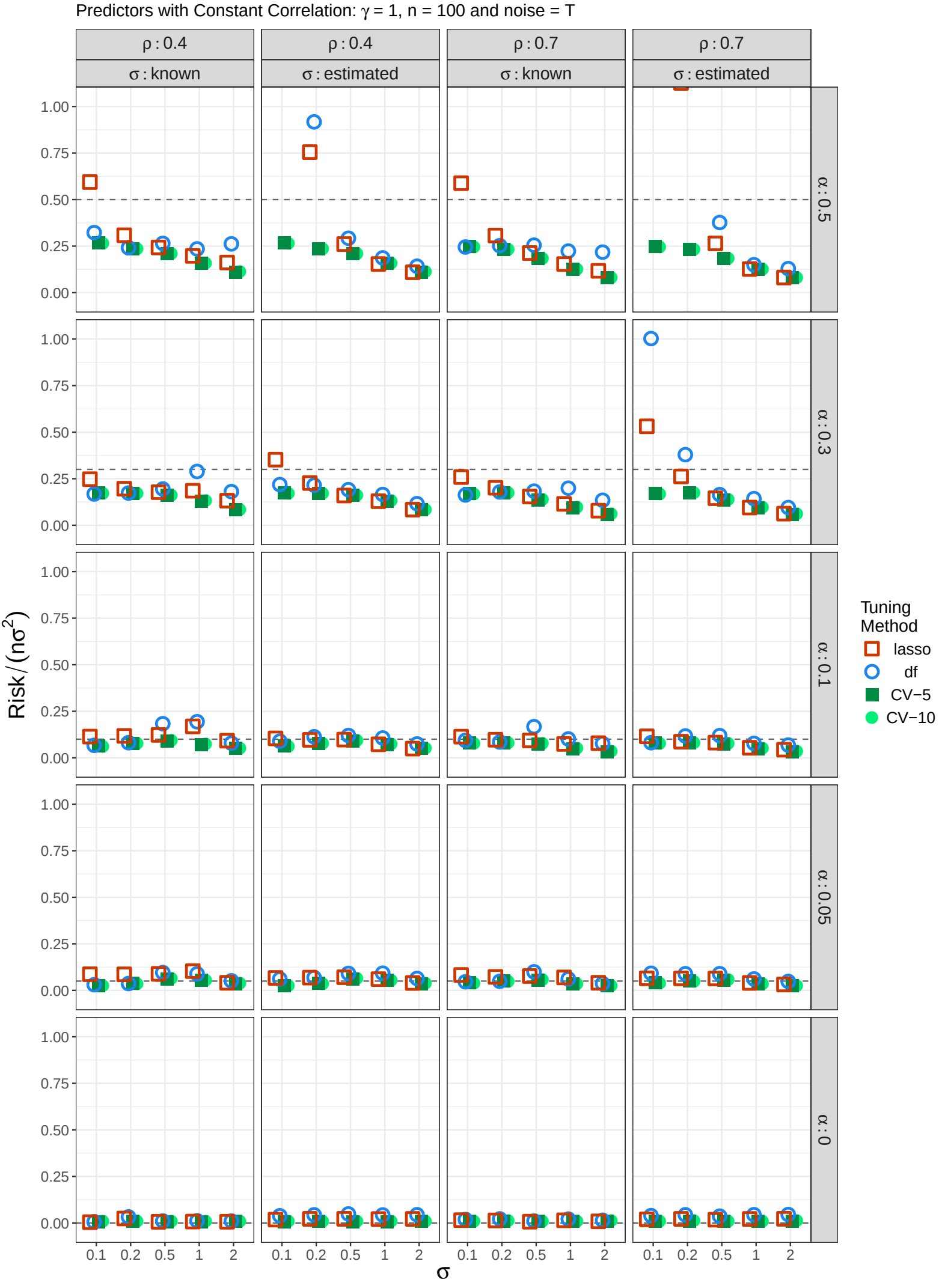
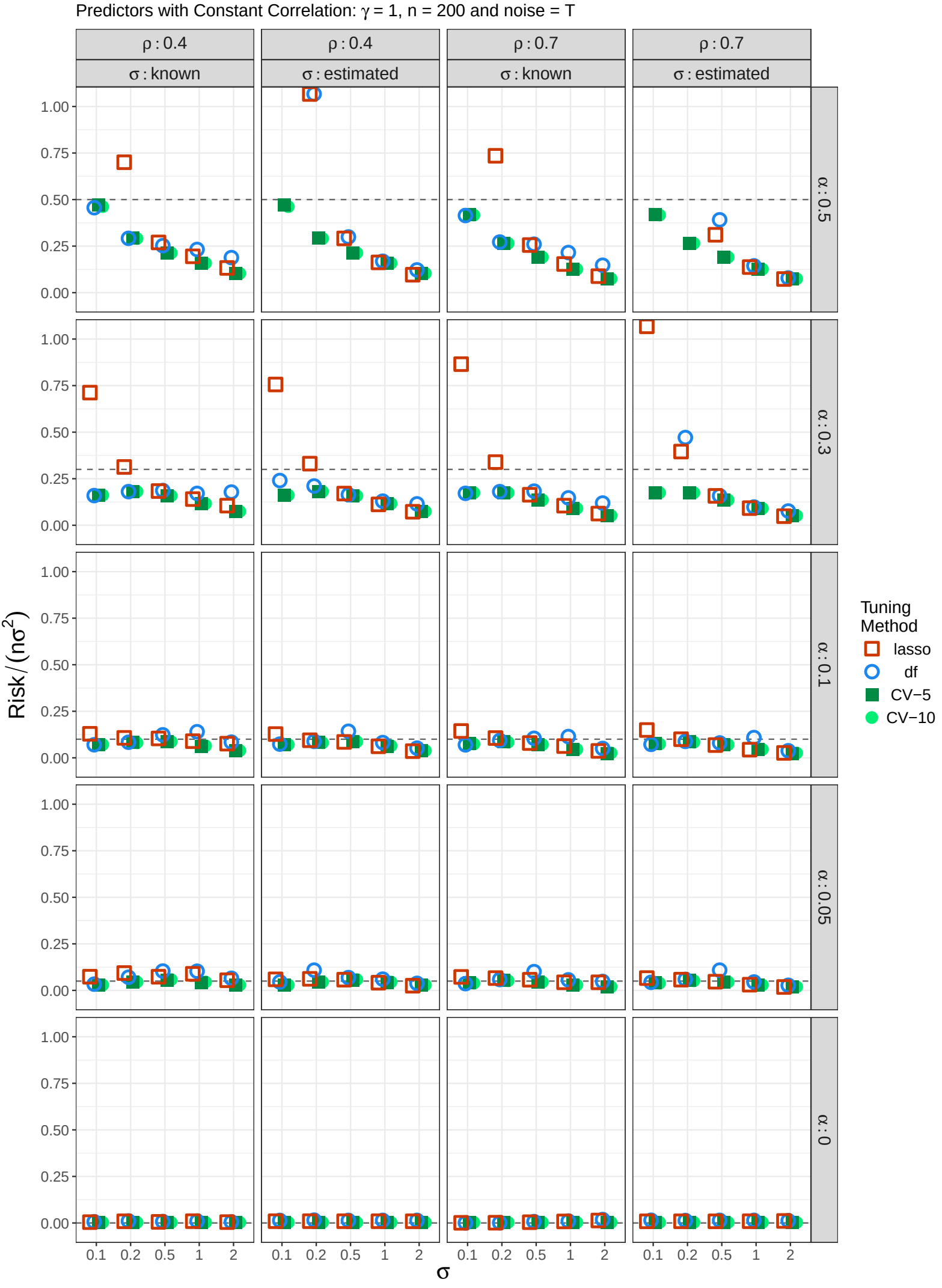
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = T
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = T
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
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1
2
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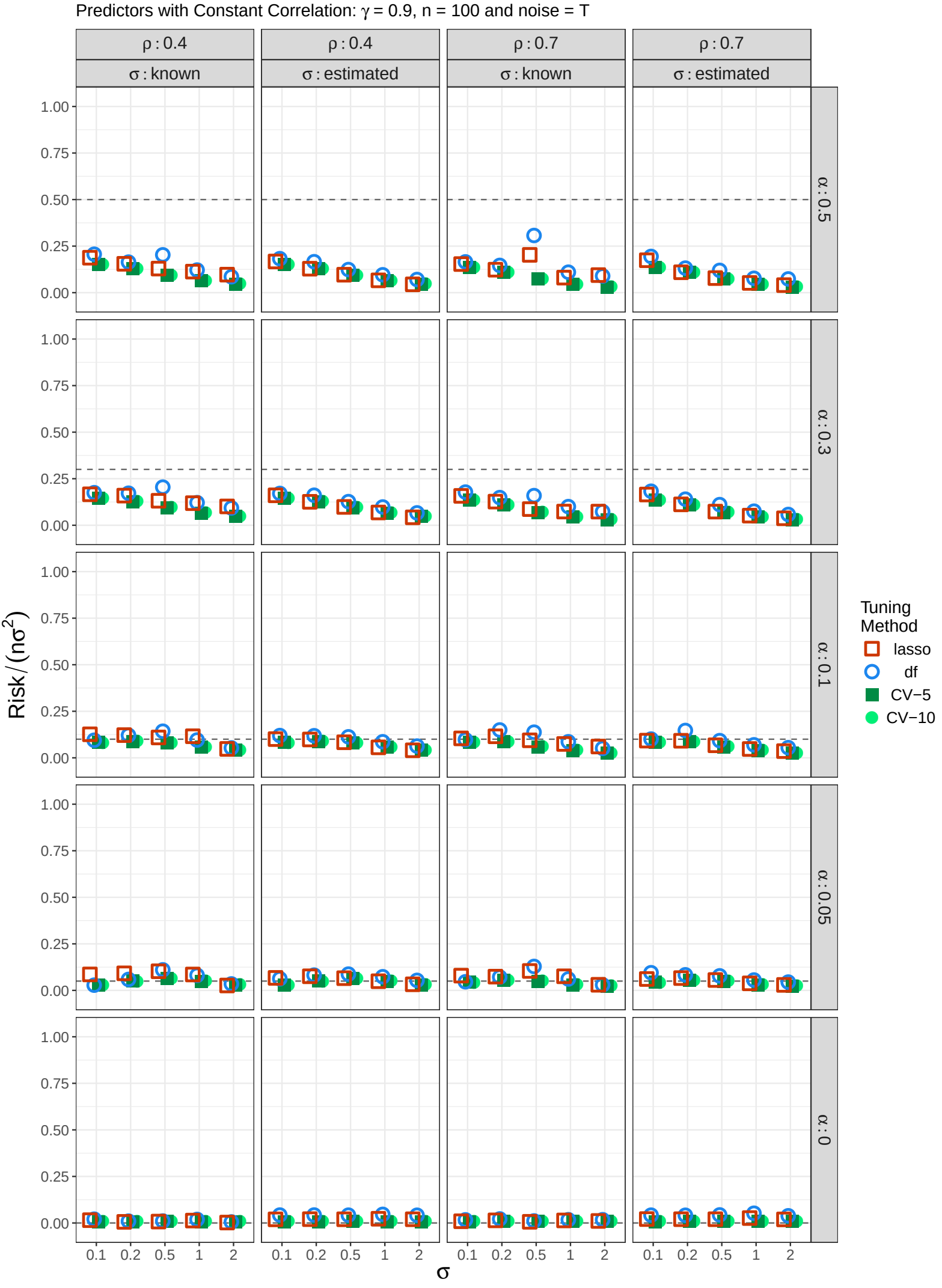
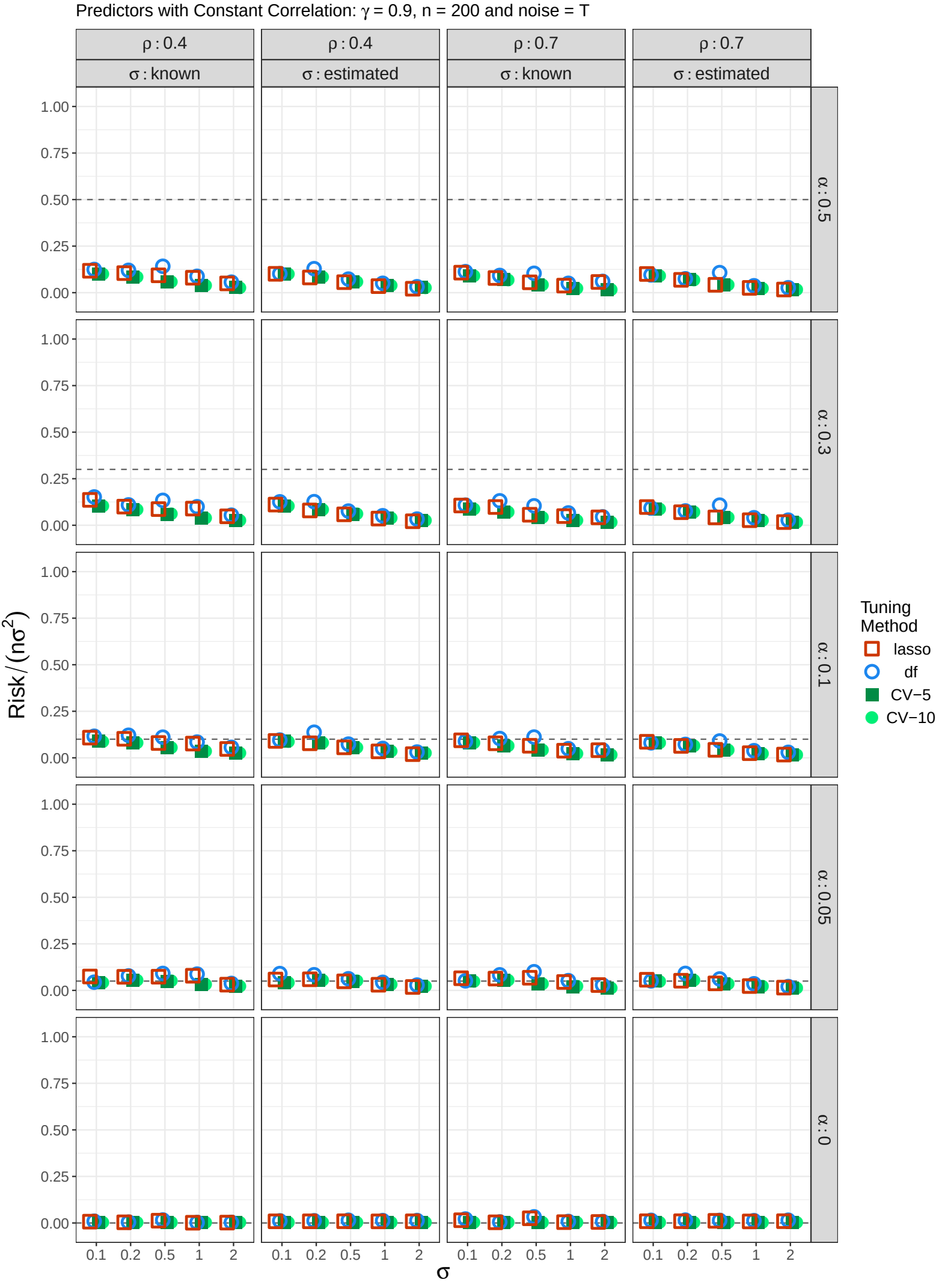
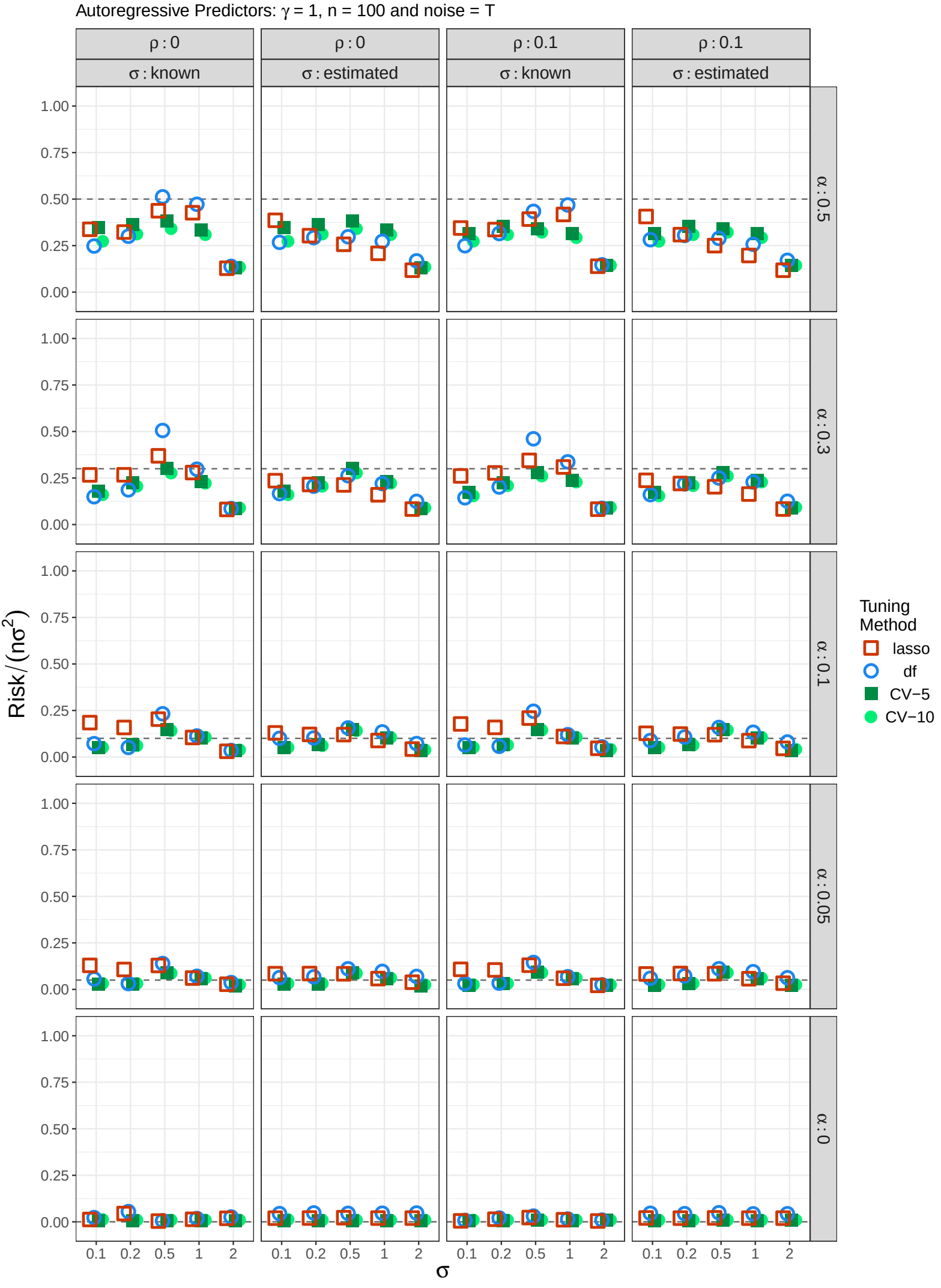
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = T
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = T
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
1.00
0.75
0.50
0.25
0.00
Risk/(nσ²)
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 100 and noise = T
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
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0.1
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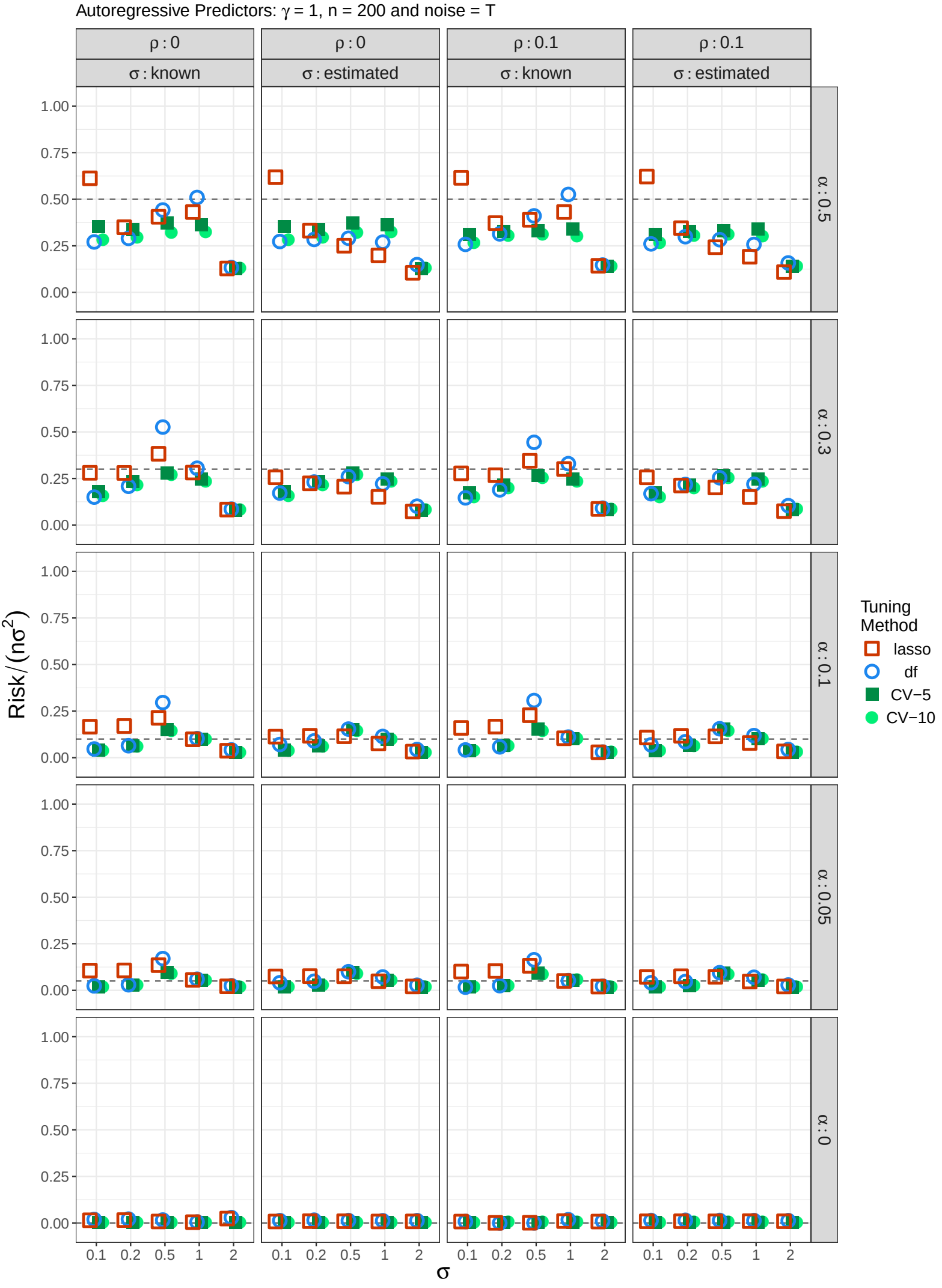
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = T
ρ : 0
σ : known
ρ : 0
σ : estimated
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σ : known
ρ : 0.1
σ : estimated
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α : 0.05
α : 0
Risk/(nσ²)
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0.25
0.00
0.1
0.2
0.5
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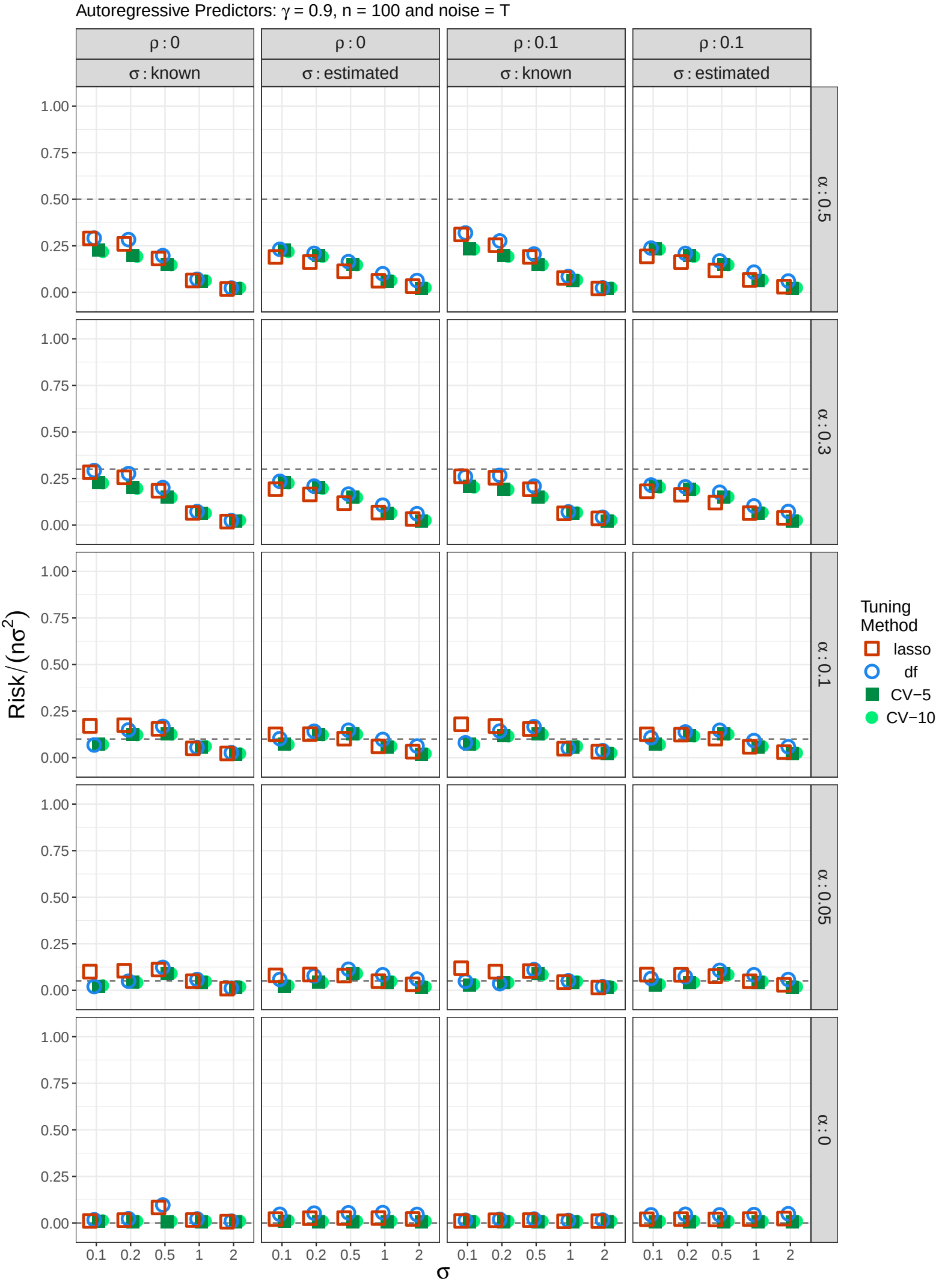
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 100 and noise = T
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σ : known
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σ : estimated
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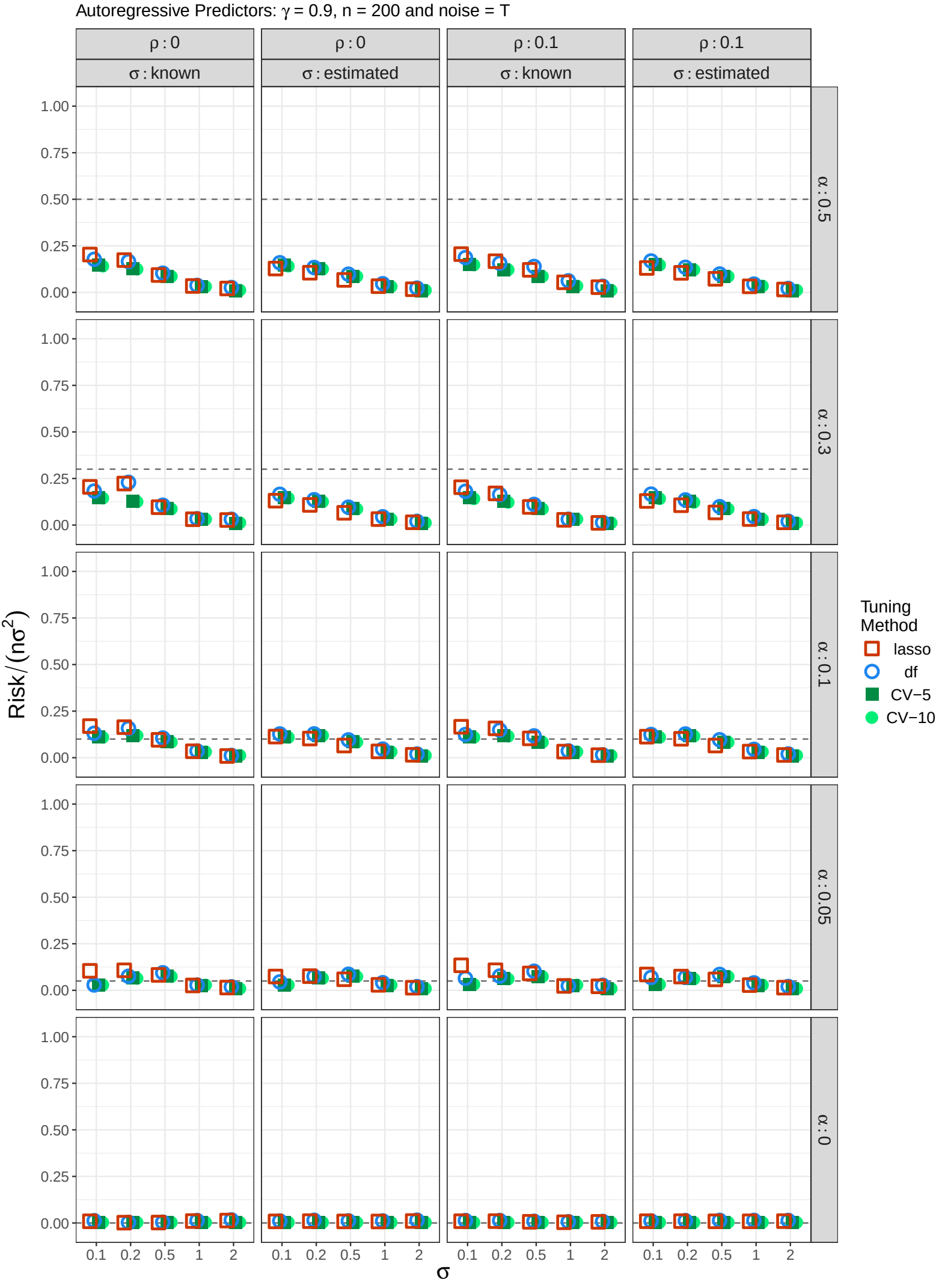
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = T
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ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
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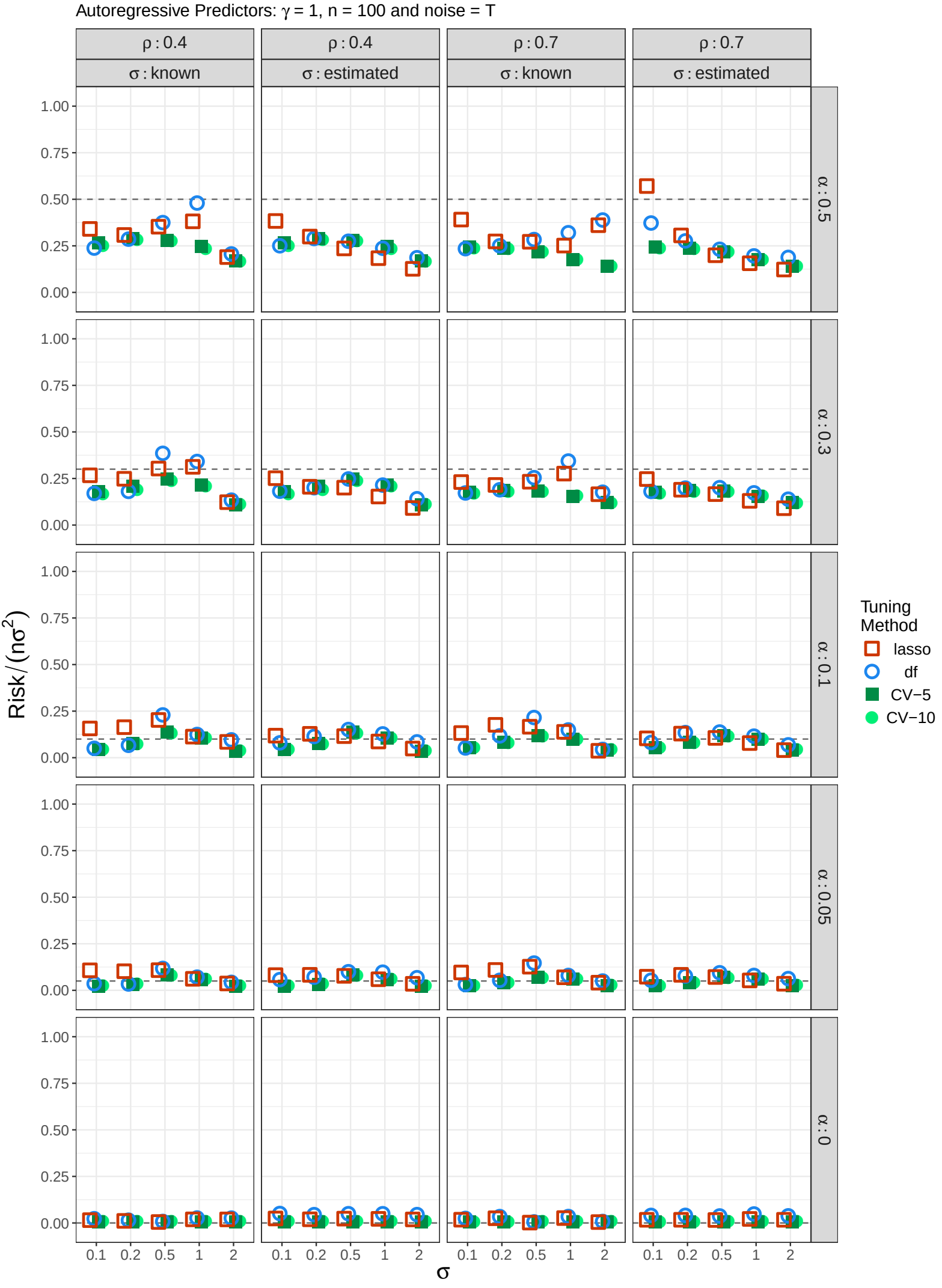
Autoregressive Predictors: γ = 1, n = 100 and noise = T
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ρ : 0.7
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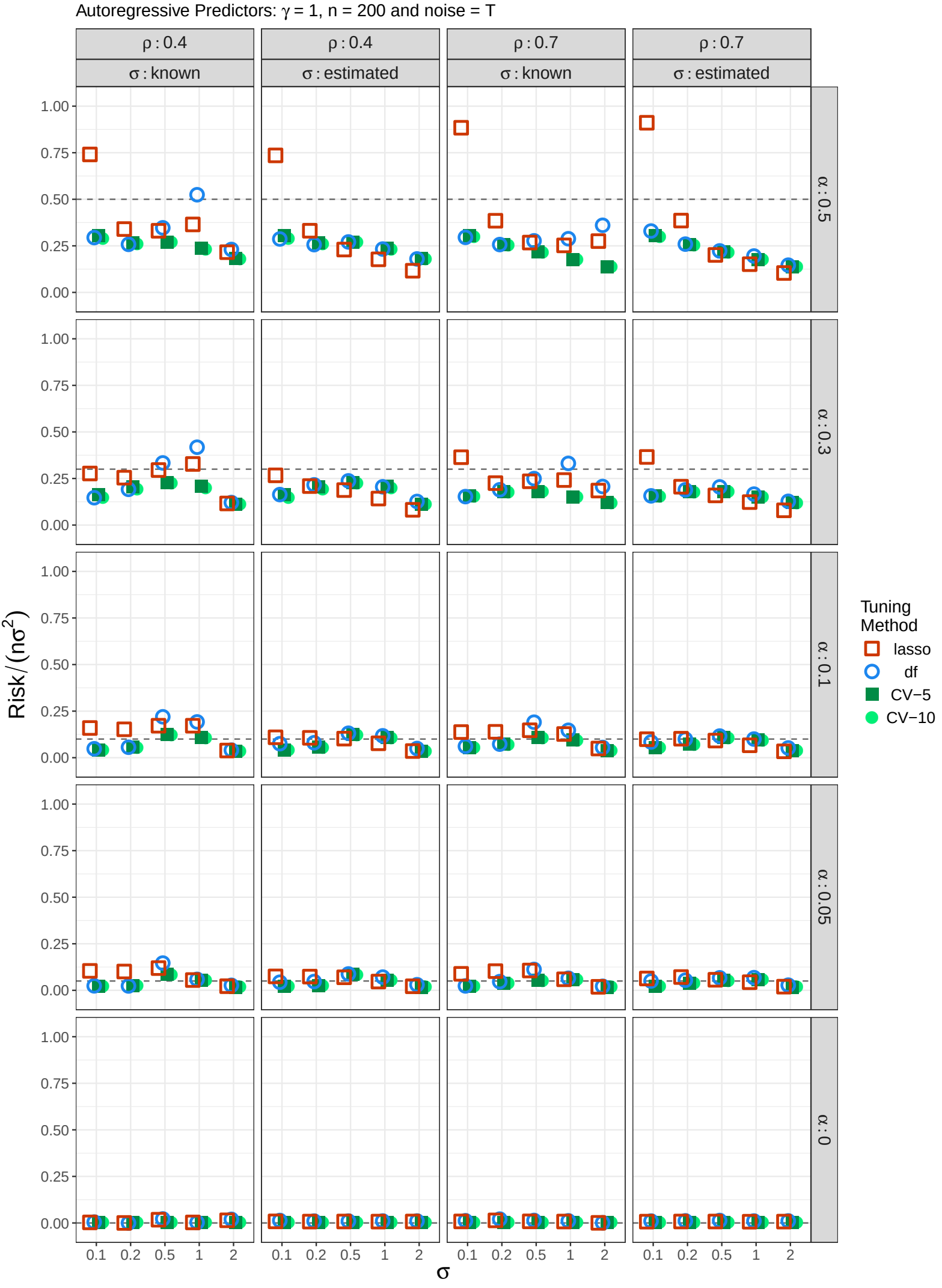
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = T
ρ : 0.4
σ : known
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σ : estimated
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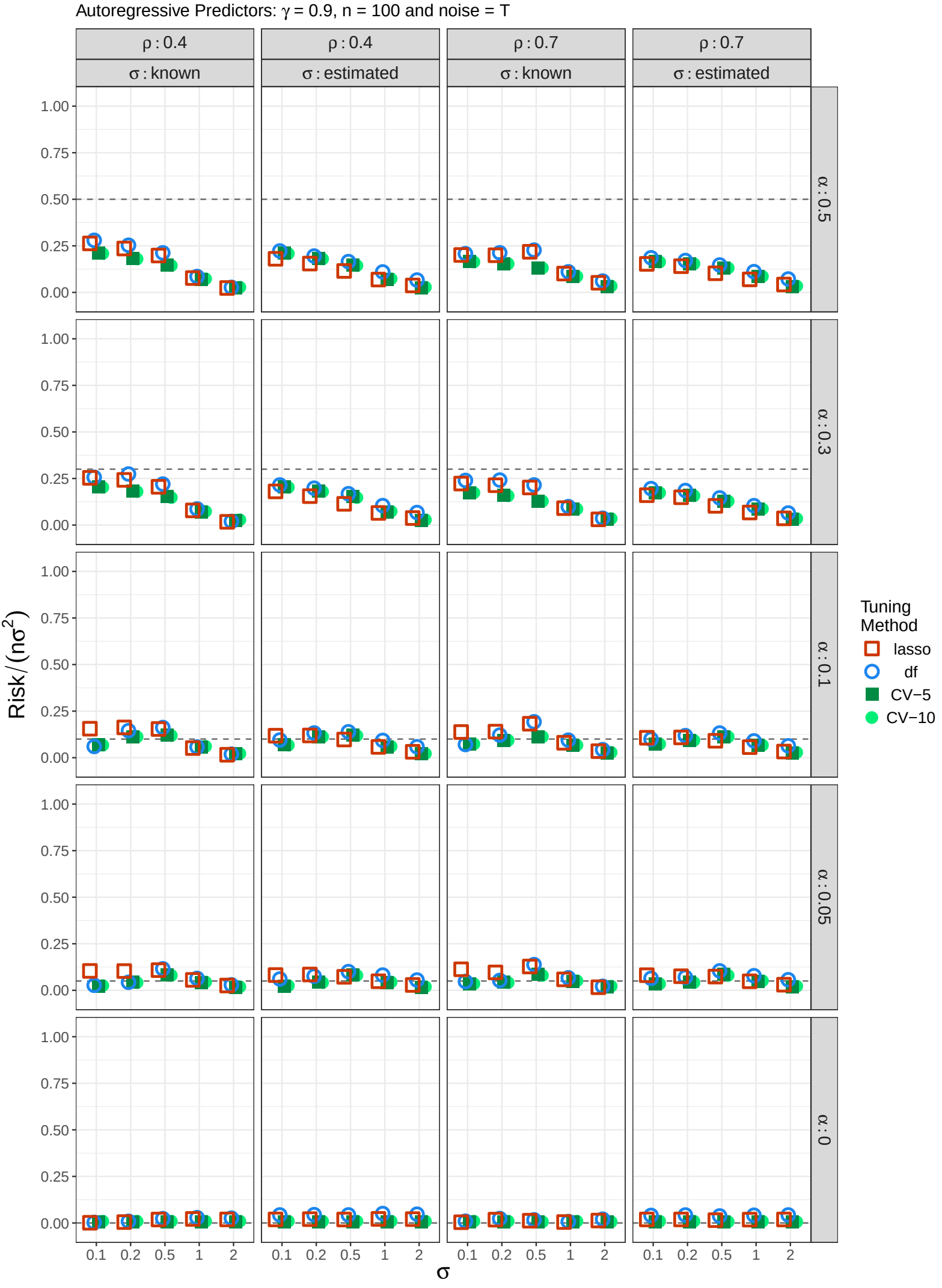
Autoregressive Predictors: γ = 0.9, n = 100 and noise = T
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ρ : 0.4
σ : estimated
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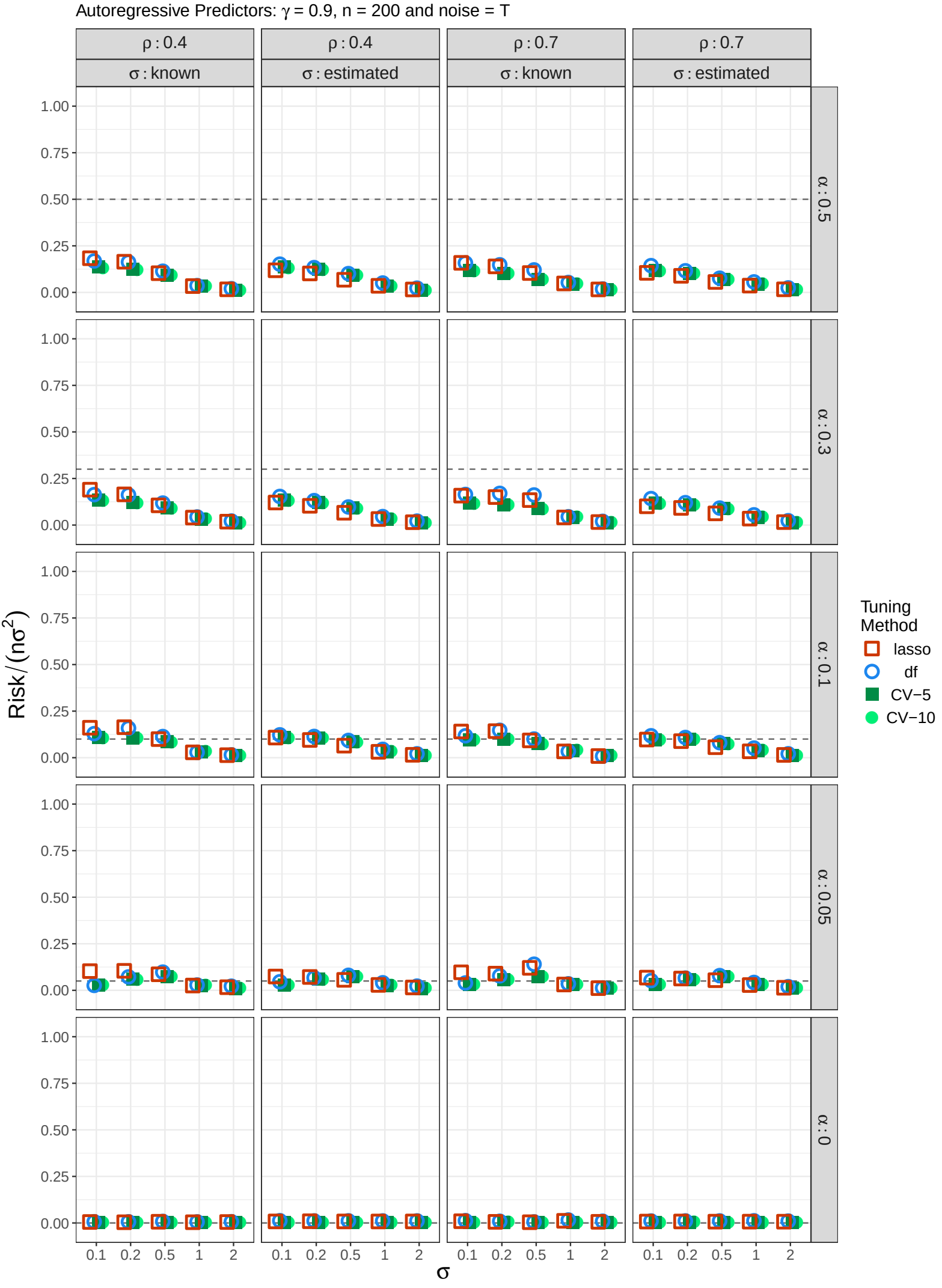
Autoregressive Predictors: γ = 0.9, n = 200 and noise = T
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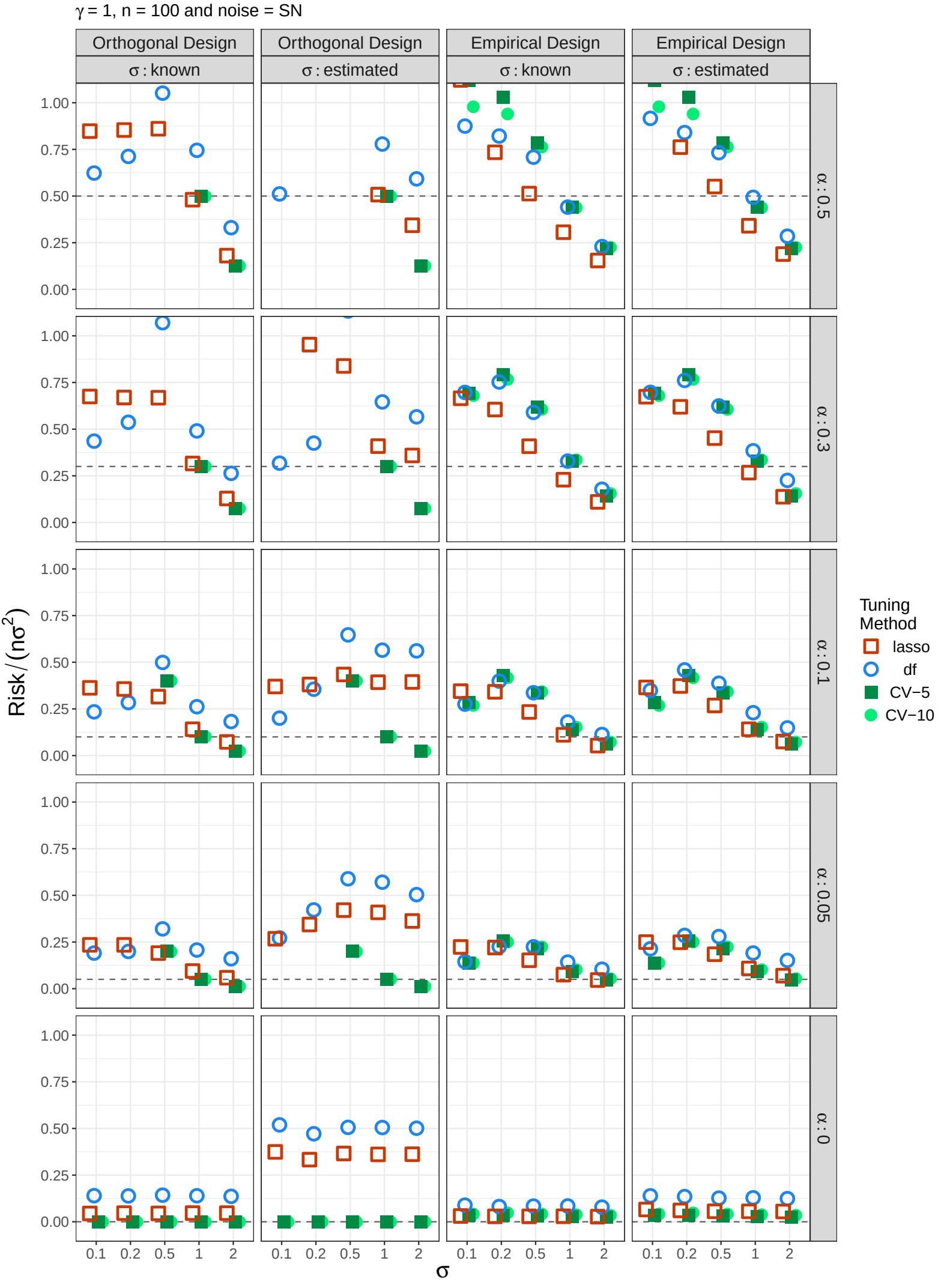
γ = 1, n = 100 and noise = SN
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
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2
0.00
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Tuning Method
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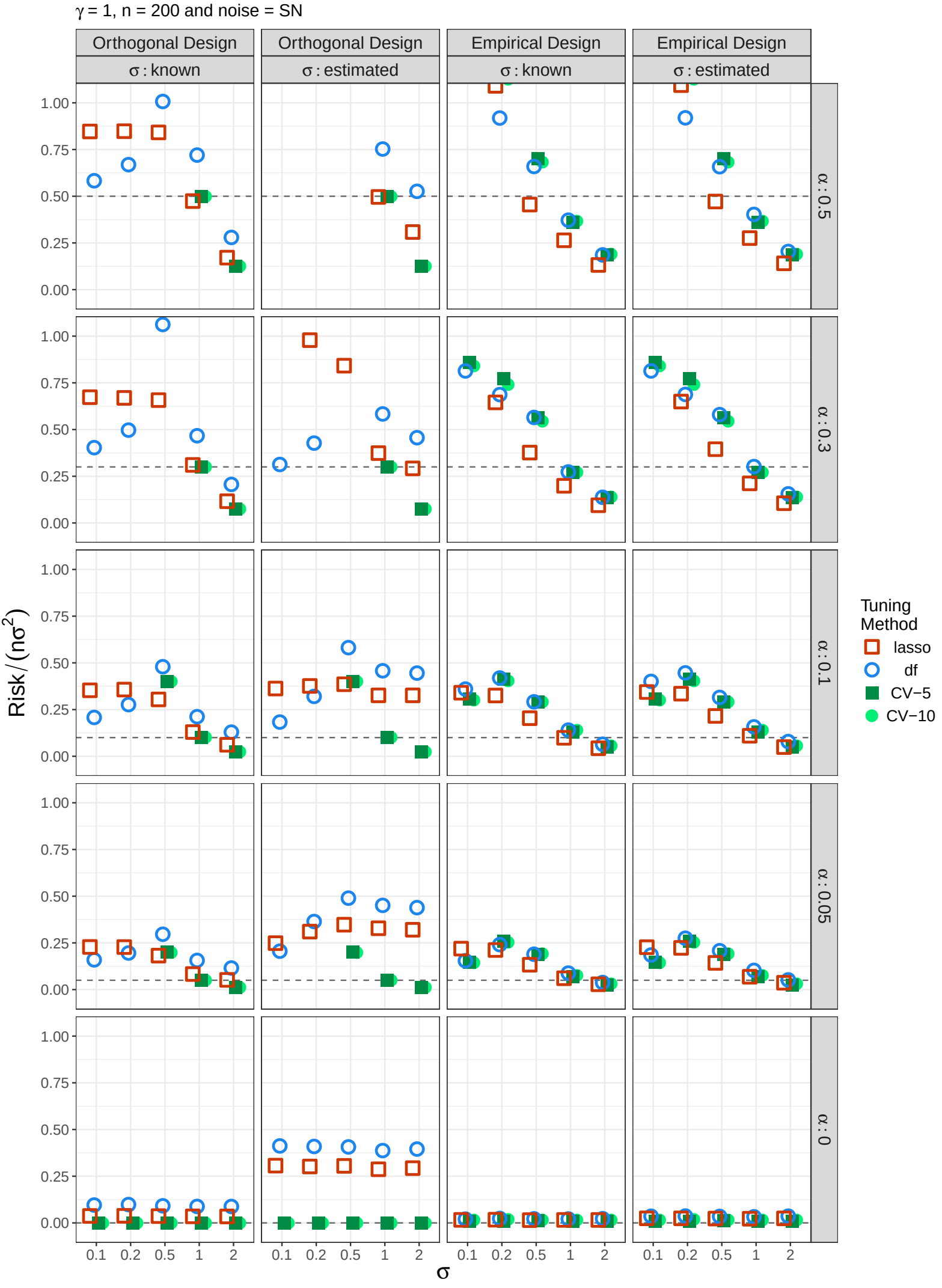
df
CV−5
CV−10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Orthogonal Design
σ : estimated
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df
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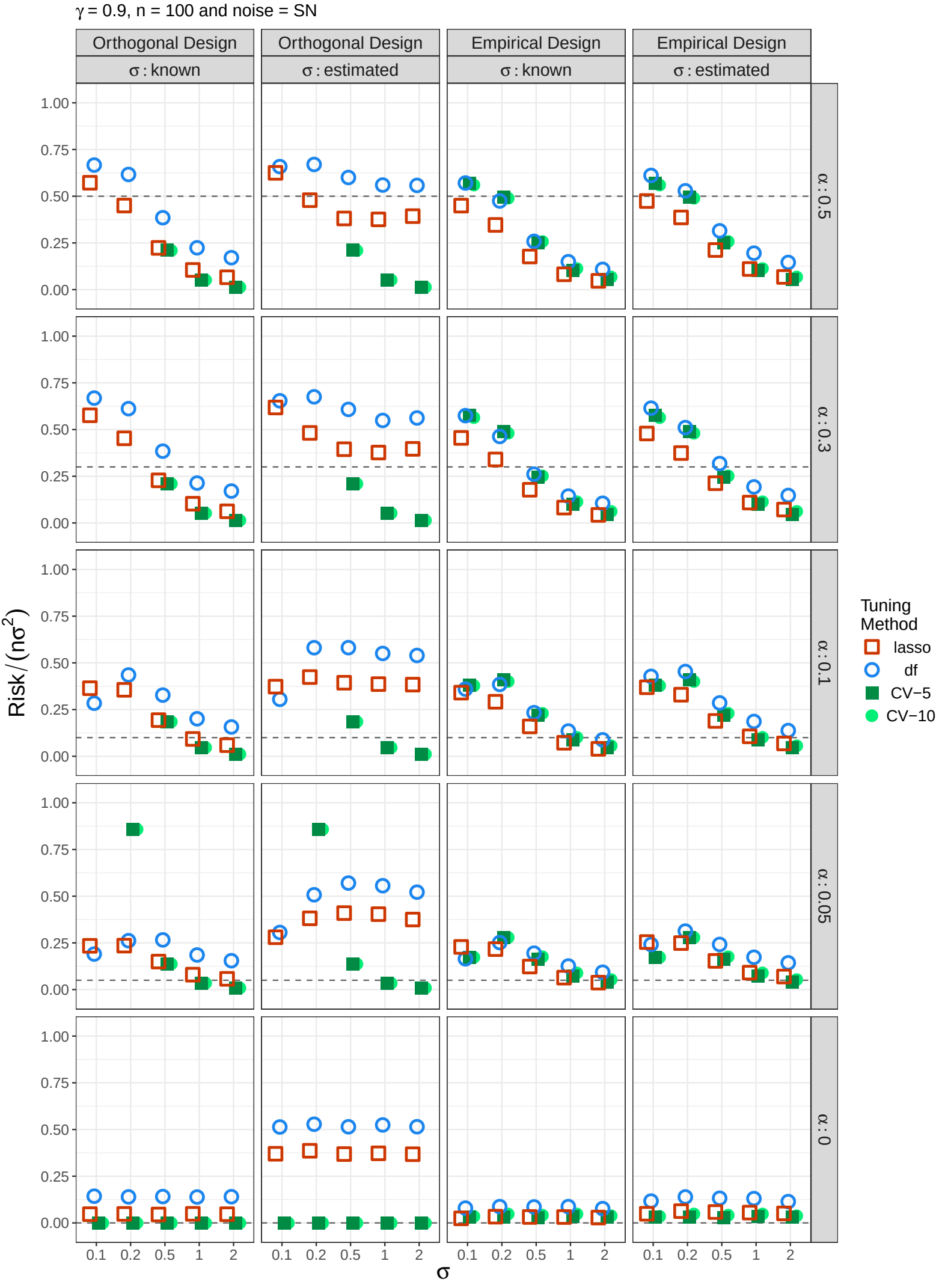
CV−10


γ = 0.9, n = 100 and noise = SN
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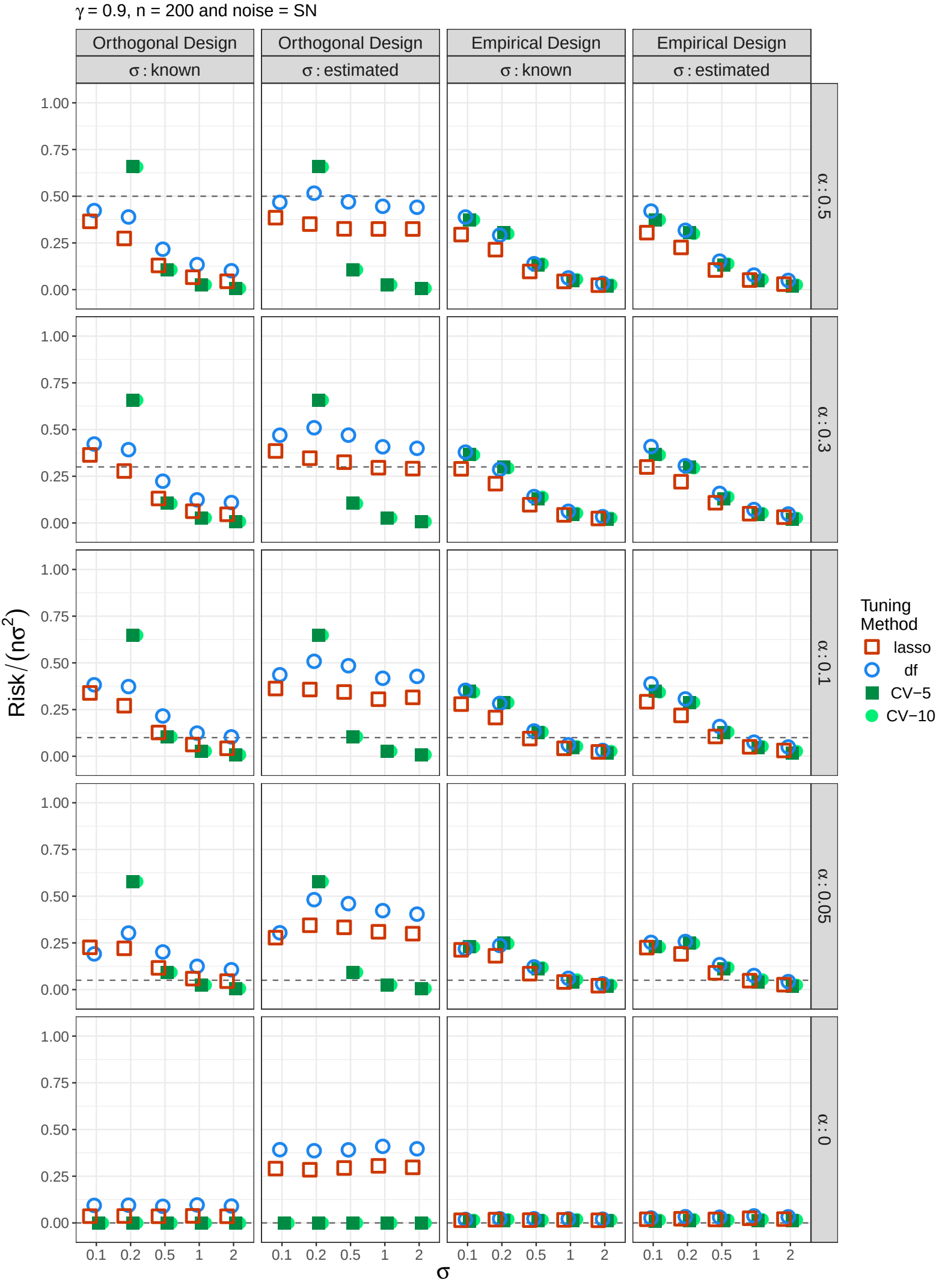
γ = 0.9, n = 200 and noise = SN
Orthogonal Design
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Risk/(nσ²)
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Tuning Method
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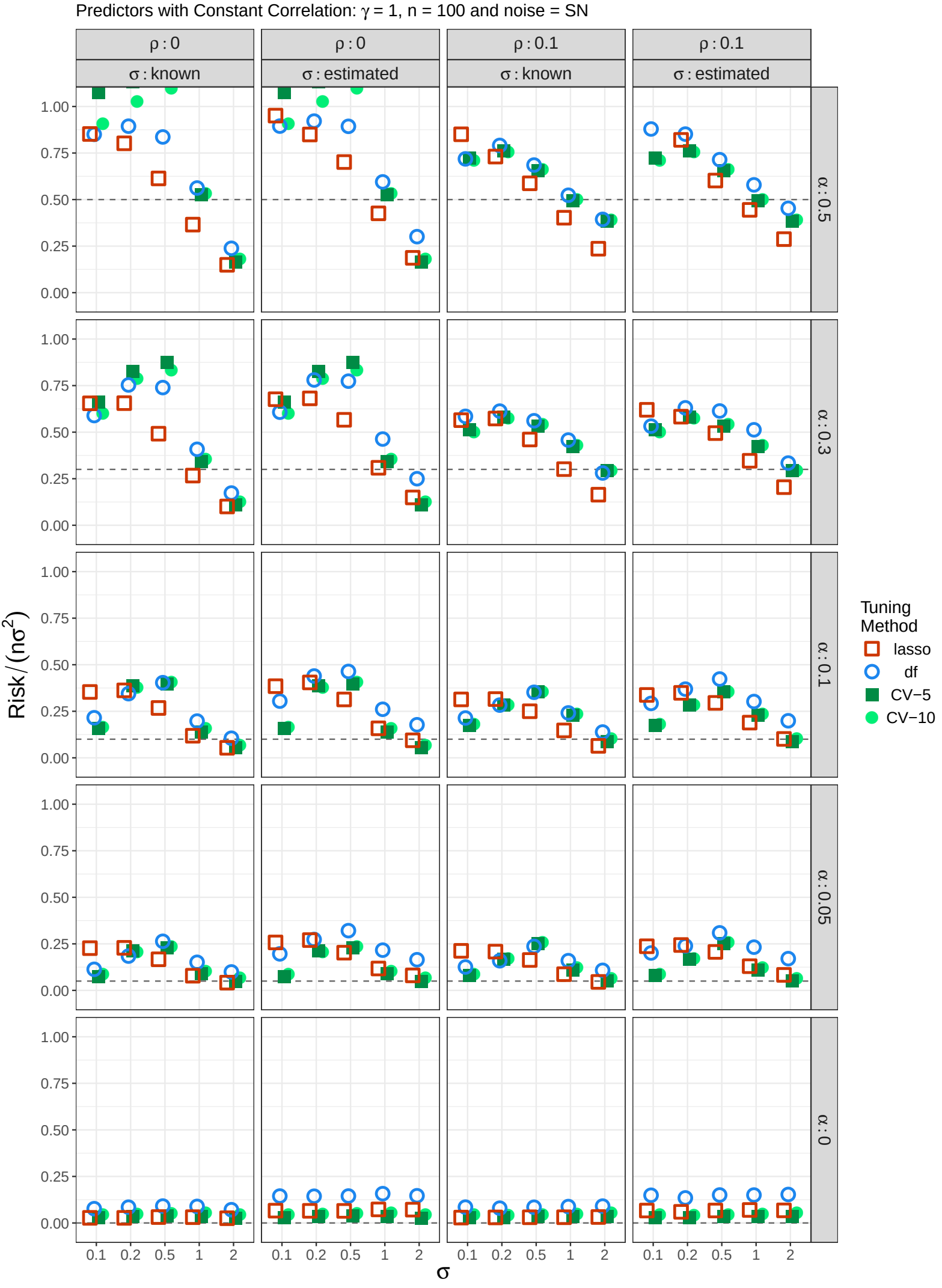
Predictors with Constant Correlation: γ = 1, n = 100 and noise = SN
ρ : 0
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Risk/(nσ²)
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Tuning Method
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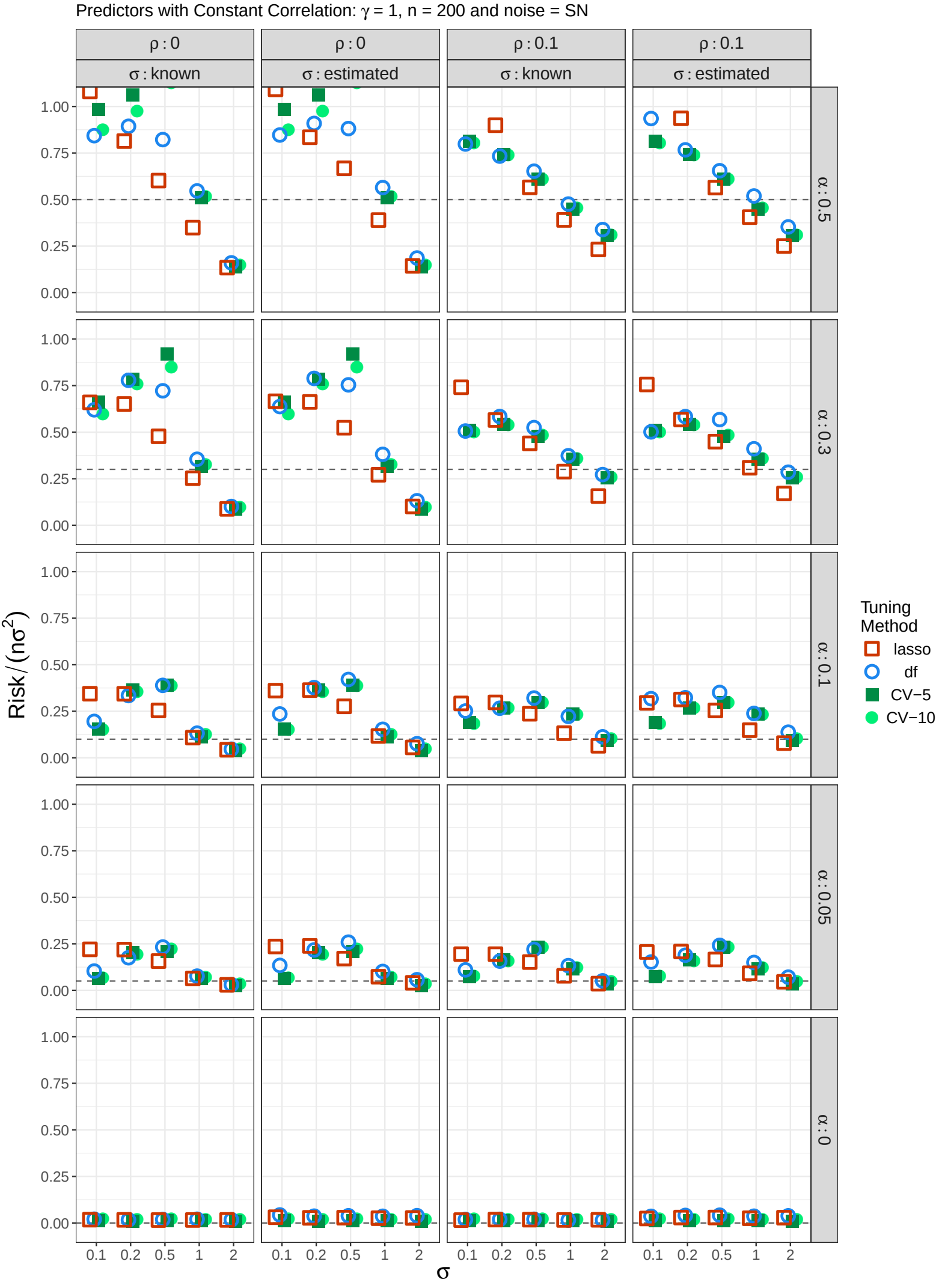
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = SN
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Tuning Method
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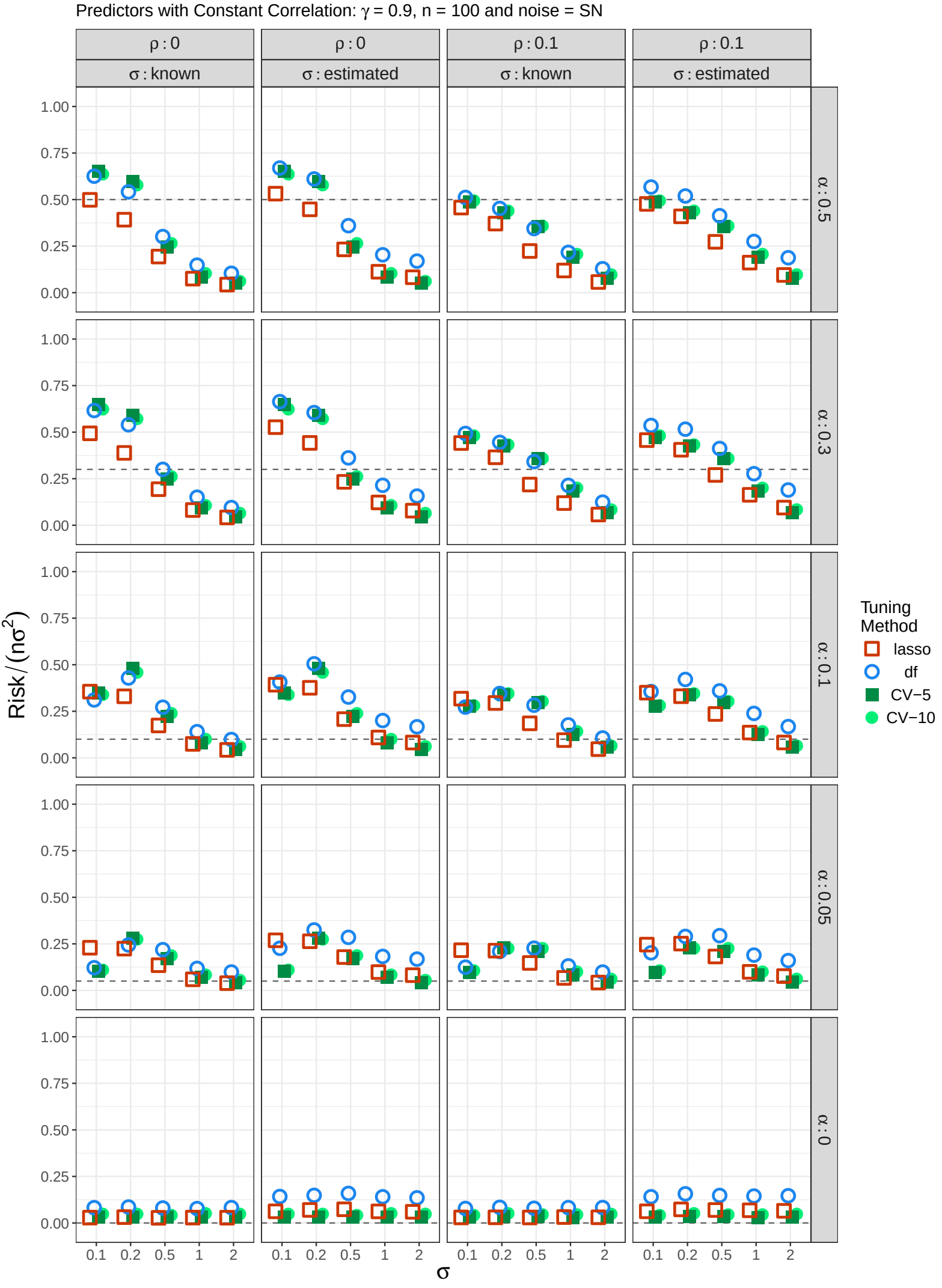
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = SN
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Risk/(nσ²)
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Tuning Method
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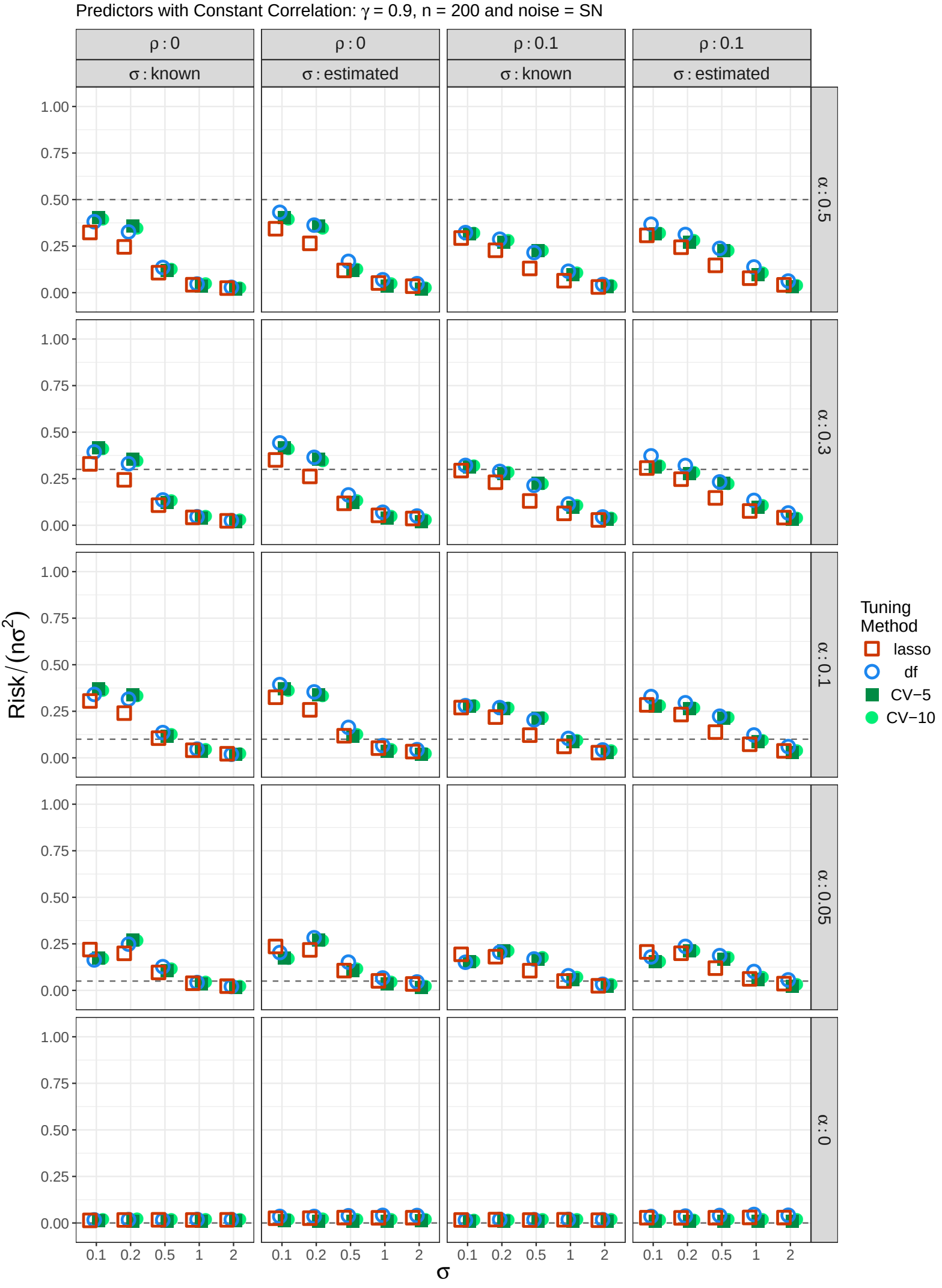
Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = SN
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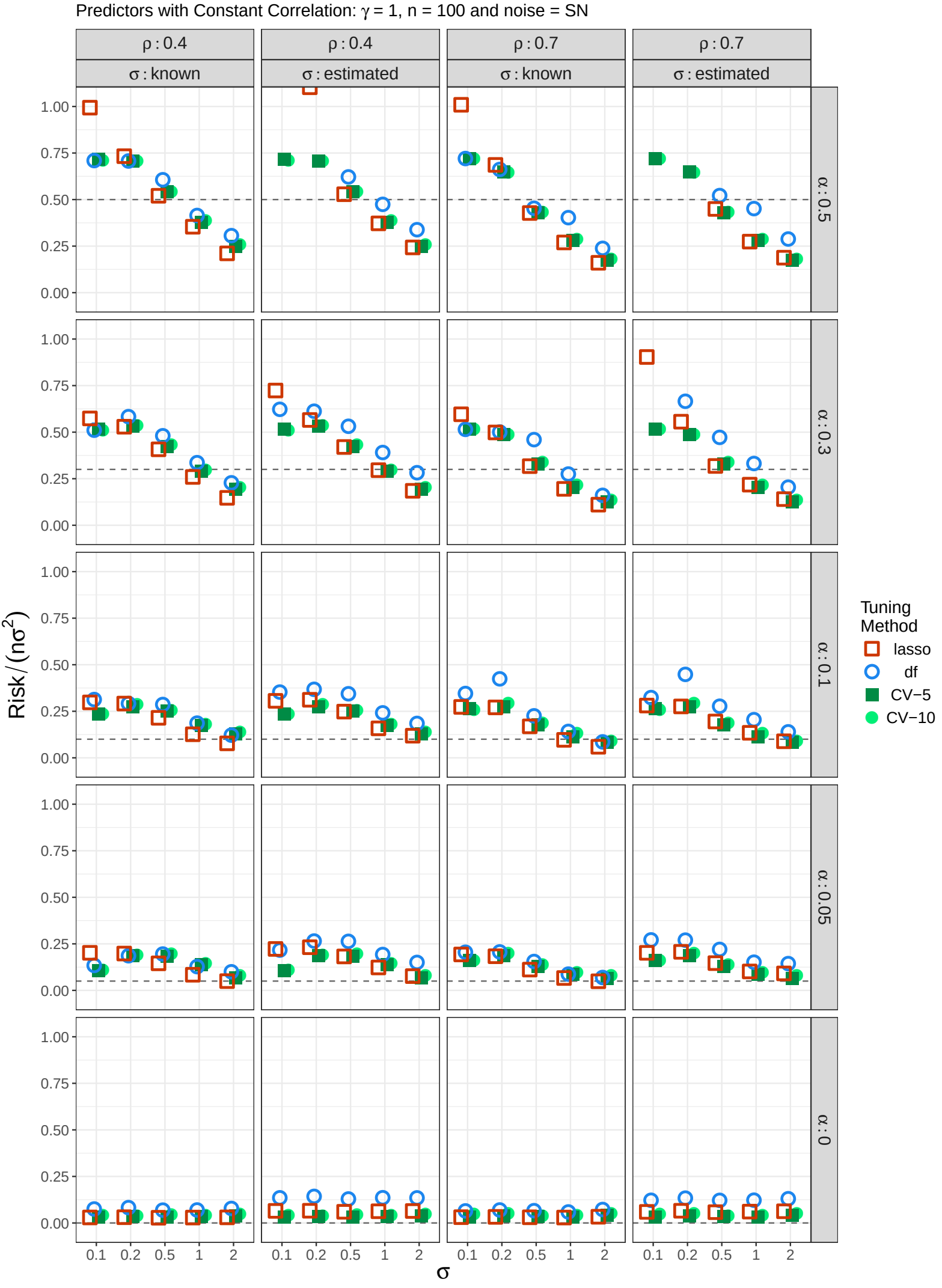
Predictors with Constant Correlation: γ = 1, n = 100 and noise = SN
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ρ : 0.7
σ : known
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Risk/(nσ²)
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Tuning Method
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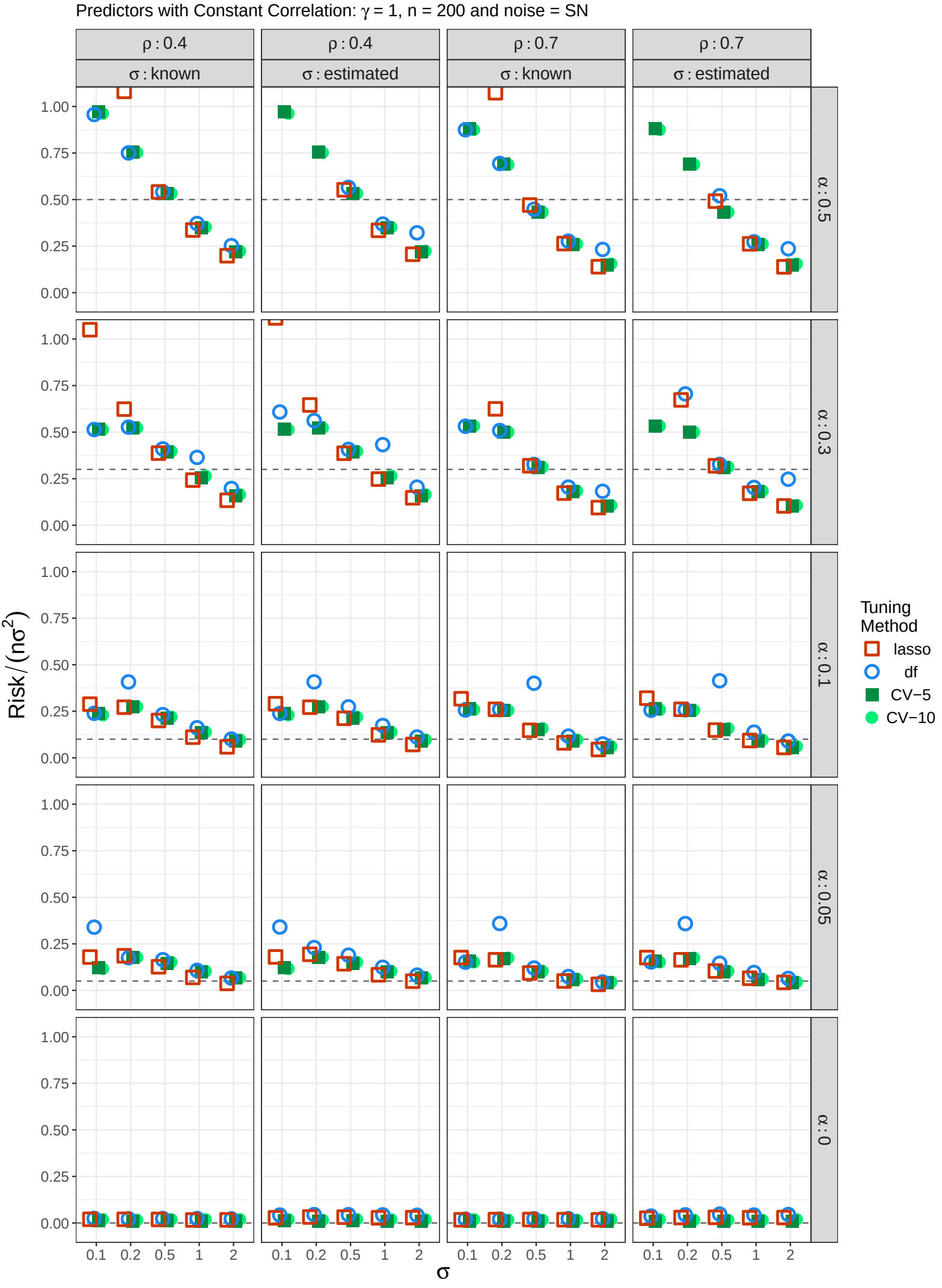
Predictors with Constant Correlation: γ = 1, n = 200 and noise = SN
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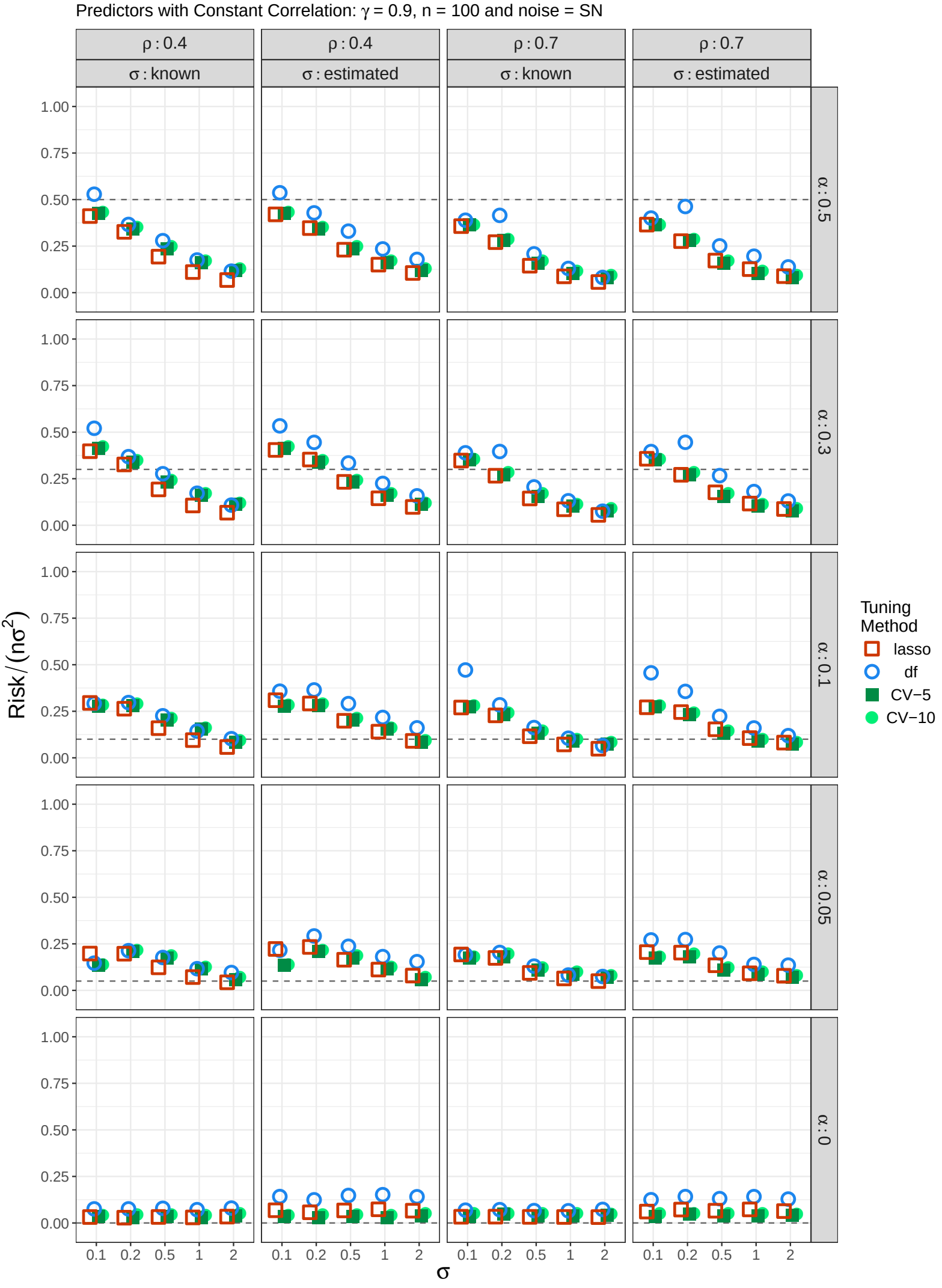
Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = SN
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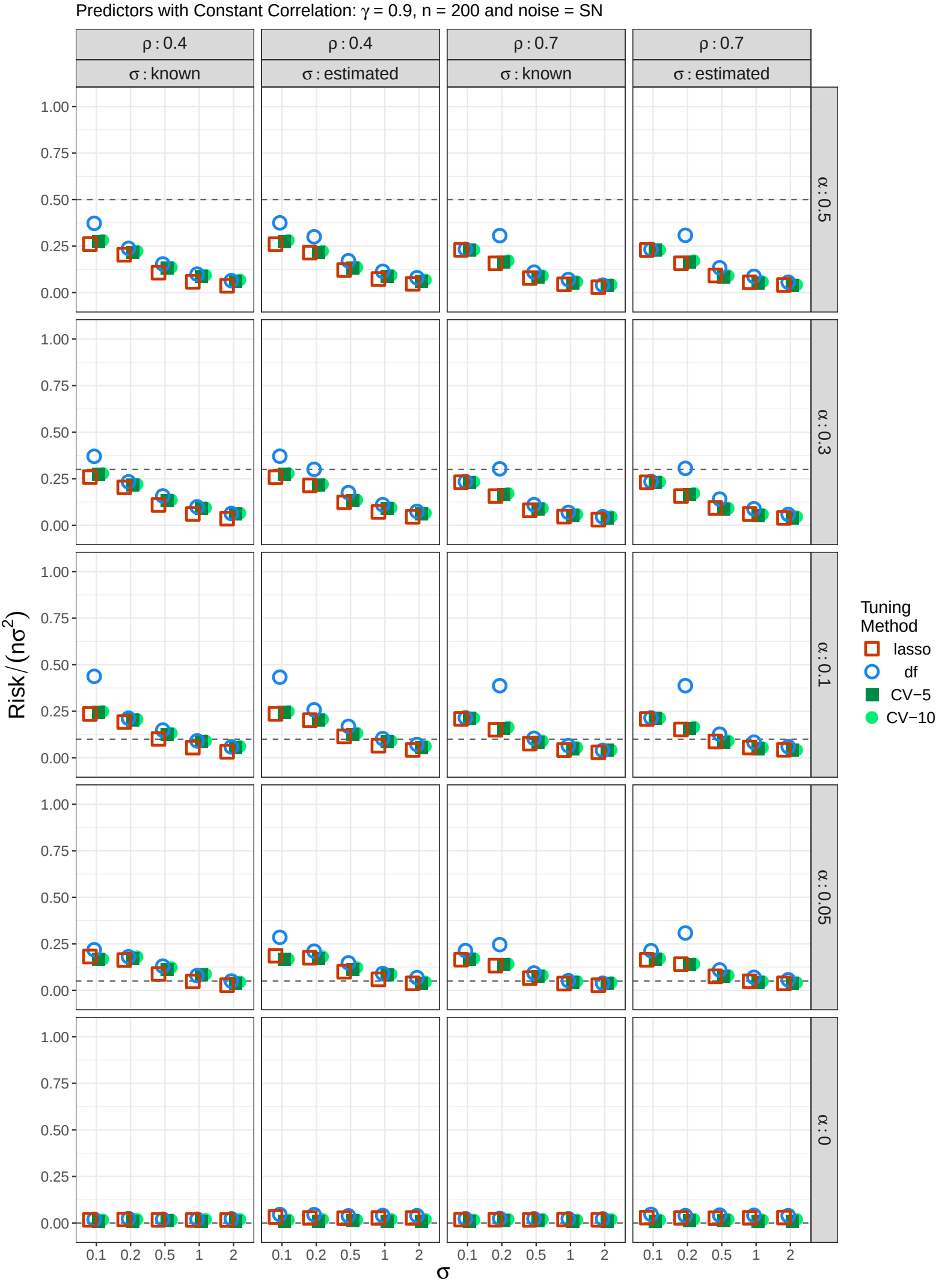
Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = SN
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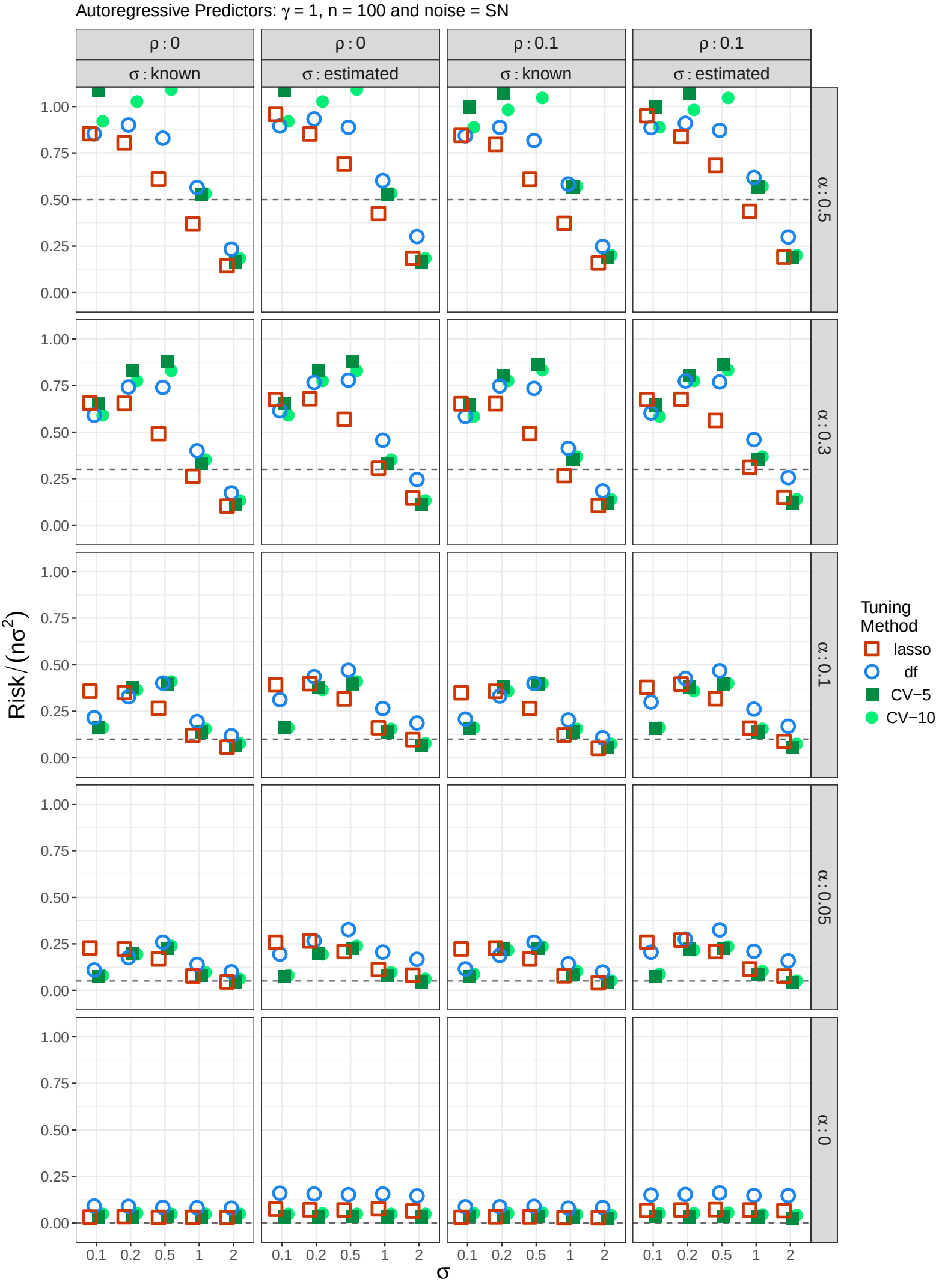
Autoregressive Predictors: γ = 1, n = 100 and noise = SN
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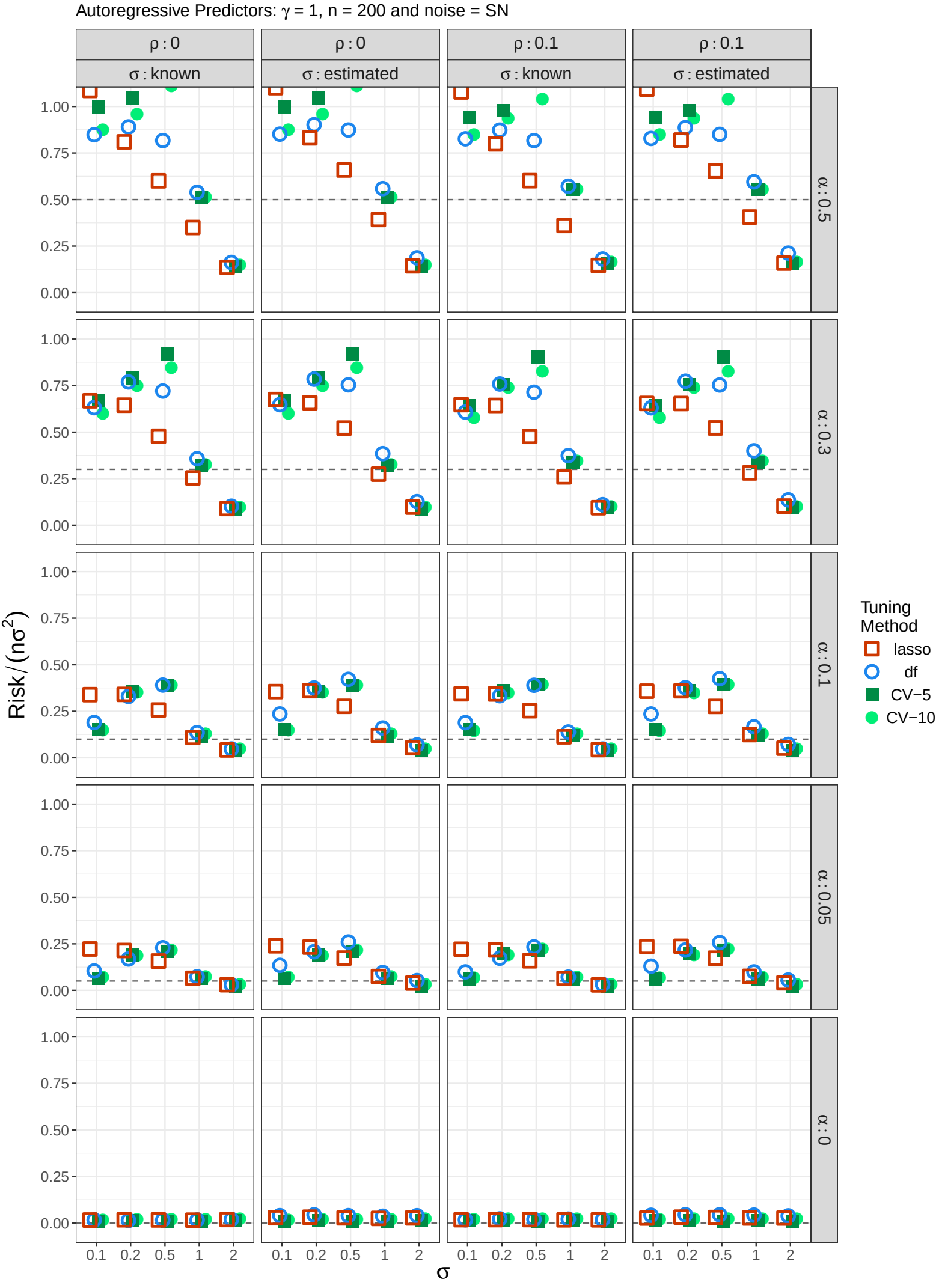
Autoregressive Predictors: γ = 1, n = 200 and noise = SN
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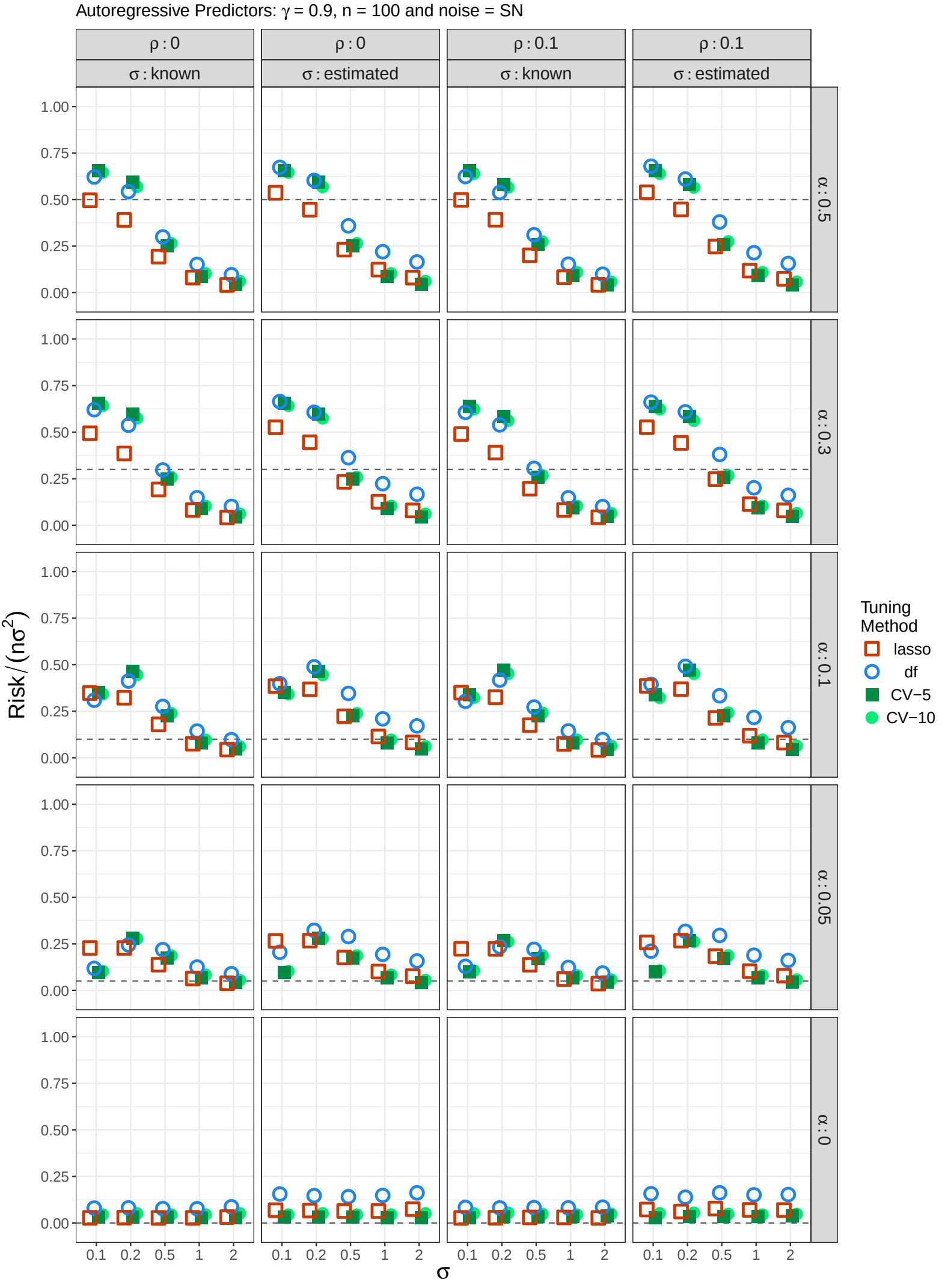
Autoregressive Predictors: γ = 0.9, n = 100 and noise = SN
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CV−5
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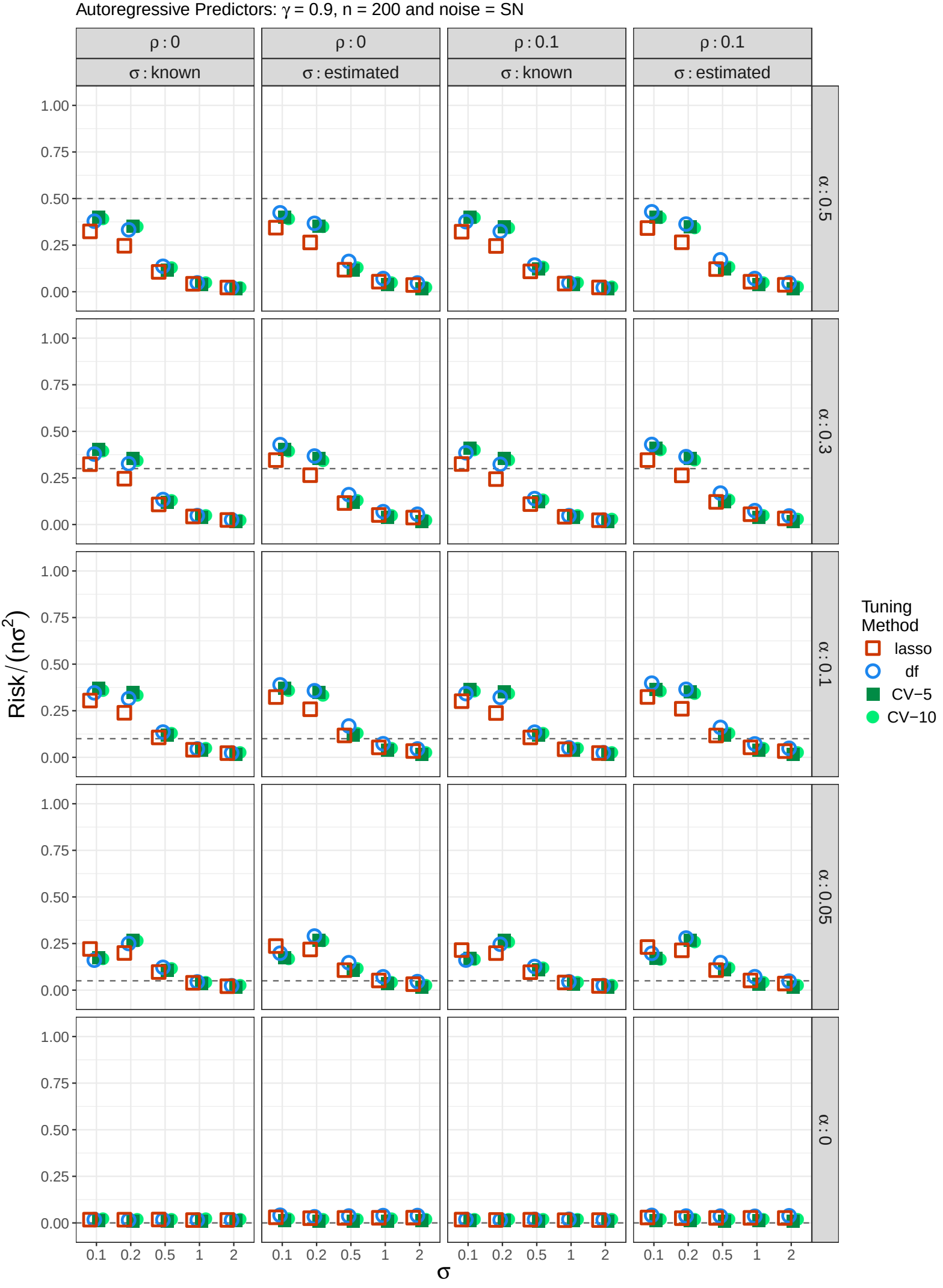
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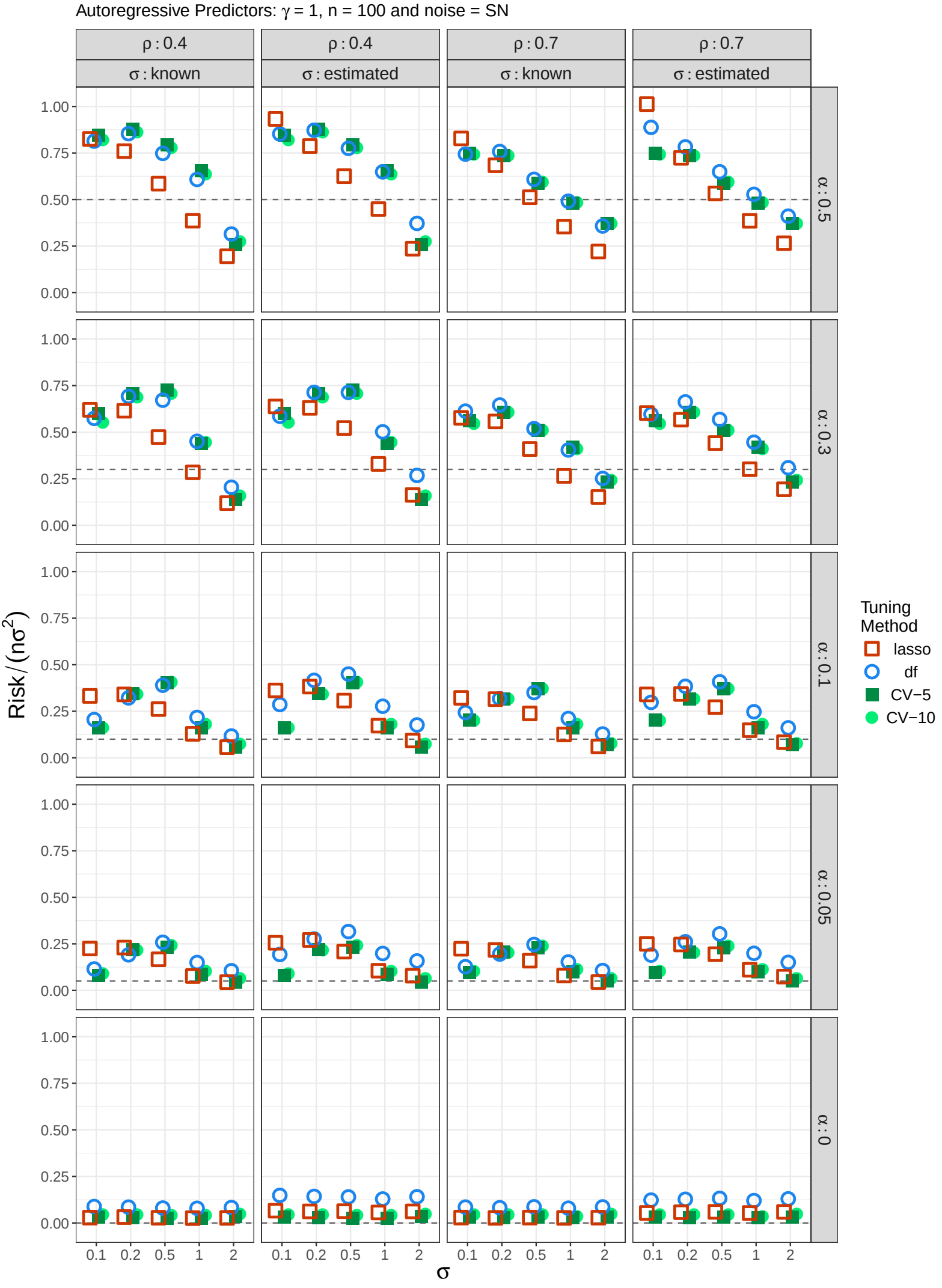
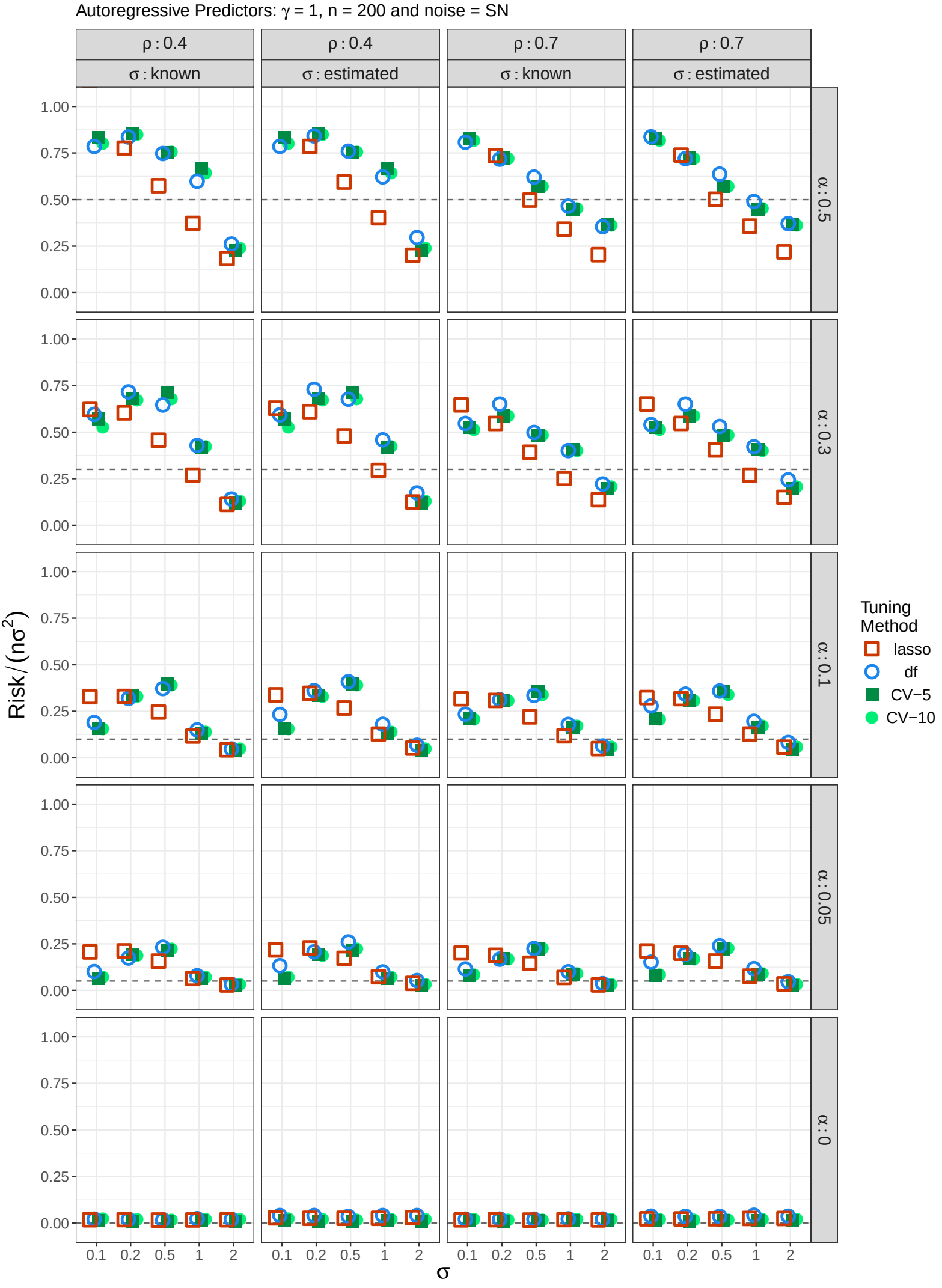
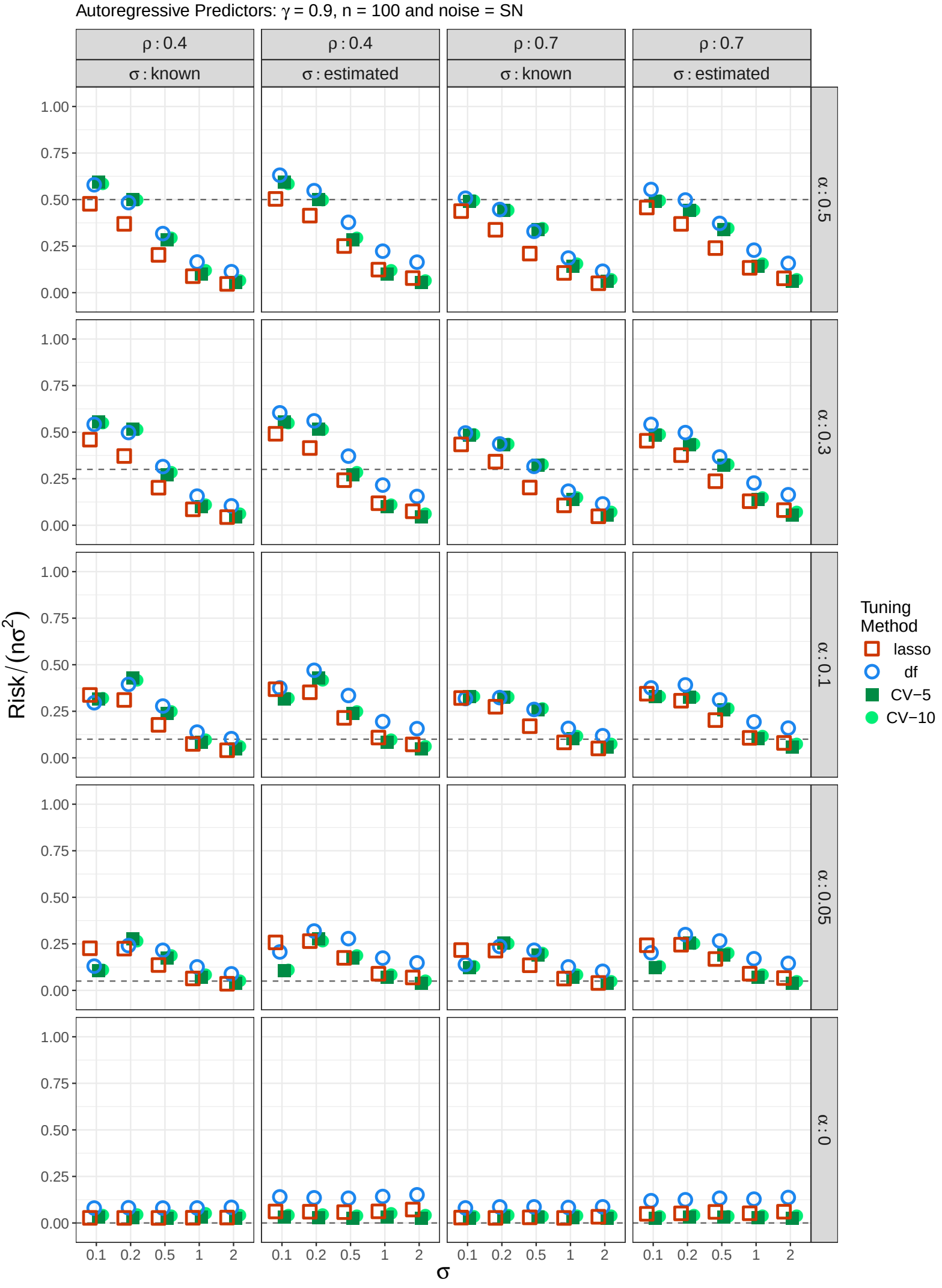
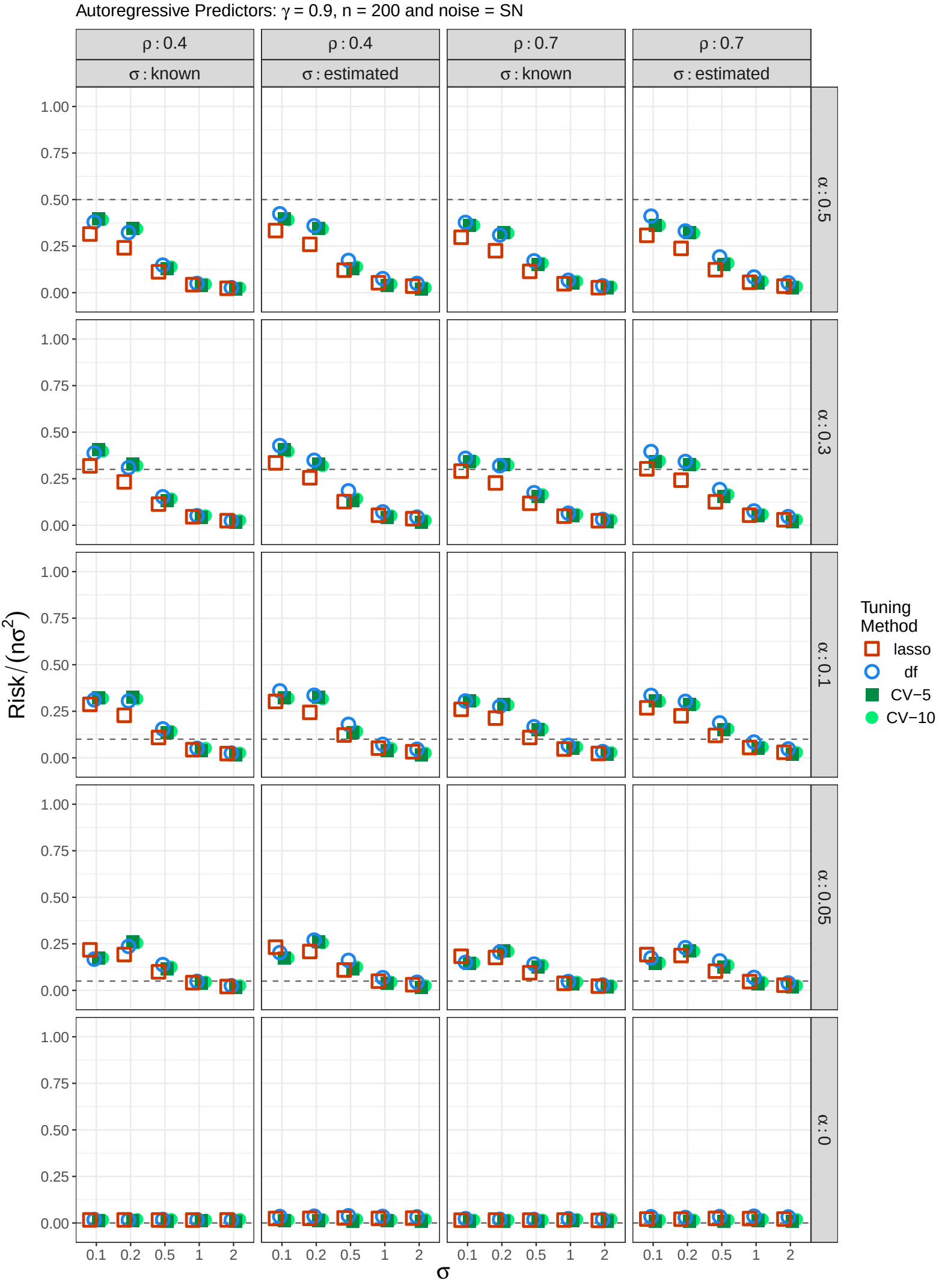
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 100 and noise = SN
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = SN
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 100 and noise = SN
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = SN
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

SUPPL. MAT. DF FOR PIECEWISE LIPSCHITZ ESTIMATORS

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