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Mathematical Model of a Tiny Fluid in Porous Media. Spilled Oil as a Case Study

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ABSTRACT

A dynamic model for the movement of spilled oil through a porous medium was developed. The equation being non-linear partial differential equation and parabolic forms, was thus solved numerically with the adoption of the Crank Nicholson finite difference scheme. A mathematical algorithm to solve this system of equation was developed and implemented into simulation program. The graph of the concentration against axial distance (height), concentration against radial distance at different time were plotted from simulation results obtained which shows decrease in the concentration of solute (contaminant) through the media.

Keyword: Concentration; contaminants; distance; mathematical model; porous media.

INTRODUCTION

Groundwater is a major source of water supply, especially in arid or semi-arid areas where surface water is limited. Because groundwater is filtered by flow through the formation, it generally requires little treatment for use as a water supply. Groundwater can be considered as subsurface storage of water with limited evaporation [1]. However, in recent years, contaminant of groundwater by industrial products has become a problem of growing concern. Nigeria being one of the oil producing countries, currently produces, transports, uses 3.0mbpd of crude oil and eventually disposes of in the region where it is being produced. Many of the crude oil components are highly toxic and partially water-soluble. Thus, they pose a potential threat to large volumes of groundwater if they are introduced into the subsurface [2]. Furthermore, many toxic contaminants commonly found in the subsurface occur as slightly soluble and highly volatile fluids that are immiscible with water. Despite their low volatility, these fluids pose a widespread potential threat to groundwater and soil resources [3].

Introduction of contaminants into the marine environment have been found to have serious impact and indigenous marine population [1]. The effects of groundwater contamination ranges from health, social to economic. Some natural processes may be employed to control the movement of contaminants (pollutants). These processes include biodegradation in the groundwater zone, though it proceeds very slowly.

Mathematical models have been developed in an effort to qualify some of the physical processes described above. Many experimental studies had also been carried out to help in the understanding of these processes and to help in model verification [4]. Most of these experiments and models were designed to assess the impact of crude oil components contaminants on the subsurface, since this problem has been recognized for much longer time than that of other types of chemical contaminations. One method to examine microscopic flow is to create a synthetic model of the pore space from clear fluid through the synthetic medium so that flow can be directly observed [5]. Another method is to study thin sections from the medium, and create relationships between the fractal dimension of the pores and permeability, porosity and conductivity [6]. A third method is to use NMR to obtain average velocity fields of the fluid as it moves through a porous medium [7]. Finally, computer simulations have recently been shown to compare well with laboratory experiments and therefore may be useful for studying microscopic flow [8].

There are two main ways to do computer simulations of fluid flow- the finite difference or finite element method and the lattice-gas method [9]. The finite difference method has been used extensively and can be applied to problems such as modeling the flow of air around a space and the calculation of water in an ocean basin [10]

Eddie (1999) modelled fluid flow in porous media using NMR imaging and numerical simulation. She used nuclear magnetic resonance (NMR) imaging to obtain a three-dimensional image of the pore structure in a limestone core. This image was converted into boundary conditions for simulation of fluid flow through the rock using the lattice gas method.

Bear developed numerical models for the transport of solutes in porous media. A major concern with numerical models is numerical dispersion, because the average concentration is often used in a grid element. He then solved the partial differential equation describing unsteady transport of solute in a two- dimensional, isentropic homogeneous porous media using integral method [11].

In this research work, a study to ascertain the extent to which soil sample has been affected by spilled oil i.e. the radius of influence and the concentration at such point, the depth to which the spilled oil has travelled.

Mathematical Model

In order to develop a model able to describe the transport of contaminants (solutes) in porous media, the following assumptions are made:

(i) The elemental volume of oil is assumed to be cylindrical.
(ii) Assuming there are no losses caused by adsorption.
(iii) Only oil flow within the pores.
(iv) Mass flux with respect to angular, direction is negligible compared to that in axial and radial directions.
(v) Mass transfer only in radial, r and axial, z directions.
(vi) Molecular diffusion, porosity, and permeability are assumed to be constants.
(vii) It is also assumed that the diffusivity in the r and z directions is approximately equal to each other.

The mathematical model is based on the continuity equation.

Rate of solute In – Rate of solute Out + Rate of mass generated by reaction = Rate of accumulation.

Subject to those aforementioned assumptions and continuity equation written above, the following equations represent the transport of solute in porous media.

$$\frac{\partial c}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rN_r)+\frac{1}{r}\frac{\partial N_\theta}{\partial \theta}+\frac{\partial N_z}{\partial z}=0 \quad (1)$$

Since angular direction (θ) is negligible, equation 1 reduces to

$$\frac{\partial c}{\partial t}+\frac{1}{r}\frac{\partial (rN_r)}{\partial r}+\frac{\partial N_z}{\partial z}=0 \quad (2)$$

Applying Fick's law to a homogeneous soil layer, where there is a linear concentration gradient that is steady state in nature.

$$N_A = J_A + X_A(N_A+N_B) \quad (3)$$

Where $X_A=\frac{C_A}{C}$

$$J_A=-D_{AB}\frac{\partial C_A}{\partial X} \quad [12] \quad (4)$$

$$X_A(N_A+N_B)=VC_A \quad (5)$$

Putting equations 4 and 5 in equation 3 to obtain:

$$N_A=VC_A-D_{AB}\frac{\partial C_A}{\partial X} \quad (6)$$

Mass transfer in both r and z directions are written as:

$$N_r=CV_r-D_r\frac{\partial C}{\partial r} \quad (7)$$

$$N_z=CV_z-D_z\frac{\partial C}{\partial z} \quad (8)$$

Differentiating and simplifying second term of equation 1,

$$\frac{\partial (rN_r)}{\partial r}=N_r+r\frac{\partial N_r}{\partial r} \quad (9)$$

Putting equation 7 in equation 9 to obtain equations 10 and 11,

$$\frac{\partial (rN_r)}{\partial r}=CV_r-D_r\frac{\partial C}{\partial r}+r\frac{\partial N_r}{\partial r} \quad (10)$$

$$=CV_r-D_r\frac{\partial C}{\partial r}+rV_r\frac{\partial C}{\partial r}-rD_r\frac{\partial^2 C}{\partial r^2} \quad (11)$$

Differentiate N_z with respect to z in equation (8) to get equation 12,

$$\frac{\partial N_z}{\partial z}=V_z\frac{\partial C}{\partial z}-D_z\frac{\partial^2 C}{\partial z^2} \quad (12)$$

Inculcate equations 11 and 12 in equations 2 to obtain equation 13

$$\frac{\partial C}{\partial t}+\frac{1}{r}(CV_r-D_r\frac{\partial C}{\partial r}+rV_r\frac{\partial C}{\partial r}-rD_r\frac{\partial^2 C}{\partial r^2})+V_z\frac{\partial C}{\partial z}-D_z\frac{\partial^2 C}{\partial z^2}= \quad (13)$$

It is assumed that the diffusivity in the r and z directions are approximately equal to each other, therefore,

$$D_z=D_r=D_e \quad (14)$$

Equation 13 becomes

$$\frac{\partial C}{\partial t} = D_e \frac{\partial^2 C}{\partial z^2} + D_e \frac{\partial^2 C}{\partial r^2} - V_z \frac{\partial C}{\partial z} - (V_r - \frac{D_e}{r})\frac{\partial C}{\partial r} - \frac{CV_r}{r} \quad (15)$$

Initial condition:

$C = 0$ @ $t = 0$ for all r > 0 and z >0 (16)

Boundary condition:

$C = C_o = 1 @ r = 0, z = 0$ and $t \geq 0$ (17)

$C = 0 @ z = \infty, 0 \leq r \leq R$ (18)

Method of Numerical Solution

The method adopted for the solution of the system of partial differential equation (18) for the model of tiny fluid (oil) in porous media is the Crank-Nicholson Scheme. A mathematical algorithm to solve these system of equation was developed and implemented into computer program using FORTRAN Software. The parameters reported by [13] were used for simulation in this study, which are given below:

$$D_e = 4.8x10^{-4} m^2/s, V_z = 3.5 m/s, \ A = 3.7m^2$$
$$V_r^* = 2.0m$$

SIMULATION RESULT AND DISCUSSION

In the modeling process, various assumptions were made in order to simplify the model equation. Only oil flow within the pores of the porous medium and the mass flux with respect to angular direction considered negligible compared to that in axial and radial direction. A stationary porous medium was assumed against a continuous flow since this is preferred in real life situation due to the high volume of spillage in most cases. A material balance equation was obtained which result into non-linear partial differential equations, which describe the change in concentration of contaminant as a function of time, radial distance and axial distance. Crank-Nicolson finite difference scheme was employed to solve the equation. The final equation obtained was then used in the design of the simulation program. The results of the numerical solution of the model are presented in figure 1 to 3.

Since no experimental studies were carried out, the input data used for the simulation program were obtained from Conningham and Geenkoplis, [13] work, however, some values of parameters were assumed to suit the purpose of the work.

The simulation of the model gives a solution for the plot of concentration against axial distance (height) at 24 and 48 seconds and concentration against radial distance at 48seconds. The profile shows decrease in the concentration of the contaminant as diffusion progresses.

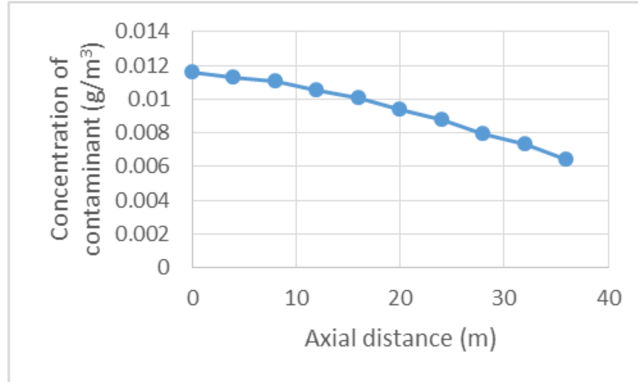

Figure 1: Concentration of contaminant against axial distance (m) for 24s.

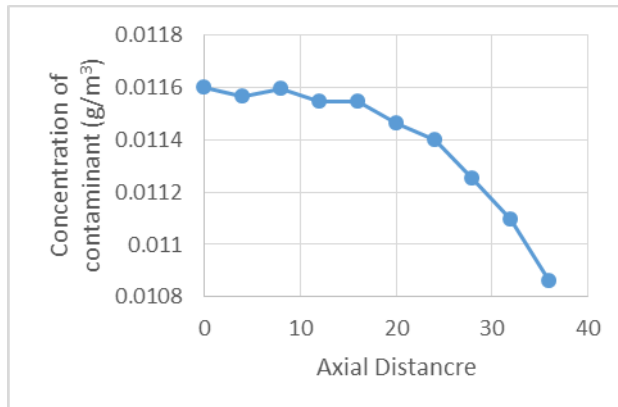

Figure 2: Concentration of contaminant against axial distance (m) for 48s.

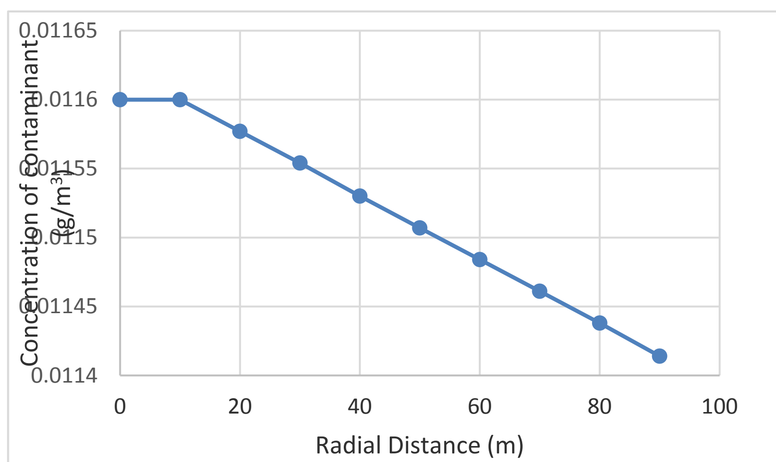

Figure 3 : Concentration of contaminant against radial distance (m) for 48s

CONCLUSION

Groundwater contamination has become an important issue, which poses a serious threat to drinking water quality. The simulation of contaminant transport through the subsurface is necessary in order to effectively design mitigaham methods for clean-up and prevention of the deterioration of groundwater. It can be concluded that decrease in concentration of contaminants in downward distance and radially and contaminant speed has no effect on the result.

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Nomenclature

A_v	Absorbed surface area , m^2
C	Concentration of spilled oil, g/m^3
C_A	Concentration of the absorbed surface, g/m^3
D_e	Effective diffusivity within the medium, m^2/s
J	Flux of diffusion relative to molar averege, mol/m^2s
N_A	A-direction flux, mol/m^2s
N_B	B-direction flux, mol/m^2s
N_r	r-direction flux, mol/m^2s
N_z	z-direction flux, mol/m^2s
r	Radial distance, m
V	Contaminant speed, m/s
V_r	Velocity in radial direction, m/s
Z	Axial distance, m.
Vz	Velocity in axial direction, m/s

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