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CERTAIN THRESHOLDS OF SOFT SUBSTRUCTURES OF RINGS FOCUSED ON IDEALS

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Abstract:

In this paper, we introduce a new kind of soft ring called (α,β)-soft ring. We then focused on the concepts of (α,β)-soft ideal, sum, difference, product of two soft sets, negation of a soft set. Also, we derive its various related properties. We then study and discuss its structural characteristics.

Key Words: Soft Sets, (α,β)-Soft Sub Groupoids, (α,β)-Soft Ring, (α,β)-Soft Ideal & t-Inclusion

1. Introduction:

The notion of soft set was introduced in 1999 by Molodtsov [1] as a new mathematical tool for dealing with uncertainties. Since its inception, it has received much attention in the mean of algebraic structures such as groups [2], semirings [3], rings [4], BCK/BCI-algebras [5–7], normalistic soft groups [8], BL-algebras [9], BCH-algebras [10] and near-rings [11]. Atagu¨n and Sezgin [12] defined the concepts of soft subrings and ideals of a ring, soft subfields of a field and soft submodules of a module and studied their related properties with respect to soft set operations also union soft substructes of near-rings and near-ring modules are studied in [13]. Cag˘man et al. defined two new type of group action on a soft set, called group SI-action [14] and group SU-action [14], which are based on the inclusion relation and the intersection of sets and union of sets, respectively. Algebraic structures of soft sets have been studied by some authors. Maji et al. [15] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [16] introduced several operations of soft sets and Sezgin and Atagu¨n [17] studied on soft set operations as well. Soft set relations and functions [18] and soft mappings [19] were proposed and many related concepts were discussed, too. Moreover, the theory of soft set has gone through remarkably rapid strides with a wide ranging applications especially in soft decision making as in the following studies: [20–22] and some other fields such as [23–26]. Cag˘man and Enginog˘lu [21] redefined the operations of soft sets to develop the soft set theory. In this paper, we introduce a new kind of soft ring called (α,β)-soft ring. We then focused on the concepts of (α,β)-soft ideal, sum, difference, product of two soft sets, negation of a soft set. Also, we derive its various related properties. We then study and discuss its structural characteristics.

2. Preliminaries:

In this section, we recall some basic notions relevant to near-ring modules (N-modules) and fuzzy soft sets. By a near-ring, we shall mean an algebraic system (N,+,.),
where

(N_1) (N, +) forms a group (not necessarily abelian)

(N_2) (N, .) forms a semi group and

(N_3) (x + y)z = xz + yz for all x,y,z ∈ N. (that is we study on right Near-ring modules)

Throughout this paper, N will always denote right near-ring. A normal subgroup H of N is called a left ideal of N if n(s+h)-ns ∈ H for all n,s ∈ N and h ∈ I and denoted by $H \lhd_{\ell} N$. For a near-ring N, the zero-symmetric part of N denoted by N_0 is defined by N_0={n∈ S / n0=0}.

Let (S,+) be a group and A: N×S →S, (n,s)→s.

(S,A) is called N-module or near-ring module if for all x,y ∈ N,for all s ∈ S.

(i) x(ys) = (xy)s

(ii) (x+y)s = xs+ys. It is denoted by N^S. Clearly N itself is an N-module by natural operations. A subgroup T of N^S with NT⊆T is said to be N-sub module of S and denoted by
$T \leq_N S$. A normal subgroup T of S is called an N-ideal of N^S and denoted by a near-ring, S and χ two N-modules. Then h: S→χ is called an N-homomorphism if s,δ ∈ S, for all n∈ N,

(i) h(s+δ) = h(s)+h(δ) and

(ii) h(ns) = nh(s).

For all undefined concepts and notions we refer to (28). From now on, U refers to on initial universe, E is a set of parameters P(U) is the power set of U and A,B,C⊆ E.

Throughout this section, Ω denotes on arbitrary ring with the additive identity element 0_R. If R is a division ring, then the multiplicative identity element of Ω will be denoted by 1_Ω.

2.1 Definition [1]:

A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11, 17, 18, 26, 29, 7].

2.2 Definition [21]:

The relative complement of the soft set F_A over U is denoted by $F^r{}_A$, where $F^r{}_A : A \to P(U)$ is a mapping given as $F^r{}_A(a) = U \setminus F_A(a)$, for all $a \in A$.

2.3 Definition [21]:

Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted intersection of F_A and G_B is denoted by $F_A \Cap G_B$, and is defined as $F_A \Cap G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

2.4 Definition [21]:

Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B = (H,C)$,where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

2.5 Definition [12]:

Let F_A and G_B be soft sets over the common universe U and ψbe a function from A to B. Then we can define the soft set $\psi(F_A)$ over U, where $\psi(F_A) : B \to P(U)$ is a set valued function defined by $\psi(F_A)(b) = \cup\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$, $= 0$ otherwise for all $b \in B$. Here, $\psi(F_A)$ is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B) : A \to P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all $a \in A$. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

2.6 Definition [13]:

Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A) : B \to P(U)$ is a set-valued function defined by $\psi^\star(F_A)(b) = \cap\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$,
$=0$ otherwise for all $b \in B$. Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

2.7 Definition:

Let Ω be a ring with respect to two binary operations '+', '.' and $f_\Omega \in S(U)$. f_Ω is called a (α,β)-soft ring over U, if f_Ω is a (α,β)-soft groupoid over U for the binary operation '+' in S(U) induced by '+' in Ω, and f_Ω is a soft groupoid over U for the binary operation '.' in S(U) induced by '.' in Ω.

3. Properties of (α,β)-Soft Ring and (α,β)-Soft Ideal:

3.1 Theorem:

Let Ω be a ring and $f_\Omega \in S(U)$, then f_Ω is called (α,β)-soft ring over U iff

(i) $f_\Omega(x-y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$,

(ii) $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$ for all $x,y \in \Omega$.

Proof:

Suppose that f_Ω is (α,β)-soft ring over U. Then we have $f_\Omega(x-y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$ and $f_\Omega(-x) \cap \alpha = f_\Omega(x) \cup \beta$. Hence $f_\Omega(x-y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(-y) \cup \beta = f_\Omega(x) \cap f_\Omega(y) \cup \beta$. Moreover, as f_Ω is a (α,β)-soft groupoid over U, then we have $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$.

Conversely, suppose that $f_\Omega(x-y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$, $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$ for all $x,y \in \Omega$. Choosing $x = 0_\Omega$ yields $f_\Omega(0_\Omega - y) \cap \alpha = f_\Omega(-y) \cap \alpha = f_\Omega(y) \cup \beta$. And $f_\Omega(y) \cap \alpha = f_\Omega(-(-y)) \cap \alpha \supseteq f_\Omega(-y) \cup \beta$ for all $y \in \Omega$. Thus $f_\Omega(-x) \cap \alpha = f_\Omega(x) \cup \beta$ for all $x \in \Omega$. Also, $f_\Omega(x+y) \cap \alpha = f_\Omega(x-(-y)) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(-y) \cup \beta = f_\Omega(x) \cap f_\Omega(y) \cup \beta$. Therefore, f_Ω is called a (α,β)-soft ring over U.

3.1 Definition:

Let Ω be a ring. Then, (α,β)-soft ring f_Ω is called a (α,β)-soft left ideal over U, if $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(y) \cup \beta$ for all $x,y \in \Omega$ and f_Ω is called a (α,β)-soft right ideal over U, if $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup \beta$ for all $x,y \in \Omega$. If f_Ω is a (α,β)-soft left and right ideal over U, then f_Ω is called to be a (α,β)-soft ideal over U.

3.2 Theorem:

Let Ω be a ring and $f_\Omega \in S(U)$, then f_Ω is called (α,β)-soft ideal over U iff

(i) $f_\Omega(x-y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$,

(ii) $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(y) \cup \beta$ for all $x,y \in \Omega$.

Proof:

Suppose that f_Ω is called (α,β)-soft ideal over U. Then, we have $f_\Omega(x-y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$. Moreover, since $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup \beta$ and $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(y) \cup \beta$, it follows that $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(y) \cup \beta$.

Conversely, suppose that $f_\Omega(x-y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$ and $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(y) \cup \beta$ for all $x,y \in \Omega$. Thus

$f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(y) \cup \beta \supseteq f_\Omega(x) \cup \beta$, $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(y) \cup \beta \supseteq f_\Omega(y) \cup \beta$ and $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$.Therefore, f_Ω is called (α,β)-soft ideal over U.

3.1 Proposition:

If f_Ω is (α,β)-soft ideal over U, then $f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(x) \cup \beta$,for all $x \in \Omega$.

Proof:

Suppose that f_Ω is called (α,β)-soft ideal over U. Then, for all $x \in \Omega$,

$$f_\Omega(0_\Omega) \cap \alpha = f_\Omega(x - x) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(x) \cup \beta \supseteq f_\Omega(x) \cup \beta.$$

3.2 Proposition:

Let Ω be a ring with identity. If f_Ω is (α,β)-soft ideal over U, then $f_\Omega(x) \cap \alpha \supseteq f_\Omega(1_\Omega) \cup \beta$,for all $x \in \Omega$.

Proof:

Suppose that f_Ω is (α,β)-soft ideal over U. Then, for all $x \in \Omega$, $f_\Omega(x) \cap \alpha = f_\Omega(x1_\Omega) \cap \alpha \supseteq f_\Omega(1_\Omega) \cup \beta$.

3.3 Theorem:

Let R be a division ring and $f_\Omega \in S(U)$. Then f_Ω is (α,β)-soft ideal over U iff $f_\Omega(x) \cap \alpha = f_\Omega(1_\Omega) \cap \alpha \subseteq f_\Omega(0_\Omega) \cup \beta$ for all $0_\Omega \neq x \in \Omega$.

Proof:

Suppose that f_Ω is (α,β)-soft ideal over U. Since $f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(x) \cup \beta$ for all $x \in \Omega$, then in particular $f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(1_\Omega) \cup \beta$. Now let $0_\Omega \neq x \in \Omega$,

$$f_\Omega(x) \cap \alpha = f_\Omega(x1_\Omega) \cap \alpha \supseteq f_\Omega(1_\Omega) \cup \beta \text{ and } f_\Omega(1_\Omega) \cap \alpha = f_\Omega(xx^{-1}) \cap \alpha \supseteq f_\Omega(x) \cup \beta.$$

It follows that $f_\Omega(x) \cap \alpha = f_\Omega(1_\Omega) \cap \alpha \subseteq f_\Omega(0_\Omega) \cup \beta$.

Conversely,

(i) Let $x,y \in \Omega$. If $x\text{-}y \neq 0_\Omega$, then $f_\Omega(x - y) \cap \alpha = f_\Omega(1_\Omega) \cap \alpha = f_\Omega(x) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$ and if $x\text{-}y = 0_\Omega$, then $f_\Omega(x - y) \cap \alpha = f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(x) \cup \beta \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta$.

(ii) Let $x,y \in \Omega$. If $x \neq 0_\Omega$ and $y = 0_\Omega$, then $f_\Omega(xy) \cap \alpha = f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(1_\Omega) \cup \beta = f_\Omega(x) \cup \beta$ and $f_\Omega(xy) \cap \alpha = f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(1_\Omega) \cup \beta = f_\Omega(y) \cup \beta$. Thus $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(y) \cup \beta$.

(iii) Let $x,y \in \Omega$. If $x \neq 0_\Omega$ and $y \neq 0_\Omega$, then either $xy \neq 0_\Omega$ or $xy = 0_\Omega$.

If $xy \neq 0_\Omega$, then $f_\Omega(xy) \cap \alpha = f_\Omega(1_\Omega) \cap \alpha = f_\Omega(x) \cup \beta$ and $f_\Omega(xy) \cap \alpha = f_\Omega(1_\Omega) \cap \alpha = f_\Omega(y) \cup \beta$.

If $xy = 0_\Omega$, then $f_\Omega(xy) \cap \alpha = f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(x) \cup \beta$ and $f_\Omega(xy) \cap \alpha = f_\Omega(0_\Omega) \cap \alpha \supseteq f_\Omega(y) \cup \beta$.

Thus $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cup f_\Omega(y) \cup \beta$ implying that f_Ω is (α,β)-soft ideal over U.

Remark:

The above theorem 3.3 shows that in a division ring a (α,β)-soft left ideal in a (α,β)-soft ideal.

3.4 Theorem:

Let f_Ω be (α,β)-soft ring / ideal over U. If $f_\Omega(x\text{-}y) = f_\Omega(0_\Omega)$ for any $x,y \in \Omega$, then $f_\Omega(x) \cap \alpha = f_\Omega(y) \cup \beta$.

Proof:

Assume that $f_\Omega(x\text{-}y) = f_\Omega(0_\Omega)$ for any $x,y \in \Omega$. Then $f_\Omega(x) \cap \alpha = f_\Omega(x - y + y) \cap \alpha \supseteq f_\Omega(x\text{-}y) \cup f_\Omega(y) \cup \beta = f_\Omega(0_\Omega) \cup f_\Omega(y) \cup \beta = f_\Omega(y) \cup \beta$.

Similarly, using $f_\Omega(x\text{-}y) \cap \alpha = f_\Omega(-(y\text{-}x)) \cap \alpha = f_\Omega(y\text{-}x) = f_\Omega(0_\Omega)$, we have $f_\Omega(y) \cap \alpha \supseteq f_\Omega(x) \cup \beta$.

Thus the proof is completed.

3.3 Proposition:

Let f_Ω be (α,β)-soft ring / ideal over U such that the image of f_Ω is ordered by inclusion for all $x \in \Omega$. If $f_\Omega(y) \cap \alpha \supset f_\Omega(x) \cup \beta$ for $x,y \in \Omega$, then $f_\Omega(x\text{-}y) = f_\Omega(x) = f_\Omega(y\text{-}x)$.

Proof:

Assume that $f_\Omega(y) \cap \alpha \supseteq f_\Omega(x) \cup \beta$ for $x,y \in \Omega$. Then $f_\Omega(x\text{-}y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta = f_\Omega(x) \cup \beta$ and $f_\Omega(x) \cap \alpha = f_\Omega(x - y + y) \cap \alpha \supseteq f_\Omega(x\text{-}y) \cup f_\Omega(y) \cup \beta$

Since, $f_\Omega(y) \cap \alpha \supset f_\Omega(x) \cup \beta$ and $f_\Omega(x) \cap \alpha \supseteq f_\Omega(x\text{-}y) \cup f_\Omega(y) \cup \beta$, for $x,y \in \Omega$, then $f_\Omega(x\text{-}y) \cap \alpha \subseteq f_\Omega(x) \cup \beta$.

It follows that $f_\Omega(x\text{-}y) = f_\Omega(x) = f_\Omega(y\text{-}x)$.

3.5 Theorem:

Let f_Ω be (α,β)-soft ring / ideal over U with Im $f_\Omega = (\phi, \alpha)$, where $\phi \neq \alpha \subseteq U$. If $f_\Omega = g_\Omega \tilde{\cup} h_\Omega$ where g_Ω and h_Ω are (α,β)-soft ideal over U then either $g_\Omega \subseteq h_\Omega$ or $h_\Omega \tilde{\subseteq} g_\Omega$.

Proof:

To obtain a proof by contradiction, assume that $g_\Omega(x) \cap \alpha \supset h_\Omega(x) \cup \beta$ and $h_\Omega(y) \cap \alpha \supset g_\Omega(y) \cup \beta$ for $x,y \in \Omega$.

As $f_\Omega = g_\Omega \tilde{\cup} h_\Omega$, therefore $f_\Omega(x) = g_\Omega(x) \cap \alpha \supset h_\Omega(x) \cup \beta \supseteq \phi$.

And $f_\Omega(y) = h_\Omega(y) \cap \alpha \supset g_\Omega(y) \cup \beta \supseteq \phi$. Since Im $f_\Omega = (\phi, \alpha)$, it follows that $f_\Omega(x) = \alpha = f_\Omega(y) = g_\Omega(x) = h_\Omega(y) = f_\Omega(x\text{-}y)$.

From proposition 3.3 and the facts that $g_\Omega(y) \subseteq m = g_\Omega(x)$ and $h_\Omega(x) \subseteq m = h_\Omega(y)$.

Thus $g_\Omega(x - y) = g_\Omega(y)$ and $h_\Omega(x - y) = g_\Omega(x)$.

So that $f_\Omega(x - y) \cap \alpha = g_\Omega(y) \cup h_\Omega(x) \subseteq m$, the desired contradiction.

4. Properties of Product of (α,β)-Soft Ring and (α,β)-Soft Ideal:

4.1 Theorem:

Let f_Ω and f_χ be two (α,β)-soft rings over U. Then $f_\Omega \wedge f_\chi$ is (α,β)-soft ring over U.

Proof:

Let $(x_1, y_1), (x_2, y_2) \in \Omega \times \chi$. Then

$$\begin{aligned} f_{\Omega\wedge\chi}((x_1,y_1)-(x_2,y_2)) \cap \alpha &= f_{\Omega\wedge\chi}(x_1-x_2, y_1-y_2) \cap \alpha \\ &= f_\Omega(x_1-x_2) \cap f_\chi(y_1-y_2) \cap \alpha \\ &\supseteq (f_\Omega(x_1) \cap f_\Omega(x_2)) \cap (f_\chi(y_1) \cap f_\chi(y_2)) \cup \beta \\ &= ((f_\Omega(x_1) \cap f_\chi(y_1)) \cup \beta) \cap ((f_\Omega(x_2) \cap f_\chi(y_2)) \cup \beta) \\ &= f_{\Omega\wedge\chi}(x_1,y_1) \cap f_{\Omega\wedge\chi}(x_2,y_2) \cup \beta \end{aligned}$$

And

$$\begin{aligned} f_{\Omega\wedge\chi}((x_1,y_1)(x_2,y_2)) \cap \alpha &= f_{\Omega\wedge\chi}(x_1x_2, y_1y_2) \cap \alpha \\ &= f_\Omega(x_1x_2) \cap f_\chi(y_1y_2) \cap \alpha \\ &\supseteq (f_\Omega(x_1) \cap f_\Omega(x_2)) \cap (f_\chi(y_1) \cap f_\chi(y_2)) \cup \beta \\ &= ((f_\Omega(x_1) \cap f_\chi(y_1)) \cup \beta) \cap ((f_\Omega(x_2) \cap f_\chi(y_2)) \cup \beta) \\ &= f_{\Omega\wedge\chi}(x_1,y_1) \cap f_{\Omega\wedge\chi}(x_2,y_2) \cup \beta \end{aligned}$$

Therefore, $f_\Omega \wedge f_\chi$ is (α,β)-soft ring over U. Note that $f_\Omega \vee f_\chi$ is not (α,β)-soft ring over U.

4.1 Example:

Assume that $U = S_3$ is the universal set. Let $\Omega = Z_5$ and $\chi = \left\{\begin{pmatrix} a & a \\ b & b \end{pmatrix} / a, b \in Z_2\right\}$, 2×2 matrices with Z_5 terms, be sets of parameters.

We define (α,β)-soft ring f_Ω over $U = S_3$ by

$f_\Omega(0) = S_3$, $f_\Omega(1) = \{(1), (1\ 2), (1\ 3\ 2)\}$, $f_\Omega(2) = \{(1), (1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}$,
$f_\Omega(3) = \{(1), (1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}$, $f_\Omega(4) = \{(1), (1\ 2), (1\ 3\ 2)\}$

We define (α,β)-soft ring f_χ over $U = S_3$ by

$f_\chi\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) = S_3$, $f_\chi\left(\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\right) = \{(1), (1\ 2), (1\ 3\ 2)\}$,

$f_\chi\left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\right) = \{(1), (1\ 3), (1\ 3\ 2)\}$,

$f_\chi\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right) = \{(1), (1\ 2\ 3), (1\ 3\ 2)\}$.

Then $f_\Omega \vee f_\chi$ is (α,β)-soft ring over U.

4.2 Theorem:

Let f_Ω and f_χ be two (α,β)-soft ideals over U. Then $f_\Omega \wedge f_\chi$ is (α,β)-soft ideal over U.

Proof:

We showed that if f_Ω and f_χ are two (α,β)-soft rings over U. Then $f_\Omega \wedge f_\chi$ is (α,β)-soft ring over U in the previous theorem. Let $(x_1, y_1), (x_2, y_2) \in \Omega \times \chi$. Then,

$$\begin{aligned} f_{\Omega\wedge\chi}((x_1,y_1)(x_2,y_2)) \cap \alpha &= f_{\Omega\wedge\chi}(x_1x_2, y_1y_2) \cap \alpha = f_\Omega(x_1x_2) \cap f_\chi(y_1y_2) \cap \alpha \\ &\supseteq f_\Omega(x_1) \cap f_\chi(y_1) \cup \beta = f_{\Omega\wedge\chi}(x_1, y_1) \end{aligned}$$

And

$$\begin{aligned} f_{\Omega\wedge\chi}((x_1,y_1)(x_2,y_2)) \cap \alpha &= f_{\Omega\wedge\chi}(x_1x_2, y_1y_2) \cap \alpha = f_\Omega(x_1x_2) \cap f_\chi(y_1y_2) \cap \alpha \\ &\supseteq f_\Omega(x_2) \cap f_\chi(y_2) \cup \beta = f_{\Omega\wedge\chi}(x_2, y_2) \end{aligned}$$

Therefore, $f_\Omega \wedge f_\chi$ is (α,β)-soft ideal over U. Note that $f_\Omega \vee f_\chi$ is not (α,β)-soft ideal over U.

4.3 Theorem:

Let f_Ω and g_Ω be two (α,β)-soft rings over U. Then $f_\Omega \tilde{\cap} g_\Omega$ is (α,β)-soft ring over U.

Proof:

Let $x, y \in \Omega$. Then,

$$\begin{aligned} (f_\Omega \tilde{\cap} g_\Omega)(x-y) \cap \alpha &= f_\Omega(x-y) \cap g_\Omega(x-y) \cap \alpha \supseteq (f_\Omega(x) \cap f_\Omega(y) \cup \beta) \cap (g_\Omega(x) \cap g_\Omega(y) \cup \beta) \\ &= (f_\Omega(x) \cap g_\Omega(x) \cup \beta) \cap (f_\Omega(y) \cap g_\Omega(y) \cup \beta) \\ &= (f_\Omega \tilde{\cap} g_\Omega)(x) \cap (f_\Omega \tilde{\cap} g_\Omega)(y) \cup \beta. \end{aligned}$$

$$\begin{aligned} (f_\Omega \tilde{\cap} g_\Omega)(xy) \cap \alpha &= f_\Omega(xy) \cap g_\Omega(xy) \cap \alpha \supseteq (f_\Omega(x) \cap f_\Omega(y) \cup \beta) \cap (g_\Omega(x) \cap g_\Omega(y) \cup \beta) \\ &= (f_\Omega(x) \cap g_\Omega(x) \cup \beta) \cap (f_\Omega(y) \cap g_\Omega(y) \cup \beta) \\ &= (f_\Omega \tilde{\cap} g_\Omega)(x) \cap (f_\Omega \tilde{\cap} g_\Omega)(y) \cup \beta. \end{aligned}$$

Therefore, $f_\Omega \tilde{\cap} g_\Omega$ is (α,β)-soft ring over U.

4.4 Theorem:

Let f_Ω and g_Ω be two (α,β)-soft ideals over U. Then $f_\Omega \tilde{\cap} g_\Omega$ is (α,β)-soft ideal over U.

Proof:

In the above theorem 4.1, we showed that f_Ω and g_Ω are two (α,β)-soft rings over U, Then $f_\Omega \tilde{\cap} g_\Omega$ is (α,β)-soft ring over U.

Let x,y ∈ Ω. Then,

$$(f_\Omega \tilde{\cap} g_\Omega)(xy) \cap \alpha = f_\Omega(xy) \cap g_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cap g_\Omega(x) \cup \beta$$
$$= (f_\Omega \tilde{\cap} g_\Omega)(x) \cup \beta$$

And $$(f_\Omega \tilde{\cap} g_\Omega)(xy) \cap \alpha = f_\Omega(xy) \cap g_\Omega(xy) \cap \alpha \supseteq (f_\Omega(x) \cap f_\Omega(y) \cup \beta) \cap (g_\Omega(x) \cap g_\Omega(y) \cup \beta)$$
$$= (f_\Omega \tilde{\cap} g_\Omega)(x) \cap (f_\Omega \tilde{\cap} g_\Omega)(y) \cup \beta$$

Therefore, $f_\Omega \tilde{\cap} g_\Omega$ is (α,β)-soft ideal over U.

5. Homomorphisms of (α,β)-Soft Ring and (α,β)-Soft Ideal:

5.1 Theorem:

Let f_Ω be (α,β)-soft ring over U and 'h' be a surjective homomorphism from Ω to χ. Then $h(f_\Omega)$ is a (α,β)-soft ring over U.

Proof:

Since 'h' is a surjective homomorphism from Ω to χ , there exist x,y ∈ Ω such that u=h(x) and v=h(y) for all u,v ∈ χ. Then

$$(h(f_\Omega))(u-v) \cap \alpha = \cup\{f_\Omega(z) \cap \alpha, z \in \Omega, h(z) = u - v\}$$
$$= \cup\{f_\Omega(x-y) \cap \alpha; x,y \in \Omega, u = h(x), v = h(y)\}$$
$$\supseteq \cup\{f_\Omega(x) \cap f_\Omega(y) \cup \beta; x,y \in \Omega, u = h(x), v = h(y)\}$$
$$= \{\cup\{f_\Omega(x); x \in \Omega, u = h(x)\}\} \cap \{\cup\{f_\Omega(y); y \in \Omega, v = h(y)\}\} \cup \beta$$
$$= (h(f_\Omega))(u) \cap (h(f_\Omega))(v) \cup \beta$$

And ,

$$(h(f_\Omega))(uv) \cap \alpha = \cup\{f_\Omega(z) \cap \alpha, z \in \Omega, h(z) = uv\}$$
$$= \cup\{f_\Omega(xy) \cap \alpha; x,y \in \Omega, u = h(x), v = h(y)\}$$
$$\supseteq \cup\{f_\Omega(x) \cap f_\Omega(y) \cup \beta; x,y \in \Omega, u = h(x), v = h(y)\}$$
$$= \{\cup\{f_\Omega(x); x \in \Omega, u = h(x)\}\} \cap \{\cup\{f_\Omega(y); y \in \Omega, v = h(y)\}\} \cup \beta$$
$$= (h(f_\Omega))(u) \cap (h(f_\Omega))(v) \cup \beta$$

Hence, $h(f_\Omega)$ is a (α,β)-soft ring over U.

5.2 Theorem:

Let f_Ω be (α,β)-soft ideal over U and 'h' be a surjective homomorphism from Ω to χ. Then $h(f_\Omega)$ is a (α,β)-soft ideal over U.

Proof:

We know that $h(f_\Omega)$ is a (α,β)-soft ring over U, under these conditions as shown in the above theorem. Suppose that u=h(x) and v=h(y) for some x,y ∈ Ω such that u,v ∈ χ. Then

$$(h(f_\Omega))(uv) \cap \alpha = \cup\{f_\Omega(z) \cap \alpha, z \in \Omega, h(z) = uv\}$$
$$= \cup\{f_\Omega(xy) \cap \alpha; x,y \in \Omega, u = h(x), v = h(y)\}$$
$$\supseteq \cup\{f_\Omega(x) \cup \beta; x \in \Omega, u = h(x)\}$$
$$= (h(f_\Omega))(u)$$

and

$$(h(f_\Omega))(uv) \cap \alpha = \cup\{f_\Omega(z) \cap \alpha, z \in \Omega, h(z) = uv\}$$
$$= \cup\{f_\Omega(xy) \cap \alpha; x,y \in \Omega, u = h(x), v = h(y)\}$$
$$\supseteq \cup\{f_\Omega(y) \cup \beta; y \in \Omega, v = h(y)\}$$
$$= (h(f_\Omega))(v)$$

Hence, $h(f_\Omega)$ is a (α,β)-soft ideal over U.

5.3 Theorem:

Let f_χ be (α,β)-soft ring over U and 'h' be a homomorphism from Ω to χ. Then $h^{-1}(f_\chi)$ is a (α,β)-soft ring over U.

Proof:

Let x,y ∈ Ω. Then

$$h^{-1}(f_\chi)(x-y) \cap \alpha = f_\chi(h(x-y)) \cap \alpha = f_\chi(h(x)-h(y)) \cap \alpha$$
$$\supseteq f_\chi(h(x)) \cap f_\chi(h(y)) \cup \beta = h^{-1}(f_\chi)(x) \cap h^{-1}(f_\chi)(y) \cup \beta \text{ and,}$$
$$h^{-1}(f_\chi)(xy) \cap \alpha = f_\chi(h(xy)) \cap \alpha = f_\chi(h(x)h(y)) \cap \alpha$$
$$\supseteq f_\chi(h(x)) \cap f_\chi(h(y)) \cup \beta = h^{-1}(f_\chi)(x) \cap h^{-1}(f_\chi)(y) \cup \beta$$

Hence, $h^{-1}(f_\chi)$ is a (α,β)-soft ring over U.

5.4 Theorem:

Let f_χ be (α,β)-soft ideal over U and 'h' be a homomorphism from Ω to χ. Then $h^{-1}(f_\chi)$ is a (α,β)-soft ideal over U.

Proof:

We know that $h^{-1}(f_\chi)$ is a (α,β)-soft ring over U, under these conditions as shown in the above theorem 5.3. Then for all x,y ∈ Ω, $h^{-1}(f_\chi)(xy) \cap \alpha = f_\chi(h(xy)) \cap \alpha \supseteq f_\chi(h(x)) \cup \beta = h^{-1}(f_\chi)(x) \cup \beta$

And

$$h^{-1}(f_\chi)(xy) \cap \alpha = f_\chi(h(xy)) \cap \alpha \supseteq f_\chi(h(y)) \cup \beta = h^{-1}(f_\chi)(y) \cup \beta$$

Hence, $h^{-1}(f_\chi)$ is a (α,β)-soft ideal over U.

5.1 Definition:

Let Ω be a ring and $f_\Omega, g_\Omega \in S(U)$. Then $f_\Omega \mp g_\Omega$, $-f_\Omega$, $f_\Omega g_\Omega \in S(U)$ are defined as follows;

$(f_\Omega \mp g_\Omega)(x) \cap \alpha = \cup\{f_\Omega(y) \cap g_\Omega(z) \cup \beta \ / \ y,z \in \Omega, y \mp z = x\}$

$(-f_\Omega)(x) \cap \alpha = f_\Omega(-x) \cup \beta$

$(f_\Omega g_\Omega)(x) \cap \alpha = \cup\{f_\Omega(y) \cap g_\Omega(z) \cup \beta \ / \ y,z \in \Omega, yz = x\}$ for all x ∈ Ω.

$f_\Omega + g_\Omega$, $f_\Omega - g_\Omega$, $f_\Omega g_\Omega$ are called sum, difference and product of f_Ω and g_Ω, respectively, and $-f_\Omega$ is called the negative of f_Ω.

5.5 Theorem:

Let Ω be a ring and $f_\Omega, g_\Omega, h_\Omega \in S(U)$. Then $f_\Omega(g_\Omega + h_\Omega) \subseteq (f_\Omega g_\Omega + f_\Omega h_\Omega)$.

Proof:

Assume that w∈ Ω and u, v ∈ Ω such that uv=w. Then

$f_\Omega(g_\Omega + h_\Omega)(w) = \cup\{f_\Omega(u) \cap (g_\Omega + h_\Omega)(v) \ / \ u,v \in \Omega, uv=w\}$ and

$$\begin{aligned} f_\Omega(u) \cap (g_\Omega + h_\Omega)(v) &= f_\Omega(u) \cap \{\cup\{g_\Omega(y) \cap h_\Omega(z) \ / \ y,z \in \Omega, y+z=v\} \\ &= \cup \{(f_\Omega(u) \cap g_\Omega(y)) \cap (f_\Omega(u) \cap h_\Omega(z)) \ / \ y,z \in \Omega, y+z=v\} \\ &= \cup \{(f_\Omega(u) \cap g_\Omega(y)) \cap (f_\Omega(u) \cap h_\Omega(z)) \ / \ y,z \in \Omega, uy+uz=uv\} \\ &\subseteq \cup \{(f_\Omega g_\Omega)(uy) \cap (f_\Omega h_\Omega)(uz)) \ / \ y,z \in \Omega, uy+uz=uv\} \\ &= (f_\Omega g_\Omega + f_\Omega h_\Omega)(w) \end{aligned}$$

Thus $f_\Omega(g_\Omega + h_\Omega)(w) \subseteq (f_\Omega g_\Omega + f_\Omega h_\Omega)(w)$ for all w∈R. Hence, $f_\Omega(g_\Omega + h_\Omega) \subseteq (f_\Omega g_\Omega + f_\Omega h_\Omega)$.

5.6 Theorem:

Let f_Ω is (α,β)-soft right ideal and g_χ is (α,β)-soft left ideal over U. Then $f_\chi g_\chi \subseteq f_\chi \tilde{\cap} g_\chi$.

Proof:

If $(f_\chi g_\chi)(x) = \emptyset$, then it is clear that $f_\chi g_\chi \subseteq f_\chi \tilde{\cap} g_\chi$.

Suppose $(f_\chi g_\chi)(x) \neq \emptyset$ and

$(f_\chi g_\chi)(x) = \cup\{f_\chi(y) \cap g_\chi(z) \cap \alpha \ / \ y,z \in \Omega, x=yz\}$

Since, f_Ω is (α,β)-soft right ideal and g_χ is (α,β)-soft left ideal over U, we have

$f_\chi(x) = f_\chi(yz) \cap \alpha \supseteq f_\chi(y) \cup \beta$ and $g_\chi(x) = g_\chi(yz) \cap \alpha \supseteq g_\chi(z) \cup \beta$. Hence

$$\begin{aligned} (f_\chi g_\chi)(x) \cap \alpha &= \cup\{f_\chi(y) \cap g_\chi(z) \cap \alpha \ / \ y,z \in \Omega, x=yz\} \\ &\subseteq f_\chi(x) \cap g_\chi(x) \cup \beta \\ &= (f_\chi \tilde{\cap} g_\chi)(x) \cup \beta \end{aligned}$$

Therefore, $f_\chi g_\chi \subseteq f_\chi \tilde{\cap} g_\chi$.

5.2 Definition:

Let f_Ω is (α,β)-soft ring over U. Then centre-set of f_Ω, denoted by λf_Ω ,is defined as $\lambda f_\Omega = \{x \in \Omega \ ; f_\Omega(x) = f_\Omega(0_\Omega)\}$.

5.7 Theorem:

Let f_Ω be a (α,β)-soft ring over U. Then λf_Ω is a sub ring of Ω.

Proof:

It is clear that $0_\Omega \in \lambda f_\Omega \subseteq \Omega$. Let x,y ∈ λf_Ω. Then we have $f_\Omega(x) = f_\Omega(y) = f_\Omega(0_\Omega)$.

It follows that , $f_\Omega(x - y) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta = f_\Omega(0_\Omega) \cap f_\Omega(0_\Omega) \cup \beta = f_\Omega(0_\Omega)$

And $f_\Omega(xy) \cap \alpha \supseteq f_\Omega(x) \cap f_\Omega(y) \cup \beta = f_\Omega(0_\Omega) \cap f_\Omega(0_\Omega) \cup \beta$

$= f_\Omega(0_\Omega)$ implying that $x - y, xy \in \lambda f_\Omega$. Therefore , λf_Ω is a sub ring of Ω.

5.8 Theorem:

Let f_Ω be a (α,β)-soft ideal over U. Then λf_Ω is a ideal of Ω.

Proof:

The proof can be made by using theorem 5.7

5.9 Theorem:

Let f_Ω be a (α,β)-soft ring over U and $t \subseteq f_\Omega(0_\Omega)$. Then $f_\Omega{}^t$ is a sub ring of Ω.

Proof:

It is clear that $0_\Omega \in f_\Omega{}^t \subseteq \Omega$. Let x,y∈ $f_\Omega{}^t$, then $f_\Omega(x) \supseteq t$ and $f_\Omega(y) \supseteq t$. z

It follows that , $f_\Omega(x-y) \supseteq f_\Omega(x) \cap f_\Omega(y) \supseteq \alpha$ and $f_\Omega(xy) \supseteq f_\Omega(x) \cap f_\Omega(y) \supseteq \alpha$. Thus $x-y,\ xy \in f_\Omega{}^t$. Therefore , $f_\Omega{}^t$ is a sub ring of Ω.

5.10 Theorem:

Let f_Ω be a (α,β)-soft ideal over U and $t \subseteq f_\Omega(0_\Omega)$. Then $f_\Omega{}^t$ is an ideal of Ω.

Proof:

The proof can be made by using theorem 5.9

6. Conclusion:

Here, we define (α,β)-soft ring that as alternative definition to soft rings. We then focused on the concepts of (α,β)-soft ideal, sum, difference, product of two soft sets, negation of a soft set and study their properties. To extend over work, further research could be done in other algebraic structures such as fields as in the case of (α,β)-soft ring.

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