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COMPACT HAUSDORFF SPACES AND INVERSE LIMIT SPACES

By

Mitsunobu Shiraki

(Received September 30, 1970)

The purpose of this note is to show that any compact Hausdorff space is represented as the inverse limit space of an inverse limit system in which each space is a compact subspace of a finite dimensional Euclidean cube.

Let \( X \) be a compact Hausdorff space and let \( I = [0, 1] \) be the closed interval with the usual topology. Suppose that \( C(X) = \{ \varphi_\mu : \mu \in M \} \) is the family of all continuous mappings from \( X \) to \( I \). Now we consider a family \( D = \{ \{ \varphi_\mu \}_\mu \in \alpha : \alpha \) is a finite subset of \( M \}. \)

And we define an order relation \( \prec \) in \( D \) by saying \( f \prec g \), where \( f = \{ \varphi_\mu \}_\mu \in \alpha \) and \( g = \{ \varphi_\mu \}_\mu \in \beta \), if \( \alpha \subset \beta \). Then \( (D, \prec) \) is a directed set. Because, given \( f = \{ \varphi_\mu \}_\mu \in \alpha \in D \) and \( g = \{ \varphi_\mu \}_\mu \in \beta \in D \), we take \( h = \{ \varphi_\mu \}_\mu \in \alpha \cup \beta \). Of course \( h \) is in \( D \) (Such the \( h \) is denoted by \( f \lor g \)). Then we have obviously \( h \succ f \) and \( h \succ g \), hence \( (D, \prec) \) is a directed set.

For each \( \varphi_\mu \in C(X) \), setting \( H_\mu = \varphi_\mu(X) \), \( H_\mu \) is a compact subset of \( I \). And for each \( f = \{ \varphi_\mu \}_\mu \in \alpha \in D \), we define a mapping \( f : X \rightarrow \prod(H_\mu : \mu \in \alpha) \) by

\[
  f(x) = (\varphi_\mu(x) : \mu \in \alpha),
\]

and set \( X_f = f(X) \). Then \( f \) is continuous and \( X_f \) is a compact subspace of a finite dimensional Euclidean space.

Next, we consider the family \( \{ X_f : f \in D \} \). For each \( f, g \in D \) with \( f \prec g \), a mapping \( \pi_{fg} : X_g \rightarrow X_f \) is defined by

\[
  \pi_{fg}(x) = f(x).
\]

Then \( \pi_{fg} \) has the following properties:

1. \( \pi_{fg} \) is well defined.
2. \( \pi_{fg} \) is continuous onto.
3. \( \pi_{ff} \) is identity.
4. If \( f \prec g \prec h \), then \( \pi_{fg} \pi_{gh} = \pi_{fh} \).

In fact, suppose \( f \prec g \), where \( f = \{ \varphi_\mu \}_\mu \in \alpha \) and \( g = \{ \varphi_\mu \}_\mu \in \beta \). If \( g(x) = g(y) \), then \( \varphi_\mu(x) = \varphi_\mu(y) \) for \( \mu \in \beta \). Since \( f \prec g \), we have \( \alpha \subset \beta \), so that \( \varphi_\mu(x) = \varphi_\mu(y) \) for \( \mu \in \alpha \). Hence \( f(x) = f(y) \), and we have (1). (2) is evident since \( \pi_{fg} \) is a projection of the product space onto its factor space. (3) and (4) follow immediately from the definition of the mapping \( \pi_{fg} \). Therefore we can conclude that the family \( \{ X_f, \pi_{fg} \} \) is an inverse limit system over the directed set \( D \).

Moreover, since \( X_f \) is a non empty compact Hausdorff space, the inverse limit space
$X_\infty$ of the inverse limit system $(X_f, \pi_{f\beta})$ is non empty compact Hausdorff [1].

Now, the evaluation mapping $e : X \rightarrow \Pi(X_f : f \in D)$ is continuous [2]. And $e(x) \in X_\infty$ since $e(x) = (f(x) : f \in D)$ and $\pi_{f\beta}f(x) = h(x)$ whenever $h < f$.

The mapping $e$ is injective. To prove this, suppose that $x$ and $y$ are two distinct points of $X$. Since $X$ is a compact Hausdorff space, there exists a mapping $\varphi, \epsilon O(X)$ such that

$$\varphi_*(x) = 0 \text{ and } \varphi_*(y) = 1.$$  

Take $h = \{\varphi_\beta\}$ consisting of only one element $\varphi_\beta$. Then $h$ is a member of $D$ and $h(x) \neq h(y)$. Thus $e(x) \neq e(y)$, so that $e$ is injective.

Next, we shall show that $e(X)$ is dense in $X_\infty$. For this, it is sufficient to prove that every open neighborhood of any point of $X_\infty$ contains a point of $e(X)$. Let $(x_f) \in X_\infty$, and suppose that $\Pi(U_f : f \in D)$ is an arbitrary open neighborhood of $(x_f)$, where each $U_f$ is an open neighborhood of $x_f$ in $X_f$, and $U_f = X_f$ for all but a finite number of $f \in D$. Let the finite elements of $D$ be $(g, \cdots, h)$, and take $\psi\epsilon g \vee \cdots \vee h$. Then there exists a $y \in X$ such that $x_\psi = \psi(y) \in X_\psi$, since $x_\psi \in X_\psi$ and $X_\psi = \psi(X)$. When considering $(f(y) : f \in D) \in X_\infty$,

$$\pi_{\psi\beta} \psi(y) = g(y), \cdots, \pi_{\psi h} \psi(y) = h(y),$$

and since $\psi(y) = x_\psi$ and for $l < \psi \pi_{f\beta}x_\psi = x_l$, we have

$$g(y) = x_g, \cdots, h(y) = x_h.$$  

It follows that $(f(y)) \epsilon \Pi(U_f : f \in D)$. This proves that $e(X) = X_\infty$.

Since $X$ is a compact Hausdorff space and $e$ is a continuous mapping, $e(X)$ also is a compact Hausdorff space. Moreover since $X_\infty$ is Hausdorff, $e(X)$ is closed in $X_\infty$. Consequently,

$$e(X) = e(X) = X_\infty,$$

and therefore $e$ is homomorphism.

Thus we have established the following theorem.

**Theorem.** Every compact Hausdorff space is homeomorphic to the inverse limit space of an inverse limit system in which each space is a compact subspace of a finite dimensional Euclidean space.

**References**
