Metacognition and mathematics education

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Abstract

The role of metacognition in mathematics education is analyzed based on theoretical and empirical work from the last four decades. Starting with an overview on different definitions, conceptualizations and models of metacognition in general, the role of metacognition in education, particularly in mathematics education, is discussed. The article emphasizes the importance of metacognition in mathematics education, summarizing empirical evidence on the relationships between various aspects of metacognition and mathematics performance. As a main result of correlational studies, it can be shown that the impact of declarative metacognition on mathematics performance is substantial (sharing about 15–20% of common variance). Moreover, numerous intervention studies have demonstrated that "normal" learners as well as those with especially low mathematics performance do benefit substantially from metacognitive instruction procedures.

Keywords: Metacognition, Mathematics achievement. Training effects

1 Conceptualizations of metacognition

Although the concept of "metacognition" is now frequently used and investigated in various fields of psychology and education, its history is rather short. The first studies on metacognition were developmental in nature and initiated in the early 1970s by Ann Brown and John Flavell et al. (for reviews, see Brown, Bransford, Ferrara, & Campione, 1983; Flavell, Miller, & Miller, 2002; Goswami, 2008; Schneider & Pressley, 1997). Although various conceptualizations of the term "metacognition" have been used in literature on cognitive development, the concept has usually been broadly defined as any knowledge or cognitive activity that takes as its object, or regulates, any aspect of any cognitive enterprise (cf. Flavell, Miller, & Miller, 2002). According to this conceptualization, metacognition refers to people's knowledge of their own information-processing skills, as well as knowledge about the nature of cognitive tasks, and of strategies for coping with such tasks. Moreover, it also includes executive skills related to monitoring and self-regulation of one's own cognitive activities.

In a now classic paper, Flavell (1979) described three major facets of metacognition, namely metacognitive knowledge, metacognitive experiences, and metacognitive skills, that is, strategies controlling cognition. Declarative metacognitive knowledge refers to the segment of "world knowledge" that has to do with the human mind and its doings. According to Flavell et al. (2002), metacognitive knowledge about memory includes explicit, conscious, and factual knowledge about the importance of person, task, and strategy variables for memorizing and recalling information. For instance, knowing that an older child typically recalls more than a younger child is an indicator of person knowledge, and knowing that the time available for solving a mathematical problem is important for subsequent success indicates task-related metacognitive knowledge. Metacognitive experiences refer to a person's awareness and feelings elicited in a problem-solving situation (e.g., feelings of knowing), and metacognitive skills are believed to play a role in many types of cognitive activities such as oral communication of information, reading comprehension, attention, and memory. These facets of metacognition refer to a person's procedural knowledge, which Brown et al. (1983) referred to as "knowing how" and which can be further subdivided into monitoring and self-regulatory functions (see below). For an excellent discussion of more subtle distinctions among various aspects of metacognition, see Kuhn (1999, 2000).

From a historical perspective, the concept of "metamemory", that is, children's knowledge about memory, was explored first. Flavell's (1971) conception of metamemory was global, encompassing knowledge of all possible aspects of information storage and retrieval. Accordingly, metamemory included (but was not limited to) knowledge about memory functioning, difficulties, and strategies. Flavell and Wellman (1977) distinguished between two main metamemory categories, "sensitivity" and "variables".

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The “sensitivity“ category referred to mostly implicit, unconscious behavioral knowledge of when memory is necessary and thus was very close to subsequent definitions of *procedural metacognitive knowledge*. The “variables“ category referred to explicit, conscious, and factual knowledge about the importance of person, task, and strategy variables for memory performance (see above).

One impression that could be gleaned from the early research carried out by Flavell et al. was that a lot of metacognitive development was complete by age 8 or 9 years (e.g., Kreutzer, Leonard, & Flavell, 1975). Subsequently, Ann Brown et al. (Brown, 1978; Brown et al., 1983) counteracted this impression by focusing on procedural metamemory (“here-and-now-memory monitoring“) and children’s text processing. Research carried out by Brown et al. was able to demonstrate that metacognitive abilities develop quite slowly during the school years and that there is room for improvement even in adolescents and adults (see Brown et al., 1983).

The taxonomy of metamemory presented by Flavell and Wellman (1977) was not intended to be exhaustive. Since the late 1970s, a number of additions and changes have been suggested (for comprehensive reviews, see Holland Joyner & Kurtz-Costes, 1997; Schneider, 1999, 2010; Schneider & Pressley, 1997). For instance, Paris and Oka (1986) introduced a component labeled *conditional metacognitive knowledge* that focused on children’s ability to justify or explain their decisions concerning memory activities, whereas the declarative metamemory component first introduced by Flavell and Wellman (1977) focused on “knowing that“ conditional metamemory referred to “knowing when and why“. The procedural metamemory component emphasized by Brown et al., that is, children’s ability to monitor and self-regulate their memory-related behavior, refers to “knowing how“ and plays a major role in complex cognitive tasks such as comprehending and memorizing text materials and mathematical problem solving.

Although subsequent conceptualizations of metacognition expanded the scope of this theoretical construct, they also made use of the basic distinction between declarative and procedural knowledge. For instance, Pressley, Borkowski, and Schneider (1989) systematically considered declarative and procedural components of metacognition in developing a theoretical model that emphasized the dynamic interrelations among strategies, monitoring abilities, and motivation (e.g., Pressley, Borkowski, & O’Sullivan, 1985; Pressley, Borkowski, & Schneider, 1987, 1989). In their extension of the theoretical framework of metacognition, Pressley et al. proposed an elaborate model, the *Good Information Processing Model,* which linked aspects of procedural and declarative metacognitive knowledge to other features of successful information processing. According to this model, sophisticated metacognition is closely related to the learner’s strategy use, domain knowledge, motivational orientation, general knowledge about the world, and automated use of efficient learning procedures. All of these components are assumed to interact.

Overall, the distinction between declarative and procedural metacognitive knowledge is widely accepted in developmental and educational psychology. Although these components are generally conceived of as relatively independent, empirical findings suggest that they can mutually influence each other (see Schneider, Körkel, & Weinert, 1987; Schraw, 1994). For instance, knowing about one’s own tendency to commit easy errors may lead to increased self-regulatory activities in test situations.

Taken together, the popularity of the metacognition construct is mainly due to the fact that it seems crucial to concepts of everyday reasoning and those assessing scientific thinking as well as social interactions.

2 Assessment of metacognition

Currently, a large number of different measures are being used to assess metacognitive knowledge and metacognitive skills (also referred to as procedural metacognitive knowledge). The most important measures are briefly summarized below.

2.1 Measures of declarative metacognitive knowledge

There are a variety of measures that have been used to capture what children know about cognitive activities. As noted by Cavanaugh and Perlmutter (1982), measures assessing *declarative metacognitive knowledge* are taken without concurrent memory or problem-solving assessment (independent measures), whereas measures of *procedural metacognitive knowledge* are collected simultaneously with the measurement of memory activity (concurrent measures).

Most measurements of *declarative metacognitive knowledge* in children have used interviews or questionnaires. One of the earliest and best-known interview studies on declarative metamemory was carried out by Kreutzer, Leonard, and Flavell (1975) who assessed children’s knowledge about person, task, and strategy variables relevant to memory performance in different settings. Given young children’s problems with questionnaires and interviews, alternative nonverbal assessment procedures such as videotape illustrations of cognitive strategies were also used (for details, see the overviews in Holland Joyner & Kurtz-Costes, 1997; Schneider & Pressley, 1997).

To insure that children provide all their available metacognitive knowledge in a test situation, Best and Ornstein (1986) used a peer tutoring assessment procedure where older children (e.g., third or sixth graders) were asked to teach a memory strategy such as sorting items into semantic categories to younger children (e.g., first graders). Tutors’ instructions were taped and subjected to content analyses. The measure of metacognitive knowledge was the extent to which the instructions include appropriate strategy instructions.

Overall, these alternative methods alleviated some of the problems usually related to the use of questionnaire measures. However, these measures still created difficulties when applied to older children and adolescents, particularly when knowledge about text processing was assessed. There is the risk that social desirability factors reduce the validity of outcomes for this target group. Accordingly, more sophisticated measures of metacognition have to be used with older children and adolescents. For instance, Schlagmüller and Schneider (2007) came up with a standardized measure of metacognition that was based on a revised test instrument developed for PISA 2000 (see Artelt, Schiefele, & Schneider, 2001). This instrument taps adolescents' knowledge of strategies that are relevant during reading and for comprehension as well as recall of text information. For each of the six scenarios, students have to evaluate the quality and usefulness of five different strategies available for reaching the intended learning or memory goal. The rank order of strategies obtained for each scenario is then compared with an optimal rank order provided by experts in the field of text processing. The correspondence between the two rankings is expressed in a metacognition score, indicating the degree to which students are aware of the best ways to store and remember text information.

2.2 Measures of procedural metacognitive knowledge

Concurrent measures of metacognitive skills are characterized by the presence of a simultaneous cognitive activity. For instance, in the area of memory research, children and adolescents are asked to judge their memory performance shortly before, during, or after working on a memory task. The most studied type of procedural metacognitive knowledge is *self-monitoring,* that is, evaluating how well one is progressing. The developmental literature has focused on performance prediction or ease-of-learning (EOL) judgments, judgments of learning (JOL), and feeling-of-knowing (FOK) judgments. See Brown et al. (1983), Flavell et al. (2002), and Schneider and Lockl (2008) for overviews.

EOL judgments occur in advance of the learning process, are largely inferential, and refer to items that have not yet been learned (Nelson & Narens, 1994). The corresponding memory paradigm is performance prediction. In comparison, *JOLs* occur, during, or soon after the acquisition of memory materials and are predictions about future test performance on recently studied (and probably still recallable) items. Typically, paired-associate learning tasks are used in this context. After completion of a learning trial, participants are shown the stimuli of a given pair and have to indicate how confident they are about whether they will remember the correct item response, either immediately or 10 min later. A number of developmental studies also explored children's *feeling-of-knowing (FOK) judgments* (e.g., Lockl & Schneider, 2002). These judgments occur either during or after a learning procedure and are judgments about whether a currently unrecallable item will be remembered at a subsequent retention test.

Whereas self-monitoring involves knowing where you are with regard to your goal of understanding and memorizing task materials, *self-regulation* includes planning, directing, and evaluating one's mnemonic activities (cf. Flavell et al., 2002). Some developmental studies addressed aspects of children's *control and self-regulation processes* such as termination of study (recall readiness) and allocation of study time (see the review by Schneider & Pressley, 1997). *Recall readiness* assessments are made after learning materials have been studied at least once. Typically, participants are asked to continue studying until their memory of the materials to be learned is perfect. Another example of self-regulation skills concerns the allocation of study time. This research observes how learners deploy their attention and effort when studying lists of items. For instance, developmental studies on the *allocation of study time* examined whether school children and adults were more likely to spend more time on less well-learned material. After a first free recall trial, participants had to distinguish between recalled and non-recalled items (monitoring component) and were then asked to select half of the items for additional study (self-regulation component). One problem with the paradigm of the allocation of study time is that it may not only tap metacognitive processes, but also be influenced by motivational variables (see Schneider & Lockl, 2002).

2.3 Metacognition and education

The importance of educational contexts for the development of metacognitive knowledge was first highlighted in the field of memory development. In particular, findings from studies that focused on children's strategy development indicated that most of the memory and metamemory development is not so much a product of age, but of education and practice. For instance, in a recent longitudinal study on memory development from kindergarten age to the end of elementary school, Kron-Sperl, Schneider, and Hasselhorn (2008) repeatedly presented the children of their sample with a semantic organization (sort–recall) task without giving any specific strategy cues. When performance of these children was compared with that of random samples of school children of the same age who received this task for the first time, substantial practice effects were found. Children of the longitudinal sample not only outperformed the control children regarding strategy use and memory performance, but also showed considerably better task-specific metamemory. Obviously, it does not require much effort to improve children's strategy knowledge in school. There is broad agreement that one way in which parents and teachers can facilitate cognitive development is by the development of children's metacognition (see Carr, Kurtz, Schneider, Turner, & Borkowski, 1989; Coffman, Ornstein, McCall, & Curran, 2008). Both classic and recent studies on this issue show that there is still room for improvement in this regard, but that considerable progress can be found.

One of the most important outcomes of educational research on school learning in the 1980s was the documentation of metacognitive processes that serve to guide students through learning tasks (see the comprehensive meta-analysis by Wang, Haertel, & Walberg, 1993). During the last three decades, several attempts have been made to apply metacognitive theory to educational settings (see Desoete & Veenman, 2006a; Paris & Oka, 1986; Moely, Santulli, & Obach, 1995; Palincsar, 1986; Pressley, 1995). One interesting and effective approach to teaching knowledge about strategies was developed by Palincsar and Brown (1984). The "reciprocal teaching" procedure requires that teachers and students take turns executing reading strategies that are taught with instruction occurring in true dialog. Strategic processes are made very overt, with plenty of exposure to modeling of strategies and opportunities to practice these techniques over the course of a number of lessons. The goal is that children discover the utility of reading strategies, and that teachers convey strategy–utility information as well as information about when and where to use particular strategies. Teachers using reciprocal instruction assume more responsibility for strategy implementation early in instruction, gradually transferring control over to the student (see Palincsar, 1986, for an extensive description of the implementation of reciprocal instruction; see National Institute of Child Health and Human Development, 2000, for a realistic appraisal of its benefits).

During the 1980s and 1990s of the last century, numerous studies explored the efficiency of strategy training approaches in school (for a review, see Schneider & Pressley, 1997). The basic assumption was that although children in most cases do not efficiently monitor the effectiveness of strategies they use, they can be trained to do so. For instance, in a training program carried out by Ghatala et al. (e.g., Ghatala, Levin, Pressley, & Goodwin, 1986), elementary school children were presented with paired-associate learning tasks. Before studying these lists, some children received a three-component training. They were taught (a) to assess their performance with different types of strategies, (b) to attribute differences in performance to use of different strategies, and (c) to use information gained from assessment and attribution to guide selection of the best strategy for a task. As a major result, it was shown that even children 7–8 years of age can be taught to monitor the relative efficacy of strategies that they use, and to use utility information gained from monitoring in making future strategy selections.

Another more large-scale approach concerns the implementation of comprehensive evaluation programs that aim at assessing the systematic instruction of metacognitive knowledge in schools. As emphasized by Holland Joyner and Kurtz-Costes (1997), both Moely et al. and Pressley et al. have conducted very ambitious programs of evaluating effective instruction in public school systems. For instance, Pressley et al. found that effective teachers regularly incorporated strategy instruction and metacognitive information about effective strategy selection and modification as a part of daily instruction. It seems important to note that strategy instruction was not carried out in isolation, but integrated in the curriculum and taught as part of language arts, mathematics, science, and social studies (see also Pressley, 2002). In accord with the assumption of the Good Information Processing Model outlined above (cf. Pressley, Borkowski, & Schneider, 1989), effective teachers did not emphasize the use of single strategies but taught the flexible use of a range of procedures that corresponded to subject matter, time constraints, and other task demands. On most occasions, strategy instruction occurred in groups, with the teachers demonstrating appropriate strategy use. By comparison, the work by Moely et al. (e.g., Moely, Santulli, & Obach, 1995) illustrated that the effective teaching process described by Pressley et al. does not necessarily constitute the rule, and that effective teachers may represent a minority group in elementary school classrooms. Taken together, the careful documentation of instructional procedures carried out by Pressley et al. and Moely et al. has shown that there is a lot of potential for metacognitively guided instructional processes in children's everyday learning.

3 Metacognition and mathematics education

3.1 Early studies on the relationship

From the early 1980s on, researchers interested in mathematical problem solving became interested in the concept of metacognition. Questions frequently asked by scientists and mathematics educators included "can problem solving be taught?", "what is the role of understanding in problem solving?", and "what is the role of metacognitive behavior in problem solving?" (e.g., see Lester, 1982; Silver, 1982). International studies on students' problem solving in the context of mathematics education repeatedly demonstrated that children did not perform well in tasks that required more than one step, and that mathematics teachers seemed to have difficulty in planning and implementing lessons that build students' problem-solving skills (for recent examples, see Kramarski, 2008). The new concept of metacognition appeared useful in improving the situation in this regard. For instance, Lester (1982) chose to include metacognition in his list of questions, because he firmly believed that a person's knowledge about one's own cognitions before, during, and after a problem-solving period as well as his or her ability to maintain executive control in the sense of monitoring and self-regulation should significantly affect successful problem solving in mathematics. Thus, metacognitive activities were involved in at least two of the five necessary components for successful problem solving, with mathematical knowledge and experience, skills in generating relevant "tool" skills such as separating relevant from irrelevant information, and the ability to use a variety of heuristics representing the remaining components for successful mathematics performance. Verschaffel (1999) also pointed out that metacognition is of particular importance in the process of mathematical problem solving. He assumes that metacognition in the sense of prediction is instrumental during the initial stage of mathematical problem solving, when problem solvers try to build an appropriate representation of the problem. For the final stage of mathematical problem solving, in which the calculation outcomes need to be checked, he highlights the importance of metacognition in the sense of evaluation. In a similar vein, Brophy (1986) emphasized the importance of metacognition when teaching cognitive skills. In his view, one needs to address students' metacognition as well as their cognition in this process, and to supplement instruction with *figurative knowledge* about the skills (which skills are relevant) with instruction in operative knowledge (how to use the skills) and *conditional knowledge* (when and why to use the skills). Silver (1982) highlighted the importance of the problem solver's decision making in the process of solving a mathematical problem, for instance, when choosing between different cognitive strategies to assist the solution. These decision processes are not only metacognitive in nature, but also influenced by one's beliefs and values. Accordingly, in the context of mathematical problem solving, a person's beliefs about learning and problem solving, in general, and beliefs about mathematical problem solving, in particular, can act as important guides in the encoding and retrieval of mathematical material.

In a similar vein, Garofalo and Lester (1985) claimed that a pure cognitive analysis of mathematical performance is inadequate. They emphasized the importance of metacognition for the analysis and understanding of mathematical performance. Referring to the distinction between knowledge about and regulation of cognition, they argued that not only regulatory metacognitive behaviors but also person, task, and strategy categories of metacognitive knowledge are important in mathematical performance (see also Schoenfeld, 1983). Although the latter categories were originally devised to classify metacognitive knowledge about memory, they also seemed appropriate in the context of metacognitive influences on mathematical achievement. According to Garofalo and Lester (1985), person knowledge in the domain of mathematics includes one's assessment of one's own capabilities and limitations in this domain, both in general and with respect to particular mathematical topics or tasks. Task knowledge in the domain of mathematics consists of one's beliefs about the subject of mathematics as well as beliefs about the nature of mathematical tasks. This knowledge also includes an awareness of the effects of task features such as content, context, structures, and syntax on task difficulty. It seems that third and fifth graders often possess rather inadequate or immature task knowledge. They "believe that verbal problems can be solved by a direct application of one or more arithmetic operations and that the correct operations to use can be determined merely by identifying the key words; little planning or searching for meaning is necessary" (Garofalo & Lester, 1985, p. 167). More mature forms of task knowledge may include the notion that there usually is more that one way to solve a problem, or that two different methods of solutions can yield the same correct results. Knowledge of this kind seems to have profound positive effects on children's problem-solving attempts. It is important to note that correctness or veridicality of task knowledge can be regarded not only from a developmental perspective, but also from a structural point of view. Holding false beliefs about task characteristics can serve as one explanation for the phenomenon of "inert knowledge", a term used for the fact that knowledge acquired in school contexts may not be used to solve out-of-school problems (e.g., "street mathematics", see Nunes, Schliemann, & Carraher, 1993). Components of school knowledge and "out-of-school" knowledge are often encapsulated and seldom processed together. The problem with "inert knowledge" in the domain of mathematics may be related to the fact that students are not sufficiently confronted with "real-life" mathematical problems, leading to the phenomenon of knowledge compartmentalization (in school vs. out of school). Another explanation for this phenomenon may be related to metacognitive deficits in the sense of a lack of adequate regulation or comprehension monitoring (for a review, see Renkl, 1996).

When discussing the relevance of Flavell and Wellman's (1977) strategy knowledge component for the domain of mathematics, Garofalo and Lester (1985) referred to knowledge of algorithms and heuristics. Moreover, they included in their list a person's awareness of strategies to aid in comprehending problem statements, organizing information of data, planning solution attempts, executing plans, and checking results.

Although students' tactical behaviors have been studied extensively in mathematics education, much less is known (and taught in school) about metacognitive monitoring and regulation. There is a need to focus on behavior relevant to strategy selection, cognitive monitoring, and evaluation of cognitive processes. As noted by Silver (1982), many of the driving forces that determine success or failure are metacognitive in nature. With this in mind, Garofalo and Lester (1985) reformulated Polya's influential four-phase description of problem-solving activities to properly incorporate metacognitive activities. Their cognitive– metacognitive framework for studying mathematical performance consists of an orientation phase (strategic behavior to assess and understand a problem), an organization phase (planning of behavior and choice of actions), an execution phase (regulation of behavior to conform to plans), and a verification phase (evaluations of decisions made and of outcomes of executed plans), each being filled with cognitive as well as metacognitive activities. The framework developed by Garofalo and Lester was intended to serve as a tool for analyzing metacognitive aspects of mathematical performance. The authors also emphasized the importance of developing instructional treatments, which aim at incorporating metacognitive activities into mathematics instruction. This was seen as a new and important development at that time, since mathematics instruction had overemphasized the development of heuristic skills and virtually ignored the managerial skills necessary to regulate one's activities. For a more recent application of Garofalo and Lester's framework, see Stillman and Galbraith (1998).

One of the early attempts to highlight the importance of metacognition for mathematics education was presented by Schoenfeld (1987). He wrote his chapter in response to several mathematicians (e.g., Anna Henderson and Henry Pollak), who asked him to explain the surplus value of metacognition for mathematics instruction. In his response, Schoenfeld emphasized the fact that the prevalent belief that classroom mathematics should consist of mastering formulas seemed questionable and wrong, mainly because it prevented students from understanding that mathematics can be meaningful. In his view, metacognition has the potential to increase the meaningfulness of students' classroom learning, and the creation of a "mathematics culture" best fosters metacognition. More specifically, such a "mathematics culture" implies that students learn to think of mathematics as an integral part of their everyday lives, helping them to make connections between mathematical concepts in different contexts. According to Schoenfeld, the most important contribution of metacognition to the learning of mathematics can be seen in the acquisition of knowledge about one's own thought processes and the development of adequate monitoring and self-regulation activities. In his seminal chapter on learning to think mathematically, Schoenfeld (1992) provided a theoretically well-elaborated overview on problem solving, metacognition, and sense making in mathematics. In his conceptualization of mathematical thinking, metacognition, beliefs, and mathematical practices play a crucial role. Schoenfeld also elaborated on missing links with respect to theoretical model building for metacognition in the domain of mathematics. With respect to the "control" component, he described the theoretical state of the art as not yet well developed. In his view, more thought should be given to metacognitive activities, and the mechanism of control seems unclear: "we do not have good theoretical models of what control is, and how it works" (p. 364). The conceptual problems related to the mechanism as well as to the developmental pathways of control also include the question of whether control is domain independent or domain dependent, and which mechanisms may relate control decisions to domain knowledge.

What is also apparent from Schoenfeld's work is the clear need for implementing empirical research dealing with metacognition in mathematics education: Although the situation has improved somewhat during the past few decades, much work is still required on practical and implementational levels. After extensive discussion of findings on the development of self-regulatory skills in complex subject matter domains, such as mathematics, Schoenfeld concluded that such development is difficult to obtain, and that it often involves behavior modifications, including the unlearning of inappropriate control behaviors developed through prior instruction (see also Flavell, Miller, & Miller, 2002). He also states that concrete descriptions and studies regarding the teaching and learning of mathematics are still missing: "Here, in what may ultimately turn out to be one of the most important arenas for understanding the development of mathematical thinking, we seem to know the least" (Schoenfeld, 1992, p. 365). As will be shown below, Schoenfeld's critical analyses stimulated subsequent empirical research in this area.

At about the same time, Pressley (1986) used the *Good Strategy User Model* (GSU, Pressley et al., 1987) described above to explain his approach concerning appropriate mathematics instruction. In his view, "strategy" is a broad term and very similar to "procedural knowledge". Accordingly, mathematical algorithms and problem-solving routines qualify as strategies, and insights derived from the GSU can also be applied to the teaching of mathematics. Pressley (1986) described five broad principles of mathematics instructions: first, explicit *teaching of mathematics strategies* to youngsters is recommended. These strategies also include self-testing and other monitoring strategies, which have proven helpful in many educational contexts. The second principle referred to the teaching of specific strategy knowledge, meaning that learners need to know when, where, and how to apply specific strategies. The third instructional principle emphasized the need for acquiring general strategy knowledge. For instance, children should be taught that errors are often the result of applying incorrect strategies rather than due to simple shortcomings in effort. The fourth instructional principle referred to the *enrichment of the knowledge base.* Here, the basic assumption is that relying on the knowledge base is adaptive because increasing demands are made on children's arithmetic abilities with increasing age. Repeated practicing of elementary arithmetic operations should help in increasing the speed of mathematics operations, eventually leading to automatic, fast fact retrieval. Finally, the fifth instructional principle emphasized the need for first practicing each component separately, before attempts are made to coordinate the components. This sequential approach seems to cope best with the problem of limited attentional resources and resource allocation. Repeated practicing of relevant mathematics strategies eventually reduces the attentional resources that are consumed by each component, thus facilitating the coordination of components in a second step. It seems interesting to note that both Pressley (1986) and Silver (1982) relate their metacognitive approaches to the work of Polya (1957, 1973), emphasizing that Polya's heuristic suggestions and analyses of good mathematical problem solving can be conceived of as metacognitive prompts.

Early research on the importance of metacognition in mathematics education was not restricted to "normal" student populations. Theoretical analyses and empirical research reflecting school problems of learning-disabled children also addressed the issue of metacognition. For instance, Allardice and Ginsburg (1983) explored the role of *executive processes* in mathematics difficulties. In their view, a lack of, or failing to use, cognitive strategies to control the processes of thinking, remembering, and understanding may result in children failing to complete satisfactorily various academic tasks, including mathematical ones. Their case studies revealed that children with mathematics difficulties lack effective procedures for learning number facts, but can be taught reasoning methods that turn out to be effective (for more details, see Russell & Ginsburg, 1981, see Braten & Throndsen, 1998 for a similar case study).

3.2 More recent research on metacognition and mathematics education

Research carried out in the 1990s and after the millennium continued to use these core categories of metacognition to explore the utility of the concept in research with children and adolescents, assessing the predictive potential of metacognitive knowledge and skillfulness in mathematics (e.g., Carr & Jessup, 1995; Lucangeli & Cornoldi, 1997; Desoete & Veenman, 2006a). As can be inferred from the overview presented by Desoete and Veenman (2006b), a large proportion of the relevant studies focused on the importance of metacognitive skills, thus emphasizing the relevance of procedural metacognitive knowledge (e.g., planning, monitoring, self-control) on problem solving in mathematics. In the following, the relevant evidence from correlational and intervention studies is summarized.

4 Relationships between metacognition and mathematics performance

4.1 Correlational evidence

Although most correlational studies on the relationship between metacognition and mathematics performance were carried out with older students and adolescents, a few explored the importance of metacognitive knowledge for mathematics performance of young elementary school students. For instance, Carr, Alexander, and Folds-Bennett (1994) designed a longitudinal study to examine the role of second graders' metacognitive knowledge in their mathematics strategy use over a 5-month period. As a main result, it was shown that the use of internal strategies (e.g., counting in the head), but not external strategies (e.g., counting on fingers), was related to metacognition and effort attribution. Thus, Carr et al. (1994) were able to confirm the outcome of previous research by Garofalo and Lester (1985), which already had indicated that elementary children possess knowledge about mathematics strategies and can use this knowledge to their advantage. Moreover, the significant relations between metacognitive knowledge and motivation (effort attribution) over time confirmed that both concepts contributed to increases in mathematics performance. In a follow-up study, Carr and Jessup (1995) replicated the positive results by Carr et al. (1994), showing that metacognitive knowledge significantly influenced young elementary children's developing strategy use.

Similar evidence supporting the relative importance of metacognitive knowledge for mathematics performance comes from a recently conducted first assessment of our own ongoing longitudinal research project, which deals with the developmental pathways and interrelations of metacognitive knowledge and prior knowledge in different school subject domains and will be described in some detail below. The first assessment took place in 2008 at the beginning of secondary school (fifth grade, 9–10-year-old children) in Germany. A total number of 763 fifth graders from three different educational tracks (low, middle, and high) in Northern Bavaria, Germany, participated in this study. By taking into account age differences and curriculum-specific particularities, a test for metacognitive knowledge related to mathematics was constructed according to the rationale already described for the instrument on metacognitive knowledge related to reading. To establish content validity of the metacognitive knowledge test, 19 experts from German university departments of didactics of mathematics were asked to examine and verify the ratings of the appropriateness of suggested strategies for the described tasks. A total score summarizing students' metacognitive knowledge was computed using those strategy pairs with clear superiority/inferiority relations based on expert ratings. Overall, Cronbach's alpha of the metacognition test was 0.80, indicating sufficient internal consistency.

Mathematics achievement, on the other hand, was measured using a newly developed test in accordance with the current Bavarian mathematics curricula for grade five. When analyzing performance differences as a function of academic track, it became apparent that students from the higher academic track not only performed better on the mathematics test, but also knew more about cognitive and metacognitive strategies. Metacognitive knowledge and mathematics performance shared about 17% common variance, with a bivariate correlation of $r = 0.41$. Even when controlling for the effects of general cognitive abilities, the correlations remained significant (partial $r = 0.31$). When looking at gender differences in mathematics achievement, 10-year-old girls scored lower on the mathematics test than the boys. On the other hand, however, girls' metacognitive knowledge was comparable to that of the boys, with girls even tending to be more knowledgeable than boys. This finding seems to indicate that girls do not take sufficient advantage of their metacognitive knowledge when working on mathematics problems.

Findings from the German extension of the 2003 PISA study seem to confirm this interpretation for older students. A very similar instrument measuring metacognitive knowledge for mathematics (students' knowledge base about mathematical strategies, i.e., cognitive and metacognitive strategies related to mathematics) was used with 15-year-old students in the German extensions of the international large-scale study PISA 2003 (OECD, 2004). Like the knowledge test for the 10-year-olds and similar to the metacognitive knowledge test for reading described above, the mathematics-related metacognitive knowledge test consisted of scenarios, each offering different strategic approaches that the students had to judge concerning their appropriateness and effectiveness for the specific learning or problem-solving scenario at hand. The internal consistency of the metacognition scale was sufficient, with Cronbach's alpha equaling 0.78. Given that in the PISA 2003 assessment, mathematics was the major assessment domain of the study and particularly because the operationalization of mathematics performance is based on the notions of applying mathematics in everyday life and mathematical problem solving, students' performance on this test is an interesting criterion for studying the effects of metacognition. It is highly likely that mathematical problem-solving tasks are more susceptible to metacognition than routine applications of algorithm or highly automated processes, leading to the assumption that the relations between metacognition and mathematics performance in PISA 2003 should be substantial. Based on data from 1,433 15-year-old students, this assumption could be confirmed. As for the 10-year-olds in the aforementioned study, significant differences in mathematics performance as well as regarding metacognitive knowledge were found as a function of academic track. Moreover, mathematics performance and metacognitive knowledge were substantially correlated ($r = 0.43$), indicating that roughly 18% of the variance of mathematics performance in the PISA 2003 test could be explained by the metacognition indicator. This finding illustrates that students' metacognitive knowledge (related to mathematics) is not only a significant predictor of mathematics achievement, but also of high practical relevance, since metacognitive knowledge is modifiable and it is likely (although longitudinal and/or experimental studies are needed to prove this) that growth in metacognitive knowledge results in improved mathematics performance. Against this background, it seems interesting to note (again) that although boys clearly outperformed girls on the mathematics achievement test, girls scored significantly higher on the metacognitive knowledge test for mathematics than boys. This finding probably indicates that girls do have a high potential for mathematics, which for some unknown reason cannot be properly transferred into mathematics performance.

The utility of metacognition for mathematics performance is not restricted to the (declarative) knowledge component. In their comprehensive research on elementary school children's metacognitive skills (in the sense of procedural metacognitive knowledge), Lucangeli and Cornoldi (1997) demonstrated that children's monitoring and evaluation attempts were closely related to mathematical performance. Individual differences in metacognitive skills seem even more important in secondary school students. In a more recent study with older school children, Veenman (2006) explored the contribution of metacognition skills and general intelligence to the development of mathematical learning performance. Metacognitive skillfulness was measured through systematic observation, and mathematics learning was assessed by a mathematics test. Overall, the findings showed that both intelligence and metacognitive skills influenced mathematics performance. Interestingly, metacognition outweighed intelligence as predictor of mathematics learning performance. Although related to metacognition, intelligence did not play a major role in secondary school students' mathematics learning.

4.2 Findings from intervention studies

Given the consistently positive outcomes from correlational studies, several intervention programs have been developed that aim at improving children's metacognitive knowledge as well as their metacognitive skills. Looking at the relationship between metacognition and mathematics performance based on data from intervention studies is an important step: given that data from cross-sectional studies are not sufficient to exclude the possibility that effects of metacognition on mathematics achievement are caused by other unknown factors. Although correlational longitudinal designs are more appropriate in this respect, it appears that intervention studies with an experimental design are more adequate tools to confirm the assumption that metacognition indeed causes gains in mathematics achievement, by showing that fostering metacognitive knowledge results in improved performance. A few selected intervention studies focusing on this issue will be summarized below.

For instance, Cornoldi, Lucangeli, Caponi, Falco, Focchiatti, and Todeschini (1995) developed a systematic program that focused on the training of metacognitive awareness and control processes, which was presented to children with and without learning difficulties. In a first study carried out with normally achieving children, increases in metacognition were related to improvement in some aspects of mathematical performance (problem solving and logical reasoning), but not in geometry. Findings of the second study, which focused on children with mathematics difficulties, were generally more impressive, showing that the learning-disabled children benefitted considerably from the training program. This was also true for those children who were considered to be severely learning disabled by their teachers.

In Germany, Cohors-Fresenborg and Kaune (2001; Kaune, 2006) developed a special mathematics curriculum that took metacognitive activities into account, using a categorical system that focused on reflection processes and demonstrating the relevance of a discourse-based teaching culture for children's understanding of mathematical problems. If we see it correctly, the value of this approach has been proven in numerous case studies. More comprehensive evaluation approaches are needed to illustrate its general utility.

Another set of training studies carried out in the Netherlands and based on the MASTER program (Mathematics Strategy Training for Educational Remediation; Van Luit & Kroesbergen, 2006) focused on self-instruction in mathematical problem solving and was particularly designed for children with mathematics disabilities. In a recent intervention study, Van Luit and Kroesbergen (2006) trained children with severe mathematics disabilities in groups of five. During the 16-week intervention, the children did not work on any other math program, whereas children in the control group received a mathematics training based on the standard curriculum. The children in the training group received several lessons in multiplication and division. The major goals of the MASTER program included an increase in children's orientation toward the problem (planning), a better understanding of the number system, an increase in control activities (checking the chosen solution strategy and the answer), and improvement of memorization of multiplication and division facts below 100. Parallel versions of a mathematics test were used at pretest, posttest, and delayed posttest. As a main result, children in the training group gained more between pre- and posttest assessments than the children of the control group, a finding which turned out to be stable throughout the follow-up period (for more details, see Van Luit & Kroesbergen, 2006).

An intervention study with elementary school children was undertaken by Desoete, Roeyers, and De Clercq (2003) who studied the effects of so-called "off-line" metacognition on mathematical problem solving (i.e., prediction and evaluation assessments, measured before or after the solving of mathematics exercises). This study showed that third graders participating in five metacognitive strategy instruction sessions (compared to students of four other experimental groups) achieved significant gains in the trained metacognitive skills. Their advantage continued to be significant in follow-up measures on domain-specific mathematics problem-solving knowledge. In addition, Desoete, Roeyers, and De Clercq (2001) were able to show that individual differences in students' off-line metacognition (prediction and evaluation) differentiated among good performers, moderate performers, and children with mathematics learning disabilities.

Another metacognitive training program suited for secondary school students was developed in Israel by Mevarech and Kramarski (1997, 2003; Kramarski & Mevarech, 2003). Their instructional method was called IMPROVE, which is the acronym of Introducing new material, Metacognitive questioning, Practicing, Reviewing, Obtaining mastery on higher and lower cognitive processes, Verification, and Enrichment and remedial. The metacognitive questioning included comprehension questions ("What is the problem all about?"), connecting questions ("How does the problem relate to others already solved in the past?"), strategic questions ("What kinds of strategies are appropriate for solving the problem, and why?"), and, finally, reflection questions ("Does the solution make sense? Can the problem be solved in a different way?"). In a recent training study with eighth graders using a pretest–posttest design, Mevarech, Tabuk, and Sinai (2006) compared the effects of the IMPROVE program in educational settings with and without cooperative learning environments. Their results showed that IMPROVE had the potential to enhance students' mathematics problem solving. Students who were instructed in cooperative settings and additionally exposed to IMPROVE outperformed students who only experienced cooperative learning. Thus, just practicing mathematical problem solving in cooperative settings was not sufficient. The IMPROVE method had important effects on students' metacognitive knowledge and skills. In particular, planning and comprehension processes as well as students' reflection skills were positively affected by the program.

Recently, Kramarski (2008) reported about the effects of the IMPROVE program embedded in a 3-year training aimed at enhancing elementary school teachers' mathematical knowledge. By contrasting a group of teachers who either received a professional development program alone or in combination with IMPROVE metacognitive questioning, she found that teachers in the combined program scored higher on various algebraic procedural and real-life tasks regarding conceptual mathematical explanations as well as in using self-monitoring and evaluation strategies in algebraic problem solving.

A rather promising approach for implementing metacognitive training is the use of a computer. The possibilities in terms of adaptive feedback and prompts are manifold and bear potential for research on metacognition as well as practical implementations (see Zimmerman & Tsikalas, 2005). A few such programs already exist, although not necessarily specific to learning mathematics (e.g. AutoTutor, iStart, Point & Query, see Graesser, McNamara, & VanLehn, 2005). For example, Teong (2003) used a program specific to metacognitive training on mathematical word-problem solving in a cognitive-apprenticeship computer-based environment. Forty 11–12-year-old low achievers were trained in this environment, and results showed that these children outperformed control students on mathematical achievement tests (solving word problems). In addition, the trained students developed the ability to ascertain when to make metacognitive decisions and elicited more appropriate metacognitive decisions than control students, indicating that the cognitive-apprenticeship computer-based environment helps to amplify low achievers' metacognitive and cognitive behaviors during word problem solving.

A different intervention approach was taken by Clarke, Waywood, and Stephens (1993), who worked on "journal writing" in mathematics, thereby also fostering metacognitive processes. One major finding of their long-term journal writing study was that students convincingly explained why they used journal writing: "Sixty percent of the students gave as the main reason for writing in their journal, because it helps me (…), the most popular justification for journal use was To help me learn (…) half of the student sample reported that the most important thing learned from journal completion was To be able to explain what I think." (p. 241). According to the authors, "these perceptions of the nature of journal use relate quite closely to the stated goals of the school program and suggest both cognitive and metacognitive consequences" (p. 241). Nevertheless, Clarke et al. (1993, p. 247) also reported difficulties, especially at the beginning of journal writing, in the sense that students often write descriptions of what had been done in classes ("We did the middle of chapter 3") and were satisfied with it. It takes time and modeling to develop journal writing practices that have the potential to become a metacognitive learning tool.

5 Concluding remarks

Overall, the numerous studies described in this paper confirm the view that metacognitive knowledge and self-regulated, insightful use of learning strategies predict mathematics performance in primary and secondary school settings even after differences in intellectual abilities have been taken into account. They also give evidence that metacognitive knowledge relevant to school-related domains normally develops during the course of primary school, but is not at peak in adolescence. Findings from various intervention approaches showed that instructional settings, including the training of metacognitive skills, can be successful from early school age on, and can still be effective in late childhood and early adolescence.

It is obvious that the amount of research on metacognition in the domain of language arts and reading still exceeds the amount of research on metacognition in the domain of mathematics. Nevertheless, Schoenfeld's (1992) rather negative evaluation needs to be modified. There has been important empirical research on metacognition ever since, and there are also studies available that emphasize on practical approaches for teachers (e.g., Clarke, Waywood, & Stephens 1993; Cohors-Fresenborg & Kaune, 2001; Sjuts, 2002; Stillman & Galbraith, 1998).

Nevertheless, empirical research systematically and thoroughly addressing the developmental relations between metacognition, general intellectual abilities, and prior knowledge is still rare. More longitudinal and intervention studies are needed to disentangle the longitudinal effects and interactions of metacognition, general cognitive abilities, and subject matter knowledge in mathematics as well as in other school subject domains. Such studies should help us gain a better understanding of their relative importance over time, the developmental mechanisms as well as stage-specific effects. Besides, another research desideratum for metacognition research applies to replicable empirical evidence related to the transferability of metacognition across situations, tasks, or domains (see also Pintrich, Wolters, & Baxter, 2000).

References

Allardice, B. S., & Ginsburg, H. P. (1983). Children's psychological difficulties in mathematics. In H. P. Ginsburg (Ed.), *The development of mathematical thinking.* New York: Academic Press.

Artelt, C., Schiefele, U., & Schneider, W. (2001). Predictors of reading literacy. *European Journal of Psychology of Education, 16,* 363–383.

Best, D. L., & Ornstein, P. A. (1986). Children's generation and communication of mnemonic organizational strategies. *Developmental Psychology, 22,* 845–853.

Braten, I., & Throndsen, I. S. (1998). Cognitive strategies in mathematics. Part II: Teaching a more advanced addition strategy to an eight-year-old girl with learning difficulties. *Scandinavian Journal of Educational Research, 42,* 151–171.

Brophy, J. (1986). Teaching and learning mathematics: Where research should be going. *Journal for Research in Mathematics Education, 17,* 323–346.

Brown, A. L. (1978). Knowing when, where, and how to remember: A problem of metacognition. In R. Glaser (Ed.), *Advances in instructional psychology* (pp. 77–165). Hillsdale, NJ: Erlbaum.

Brown, A. L., Bransford, J. D., Ferrara, R. A., & Campione, J. C. (1983). Learning, remembering, and understanding. In J. H. Flavell & E. M. Markham (Eds.), *Handbook of child psychology: Vol. 3. Cognitive development* (pp. 77–166). New York: Wiley.

Carr, M., Alexander, J., & Folds-Bennett, T. (1994). Metacognition and mathematics strategy use. *Applied Cognitive Psychology, 8,* 583–595.

Carr, M., & Jessup, D. L. (1995). Cognitive and metacognitive predictors of mathematics strategy use. *Learning and Instruction, 7,* 235–247.

Carr, M., Kurtz, B. E., Schneider, W., Turner, L. A., & Borkowski, J. G. (1989). Strategy acquisition and transfer: Environmental influences on metacognitive development. *Developmental Psychology, 25,* 765–771.

Cavanaugh, J. C., & Perlmutter, M. (1982). Metamemory: A critical examination. *Child Development, 53,* 11–28.

Clarke, D. J., Waywood, A., & Stephens, M. (1993). Probing the structure of mathematical writing. *Educational Studies in Mathematics, 25*(3), 235–250.

Coffman, J. L., Ornstein, P. A., McCall, L. E., & Curran, P. J. (2008). Linking teachers' memory-relevant language and the development of children's memory skills. *Developmental Psychology, 44,* 1640–1654.

Cohors-Fresenborg, E., & Kaune, C. (2001). Mechanisms of the taking effect of metacognition in understanding processes in mathematics teaching. In G. Törner, R. Bruder, N. Neill, A. Peter-Koop, & B. Wollring (Eds.), *Developments in mathematics education in German-speaking countries, selected papers from the annual conference on didactics of mathematics, Ludwigsburg* (pp. 29–38). Hildesheim: Franzbecker.

Cornoldi, C., Lucangeli, D., Caponi, B., Falco, G., Focchiatti, R., & Todeschini, M. (1995). *Matematica e Metacognizione.* Trento: Erickson.

Desoete, A., Roeyers, H., & De Clercq, A. (2001). EPA2000: Een instrument om metacognitieve en rekenvaardigheden te meten. [EPA2000: An instrument for measuring metacognitive and arithmetic skills]. *Kind en Adolescent, 22*(2), 85–94.

Desoete, A., Roeyers, H., & De Clercq, A. (2003). Can offline metacognition enhance mathematical problem solving? *Journal of Educational Psychology, 95*(1), 188–200.

Desoete, A., & Veenman, M. (Eds.). (2006a). *Metacognition in mathematics education.* Haupauge, NY: Nova Science.

Desoete, A., & Veenman, M. (2006b). Metacognitions in mathematics: Critical issues on nature, theory, assessment and treatment. In A. Desoete & M. Veenman (Eds.), *Metacognition in mathematics education* (pp. 1–10). Haupauge, NY: Nova Science.

Flavell, J. H. (1971). First discussant's comments: What is memory development the development of? *Human Development, 14,* 272–278.

Flavell, J. H. (1979). Metacognition and cognitive monitoring. A new area of cognitive-developmental inquiry. *American Psychologist, 34,* 906–911.

Flavell, J. H., Miller, P. H., & Miller, S. A. (2002). *Cognitive development* (4th ed.). Englewood Cliffs, NJ: Prentice-Hall.

Flavell, J. H., & Wellman, H. M. (1977). Metamemory. In R. Kail & J. Hagen (Eds.), *Perspectives on the development of memory and cognition* (pp. 3–33). Hillsdale, NJ: Erlbaum.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal of Research in Mathematics Education, 16,* 163–176.

Ghatala, E. S., Levin, J. R., Pressley, M., & Goodwin, D. (1986). A componential analysis of effects of derived and supplied strategy–utility information on children's strategy selections. *Journal of Experimental Child Psychology, 41,* 76–92.

Goswami, U. (2008). *Cognitive development—The learning brain* (2nd ed.). Hove, UK: Psychology Press.

Graesser, A. C., McNamara, D. S., & VanLehn, K. (2005). Scaffolding deep comprehension strategies through Point & Query, AutoTutor, and iSTART. *Educational Psychologist, 40*(4), 225–234.

Holland Joyner, M. H., & Kurtz-Costes, B. (1997). Metamemory development. In N. Cowan (Ed.*), The development of memory in childhood* (pp. 275–300). Hove, UK: Psychology Press.

Kaune, C. (2006). Reflection and metacognition in mathematics education—tools for the improvement of teaching quality. *Zentralblatt für Didaktik der Mathematik, 38*(4), 350–360.

Kramarski, B. (2008). Promoting teachers' algebraic reasoning and self-regulation with metacognitive guidance. *Metacognition and Learning, 3*(2), 83–99.

Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and meta-cognitive training. *American Educational Research Journal, 40,* 281–310.

Kreutzer, M. A., Leonard, C., & Flavell, J. H. (1975). An interview study of children's knowledge about memory. *Monographs of the Society for Research in Child Development, 40* (serial no. 159).

Kron-Sperl, V., Schneider, W., & Hasselhorn, M. (2008). The development and effectiveness of memory strategies in kindergarten and elementary school: Findings from the Würzburg and Göttingen longitudinal studies. *Cognitive Development, 23,* 79– 104.

Kuhn, D. (1999). Metacognitive development. In L. Balter & C. S. Tamis-LeMonda (Eds.), *Child Psychology: A Handbook of Contemporary Issues* (pp. 259–286). Philadelphia, PA: Psychology Press.

Kuhn, D. (2000). Theory of mind, metacognition, and reasoning: A life-span perspective. In P. Mitchell & K. J. Riggs (Eds.), *Children's reasoning and the mind* (pp. 301–326). Hove, UK: Psychology Press.

Lester, F. K. (1982). Building bridges between psychological and mathematics education research on problem solving. In F. K. Lester & J. Garofalo (Eds.), *Mathematical problem solving* (pp. 55–85). Philadelphia: The Franklin Institute Press.

Lockl, K., & Schneider, W. (2002). Developmental trends in children's feeling-of-knowing judgements. *International Journal of Behavioral Development, 26,* 327–333.

Lucangeli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition, 3,* 121–139.

Mevarech, Z. R., & Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal, 34,* 365– 394.

Mevarech, Z. R., & Kramarski, B. (2003). The effects of metacognitive training versus worked-out examples on students' mathematical reasoning. *British Journal of Educational Psychology, 73*(4), 449–471.

Mevarech, Z. R., Tabuk, A., & Sinai, O. (2006). Metacognitive instruction in mathematics classrooms: Effects on the solution of different kinds of problems. In A. Desoete & M. Veenman (Eds.), *Metacognition in mathematics education* (pp. 73–81). Haupauge, NY: Nova Science.

Moely, B. E., Santulli, K. A., & Obach, M. S. (1995). Strategy instruction, metacognition, and motivation in the elementary school classroom. In F. E. Weinert & W. Schneider (Eds.), *Memory performance and competencies: Issues in growth and development* (pp. 301–321). Mahwah, NJ: Erlbaum.

National Institute of Child Health and Human Development (2000). *Report of the National Reading Panel. Teaching children to read: an evidence-based assessment of the scientific research literature on reading and its implications for reading instruction: Reports of the subgroups* (NIH Publication No. 00-4754). Washington, DC: U.S. Government Printing Office.

Nelson, T. O., & Narens, L. (1994). Why investigate metacognition? In J. Metcalfe & A. P. Shimamura (Eds.), *Metacognition. Knowing about knowing* (pp. 1–25). Cambridge, MA: MIT Press.

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics.* Cambridge, UK: Cambridge University Press.

OECD (2004). *Learning for tomorrow's world. First results from PISA 2003.* Paris: OECD.

Palincsar, A. S. (1986). The role of dialogue in providing scaffolded instruction. *Educational Psychologist, 21,* 73–98.

Palincsar, A. S., & Brown, A. L. (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction, 1,* 117–175.

Paris, S. G., & Oka, E. R. (1986). Children's reading strategies, metacognition, and motivation. *Developmental Review, 6,* 25–56.

Pintrich, P., Wolters, C. A., & Baxter, G. P. (2000). Assessing metacognition and self-regulated learning. In G. Schraw & J. C. Impara (Eds.*), Issues in the measurement of metacognition* (pp. 43–97). Lincoln, NE: Buros Institute of Mental Measurements, University of Nebraska Press.

Polya, G. (1957). *How to solve it* (2nd ed.). New York: Doubleday.

Polya, G. (1973). *Induction and analogy in mathematics.* Princeton, NJ: Princeton University Press.

Pressley, M. (1986). The relevance of the Good Strategy User model to the teaching of mathematics. *Educational Psychologist, 21,* 139–161.

Pressley, M. (1995). What is intellectual development about in the 1990s? In F. E. Weinert & W. Schneider (Eds.), *Memory performance and competencies: Issues in growth and development* (pp. 1–25). Hillsdale, NJ: Erlbaum.

Pressley, M. (2002). Comprehension strategies instruction. In C. C. Block & M. Pressley (Eds.), *Comprehension instruction: Research-based best practices* (pp. 11–27). New York: Guilford Press.

Pressley, M., Borkowski, J. G., & O'Sullivan, J. T. (1985). Children's metamemory and the teaching of memory strategies. In D. L. Forrest-Pressley, G. E. MacKinnon, & T. G. Waller (Eds.), *Metacognition, cognition, and human performance* (Vol. 1, pp. 111–153). Orlando, FL: Academic Press.

Pressley, M., Borkowski, J. G., & Schneider, W.

(1987). Cognitive strategies: Good strategy user's coordinate metacognition and knowledge. In R. Vasta & G. Whitehurst (Eds.), *Annals of Child Development* (Vol. 5, pp. 89–129). New York: JAI.

Pressley, M., Borkowski, J. G., & Schneider, W. (1989). Good information processing: What it is and what education can do to promote it. *International Journal of Educational Research, 13,* 857–867.

Renkl, A. (1996). Tra¨ges Wissen: Wenn Erlerntes nicht genutzt wird. *Psychologische Rundschau, 47,* 78–92.

Russell, R. L., & Ginsburg, H. P. (1981). *Cognitive analysis of children's mathematical difficulties.* Rochester, NY: University of Rochester.

Schlagmüller, M., & Schneider, W. (2007). *WLST-12. Würzburger Lesestrategie Wissenstest für die Klassen 7 bis 12.* Göttingen: Hogrefe.

Schneider, W. (1999). The development of metamemory in children. In D. Gopher & A. Koriat (Eds.), *Attention and performance XVII: Cognitive regulation of performance: Interaction of theory and application* (pp. 487–513). Cambridge, MA: MIT Press.

Schneider, W. (2010). Memory development in childhood and adolescence. In U. Goswami (Ed*.), The Blackwell handbook of cognitive development.* London, UK: Blackwell (in press).

Schneider, W., Körkel, J., & Weinert, F. E. (1987). The effects of intelligence, self-concept, and attributional style on metamemory and memory behavior. *International Journal of Behavioral Development, 10,* 281–299.

Schneider, W., & Lockl, K. (2002). The development of metacognitive knowledge in children and adolescents. In T. J. Perfect & B. L. Schwartz (Eds*.), Applied metacognition* (pp. 224–257). Cambridge, UK: Cambridge University Press.

Schneider, W., & Lockl, K. (2008). Procedural metacognition in children: Evidence for developmental trends. In J. Dunlosky & B. Bjork (Eds.), *A handbook of memory and metacognition.* Mahwah, NY: Erlbaum.

Schneider, W., & Pressley, M. (1997*). Memory development between 2 and 20.* Hillsdale, NJ: Erlbaum.

Schoenfeld, A. H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 345– 395). New York: Academic Press.

Schoenfeld, A. H. (1987). What's all that fuss about metacognition? In A. H. Schoenfeld (Ed*.), Cognitive science and mathematics education* (pp. 189–215). Hillsdale, NJ: Erlbaum.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Schraw, G. (1994). The effect of metacognitive knowledge on local and global monitoring. *Contemporary Educational Psychology, 19,* 143–154.

Silver, E. A. (1982). Knowledge organization and mathematical problem solving. In F. K. Lester & J. Garofalo (Eds.),*Mathematical problem solving* (pp. 15–25). Philadelphia: Franklin Institute Press.

Sjuts, J. (2002). Metacognition in mathematics lessons. In H.-G. Weigand et al. (Eds.): *Developments in mathematics education in German-speaking countries. Selected papers from the annual conference on didactics of mathematics, Bern, 1999* (pp. 76–87). Hildesheim: Verlag Franzbecker.

Stillman, G. A., & Galbraith, P. L. (1998). Applying mathematics with real world connections: Metacognitive characteristics of secondary students*. Educational Studies in Mathematics, 36*(2), 157–195.

Teong, S. K. (2003). The effect of metacognitive training on mathematical word-problem solving*. Journal of Computer Assisted Learning, 19*(1), 46–55.

Van Luit, J. E. H., & Kroesbergen, E. H. (2006). Teaching metacognitive skills to students with mathematical disabilities. In A. Desoete & M. Veenman (Eds.), *Metacognition in mathematics education* (pp. 177–190). Haupauge, NY: Nova Science.

Veenman, M. V. J. (2006). The role of intellectual and metacognitive skills in math problem solving. In A. Desoete & M. Veenman (Eds.), *Metacognition in mathematics education* (pp. 35–50). Haupauge, NY: Nova Science.

Verschaffel, L. (1999). Realistic mathematical modelling and problem solving in the upper elementary school: Analysis and improvement. In J. H. M. Hamers, J. E. H. Van Luit, & B. Csapo (Eds.), *Teaching and learning thinking skills. Contexts of learning* (pp. 215–240). Lisse: Swets & Zeitlinger.

Wang, M. C., Haertel, G. D., & Walberg, H. J. (1993). Toward a knowledge base for school learning. *Review of Educational Research, 63,* 249–294.

Zimmerman, B. J., & Tsikalas, K. E. (2005). Can computer-based learning environments (CBLEs) be used as self-regulatory tools to enhance learning? *Educational Psychologist, 40*(4), 267–271.