Risk Management in Cross-Border Electricity Markets in Europe

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Analysis and Valuation of Derivative Instruments

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To my family

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P_t	Electricity spot price at time t.
P_t^i	Electricity spot price at time t in area i.
F_t	Forward or Futures price at time t.
F_t^i	Forward or Futures price at time t for delivery in area i.
C_t	Call option price at time t.
$g(\cdot)$	Payoff function of call option.
H	Heat rate for spark spread option.
K	Strike price of electricity option.
V_t	Price of generating fuel for spark spread option.
$f(t)$	Deterministic seasonal function at time t.
X_t, Y_t, Z_t	Stochastic component of electricity price.
$W_i(t)$	Wiener process of the stochastic component i at time t.
J_t	Normally distributed jump-size.

q_t	Poisson process.
$E[\cdot]$	Expectation operator.
$Var[\cdot]$	Variance operator.
$\max[x, y]$	Maximum of x and y.
$(x)^+$	Equivalent to $\max[x, 0]$.
$\mathcal{P}$	Empirical or physical probability measure.
$\mathcal{Q}$	Risk-neutral probability measure.
Ω_t	Information set available at time t.
$\Phi(\cdot)$	Cumulative normal distribution function.
$\mathcal{N}(x_1, x_2)$	Normal distribution with mean x_1 and variance x_2.
∂	Partial differential operator.
r	Risk neutral rate of interest.
κ	Mean-reversion speed.
μ	Deterministic trend.
σ	Volatility parameter.
ρ	Correlation coefficient.
λ	Jump intensity of Poisson process q_t.
μ_i	Mean of normally distributed variable i.
γ	Mean-reversion level.
ϕ	Autoregression parameter.
c	Constant level of autoregression process.
ε_t	White noise process.
b_{ij}	Regime-switching probability.

Π	Transition matrix of regime-switching model.
R_t	Regime at time t.
$\Pr(\cdot\,\|\cdot)$	Conditional probability.
G_t	Function determining the value of a call option.
$\hat{G}_t$	Fourier transform of function G_t.
$\overline{\beta}(t), \beta(t)$	Functions in Fourier transform.
p_{ij}	Probability of an up-move in the binomial tree model at node i and time step j.
u	Up-move factor in the binomial tree model.
d	Down-move factor in the binomial tree model.

Chapter 3

Notation as in Chapter 2; additional and deviating symbols:

$\bar{P}_t^i$	Mean electricity spot price at time t in area i.
π_t	risk premium at time t.
τ_t	Time-to-maturity left at time t.
$\beta^{No}, \beta^{Fi}, \beta^{Se}$	Regression coefficient for water reservoir level in Norway, Finland and Sweden.
β	Regression coefficient for time-to-maturity of risk premium.
α	Regression coefficient for spot price variance of risk premium.

γ	Regression coefficient for spot price skewness of risk premium.

Chapter 4

Notation as in Chapters 2 and 3; additional and deviating symbols:

$\max[\cdot]$	Maximum of a set.
$\min[\cdot]$	Minimum of a set.
S_t	Stochasting diffusion process.
J_t	Jump process.
N_t	Normally distributed jump size.
p_J	Jump probability of process J_t.
ν	Mean-reversion level.
ς	Variance of J_t.
θ_J	Market price of jump risk.
$p(\cdot\,\vert\cdot)$	Conditional density.
Θ	Vector of parameters to be estimated.
X	Vector of state variables.
$\pi(\cdot)$	Posterior density operator.
$q(\cdot\,\vert\cdot)$	Conditional proposal density.
φ	Bernoulli probability.
C_t	Market price of PTR option at time t.
$PTR(\cdot)$	PTR model price.

ψ_t	Set of PTR option input factors comprising time-to-maturity and current spot price.
α	Parameter for prior distribution.
β	Parameter for prior distribution.
γ	Parameter for seasonality regression.
Δ	Difference operator.
$\mathcal{IG}\left(x_1, x_2\right)$	Inverse-Gamma distribution with parameters x_1 and x_2.
$\mathcal{B}eta\left(x_1, x_2\right)$	Beta distribution with parameters x_1 and x_2.

Chapter 1

Introduction

Over the past years, the international electricity markets have undergone an enormous restructuring. Before, these markets were characterized by generally state owned and vertically integrated utilities which offered all services along the value chain from generation over distribution up to sales. During this time, prices were determined by these monopolists and therefore indirectly by the government, based on break-even analyses. However, starting in the 1980s, electricity markets worldwide faced deregulation that reached continental Europe in the late 1990s. The former monopolistic structure broke up resulting in a disentangled value chain. Although some elements of the value chain are still subject to governmental regulation, many areas of the electricity market are now characterized by competition and open for new market participants.

Due to the deregulation and the introduction of competition, electricity prices are now determined by market forces, i.e. demand and supply, and not subject to governmental fixing. Moreover, the terms of delivery contracts between generators and wholesalers changed considerably. While they used to be characterized by long maturities with fixed prices, these contracts are now in general based on short-term market prices and therefore subject to short-term price variations. As a result of this transition, market participants are faced with considerable price risk.[1] Based on the observed volatility in electricity markets, this risk is by far larger than in any other financial or commodity market.[2]

In the course of the liberalization, the need for an organized market at the wholesale level arose. Although electricity and its derivative products can be traded Over the Counter (OTC), which is still the dominant market place, in many countries electricity exchanges emerged with the aim of introducing more competition and increasing transparency.[3] Although OTC trading is still dominant in electricity markets, exchange based trading volumes have risen continuously over the past few years highlighting the growing importance of electricity exchanges. Due to the rise of the national electricity exchanges in combination with the market opening, generators and wholesalers are not bound to their own national markets anymore. Driven by ambitions to maximize profits and minimize costs, respectively, market participants enter these new markets and thus, induce an increase in cross-border electricity flows. In 2007, 13.7% of the total electricity consumption in all UCTE countries was delivered cross-border.[4]

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[1] Following Bitz (1993) p. 642, we define risk as the danger, resulting from the uncertainty of future developments, of a negative deviation of an economic quantitiy from a defined traget value.

[2] Volatility throughout this paper is defined as the annualized standard deviation. See Weron (2006) for an overview of volatilities in different markets.

[3] Although one can distinguish between electricity pools and electricity exchanges, we refrain from discussing their difference. Throughout this thesis we only consider electricity exchanges and refer to Eydeland and Wolyniec (2003) for a detailed treatment of the topic.

[4] The Union for the Co-ordination of Transmission of Electricity (UCTE) is the association of all transmission system operators (TSO) in the continental European area. The UCTE currently comprises 29 TSOs from 24 countries.

The increased price risk in national electricity markets, as a result of the liberalization, requires an adequate risk management.[5] Moreover, cross-border markets feature distinct risk factors that deserve special consideration and thus, post an additional challenge for risk management in electricity markets. In order to meet these requirements, a thorough understanding of the characteristics of electricity prices and the relevant risk management tools is of utmost importance.

Cross-border markets refer mainly to neighboring markets and exhibit the risk of locational price differences. This risk is well known in commodity markets and has been addressed by Kamara and Siegel (1987) and Pirrong et al. (1994) in case of agricultural commodities and by Brinkmann and Rabinovitch (1995) in case of energy commodities. Research in this field generally focuses on the analysis of hedging effectiveness of exchange traded futures contracts where the actual and the underlying commodity specification differ in terms of location. Although for electricity markets some studies, such as De Vany and Walls (1999) or Hadsell and Shawky (2006) for the US and Worthington et al. (2005) for Australia, analyze the interrelations of regional electricity spot prices, there are only few that directly address locational prices and especially their spreads. Skantze et al. (2004) present a bid-based stochastic model for locational electricity spot prices in the USA, while Haldrup and Nielsen (2006a) as well as Haldrup and Nielsen (2006b) consider congestion and non-congestion periods when modeling electricity spot prices and relative prices of regional interconnected electricity markets in Scandinavia. Although their work sheds some light on the characteristics of locational electricity prices, there is almost no empirical research concerning derivative products in cross-border electricity markets. The only studies available are by Kristiansen (2004a) and Kristiansen (2004b), which, however offer only limited empirical analyzes.

[5] See Oehler and Unser (2002) p. 20 ff, for a thorough discussion of the risk management process.

This thesis, to the best of the author's knowledge, is the first work to thoroughly discuss the valuation of derivatives in the European cross-border electricity market. It covers two distinct electricity markets in Europe and discusses the two most important types of derivatives within the European cross-border electricity market. Although the focus clearly is on cross-border markets, their unique nature cannot be fully understood without a fundamental treatment of national electricity markets. Moreover, this thesis introduces the relevant risk factors associated with cross-border markets and estimates these risk-factors based on the major derivative instruments. Finally, it delivers a profound analysis of the valuation of these contracts. Overall, this thesis intends to help the reader to better comprehend cross-border electricity markets and to gain in-depth knowledge of the relevance and functioning of risk management in these markets.

The second chapter introduces the European electricity markets and discusses the economics behind electricity as a commodity. This knowledge is important for understanding the respective derivative instruments in those markets. Moreover, the second chapter also shortly explains the main types of derivatives in European cross-border electricity markets and reviews the relevant valuation models. Following the introduction, each of the subsequent two chapters discusses one the most relevant cross-border electricity derivatives. These are Contracts for Difference (CfDs) and Physical Transmission Rights (PTRs), where the latter is distinguished into day-ahead contracts with delivery of one hour and monthly contracts with delivery over an entire calendar month. CfDs are exchange traded forward contracts on the spread between two locational prices in the Scandinavian electricity market and are the only cross-border derivatives currently traded in this market. PTRs are options on the difference between two neighboring electricity prices and are, as the name suggests, physically settled. These contracts are currently the only contracts used for cross-border hedging in the German electricity market. The research in the following chapters focuses around the question of how these contracts are priced and what are the main drivers for their valuation. These questions are essential for a thorough understanding of these products and thus, for their adequate usage in cross-border risk management.

Chapter 3 empirically investigates the pricing of the aforementioned CfDs over the period between 2001 and 2006. It is shown that CfD prices contain significant risk premia. Their sign and magnitude, however, differ substantially between areas and delivery periods, because areas are subject to transmission congestion to a varying extent. While the relation between risk premia and time-to-maturity is not uniform for CfDs, there is a negative relation for implied area and system forwards, which can be explained by the relative hedging demand of market participants. In addition, we find that risk premia of CfDs and implied area forwards vary systematically with the variance and skewness of the underlying spot prices. This confirms both implications of the Bessembinder and Lemmon (2002) model.

Chapter 4 analyzes the valuation of day-ahead PTRs on the German-Dutch interconnector. From a financial perspective, PTRs are options written on the difference between the German and Dutch hourly electricity prices. Chapter 4 proposes a model for the valuation of day-ahead PTR options incorporating the unique characteristics of the underlying spread. The model is empirically testes for all PTRs between 2001 and 2008, where each hour of the day is modelled separately. Overall, especially for calm hours, the approach constitutes an adequate model for the valuation of day-ahead PTR options. Further, the estimated parameters show that during calm hours PTR options are traded at a discount, while they trade at a premium during turbulent hours. This premium can be explained by either hedging demand or speculation of market participants. Finally, we find evidence for seasonality in the residuals of hourly and monthly PTR option prices. For monthly PTRs and hourly PTRs during turbulent hours, this seasonality is strongly related to jumps in the underlying spread.

Chapter 2

Electricity Markets and Derivatives

This chapter provides the fundamentals about electricity markets and the commodity electricity itself. It begins by describing how the structure of international electricity markets has evolved during the last decades. Afterwards, this chapter discusses the main features electricity as a commodity exhibits and the implications of those unique characteristics for electricity prices. In the following, the most important derivative instruments in those markets are introduced including the currently prevailing approaches for modelling electricity prices. Finally, before the chapter closes with a short conclusion, the most important valuation models and the corresponding solution techniques for electricity derivatives are described.

2.1 Liberalization of Electricity Markets

The process of liberalization in international electricity markets and therefore the breakup of monopolistic structures has its origin in Chile. Beginning in 1982, Chile began to separate electricity generation from sales ending an era of vertical integrated utilities. Only four years later, the Chilean electricity market was shaped by extensive privatizations leading to a competitive wholesale market for electricity. During the late 1980s and early 1990s the liberalization efforts spilled over to the European electricity market. The UK was the first country to adopt these efforts. Its electricity market, which comprised until 2005 England and Wales and since then also Scotland, was restructured based on the UK Electricity Act from 1989. The disbandment of the Central Electricity Generation Board (CEGB) was at the heart of this act. The CEGB was a vertically integrated monopoly covering all steps of the value chain. The deregulation led to the England and Wales Electricity Pool, the first organized market place for electricity in the world. From the UK, the liberalization proceeded to Norway through the Norwegian Energy Act from 1991 resulting in the electricity exchange Nord Pool. While being a solely Norwegian exchange at the beginning, it quickly developed towards an international market place. In addition to Norway, Nord Pool currently comprises Sweden (since 1996), Finland (since 1998) and Denmark (since 2000).

In continental Europe, the liberalization started with the European Union Directive 96/92/EG in 1997. This directive laid down rules with the aim of a joint pan-European electricity market. Further, it postulated a clear separation of monopolistic and competitive elements within the value chain in the electricity market. While the first basically comprises all elements related to the power grid, i.e. transportation and distribution, the latter includes mainly generation and sales. This distinction is made in order to introduce as much competition as possible and to prevent the exertion of market power where competition is not feasible. Figure 2.1 shows the unregulated value chain in competitive electricity markets.[6]

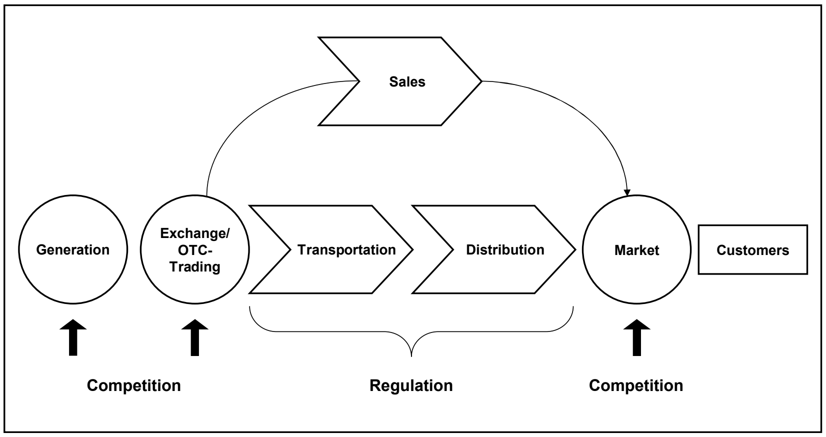

Figure 2.1: Disentangled value chain in the electricity market.

The European Commission has, however, not specified a consistent timeline for the deregulation for individual countries. Rather, it determined minimum requirements where each country defines its own rate of implementation. Although limited grid capacities hamper the complete consolidation of European electricity markets, a convergence of electricity prices is clearly visible.[7] The deregulation in continental Europe has also led to the development of organized market places. The largest electricity exchanges, sorted by the year of origin in ascending order, are the Operado del Mercado Español de Electricidad (OMEL) in Madrid (1998), the Amsterdam Power Exchange (APX) in Amsterdam (1999), the European Energy Ex-

[6] Cf. Schiffer (2005) p. 203.

[7] See Holler and Haberfeld (2006), Huisman et al. (2007) and Zachmann (2008) for an analysis of the convergence of European electricity prices.

change (EEX) in Leipzig (emerged in 2002 from the merger of the European Energy Exchange in Frankfurt and the Leipzig Power Exchange), the Powernext in Paris (2002), the Energy Exchange Austria (EXAA) in Vienna (2002) and the Italian Power Exchange (IPAX) in Rome (2004).

2.2 Electricity as a Commodity

This sections firstly describes the fundamentals of electricity from a more technical perspective a sheds some light on the underlying features that make electricity so unique as a commodity. Secondly, the characteristics of electricity that are directly derived from the aforementioned technical features of electricity, are discussed.

2.2.1 Fundamentals of Electricity

Electricity is different from other commodities in various aspects. First, electricity is always delivered over a specific period of time as its usage is not feasible at a specific point in time. Therefore, electricity is considered a flow commodity. Any agreement concerning the delivery of electricity thus, has a temporal dimension to it. The most common contracts comprise hours, weeks, months and years. Although there are further products available, both exchange traded and OTC, these usually include any combination of the above. Another characteristic of electricity is its limited storability. Although electricity is theoretically storable via pumped storage facilities, this approach is currently not feasible in most European countries on a large scale and at reasonable cost.[8] Last, electricity is grid

[8] According to the German Federal Ministry for the Environment, Nature Conservation and Nuclear Safety (2008), only 3.4% of all electricity consumption in Germany came

bound and therefore its delivery requires an existing power grid. In addition, electricity spreads across the entire network as soon as injected into the system. Therefore, a simple point to point delivery in electricity markets in not feasible. Moreover, every customer receives an electricity mix across different suppliers and various generation fuels. This however, poses no harm due to the perfect fungibility of electricity.

The grid boundness and non-storability require an independent system operator in charge of the functionality of the entire power system. These are the transmission system operators (TSOs). In Germany, there are four TSOs corresponding to the respective interconnections in the German electricity market. These include EnBW in the south-west, E.ON in northern, central and southern Germany, RWE in the west and south and Vattenfall in east Germany and Hamburg. Figure 2.2 illustrates the German electricity market and its four TSO regions.

TSOs need to ensure that at every point in time the demand and supply in the market are balanced. Since TSOs cannot simply store electricity to dampen shocks to supply or demand, they depend on consumers and generators in case of a sudden mismatch. On the supply side, electricity producers are obliged to provide a certain amount of capacity that TSOs are allowed to call on short notice. On the demand side, TSOs can cut off specific customers if needed. This procedure is called a rolling blackout. Customers who agree to accept a rolling blackout beforehand usually receive a price discount.[9]

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from hydro power utilities in 2007. One exception, however, might state Norway. There, hodro power had a share of over 98% of all power generation in 2007. See Nordel (2009) for further details on the Norwegian electricity market.

[9] See Konstantin (2009), especially chapter 9.1, for a detailed discussion of the TSO business in Europe.

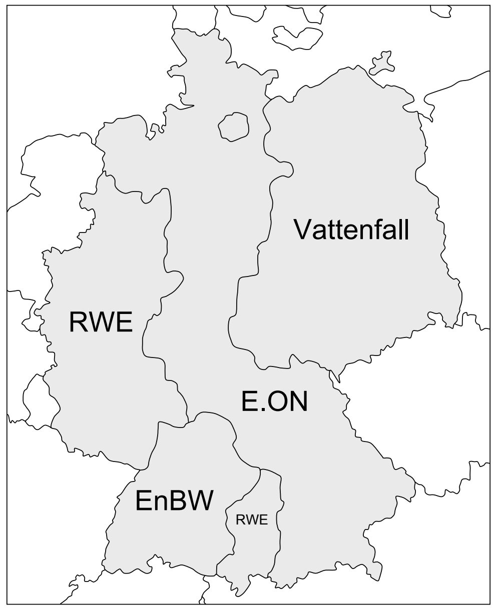

Figure 2.2: TSO map of the German electricity market.

In cross-border electricity markets, the problems for TSOs also apply. However, there is an additional challenge concerning the grid boundness of electricity. As mentioned above, electricity cannot be delivered point to point as it spreads across the entire system as soon as injected. The main problem of this characteristic can be emphasized by a simple example.[10] Assuming a three node electricity network, e.g. Germany (GER), France (FRA)

[10] Our example is based on the example of Stoft (2002) chapter 5.4. The reader is also refereed to ibid. for a detailed analysis of the technical characetristics of cross-border power grids.

and the Netherlands (NL). Further, all nodes are mutually connected by power lines with limited capacity given next to each line. Moreover, there are 150 MW supplied in Germany and there is a load (demand) of 150 in the Netherlands. Let us also assume that all lines are of the same length and share the same characteristics concerning their impedance.[11] Figure 2.3 shows the setup of our example.

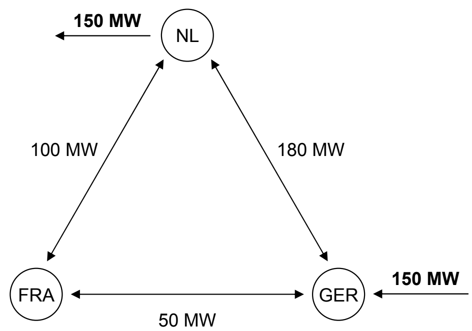

Figure 2.3: Three node power grid including Germany (GER), France (FRA) and the Netherlands (NL) with limited capacity.

In our example, 150 MW of electricity flow from Germany to the Netherlands using both routes, i.e. directly and via France. This characteristic is called loop flow and governed by Kirchhoff's Law stating that electricity spreads across an entire system via the path of least resistance. However, as the indirect path has twice the impedance, only one third of the total current flows via France, i.e. 50 MW. The other 100 MW flow directly. The problem of this feature becomes evident once we increase the load to

[11] Impedance can be seen as a generalization of resistance. It describes how difficult it is for electricity to flow over a given path.

180 MW, i.e. both supply and demand increase to 180 MW. If we supplied 180 MW in Germany, 60 MW would flow via France exceeding the given capacity. Therefore, we would need additional supply from France in order to realize this generation set up. If we assume that it is economically optimal for Germany to supply as much electricity as possible, we need to introduce another important feature of electricity in order to determine the optimal production plan in order to realize the total load of 180 MW. Opposite currents in a power system cancel out. This means that for every MW supplied in France one more MW can be supplied in Germany. Thus, it is optimal, given the above assumptions, to supply 165 MW in Germany and 15 MW in France.[12] This little example shows that limited capacity of one cable could hamper electricity flows via another connection. Although this characteristic also applies in national electricity markets, it is of particular interest in cross-border markets due to the scarcity of capacity. In fact, situations are quite common where a congested cable between Germany and France prevents the delivery of electricity via a non-congested cable between Germany and the Netherlands.

The occurrence of congestion in our example addresses two relevant topics in cross-border electricity markets. These are the calculation of locational prices (i.e. transmission pricing) and the allocation or valuation of transmission capacities in case of congested power markets. For the calculation of locational prices, there are currently two approaches applied in international electricity markets. These two are nodal transmission pricing, used, for example, in New York and the Pennsylvania-New Jersey-Maryland (PJM) market, and zonal transmission pricing, used, for example, in Aus-

[12] Given 165 MW are supplied in Germany and 15 MW in France, two-thirds of each load travel the short, i.e. direct, distance to the Netherlands. One-third travels the indirect paths. Thus, from Germany, 55 MW flow via France to the Netherlands and 5 MW flow from France via Germany to the Netherlands. Since opposite currents cancel out, the net flow between Germany and France complies with the capacity limit of 50 MW.

tralia, California and the Nordic market.[13] With nodal pricing, the locational marginal price (LMP), i.e., the price of inducing an additional MW of electricity, is calculated for each node (i.e., every point in the grid where electricity is added or removed) in the electricity network. Zonal pricing, in contrast, groups nodes that are connected by non-congested power lines into zones (or areas). Within each zone, the LMP is identical across all nodes and referred to as the zonal market clearing price (ZMCP). Figure 2.3 is an example of zonal pricing, where within each zone (country) the ZMCP is identical. While nodal pricing is considered more efficient but rather complex, zonal pricing is considered simplistic but more transparent.

Based on the distinction between zonal and nodal pricing, three approaches for the allocation of transmission rights have evolved. These comprise the contract path model, the flow based model and the point-to-point model. While the first two are usually applied in the context of zonal pricing, the latter requires nodal pricing. The contract path model is a rather naive way of allocating transmission capacity. In a first step, the total transfer capacity (TTC) is calculated as the maximum available capacity that may be transferred between two zones considering existing security standards.[14] Considering the so called transmission reliability margin (TRM), in order to account for unintended physical flows or measurement errors, leads to the net transfer capacity (NTC). After subtracting the already allocated capacity, e.g. from long-term delivery agreements, we finally receive the available transmission capacity (ATC). Based on the ATC, transmission

[13] Further approaches discussed in the literature are Chao-Peck pricing (Chao and Peck (1996)) and uniform pricing (Green (2007)). See Kristiansen (2004b) for a short discussion of these approaches.

[14] UCTE (2004) defines the so called $n-1$ criterion. It states that the breakdown of one element of the power grid, e.g. one power cable or a transformer, must be compensated by the other elements in the system.

capacity between two zones is allocated where it is assumed that the electricity flows directly between the two zones ignoring the occurrence of loop flows.[15] The flow based approach adjusts for this shortcoming. It is based on the power transfer distribution factor approach (PTDF) which incorporates the actual power flows among zones independent of the announced contract path. Once the delivery of electricity across zones is announced, the PTDF matrix transfers the intended delivery into the resulting power flows across all zones. While the contract path model only requires the cooperation of the two included TSOs, the flow-based approach requires a joint organization of all TSOs within the market. In this case, the TSOs are usually replaced by a centralized ISO (independent system operator) which allocates the path dependent transmission rights. Although the flow-based model offers more flexibility and allows loop flows across zones, intra-zone electricity flows are still not considered posing a considerable shortcoming. In the point-to-point model, which implies nodal pricing, the ISO centrally schedules all trades within the power grid. The ISO then computes the LMP for each node within the system.[16]

In order to clarify the distinction between contract path, flow-based and the poitn-to-point model, we refer to our previous example from Figure 2.3. In a contract path approach, transmission capacity between Germany and the Netherlands is allocated only based on the utilitzation of the line between those two countries. Any other restrictions occurring from loop flows are neglected. The flow based model resolves this problem by considering the loop flow from Germany via France to the Netherlands. However, as

[15] Although the contracts path methodology itself does not refer to any particular transmission pricing, it is generally applied in the context of zonal pricing.

[16] Although relevant in the context of cross-border electricity markets in principle, a thorough analysis of the power grid economics is beyond the scope of this thesis. We therefore refer to Hogan (1998), Hogan (1999) and Ruff (2001) for a comprehensive discussion of transmission pricing.

every country is assumed to be one zone, i.e. there is no congestion within those zones, intrazonal electricity flows are not considered. The point-to-point model corrects this shortcoming by not considering any zones, but by looking at each node within the entire power grid separately. Is this model, flows from every node are considered and calculated.

Although our discussion of cross-border electricity markets is based on the interconnections of three countries, cross-border electricity markets are not defined based on nationalities. Rather, cross-border electricity markets are all markets that are connected by interconnections with a limited capacity. Within Germany, there has not yet been any congestion between the four TSO areas. Because of this attribute as a single electricity market without congestion, Germany is also considered as a copperplate. However, Wawer (2007) discusses the risk of congested powerlines within the German market. This risk is driven by growing generation capacities in the north and decreasing capacities in the south of Germany leading to increasing intranational electricity flows. One reason for this development is the growing installation of wind generators in North Germany and especially its offshore regions. Considering the 22.247 MW of installed capacity by the end of 2007,[17] an estimated 10.000 MW of additionally planned capacity in offshore wind parks are a substantial contribution. Another reason is the agglomeration of new power plants in North Germany in combination with the continuing shut down of power plants in the south. Out of the 30.000 MW of new capacity planned until the year 2015, the majority of this capacity will be located around Brunsbüttel, Wilhelmshafen and the Ruhr. On the other hand, the nuclear power phase-out will result in a reduction of about 8.000 MW of installed capacity until 2015. Therefore, congested electricity markets are not limited to cross-national interconnections and the existence of cross-border electricity markets within national markets is

[17] Cf. Federal Ministry for the Environment, Nature Conservation and Nuclear Safety (2008) p. 11.

a pending threat for the German electricity market.

2.2.2 Characteristics of Electricity Prices

2.2.2.1 National Markets

The unique characteristics of electricity as a commodity have a significant impact on electricity prices. Moreover, they determine the entire setup of the electricity market. The fundamental market segment in the electricity market is, analogous to financial markets, the spot market. This market segment, however, differs from traditional financial spot markets. This distinction is based on two key aspects of electricity already introduced. First, electricity trading has always a temporal dimension to it, as described above. Therefore, the spot market partitions every day into 24 block of 60 minutes for which electricity is sold separately.[18] As the demand for electricity varies considerably during the day, choosing less than 24 partitions per day does not accommodate the specific demand characteristics. Selling electricity implies that for the respective hour, a predefined volume of electricity, usually multiples of 1 MW, is constantly delivered. Thus, one contract in the spot market implies the energy of 1 MWh. The other main feature of the electricity spot market is influenced by the fact that electricity cannot be delivered instantaneously. Rather, its supply requires a specific leadtime. Moreover, in case of sudden variations in the supply TSOs cannot guarantee the balance of the power grid. Thus, leadtime is essential from both a technical and a stability point of view.

For the above discussed reasons, the spot market for electricity is in general

[18] Although the distinction of 24 hours is most common, other setups are also possible. In the UK, for example, every day is devided into 48 blocks of 30 minutes.

defined as a day-ahead market where the day before delivery electricity is auctioned separately for each hour of the following day. In case of a weekend or holiday, the auction is held on the previous working day. The average of all 24 hourly day-ahead prices is called the baseload price.[19] In addition, the mean of the hourly prices between 8am and 20pm, i.e. the twelve hours with the highest demand, constitutes the peakload price.[20] Besides the day-ahead auction, several exchanges have established intraday market places during the last years. There, electricity is continuously traded with a leadtime between 60 and 90 minutes. At the German EEX for example, the intraday market opened on September 25, 2006 and allows trading different blocks of hours for the next day with a leadtime of 75 minutes. Although intraday markets have become more and more liquid, their volumes are still significantly below those of the day-ahead auctions.[21] Therefore, throughout this thesis, we will denote the day-ahead auction as the spot market.

In an efficient market, prices are determined by marginal costs. This also holds in electricity markets. In order to determine the marginal cost function, electric utilities align their generating plants in ascending order. This alignment also holds for various generators within a specific power plant. The resulting marginal cost function is called power stack or merit order. The power stack is a convex function of load with a small slope for lower production values. As load increases, however, marginal costs rise steeply. In those regions of the supply curve, minor changes in the demand curve result in extreme variations in the resulting price. The actual marginal

[19] Throughout this paper, the baseload price is also considered the daily price.

[20] Although baseload and peakload price are the most prominent average values and are the underlying of several exchange traded and OTC derivatives, other mean values across arbitrary hours are possible. See for example http://eex.de for other blocks of hours.

[21] Out of a total spotmarket volume of 154.4 TWh at EEX in 2008, the intraday market constituted only 2.3 TWh.

costs of a power plant are significantly determined by fuel prices, e.g. coal, natural gas, CO_2, and can even affect the order of power plants within the power stack. Moreover, shutdowns of power plants also influence the shape of the cost function. While these are in general scheduled, mostly due to maintenance measures, they can also occur unexpected leading to a shock of the supply curve.

Besides the supply curve, the demand curve is also subject to considerable variations based on varying demand. Concerning their demand patterns, we can distinguish industrial, commercial and private customers. Based on this distinction, demand varies depending on the hours of the day, the day of the week as well as working vs. non-working days. The variations are easily predictable due to their consistency. There are, however, variations in demand that are harder to predict as they are subject to uncertainty. Here, especially weather driven changes in demand are of relevance.[22] In addition to the hardly predictable weather information, its impact on electricity prices shows regional discrepancies. In the USA for example, the demand for electricity increases dramatically in the summer due to the intensive usage of air conditioning. In colder regions in contrast, electricity demand is usually higher during colder months based on longer lighting periods and electric heating devices.

Figure 2.4 shows the aggregated supply and demand curves at EEX on February 22, 2007 for hour twelve and 24.[23] For both hours, the convex shape of the supply and demand curve is clearly visible. Also the figure shows an increased demand for hour twelve compared to hour 24. The

[22] Among others, Weron (2006) and Pirrong and Jermakyan (2008) use temperature and precipitation as exogenous variables in order to model electricity prices.

[23] The supply and demand curves in Figure 2.4 are based on actual auction results observed at the EEX day-ahead auction for the respective hours, where each market participant submits combined price/volume bids.

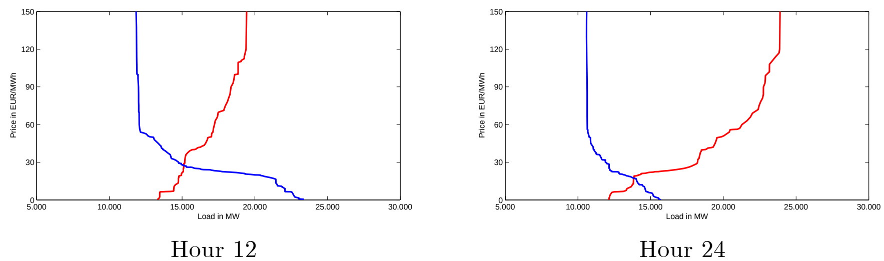

Figure 2.4: Supply (right) and demand (left) curves for the day-ahead auction at EEX on February 22, 2007.

reduced supply for hour twelve can be either due to power plant outages or, more likely, due to lesser available capacity based on prior delivery agreements. The inelastic part of the demand curve indicates the minimum volume required at any cost.

The specific characteristics of electricity are reflected by observed market prices. The non-storability and the need of a permanent match of supply and demand can result in extreme price variations. These extreme price movements are most like when demand is already high, i.e. at the steep end of the supply curve. The non-storability of electricity also induces the existence of negative electricity prices. In some cases it is more reasonable for a utility company to pay someone for the use of electricity than to shut down a power plant in order to reduce production.[24] Further, seasonal patterns in the demand are also visible in electricity prices. The seasonality

[24] The existence of negative prices depends on the costs of shutting down and firing up a plant and the required leadtime. At the EEX, negative prices were introduced September 1, 2008. The first negative electricity price was observed for hour 6 on October 5, 2008. Between September and December 2008, 15 negative prices for single hours were observed. On a daily basis, there has not yet been a negative price.

does not only appear on an annual basis but also intraday and across the week as discussed above. Figure 2.5 shows the daily spot prices at the EEX between 2001 and 2008.

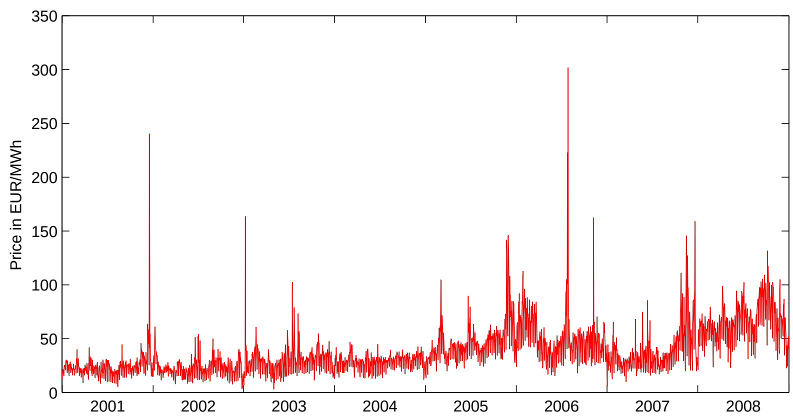

Figure 2.5: Daily spot prices at EEX between 2001 and 2008.

Figure 2.5 firstly shows that prices are not governed by a persistent trend but rather fluctuate around a certain level. The mean-reversion characteristic is symptomatic for electricity markets and indicates only a loose connection between intertemporal prices. This is based on the short-term nature of shocks to the supply and demand. Supply shocks generally occur in case of sudden breakdowns in the generation park which are resolved within a few days in most cases. Demand shock have an even shorter impact. Due to the non-storability of electricity, intertemporal electricity prices are considered as distinct assets and therefore their demand is not directly linked. Thus, supply and demand fluctuate around a given level resulting in the mean-reversion behavior of electricity prices. The mean-reversion feature was formally analyzed, among others, by Weron (2002). He shows that the Hurst exponent for prices at the California Power Exchange (CalPX) exhibit a value significantly smaller than 0.5 which indicates nonpersistent trends and therefore mean-reversion.

Another striking characteristic of electricity prices are the extreme occasional jumps. These jumps usually vanish within a few days and have an enormous impact on the overall risk in the electricity market. The standard deviation of daily returns over the above illustrated period was 47%. This value is not annualized but based on daily data. If we break down the overall standard deviation, we see major differences across hours. While the standard deviation for hour 14 is 89%, the corresponding value for hour 23 is only 18%. In addition to standard deviation, the jumps have also a considerable impact on the higher moments of the distribution of electricity prices. Between 2001 and 2008, daily spot price have a skewness (kurtosis) of 2.43 (14.32). These values confirm the impact of observed jumps and indicate significant outliers, where positive outliers are more likely than negative ones. The higher moments also vary largely across hours. For the first hour of the day, the skewness (kurtosis) is 1.06 (0.73) in contrast to values of 21.16 (627.29) for hour 19. These results support the findings of Huisman and Huurman (2003) and Weron (2005) who find fat tales in the distribution of electricity prices in the Netherlands and Germany respectively.

Besides their erratic behavior, daily prices are subject to seasonality on an hourly, daily and annual basis. Although the seasonality is not visible at first sight from figure 2.5, due to the small scale, it becomes evident when looking at aggregated mean hourly or daily prices. The intraday seasonality is shown in Figure 2.6 (left). This so called double peak pattern emphasizes that prices throughout the day differ significantly. While prices between hours three and five are around 20 EUR, their values increase 60 EUR and 50 EUR for hours twelve and 19 respectively. The intraday seasonality is confirmed for prices at the EEX by Burger et al. (2004). They show a clear autocorrelation for hourly prices at lag 24 between January 2001 and June 2002.

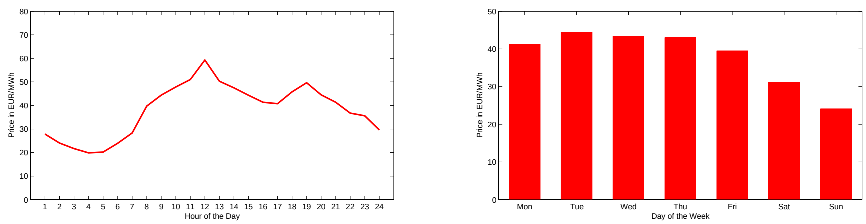

Figure 2.6: Average spot prices per hour of the day (left) and day of the week (right). Calculations are based on daily EEX prices between January 1, 2001 and December 31, 2008.

The intra-week seasonality becomes evident when looking at daily average electricity prices. Figure 2.6 (right) shows the mean baseload price per weekday at EEX between 2001 and 2008. While prices on Monday through Thursday differ only slightly, the drop towards the weekend is striking with minimum prices on Sundays. The weekly seasonality is confirmed for Dutch electricity prices by Huisman and Huurman (2003). They show that the logarithm of daily spot prices between January 2001 and July 2003 were on average 0.29 lower on Saturdays and 0.64 on Sundays compared to the overall mean price. Besides clear patterns in the intra-week spot prices, weekly seasonality is confirmed by an analysis of the autocorrelation function (ACF) of daily spot prices. Figure 2.7 shows the ACF function for daily spot prices at EEX between 2001 and 2008. The increased values for lag multiples of seven are clearly visible. The weekly seasonality in electricity prices is confirmed by Weron (2006) for prices from the CalPX between May 1998 and April 2000. He also shows a significant autocorrelation with a lag of seven days.

The last type of seasonality, i.e. on an annual level, is also evident in electricity prices. Although, due to the erratic behavior and extreme jumps in daily prices, the annual seasonality is not as pronounced in Figure 2.5.

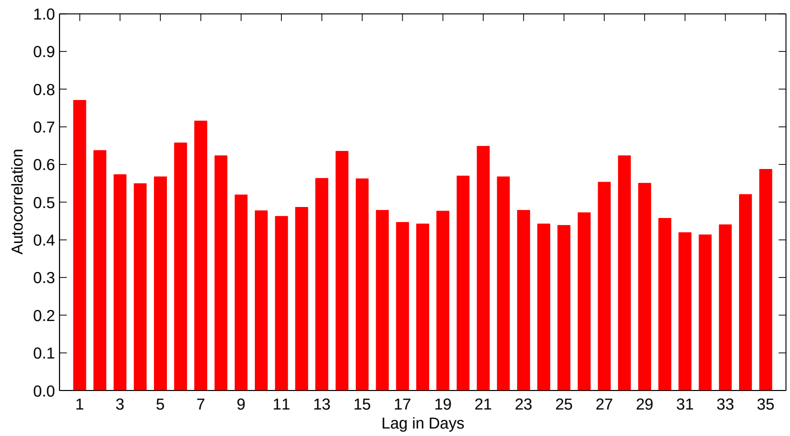

Figure 2.7: Autocorrelation function of daily spot prices at EEX between January 1, 2001 and December 31, 2008.

When looking at monthly futures contracts, however, this seasonality is emphasized. Figure 2.8 shows the futures curve for month futures at EEX for the year 2006. In order to depict an entire annual cycle, we observe futures prices at two different dates throughout the year, since only the next six month contracts are simultaneously traded. The futures prices clearly show an annual seasonality with generally lower prices in the summer and higher ones in the winter. This characteristic pattern is determined from the supply as well as the demand side. Due to longer lighting hours and electricity based heating, demand in Germany is in general higher during the winter. On the supply side, higher natural gas prices during the winter, based on extensive heating, lead to a steeper supply curve an thus, to generally higher prices.[25]

[25] See Routledge et al. (2001) and Douglas and Popova (2008) for a discussion of the relationship between natural gas and electricity prices.

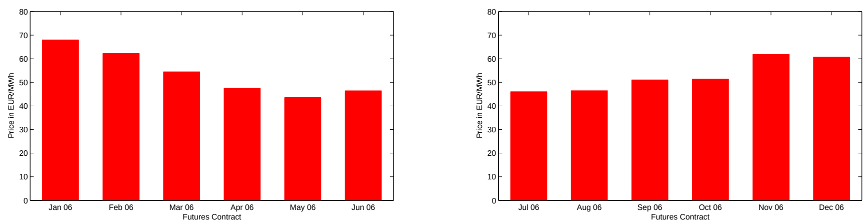

Figure 2.8: Prices of monthly futures contracts at EEX. Figures are based on closing prices from December 14, 2005 (left) and Jume 14, 2006 (right).

2.2.2.2 Cross-border markets

In cross-border markets, derivatives are generally written on the spread between two local electricity prices. This spread may either be positive or negative. Further, neighboring prices are often subject to the similar market forces, especially in Europe, and both prices exhibit the above described characteristics. Thus, most characteristics of national electricity prices carry over to cross-border markets while some, in contrast, cancel out. Adjoining market areas in Europe share distinct demand patterns. This does not only cover intraday or intra-week demand, which is probably quite similar throughout the world. As these markets are generally subject to identical climatic conditions, the demand pattern accord even on an annual basis. Therefore, seasonality in both areas is expected to be at least comparable and thus, the seasonality evident in national prices is not observed in cross-border markets.

In contrast to seasonality, mean-reversion and the characteristic jumps in national electricity prices, carry over to cross-border markets. Since national prices revert back to a long-term mean, so obviously does their spread. Moreover, as the national prices often follow similar long-term means, as discussed in the preceding paragraph, the spread generally fluc-

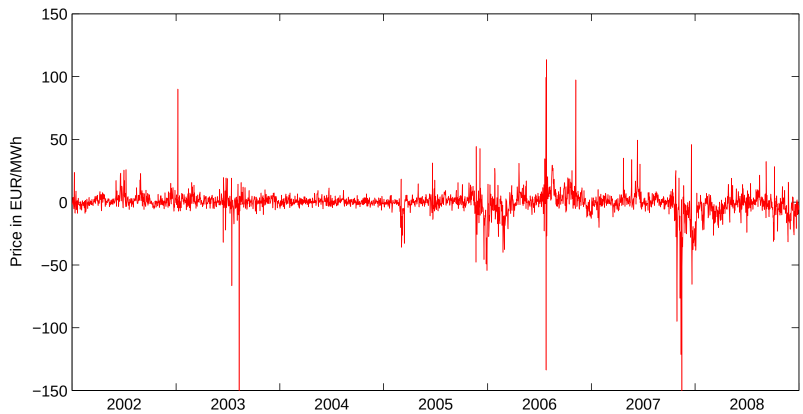

Figure 2.9: Daily spread between EEX and Powernext spot prices between 2002 and 2008.

tuates around zero. However, depending on the overall price level in adjoining markets, mostly affected by different power generation mixes, the mean-reversion level in cross-border markets may be also positive or negative. In addition to mean-reversion, the typical spikes in national electricity prices are also evident in cross-border spreads. As jumps are usually based on a temporal shock in demand or supply, they are idiosyncratic and therefore also observable in price spreads. Moreover, jumps are more frequent in cross-border markets and can be either positive or negative. Figure 2.9 shows the spread between German and French spot prices between 2002 and 2008 where a positive spread correspond to higher prices in Germany and vice versa. This figure depicts the just discussed characteristics of electricity prices in cross-border electricity markets.

2.3 Derivatives in Electricity Markets

This section introduces the basic derivative contracts in electricity markets, where we distinguish national as well as cross-border markets. In addition, we further explain the most relevant approaches in national and cross-border electricity market for modelling electricity prices. This section therefore constitutes the basis for the valuation of derivatives contracts discussed later in this thesis.

2.3.1 Derivative Products

2.3.1.1 National Electricity Markets

A derivative is defined as a "...financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables.".[26] Derivatives allow for the purchase or the sale of electricity in the future and their execution is in general binding for the issuer and may be binding or optional for the buyer. Compared to the electricity spot market, its derivatives counterpart is by far larger.[27]

In electricity markets, on a first level one generally distinguishes physically from financially settled derivatives. For physically settled derivatives, the actual good, i.e. electricity, is delivered in exchange for an ex ante agreed price. In case of financially settled derivatives, the good is not physically delivered but only payments are exchanged. Although financial derivatives are more important and the most liquid ones based on exchange traded

[26] Cf. Hull (2009), p. 1.

[27] The trading volume of the derivatives market at the EEX was about 1.165 TWh in 2008 including OTC clearing.

volumes, physical contracts are also of high relevance especially in the OTC market.

The most important derivative in the electricity market is the forward or futures contract.[28] These mutually binding contracts are either financially or physically settled, where the first are by far more liquidly traded. During their delivery period, the buyer of a forward usually pays the reference price and receives the daily spot price (either peakload or baseload) in exchange.[29] Since the delivery of electricity needs a temporal dimension, so does the forward contract. Therefore, contracts are traded at EEX on a monthly, quarterly and yearly basis, where for each contract type, baseload and peakload prices are distinguished. The payoff of a forward contract can thus be written as

$$\text{Payoff}_{Forward} = \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2} \left(P_t - F \right). \tag{2.1}$$

Here, P_t indicates the spot price at time t and F is the fix forward price. $T_1$and T_2 are the first and last day of the delivery period respectively. In case of a futures contract at EEX, the payoff slightly changes. Since the futures contract is traded during the delivery period and also marked to market every day, the payoff becomes

[28] Whether forwards or futures contracts are the dominant derivative varies with the observed market place. While at the EEX only futures contracts are traded, at Nord Pool forwards are prevailing, with the exception of day and week contracts.

[29] Since financially settled futures comprise the exchange of financial payments, they are stricitly speaking swap contracts. However, we apply the common nomenclature and refer to these contracts as futures or forward contracts. See Benth et al. (2008b) for a further discussion on the swap characterisitcs of futures contracts.

$$\text{Payoff}_{Future} = \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2} (F_t - F_{t-1}) \,. \tag{2.2}$$

F_{t-1} is the settlement price of the futures contract on the previous day. Thus, the total ex-post payoff of a futures contract on electricity is, analogous to the equity market, the sum of all daily marking to market payments.

Besides mutually binding derivatives, options contracts are also widely used in electricity markets. The buyer of such an option has the right, but not the obligation to buy or sell electricity at maturity. Analogous to equity options, European and American options are the most common contracts. Other characteristics, such as Bermudan or Asian style options, are also traded especially in the OTC market. The payoff of a plain-vanilla call option with a strike price of K is

$$\text{Payoff}_{Call} = \left(\frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2} (P_t - K) \right)^+ \,. \tag{2.3}$$

While options directly written on the electricity spot price are usually traded OTC, exchange traded options are mostly written on other derivatives such as futures contracts. Besides plain-vanilla derivatives explained above, more complex options (or exotics), are also very important in the electricity market. The two most relevant are spark-spread contracts and swing options. Spark-spread contracts are traded as forwards or options and are written on the difference between the electricity price and the price of the generating fuel. When calculating the payoff of a spark-spread option, both parties agree on a heat rate, measured in Btu/kWh.[30] Thus, the payoff of a spark-spread option can be written as

$$\text{Payoff}_{Spark-Spread} = \left(\frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2} (P_t - H \cdot V_t) \right)^+ , \qquad (2.4)$$

where V_t is the price of the generating fuel and H is the heat rate described above. While the term spark-spread refers to natural gas as the generating fuel, other fuels also involve different terms for the spark-spread contract. In case of coal, these contracts are also referred to as dark-spread contracts. For emission allowances one often finds the term clean- or green-spread. However, all of these contracts refer to the difference between the electricity price and one resource fuel needed to generate electricity. While those contracts are mostly OTC traded, there are also contracts listed at a few exchanges, e.g. New York Mercentile Exchange (NYMEX).

When exercising plain-vanilla or spark-spread contracts, the holder of the option needs to exercise the option before the delivery period starts and therefore faces the risk of uncertain price movements throughout the delivery period. Further, the amount of electricity delivered is constant for the entire delivery period once the option is exercised. Since these contracts can have delivery periods of several months, the price and volume risk for the holder might be substantial. Swing-options reduce those risks as they allow for adjustments of the delivery volume during the delivery period. A swing-option does not only define the baselevel of electricity delivery during the delivery period, but also a certain number of allowed up and down swings. These swings allow the holder to increase or decrease the

[30] A British theram unit (Btu) measures the energy required to heat one British pound (453.6 grams) of water by 1° F.

amount of electricity to be delivered or received compared to the base-level. Each swing is generally valid for one day. Therefore, these contracts offer a high level of flexibility and therefore reduce the volume risk of plain-vanilla contracts substantially. Due to their low degree of standardization, swing-options are entirely OTC traded.

2.3.1.2 Cross-Border Electricity Markets

In cross-border electricity markets, derivatives are distinguished, analogous to national markets, between financially and physically settled products. In Europe, three types of contract are of special interest.[31] Physical transmission rights (PTRs) securitize the right to physical capacity on a particular transmission path during a predefined period of time (usually multiples of one hour). PTRs are generally issued as option contracts. This means the holder of a PTR can decide whether to exercise the inherent right and inject electricity into the power grid. In case of congestion and therefore price differences in neighboring regions, the PTR allows for a profit by buying electricity in the cheaper market and selling it in the more expensive area. Since PTRs are limited to a specific direction when injecting electricity, the sign of the price difference is important for a potential profit. The resulting payoff can therefore be stated as

$$\text{Payoff}_{PTR} = \left(\frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2} \left(P_t^A - P_t^B \right) \right)^+ . \qquad (2.5)$$

[31] For a discussion of the relevant products in European cross-border electricity markets see ETSO (2006). For an introduction to relevant products in the US market see Kristiansen (2004c).

The above stated payoff corresponds to a PTR that allows to transfer electricity from region B to region A. Based on this payoff profile, the PTR can be seen as a physically settled option contract where electricity from region A can be swapped against electricity from region B, where P_t^A (P_t^B) refers to the electricity spot price in region A (B) at time t. Also, the PTR can be seen as an option on electricity in region A with the non-constant strike price P_t^B. Analogous to national contract, PTRs have also a temporal dimension. PTRs are available on an hourly, monthly or yearly basis. PTRs are currently the most widely used derivatives in the cross-border electricity markets in the UCTE area. All of Germany's cross-border capacities are managed via PTRs.

The financial equivalent to PTRs are financial transmission rights (FTRs). FTRs can either be mutually binding in form of a forward contract or equipped with an exercise option for the holder. While for the latter case the payoff is identical to (2.5), the payoff of a FTR forward contract can be written as

$$\text{Payoff}_{FTR} = \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2} \left(P_t^A - P_t^B \right), \tag{2.6}$$

which is identical to the payoff of a swap contract. In contrast to PTRs, the holder of a FTR is not required to engage in any physical transactions in order to profit from a price difference, i.e. FTR are purely financially settled. Both PTRs and FTRs are auctioned by the TSO or any organization entitled to auction these contracts by the corresponding TSOs. All cashflows occurring in the process of the auctioning are managed by these TSOs. Thus, the volume of issued PTRs and FTRs is determined by TSO and based on the available capacity.[32] Although PTRs and FTRs

[32] Although PTRs and FTRs incur the same payoff, from a financial perspective, these

are auctioned by the system operator, the method of auctioning fundamentally differs for both contracts. While FTRs are auctioned implicitly, PTRs can be auctioned implicitly as well as explicitly, where the latter is by far more common in Europe today. Explicit auctioning of PTRs relies on the contract path model for assigning transmission capacity. The two corresponding TSOs are involved in the auction which is often organized by a separate office. The TSOs receive the auction proceedings and the owner of the PTR benefits from potential price differences. FTRs require implicit auctioning, where a independent system operator (ISO) needs to organize the allocation of transmission capacity. FTRs are usually applied in a zonal pricing environment. Every generator notifies the ISO about price/volume combinations made available at each node and each load entity does the same for the demand. The ISO then calculates the nodal prices, where the difference between two nodes is the underlying for the specific FTR. Although the utilization of nodal pricing for FTRs leads to an exponential relation between nodes and possible contracts, all FTRs are related to physical flows and therefore, liquidity is no limiting factor.[33]

The third type of derivative in cross-border electricity markets in Europe are Contracts for Difference (CfDs). These products are financially settled forwards on the spread between two regional electricity prices. Therefore, they have the same payoff as FTR forwards stated in (2.6). In contrast to FTRs and PTRs, however, CfDs are exchange traded contracts. Thus, CfDs are traded bilaterally and the corresponding TSOs are not involved. Moreover, CfDs are not subject to any restrictions concerning their vol-

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two contract types differ in significant issues. See Hogan (2000) and Hogan (2003) for a discussion on FTRs and PTRs and their influence on market power. An analysis of the impact of PTRs and FTRs on the allocation and pricing of transmission rights, as well as the welfare properties of these contracts can be found in Joskow and Tirole (2000).

[33] Siddiqui et al. (2005), for example, state that in New York alone, 120,000 FTRs are possible.

ume. It is, however, not simply possible to introduce CfDs due to liquidity constraints. Since CfDs are not related to physical electricity flows, it is hard to establish sufficient liquidity to trade CfDs in a cross-border setting. In case of 100 zones (or nodes), there are almost 5,000 possible CfD connections. Therefore, the introduction of CfDs generally requires the establishment of an unconstraint system price comprising all zones (or nodes) without considering physical constraints. Then, a forward/future on the system price, and in this course the elimination of all zonal/nodal forward/futures, in combination with 100 CfDs (each contract written on the difference between the system price and one zonal or nodal price) are sufficient to hedge cross-border price risks. Currently, CfDs are employed in the Nordel area where they are the sole product used for managing cross-border risk.[34] CfDs are traded at Nord Pool where monthly, quarterly and yearly delivery periods are distinguished.[35]

2.3.2 Modelling Approaches

Electricity is, as discussed, assumed to be not storable at reasonable costs. This fact has a large affect on the modeling of electricity prices and the valuation of electricity derivatives. Simple buy-and-hold arbitrage strategies, as for example applied in equity markets or the market for CO_2 certificates, are not suitable for the electricity market.[36] Rather, in commodity

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[34] Nordel is the Nordic equivalent to UCTE. It is the organisation for the Nordic transmission system operators and comprises the countries Norway, Sweden, Finland and Denmark.

[35] See Kristiansen (2004a) for a further discussion of CfDs in the Nordic market.

[36] CO_2 certificates, or European Union Allowances (EUA), entitle its holder to the emission of one tonne of CO_2. These contracts were established in the course of the European Union Emission Trading System (EU ETS) and are governed by the Kyoto Protocoll. See Uhrig-Homburg and Wagner (2008) for a thorough introduction into the EU ETS.

markets, two methods need to be distinguished. The first explains the difference between today's spot and forward price based on the theory of storage. In addition to interest forgone through storing the commodity, storage costs for holding the commodity and a convenience yield on inventory have to be considered. Therefore, the interest rate, storage costs and convenience yield form a limit for forward prices. This approach was first introduced by Kaldor (1939) and Working (1948). However, the theory of storage is, analogous to the buy-and-hold strategy in equity markets, not directly applicable due to the non-storability of electricity.[37]

In the case of electricity, the expectations and risk preferences of market participants determine forward prices. This assumption constitutes the second approach for which the forward price is split into the expected future spot price and a risk premium . The latter represents a premium (discount) that buyers (sellers) of forward contracts are willing to pay (accept) in addition to the expected future spot price in order to eliminate the risk of unfavorable future spot price movements. Cootner (1960), Dusak (1973) and Breeden (1980) were among the first to introduce this approach. Duffie (1989) and Moulton (2005) extend the perspective to outside speculators. They explain the risk premium as a compensation for speculators to provide a form of insurance service to consumers and producers.

Therefore, the determination of the risk premium is the essential of the valuation of electricity derivatives.[38] In order to determine risk premia, there

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[37] Nevertheless, indirect storage of electricity is possible via its input fuels. Routledge et al. (2001) extend the theory of storage approach to include commodities that are not directly storable. In addition, Deng (2005) applies the theory of storage approach to Nord Pool forwards and finds a negative relation between forward prices and water reservoir levels. However, hydropower plants are rather scarce, with the exception of Norway, and this indirect storage is available only to hydropower generators. This poses a restriction on the use of indirect storage models.

[38] Among others, Longstaff and Wang (2004) and Geman (2005) show that there exist

exist three approaches in electricity markets. These comprise econometric models, equilibrium models and reduced-form models.

2.3.2.1 Econometric Models

The idea of econometric models is the prediction or forecast of future spot prices based on lagged spot prices and other exogenous factors. The latter may include all factors influencing electricity prices such as demand, weather or information about resource fuels (e.g. prices or storage data of oil or natural gas). The risk premium in econometric models is determined implicitly. Among others, Elliot et al. (2003) and Fleten and Lemming (2003) apply econometric models in order to model electricity prices. Although econometric models are practicable for the short-term forecast of prices and other factors influencing these prices, they are discussed in current literature only to a limited extend. This includes both national as well as cross-border electricity markets.

2.3.2.2 Equilibrium Models

Equilibrium models are based on the modelling of the economic drivers of electricity markets. The electricity price is then determined endogenously as a result of the equilibrium condition of the market. A large advantage of equilibrium models is the determination of structural relationships of electricity prices and their determining factors. Risk premia are determined endogenously by supply and demand. Although equilibrium models offer important insights to the impact factors for electricity prices, these models have yet not been applied to cross-border electricity markets. On a national

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significant risk premia in the market for electrictiy derivatives.

level, however, two fundamental equilibrium models need to be mentioned for the electricity market. Routledge et al. (2001) derive an equilibrium price for electricity as well as its generating fuels. The demand function for the respective commodities is given exogenously as a stochastic function. The resulting prices for electricity and other input resources are finally determined based on the utility maximization of market participants under the restriction that for commodity markets supply and demand need to be in equilibrium.

Bessembinder and Lemmon (2002) derive a static one-period model where spot and forward electricity prices as well as the risk premium are determined endogenously. In their model, the demand is also exogenously given and assumed to be stochastic. They assume a closed economy with two kinds of market participants, producers and retailers. Both can trade either in the wholesale (spot) or in the forward market. Given a total production of producer i of Q_{Pi}, the total cost function (TC) is of the form

$$TC_i = F + \frac{a}{c}\left(Q_{Pi}\right)^c, \tag{2.7}$$

where F represents the fix cost, a and c are constant and the total production volume is the sum of the total wholesale and forward sale of producer i, i.e. Q_{Pi}^W and Q_{Pi}^F respectively. Further, Bessembinder and Lemmon (2002) assume that producers and retailers are risk averse, i.e. the variance of profit enters negatively in the utility function. Each retailer can sell to consumers at a fixed retail price P_R. The equilibrium forward price is then derived by maximizing the utility function for producers and retailers assuming the market clearing condition that the total demand (Q^D) is equal to the total production, i.e. $Q^D = Q^P$. Total demand and production are simply the sum over all retailers and producers respectively. The equilibrium market price of a forward contract at time t with maturity in time T

($F_{t,T}$) follows as

$$F_{t,T} = E\left(P_T\right) + \alpha Var(P_T) + \gamma Skew(P_T). \tag{2.8}$$

where $Var(P_T)$ and $Skew(P_T)$ indicate the variance and unstandardized skewness of the spot price at maturity P_T. For reasons of tractability, we refrain from stating detailed expressions for α and γ. However, α is expected to be negative and γ is positive.[39] Thus, the forward price is negatively related to the variance of the spot price and positively related to the unstandardized skewness of the spot price.[40] Ullrich (2007) extends the Bessembinder and Lemmon (2002) model allowing for different levels of retail versus wholesale prices and thus changing signs of α. Moreover, in his model spot and forward prices are not determined by the absolute demand, but rather by the demand relative to existing capacity. This leads to more realistic trajectories for spot prices, especially in terms of the pronounced jumps.

The advantage of equilibrium models, i.e. the determination of economic drivers of spot and derivative prices, however, comes at the cost of tedious estimation and therefore only limited use for the valuation of electricity derivatives. The problem is that most underlying parameters used in equilibrium models, e.g. the cost function of generators, are latent, i.e. not directly observable and therefore hard to determine. Bühler and Müller-Merbach (2007a) derive a dynamic version of the Bessembinder and Lemmon (2002) model and use both models for the valuation of electricity futures at the Scandinavian power exchange Nord Pool. They compare

[39] While γ is always positive, the sign of α is in general negative depending on the relation between retail and wholesale prices.

[40] See Muck and Rudolf (2008) for a detailed discussion of the Bessembinder and Lemmon (2002) model.

their results to a one factor diffusion model. Although the equilibrium model better fits observed spot and futures prices, its complex implementation prevents an efficient use for the valuation of electricity derivatives, for which reduced-form models seem better suited.

2.3.2.3 Reduced-Form Models

Reduced-form models are the most prominent models for the valuation of derivatives not only in national and cross-border electricity markets, but in equity, fixed income, foreign exchange and general commodity markets as well. Here, a limited number of stochastic factors is used to adequately describe the underlying spot price process. Based on the exogenously specified spot prices and risk premia, derivative prices are derived.[41] The applied factors include both diffusion and jump components. The aim of the specification is an adequate representation of observed market prices and its characteristics while at the same time, these models provide as much tractability as possible.

A combination of reduced-form and equilibrium models are the structural models.[42] Here, the fundamental influencing factors, e.g. supply, demand, capacity or fuel prices, of electricity prices are governed by exogenously specified stochastic processes. The resulting electricity prices are then a function of these input factors. Eydeland and Geman (1999), Eydeland and Wolyniec (2003), Cartea and Villaplana (2008) and Pirrong and Jermakyan

[41] In case of the determination of forward prices, it is also possible to model forward prices or the entire forward-curve directly instead of deriving it from spot prices. Eydeland and Geman (1999) model single forward prices as a function of todays spot price, expected demand and the forward price of the generating fuel. Benth and Koekebakker (2008) use the approach from Heath et al. (1992) and model the entire forward curve directly.

[42] Cf. Pirrong (2008).

(2008) use structural models for the valuation of electricity derivatives in national markets whereas Skantze et al. (2004) applies a structural model for the valuation of financial transmission rights in the New York electricity market.

2.4 Valuation of Derivatives

This section focusses on the application of reduced form models for the valuation of electricity derivatives as they are the most widely used models for the valuation of derivatives in terms of actual pricing. Although equilibrium models are not of lesser importance as they offer important insights on the influencing factors of derivative prices, they are not applied for pricing purposes in practice due to the mentioned complexity problems. After having introduced the evolution of reduced-form models, this section discusses the application of reduced-form models to the valuation of national as well as cross-border derivatives.

2.4.1 Valuation Models

2.4.1.1 Evolution of Reduced-Form Models

A pioneer in the field of reduced form models was Louis Bachelier. Bachelier (1900) was the first to apply Wiener processes to model the movement of stock prices and even developed a solution for the valuation of put and call options.[43] Although his work was widely celebrated as a land-

[43] We refrain from a formal treatment of the financial mathematics of Wiener processes as they are extensively discussed in the literature. For a textbook treatment we refer to

mark event in modern finance, it was not before 1973, until the idea of using a Wiener process for modelling stock returns was picked up again. In their seminal work Black and Scholes (1973) as well as Merton (1973a) develop closed-form solutions for the valuation of European put and call options.[44] The solution of the Black/Scholes model is based on no-arbitrage considerations. The main advantages of their approach are the intuitive solution and the analytical tractability. Therefore, the framework of the Black/Scholes model has been applied by several authors for the valuation of contingent claims. Merton (1973b), Rubinstein and Reiner (1991), Kunitomo and Ikeda (1992) and Carr (1995) use the Black/Scholes set up for the valuation of barrier options. Stulz (1982), Johnson (1987), Boyle et al. (1989), Boyle and Tse (1990), Rubinstein (1991), Rich and Chance (1993), Kirk (1995) and Zhang (1995) apply the ideas of the Black/Scholes model to the valuation of options on several assets. In addition, lookback options were first priced using the Black/Scholes framework by Goldman et al. (1979), Conze and Viswanathan (1991) as well as Heynen and Kat (1994), while Geske (1979) investigates compound options.

The intuitive and tractable solution of the Black/Scholes formula, however, comes at the cost of a number of restricting assumptions. These include a constant interest rate at which borrowing and lending is possible, no dividends of the underlying stock, no taxes and transaction costs, the possibility of short-selling, a constant volatility of stock returns as well as continuous and normally distributed stock returns.[45] Thus, in spite of the popularity of the Black/Scholes model, empirical analyses show that it is not able to capture observed market prices. MacBeth and Merville (1979)

Musiela and Rutkowski (2002).

[44] Throughout this chapter, this model is referred to as the BlackScholes model. Strictly speaking, their solution should be considered as a quasi closed-form solution as it requires evaluating the cumulated standard normal distribution function.

[45] Cf. Black and Scholes (1973), p. 640.

and Beckers (1980) for example show, that the Black/Scholes model underestimates in-the-money options while out-of-the-money options are generally over estimated. Although Rubinstein (1983) points out some flaws in the aforementioned analysis, it is widely accepted that the Black/Scholes model is not capable of matching observed prices. Therefore, in order to better fit observed option prices, the assumptions of the Black/Scholes model are questioned and successively relaxed. In this course, research focuses on the assumption of normally distributed stock returns as well as constant volatilities, as the other assumptions, mentioned above, are either rather simple to incorporate or have only little impact on option prices.[46]

When looking at stock prices, they usually follow small erratic movements. However, from time to time, stock prices are subject to large movements questioning the reasonability of a continuous process in order to model these prices. In order to account for the jumps in stock prices, among others Merton (1976), Ball and Torous (1985) and Naik and Lee (1990) extend the Black/Scholes model by including jumps in the underlying process.[47] Although this approach helped to better match the prices of short-term options, it is not capable of adequately modelling long-term option prices.

Another important extension to the Black/Scholes model addresses the assumed constant volatility. Latané and Rendleman (1976) were the first

[46] Cox and Ross (1976) show that the interest rate has a rather small impact on the option price. Dividends can be included by either adjusting the drift in case of a continuous dividends or subtracting the present values of the dividends in case of discrete dividends (see Merton (1973a)). Finally, transaction costs and taxes can also be included in the model as done, for example, by Ingersoll (1979).

[47] Another approach follow Jarrow and Rudd (1982). They approximate the observed distribution by a normal distribution where the third and second order moment is included via series expansion. Since the authors do not offer an empirical analysis of their work and their approach is not further discussed in the literture, we refrain from a detailed discussion.

to address the problem of the Black/Scholes volatilities. As options are forward looking contracts, estimating volatilities based on historic data is not appropriate. Rather, they recommend using the volatilities implied in other options already traded. As all input factors besides volatilities are observable, there is a one-to-one relation between option prices and implied volatilities.[48] Although implied volatilities are helpful in providing the adequate parameter for option valuation, this approach does not help to resolve the problem of constant volatilities when modelling stock prices. MacBeth and Merville (1980), for example, show that stock prices are negatively correlated with volatilities and that implied volatilities vary systematically with the degree of moneyness and time-to-maturity of an option. In order to address these issues, Cox (1975) and Cox and Ross (1976) were among the first to address the issue of non-constant volatilities and introduce the constant elasticity of variance (CEV) model. Here, the stock price returns are non-centrally χ^2 distributed leading to fatter tails compared to the normal distribution assumed in the Black/Scholes framework. This approach, however, is not able to adequately model observed option prices. A different, albeit similar, approach applies the so called stochastic volatility models where the volatility is governed by a separate stochastic process. These models are extensively addressed in the literature and are discussed, among others by, Chesney and Scott (1987), Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), Hull and White (1988), Melino and Turnbull (1990), Stein and Stein (1991), Heston (1993), Ball and Roma (1994), Grünbichler and Longstaff (1996) and Schöbel and Zhu (1999).

The logical result from the above discussed modelling approaches is a combination of stochastic volatility and jumps in the underlying. Among others, Bates (1996), Bakshi et al. (1997), Bates (2000), Scott (1997) and Pan

[48] As implied volatilities provide information on the relative expensiveness of options, they are the preferred method of communicating option prices.

(2002) follow this approach, where some also include stochastic interest rates. Broadi et al. (2007) extend these models and also include jumps in the volatility. A general class of models that includes all above mentioned approaches offer Duffie et al. (2000). These models are the so called affine jump-diffusion models. This class contains an arbitrary number of state variables each governed by a jump-diffusion process. Prerequisite for these models is that all parameters, i.e. the means, jump intensities as well as the covariance matrix, need to be affine functions of the state variables. For this class of models, quasi closed-form solutions for the valuation of option contracts are provided using Fourier transform.[49]

The evolution of reduced form models shows the almost infinite opportunities these models offer while still being analytically tractable. While an increase in model parameters certainly offers a greater amount of flexibility and is therefore more likely to adequately match observed market prices, the estimation becomes also more tedious. In addition, the robustness of these models is also likely to decrease due to the inclusion of more parameters to be estimated. This trade-off needs to be analyzed thoroughly and has still yet to come.

2.4.1.2 Valuation Models in National Electricity Markets

In order to apply reduced form models to the valuation of electricity derivatives, one needs to consider the unique characteristics of electricity prices discussed above. The first characteristic considered is the seasonality. In order to account for seasonal trends in electricity prices, the observed prices are generally assumed to consist of a deterministic trend and one

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[49] All of the mentioned models rely on a vector-based specification of state-variables. A further generalization are matrix diffusion models. Leippold and Trojani (2008) or Branger and Muck (2009), for example, use Wishart processes to model stochastic covariances.

(or more) stochastic components. Let us assume the electricity price to be log-normally distributed. Then, the logarithm of the spot price at time t, i.e. $\ln P_t$, can be written as[50]

$$\ln P_t = f(t) + Y_t. \tag{2.9}$$

In the above equation, $f(t)$ is the seasonal trend. This trend is assumed to be deterministic and therefore only a function of time. Y_t is the stochastic component that is calculated as the residual after subtracting the deterministic trend from observed market prices. When determining the deterministic function, it is of utmost interest to incorporate the electricity immanent seasonality. Several approaches are currently discussed for determining the seasonal trend. Weron et al. (2004), Hikspoors and Jaimungal (2007) as well as Pilipovic (2007) use a sinusoidal function to model the deterministic trend. Huisman and De Jong (2003) and Haldrup and Nielsen (2006a) in contrast use piecewise constant functions to filter out the seasonal trend on an intra-weekly and annual basis. A combination of both, generally applied in form of a piecewise constant function for an intra-week effect and a sinusoidal function for annual seasonality, is applied by Lucia and Schwartz (2002), Bierbrauer et al. (2007), Seifert and Uhrig-Homburg (2007) and Nomikos and Soldaots (2008).

After separating the stochastic component that should not contain any seasonality, one needs to find an appropriate process to model the stochastic behavior of electricity prices. One of the first models in electricity markets was proposed by Lucia and Schwartz (2002). In a first step, they use a

[50] Another approach models the spot price directly, which then is also decomposed into a seasonal trend and a stochastic component. In this case, Equation 2.9 changes to $P_t = f(t) + Y_t$. As the following description of reduced-form models does not significantly change in this case, we refrain from discussing both approaches.

one factor diffusion model where the spot price is defined as in (2.9) to model spot prices at Nord Pool.[51] In this model the stochastic component is governed by

$$dY_t = -\kappa Y_t dt + \sigma dW(t). \tag{2.10}$$

Thus, the stochastic component is governed by an Ornstein-Uhlenbeck process, where κ is the mean-reversion speed, σ is the parameter of the volatility and $dW(t)$ is an increment of a Wiener process.[52] This model set up allows modelling the fact that electricity fluctuates in the short-term but tends towards a constant level in the long run. In addition to the one factor model, Lucia and Schwartz (2002) extend their approach and include a second factor also governed by a diffusion process. The two factor model builds on the one factor model where in (2.9) we write $Y_t = X_t + Z_t$. The latter two stochastic components are governed by

$$\begin{aligned} dX_t =& -\kappa X_t dt + \sigma_X dW_X(t), \\ dZ_t =& \mu dt + \sigma_Z dW_Z(t), \end{aligned} \tag{2.11}$$

where the two Wiener processes are correlated, i.e. $E[dW_X(t)\, dW_Z(t)] = \rho dt$. Although incorporating a second stochastic factor improves the model fit, as it offers a greater deal of flexibility, both, the one and two factor model are not able to reproduce the spikes observed in electricity prices.

[51] See Wilkens and Wimschulte (2007) for the application of the Lucia and Schwartz (2002) model to the German electricity market.

[52] A similar model was applied by Vasicek (1977) for modelling interest rates. In our approach, the mean-reversion level is zero as any constant is already considered in the deterministic function.

Thus, an application to modelling spot prices using the Lucia and Schwartz (2002) model does not seem appropriate.[53]

Based on the one and two factor diffusion models, research focuses on extending these models in order to better match the unique feature of electricity prices. Although seasonality and mean-reversion are already incorporated in those models, the striking spikes observed in market prices, i.e. discontinuities in the paths of state variables, are not matched. The implementation of jump components into reduced-form models therefore became the center of further research.

When implementing jump components, one needs to distinguish the characteristics of the jumps. Generally, there are two types of jumps observed in electricity markets. When looking at the behavior of electricity prices at the EEX (see Figure 2.5) the short-term feature of jumps is evident. These jumps are considered as spikes as they only last at most a few days until prices revert back to the mean-reversion level leading to the typical spiky trajectory of spot prices.[54] Another type of jumps in electricity prices is observed in daily spot prices at Nord Pool as shown in Figure 2.10. Although electricity prices are also subject to extreme and sudden movements, they do not immediately revert back to the long-term level. Further, although significant, the amplitude of jumps is moderate compared to EEX prices. The reason for the observed jump pattern is the large impact of hydropower in the Nordic market with a share of 58% of all electricity generation in 2008. Cheap hydropower hampers large spikes to occur while shocks to electricity prices are mostly due to shocks to the

[53] Although the Lucia and Schwartz (2002) model might seem promising to model the futures price directly, seasonality or the mean-reversion characteristic is not evident in futures prices. Thus, the fundamental Black (1976) model should be applied.

[54] Seifert and Uhrig-Homburg (2007), for example, find that jump in EEX spot prices last on average around three days.

hydropower supply. As changes in the hydropower supply have a rather medium to long-term effect, price jumps are also apparent for longer periods.[55]

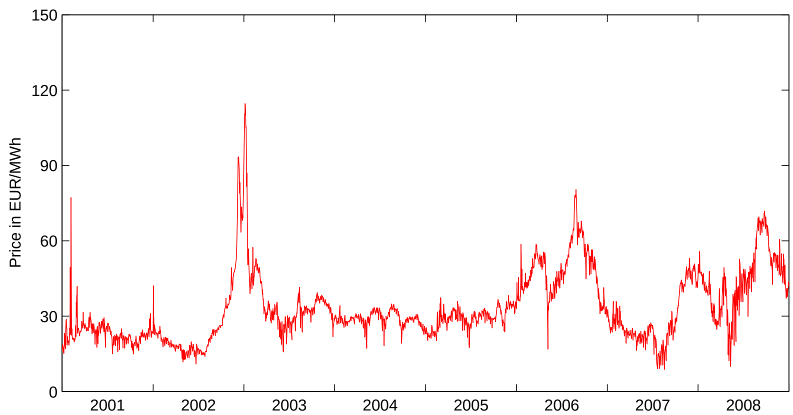

Figure 2.10: Daily spot prices at Nord Pool between 2001 and 2008.

The first approach of implementing jumps in processes for electricity prices is based on Merton (1976). Following Escribano et al. (2002), (2.10) is extended and the stochastic component of the electricity price is then governed by[56]

$$dY_t = -\kappa Y_t dt + \sigma dW(t) + J_t dq_t. \quad (2.12)$$

[55] See Nordel (2009), p. 3, for details on hydropower in Nordic countries. For a thorough analysis of hydropower on electricity prices at Nord Pool, especially during the drought between 2002 and 2003, see Von der Fehr et al. (2005).

[56] In contrast to Merton (1976), however, jump risk is explicitly priced in electricity markets. The assumption of diversifying away jumps is not applicable for electricity as electricity is not storable.

Here, q is a Poisson process with jump intensity λ. Thus intuitively, the probability of a jump during a time interval of length dt is λdt. While the Poisson process governs the occurrence of a jump, the jump size is determined by J_t which is normally distributed, i.e. $J_t \sim \mathcal{N}\left(\mu_J, \sigma_J^2\right)$. This approach to model jumps in electricity prices is, among others, also applied by Eydeland and Geman (1999), Johnson and Barz (1999), Clewlow and Strickland (2000) and Cartea and Figueroa (2005). When modelling jumps via reduced form models, the jump intensity of the above introduced Poisson process needs not to be constant. Rather, it might be useful to let the jump intensity follow a seasonal trend. In this case, one can model a higher jump intensity during Winter months when general demand for electricity is already high.

The spikes observed in the electricity prices in Figure 2.5, however, cannot be adequately matched with the above introduced jump components. Although the inclusion of the jump component allows the modelling of infrequent price jumps, the price is not immediately pulled back to the long-term mean-reversion level to form the typical spike pattern. Rather, the mean-reversion characteristic models a gradual return to the basis. While the time until the price reverts back to the long-term mean can be reduced by increasing the mean-reversion speed, it would lead to unrealistic values of the mean-reversion speed. In order to better fit the spike pattern of electricity prices, Geman and Roncoroni (2006) extend the above described model. Instead of determining the sign of the jump by the normally distributed jump size random variable, they define the sign of the jump depending on the current electricity price level. In case the electricity price is below a given threshold, the jump is positive, while jumps are strictly negative if the current price level is above the threshold. Although this model is capable of matching spikes in electricity prices, the application and estimation is rather tedious. Another approach to modelling spikes was mentioned by Simonsen et al. (2004). They disentangle the jump from the diffusion component. Given the spot price definition from (2.9), where the stochastic component Y_t is again separated, i.e. $Y_t = X_t + Z_t$, the stochastic components are governed by

$$\begin{aligned} dX_t =& -\kappa X_t dt + \sigma dW_X(t), \\ Z_t =& J_t dq_t. \end{aligned} \tag{2.13}$$

In this model, a jump that occurs at time t vanishes immediately afterwards leading to the characteristic spike pattern.

Besides the above discussed reduced-form models to describe electricity prices, another type of models has recently received a lot of attention in the literature. Instead of using one process to model electricity prices, two or more processes are used where only one of these processes describes the electricity price at one point in time. These models are referred to as regime-switching models. They are based on the idea that the behavior of electricity prices is subject to structural chances over time. Regime-switching models were firstly introduced by Hamilton (1989) in a macroeconomic context and are applied to electricity markets, among others, by Huisman and Mahieu (2003), De Jong (2005) and Bierbrauer et al. (2007). In order to describe the functioning of regime-switching models, we use a two regime model from Bierbrauer et al. (2007). We assume that the electricity price is governed by two processes one for the normal periods and the other for turbulent phases. The first process can be written as

$$Y_{t,1} = c + \phi Y_{t-1,1} + \varepsilon_t, \tag{2.14}$$

where $\varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$. Thus, the basis for our electricity price model is a simple Lucia and Schwartz (2002) one factor model.[57] In contrast, the electricity price in wild phases is simply a normally distributed variable and we can write

$$Y_{t,2} \sim \mathcal{N}\left(\mu_Y, \sigma_Y^2\right). \tag{2.15}$$

In order to identify when to switch between the two regimes, one needs to estimate the transition matrix. This matrix is not a priori evident but needs to be estimated from observed electricity prices. In case of two regimes, the transition matrix Π can be stated as

$$\Pi = \Pr\left(R_{t+1} = j \,|R_t = i\,\right)_{i,j=1,2} = \begin{pmatrix} b_{11} & 1 - b_{11} \\ 1 - b_{22} & b_{22} \end{pmatrix}. \tag{2.16}$$

Based on this notation, the transition matrix states the conditional probabilities b_{ij} that the regime in the next time step R_{t+1} is regime j, conditional on the information that the current regime is i, i.e. $R_t = i$, for all $i, j = 1, 2$. In case we want to model spikes that only last one day, we need to set $b_{22} = 0$. Given the estimated parameters of Bierbrauer et al. (2007), Figure 2.11 shows simulated electricity prices between 2001 and 2008.[58]

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[57] For demonstration purposes, we use discrete models.

[58] The parameter values for the simulation are obtained from Bierbrauer et al. (2007). Since they model the logarithm of the spot price and therefore, their stochastic component is also based on the logarithm of spot prices we convert simulated prices into EUR/MWh. The original parameter values are, in regime 1 $c = 1.105$, $\phi = 0.639$, $\sigma_\varepsilon^2 = 0.145$, regime 2 $\mu_Y = 2.916$ and $\sigma_Y^2 = 0.658$. The transition probabilites are $b_{11} = 0.953$ and $b_{22} = 0.658$.

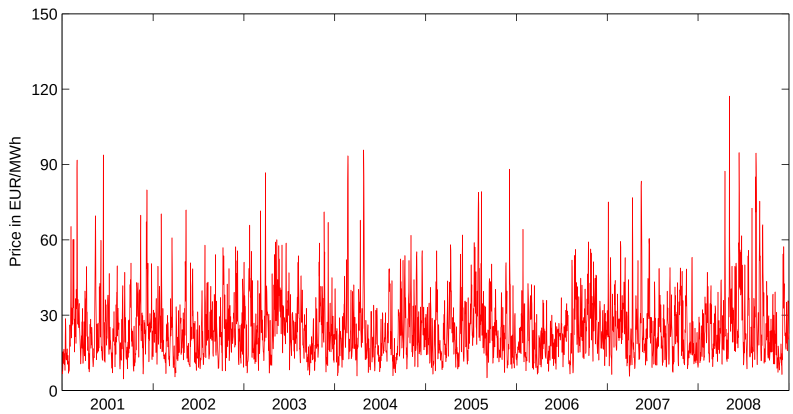

Figure 2.11: Simulated prices for the stochastic residual of a regime-switching model with two independent regimes described in Equations 2.14 through 2.16.

2.4.1.3 Valuation Models in Cross-Border Electricity Markets

The above described derivatives in cross-border electricity markets, see (2.5) and (2.6), are based on the spread between two locational electricity prices. The resulting payoff profile corresponds to the payoff of exchange products as the spread is identical to exchanging one electricity price for another. Margrabe (1978) was the first to discuss the valuation of exchange derivatives in equity markets.[59] He develops a closed-form solution for exchange options in case both assets follow Geometric Brownian Motions (GBM) as, for example, in the Black and Scholes (1973) model. Due to this setup, however, the Margrabe model has the downside that its assumed

[59] Note that at the same time as Margrabe, Fischer (1978) derives an identical formula for the valuation of exchange options. Throughout this thesis, however, we refer to the exchange option formula as the Margrabe model.

underlying processes do not reflect neither the seasonality, nor the mean-reversion characteristics nor the spiky trajectory of electricity prices. An extension of the Margrabe model to be applicable has not been introduced in the literature.

Instead of modelling both assets separately, Dempster et al. (2008) show that in case of co-integrated price processes it may be appropriate to model the spread directly. Since electricity prices in neighboring regions are usually co-integrated, modelling the spread directly is a suitable approach for electricity exchange options. Thus, the valuation of electricity exchange options reduces to modelling a single price process, i.e. the spread, and pricing the derivatives written on this underlying. In this case, the aforementioned models for national electricity markets might be applicable. The important issue, however, is that price spreads exhibit unique features that distinguish their trajectory from those of national electricity prices. Thus, it is of utmost relevance to choose an appropriate reduced-form model for the valuation of derivatives in cross-border electricity markets.[60]

2.4.2 Solution Techniques

In order to apply the above discussed valuation models for pricing electricity derivatives, this section shortly introduces several solution techniques. We hereby distinguish analytical and numerical methods and exemplify their application based on the Lucia and Schwartz (2002) one factor model described in (2.9) and (2.10). We use this model for the valuation of a plain vanilla call option with a payoff given in (2.3). In order to keep the discussion tractable, we assume that the option matures at one point in time, and not over a period, and that all parameters are the corresponding risk-

[60] We will propose a suitable model in a later chapter of this thesis.

neutral parameters.[61]

2.4.2.1 Analytical Methods

Analytical solutions are all solutions that are expressed in closed form. Here, closed form is understood in a broader sense as these solutions may contain an integral that needs to be solved numerically, as for example in the model of Black and Scholes (1973). The first and, if applicable, simplest approach of pricing a contingent claim is the risk-neutral valuation principle where the derivative is priced as its discounted expected payoff under the risk-neutral measure. Therefore, the price of a call option at time 0, i.e. C_0 with maturity at time T written on the electricity spot price P_T can be written as

$$C_0 = E^{\mathcal{Q}}\left[(P_T - K)^+\right] \cdot e^{-rT}, \tag{2.17}$$

where K is the strike price of the option and r is the constant risk free rate of interest. The super index $\mathcal{Q}$ indicates the expectation under the risk-neutral measure. As the electricity price in case of the Lucia and Schwartz (2002) one factor model is log-normally distributed, the solution to (2.17) is straight forward. The solution to the SDE in (2.10) can be stated as

$$Y_t = e^{-\kappa t} Y_0 + \sigma e^{-\kappa t} \int_0^t e^{\kappa s} dW(s). \tag{2.18}$$

[61] Harrison and Kreps (1979) and Harrison and Pliska (1981) show that in an arbitrage free market, there exists at least one risk neutral measure. As electricity markets are incomplete, we assume one of the infinitely many measures for modelling. The relevant measure must be estimated from market prices. We address the topic of measure change in detail in the subsequent chapters of this thesis.

Hence, Y_t is normally distributed and follows an Ornstein-Uhlenbeck process. Since the seasonal component in (2.9) is deterministic, $\ln P_t$ is also normally distributed with mean and variance given as[62]

$$\begin{aligned} E^{\mathcal{Q}}\left[\ln P_T\right] =& f(T) + \left(\ln P_0 - f\left(0\right)\right) \cdot e^{-\kappa T} \\ Var\left[\ln P_T\right] =& \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa T}\right). \end{aligned} \tag{2.19}$$

Using this set up, we can derive the call price as the discounted expectation under the risk-neutral measure as

$$\begin{aligned} C_0 =& e^{-rT}\left[e^{\left[E^{\mathcal{Q}}\left[\ln P_T\right] + \frac{1}{2}Var\left[\ln P_T\right]\right]} \cdot \Phi\left(d_1\right) - K \cdot \Phi\left(d_2\right)\right], \\ d_1 =& \frac{E^{\mathcal{Q}}\left[\ln P_T\right] - \ln K + Var\left[\ln P_T\right]}{\sqrt{Var\left[\ln P_T\right]}}, \\ d_2 =& d_1 - \sqrt{Var\left[\ln P_T\right]}. \end{aligned} \tag{2.20}$$

The expected value and variance are applied as in (2.19) and $\Phi\left(\cdot\right)$ indicates the operator of the cumulative standard normal distribution.

Another approach for the valuation of the call option from (2.17) is solving the respective partial differential equation (PDE). We assume that the call price C_t is a function $g\left(\cdot\right)$ of time t and the electricity price P_t, i.e.

$$C_t = g\left(t, P_t\right). \tag{2.21}$$

[62] See Mikosch (1999) for the derivation of the variance.

At the moment we do not know the SDE of the electricity price P_t but only the SDE of Y_t. Since P_t, however, can be expressed as a function of Y_t, i.e. $P_t = e^{f(t)+Y_t}$, we can apply Itô's Lemma to receive the following SDE for the electricity price

$$dP_t = \kappa \left[\gamma(t) - \ln P_t\right] P_t dt + \sigma P_t dW(t), \tag{2.22}$$

where

$$\gamma(t) = \frac{1}{\kappa}\left(\frac{1}{2}\sigma^2 + \frac{df(t)}{dt}\right) + f(t).$$

Now that we know the SDE of the electricity price, we can reapply Itô's Lemma to the call price as a function of time and the electricity price given in (2.21). We receive for the call price

$$\begin{aligned} dC_t = &\left[\frac{\partial C}{\partial t} + \kappa\left[\gamma(t) - \ln P_t\right] P_t \frac{\partial C}{\partial P} + \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 C}{\partial P^2}\right] dt \\ &+ \sigma P_t \frac{\partial C}{\partial P} dW(t), \end{aligned} \tag{2.23}$$

where $\gamma(t)$ is defined as in (2.22). In a risk-neutral setting, an investor only requires a return equal to the risk-neutral interest rate r. Therefore, the expected return when holding the option is calculated as the option price multiplied by the risk-neutral interest rate. Further, the expected change in the option price is the deterministic trend, as the expected value of the Wiener process increment is zero. Putting all parts together, we can write down the PDE of the call option as

$$\frac{\partial C}{\partial t} + \kappa \left[\gamma\left(t\right) - \ln P_t\right] P_t \frac{\partial C}{\partial P} + \frac{1}{2}\sigma^2 P_t^2 \frac{\partial^2 C}{\partial P^2} - C_t \cdot r = 0, \tag{2.24}$$

where r is again the risk-neutral interest rate. Finding a solution for the call price C_t that solves the above stated PDE, given the initial and boundary conditions, is not trivial. An heuristic approach is unfortunately not available. For details, we refer to Evans (1998) who provides in depth analysis on how to solve the most common types of differential equations.

Although solving the above stated PDE is rather tedious and not always possible, Duffie et al. (2000) derive a general solution for option prices in case of an affine model setup. This approach constitutes the third method of solving option prices analytically and works for all affine models and arbitrarily many state variables. We will only introduce the general idea of this approach as a thorough discussion is extensive and beyond the scope of this thesis. We therefore refer to Muck (2006a) for a detailed and intuitive description of affine models.

Let us assume that the spot price P is an exponentially affine function in the state variable $X\left(t\right)$ such that

$$P\left(t, X\left(t\right)\right) = e^{\bar{b}(t) + b(t) \cdot X(t)}, \tag{2.25}$$

where $\bar{b}\left(t\right) \in \mathbb{R}$ and $b\left(t\right) \in \mathbb{R}^n$. Further assume that the interest rate is constant and thus linear affine in $X\left(t\right)$. Then the price of a call option with strike L at time t and maturity in time T, i.e. $C_{t,T}$, can be expressed in terms of a function $G\left(\cdot\right)$ such that[63]

[63] See Duffie et al. (2000) for a detailed proof and Muck (2006a) for an intuitive illustration.

$$\begin{aligned} C_{t,T} &= e^{\bar{b}(t)} G\left(-\ln L + \bar{b}(t), t, T, b(t), -b(t)\right) \\ &\quad -L \cdot G\left(-\ln L + \bar{b}(t), t, T, 0, -b(t)\right). \end{aligned}$$

Solving G directly is in most cases not feasible. However, we can calculate the Fourier transform of $G(y)$, i.e. $\hat{G}(z)$ defined as

$$\hat{G}(z) = \int_{-\infty}^{\infty} e^{izy} dG(y), \tag{2.26}$$

where $i = \sqrt{-1}$. Calculating the Fourier transform explicitly involves solving a set of ordinary differential equations (ODEs). This can be done either analytically or numerically using the finite differences scheme discussed in the next section. After having calculated $\hat{G}(z)$, the function $G(y)$ can be calculated by applying the inversion formula to the Fourier transform to receive

$$G(y) = \frac{\hat{G}(0)}{2} - \frac{1}{\pi} \int_{0}^{\infty} \frac{1}{z} \operatorname{Im}\left[e^{izy}\right] dz, \tag{2.27}$$

where $\operatorname{Im}[\cdot]$ returns the imaginary part of a complex number. Since the Fourier transform $\hat{G}(z)$ is known, (2.27) determines the price of the call option. Figure 2.12 shows the general set up of the Fourier transform.

Although calculating the Fourier transform involves some algebra, the general set up remains unchanged when introducing further state variables (as long as the affine structure is guaranteed). Thus, the model of Duffie et al. (2000) offers a convenient yet flexible approach for option pricing especially in case of several state variables.

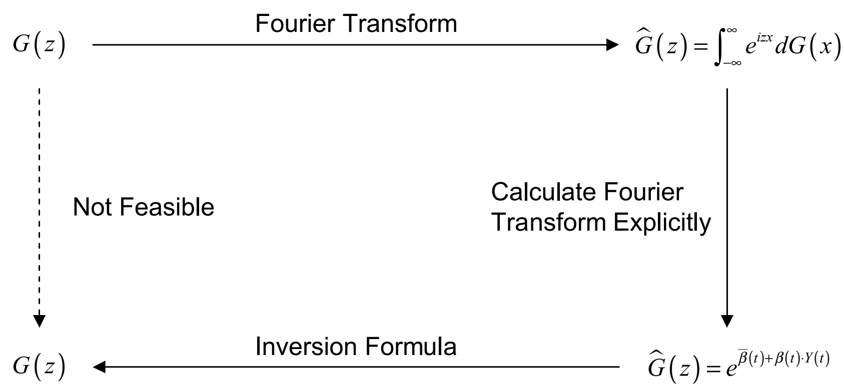

Figure 2.12: Set up of Fourier transform.

2.4.2.2 Numerical Methods

While the advantages of analytical solutions are obvious, there might not always be a way to derive them. And even in case an analytical solution can theoretically be derived, this approach might be tedious and inefficient for more complex contracts. May it be due to non-normality of prices or returns or unsolvable PDEs. And even in case of an affine model setup, the respective ODEs might not have a closed-form solution. In this case, there are several methods for calculating the call price numerically. The three most popular are shortly discussed here including Monte Carlo simulation, trees and finite differences. As the performance of numerical methods is measured with regard to the degree they match the exact solution, we compare the exact solution, given in (2.20), with the solution received by using each numerical approach.[64] The error is measured as a percentage

[64] For all three numerical methods we refer to the same option where $\kappa = 0.20$, $\sigma = 0.20$, $r = 10\%$ p.a., $K = 54$ EUR and maturity is in one year, i.e. $T = 1.0$. The current

value calculated as the absolute difference relative to the exact solution. For simplicity, we assume the seasonal component to be zero at all times such that the underlying spot price P_t is governed by the following SDE

$$dP_t = -\kappa \ln P_t \cdot P_t dt + \sigma P_t dW(t) . \tag{2.28}$$

Monte Carlo simulation is probably the easiest and most intuitive method in order to numerically evaluate an option. In the course of Monte Carlo simulation, first introduced by Boyle (1977), one draws random samples from the same distribution as the underlying electricity price. In our example, we simulate the normally distributed logarithm of the electricity price, i.e. $\ln P_t$ with expectation and variance given in (2.19), and convert each resulting value into the electricity price P_t. For every simulated price, the corresponding payoff of the option is calculated and the expected value over all resulting payoffs is calculated, where each path receives the same weight.[65] Finally, since we simulate the electricity prices under the risk-neutral measure, the expected value is discounted with the risk free rate to receive the option price. The relative error of the Monte Carlo simulation for different numbers of sample paths is shown in Figure 2.13.

Figure 2.13 emphasizes a fast convergence of the Monte Carlo approach. Even in case of several thousand sample paths, the simulation is performed within seconds. Especially in case of several state variables, this numerical procedure is advantageous. In addition to speed and simplicity, another huge advantage is that via Monte Carlo simulation one can obtain entire

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electricity price is 60 EUR.

[65] Currently, the so called Wighted Monte Carlo methods are discussed in the literature. In those models, the sample paths are weighted in order to better match observed option prices. See, for example, Avellaneda et al. (2001) and Glasserman and Yu (2005) for a discussion of those models.

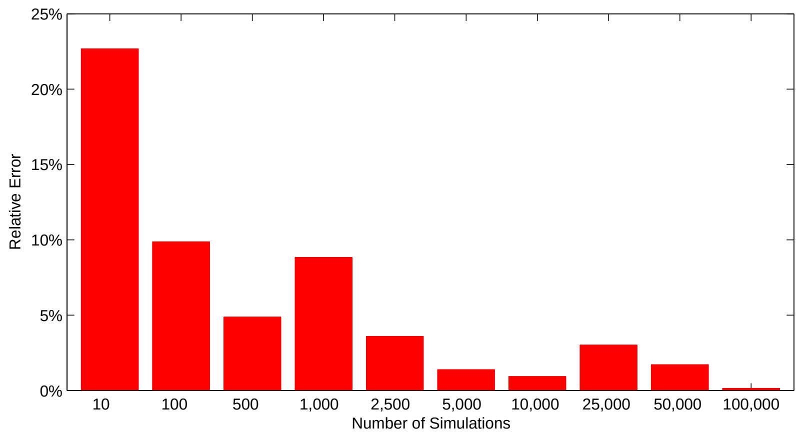

Figure 2.13: Relative pricing errors of Monte Carlo simulation.

sample paths, which offers the ability to easily price more exotic derivatives such as knock-out options.[66] Recently, Longstaff and Schwartz (2004) extend the Monte Carlo principle and derive an efficient method for the valuation of American style options via simulation. Due to the growing efficiency of Monte Carlo methods and the increasing speed of computers, Monte Carlo simulation has become one of the most widely used methods of valuing derivative products.

Another numerical method for the valuation of derivative products is the model of Cox et al. (1979), which is also called binomial tree model. In this approach, the movement of the electricity prices is described by a binomial tree model where in each step the electricity price can either go up or down.[67] The probability for an up move (down move) is p $(1-p)$

[66] See Muck (2006b) and Muck (2007) for a thorough discussion of exotic derivatives and their pricing.

[67] Besides binomial tree models, there are also approaches to use trinomial trees to describe

where the resulting electricity price is $P \cdot u$ in case of an up move and $P \cdot d$ after a down move respectively. Figure 2.14 shows the set up of the binomial tree model.

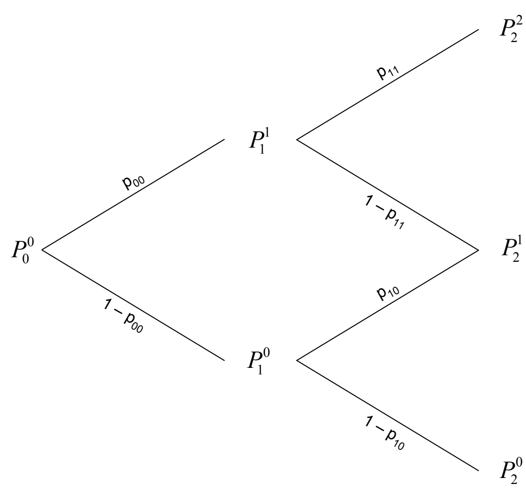

Figure 2.14: Set up of a recombining binomial tree model.

According to Figure 2.14, the electricity price after a down move followed by an up move is identical to the scenario where the price first increases and then decreases. This so called recombining set up is not required but helps to keep the calculations tractable. In case of a non recombining tree, we observe 2^n different electricity prices after n steps, while this number is

the movement of the underlying asset. The most prominent application for the implementation of a trinomial tree is short rate model described in Hull and White (1994a) and Hull and White (1994b).

reduced to $n+1$ in case of recombination. While the probability of an up and down move is constant in the original binomial tree when modelling equity prices, we need to adapt those figures to the case of mean-reversion. Analogous to the case of equity prices, the up and down factors are defined as $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$ and therefore constant throughout the tree. However, the probability of an up or down move varies. Given the SDE in (2.28) and remembering that the seasonal component is zero in our example, the probability of an up move is given by[68]

$$p_{ij} = \frac{1}{2} + \left(\kappa \frac{\frac{\sigma^2}{2\kappa \cdot \ln P_i^j} - 1}{2\sigma} - \frac{\sigma}{4} \right) \cdot \sqrt{\Delta t}, \tag{2.29}$$

for all $i, j = 0, ..., n$, where n is the number of steps. The probability is derived by identifying the first and second moment of the electricity price process and the tree to ensure the convergence of the tree solution to the exact one.[69] Δt is the length of one step in the tree defined as $\Delta t = T/n$. As the probability of an up move depends on the current value of P, the probability is not constant throughout the tree.[70] After having established our tree, we start evaluating the payoff of the option at each node at maturity. Weighting each payoff with its probability and discounting those values finally leads to today's option price. Figure 2.15 shows the relative

[68] We refer to Lari-Lavassani et al. (2001) for a derivation of the stated probability and a thorough discussion of the binomial tree model for electricity prices.

[69] We remove one degree of freedom by stating $d = 1/u$. Thus, we have two equations, i.e. one for identifying the first and another for identifying the second moment of the discrete and the continuous process, in order to determine the unknown parameters p_{ij} and u.

[70] The mean-reversion in our tree model leads to different up and down jump probabilities at each node, depending on the current spot price P_i^j. In (2.29), the probability for an up jump decreases with increasing values of P_i^j and vice versa.

pricing error of the tree for different numbers of time steps.

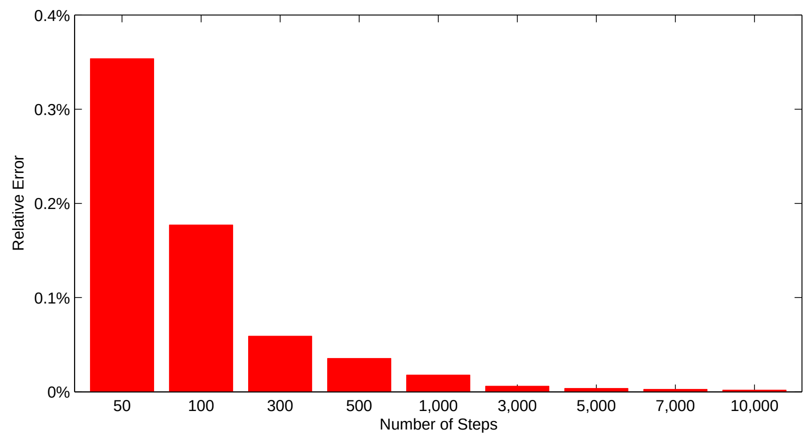

Figure 2.15: Relative pricing errors of binomial tree model.

As Figure 2.15 indicates, the binomial tree model quickly converges towards the exact solution. Moreover, the tree set up allows to evaluate the option price at each node allowing an easy valuation of American options or knock-out products. However, incorporating additional state variables, especially in case these state variables are correlated, is not straight forward.[71] Thus, the tree method is most applicable in situations where the underlying process is rather simple.

The third approach for numerically deriving the value of a call option is finite differences. This method was first used by Schwartz (1977) and focuses on the PDE where the partial derivatives are substituted by finite differences. Analogous to the other numerical procedures, we model only

[71] See, for example, Hull and White (1994b) or Muck and Rudolf (2005).

the stochastic component of the electricity price, i.e. Y_t and convert the resulting option payoff according to (2.9). To derive the PDE for Y_t, we assume the call option to be a function of the stochastic component. After applying Itô's Lemma to the call price function, we receive, analogous to (2.23) and (2.24), the following PDE

$$\frac{\partial C}{\partial t} - \kappa Y_t \frac{\partial C}{\partial Y} + \frac{1}{2}\sigma^2 \frac{\partial^2 C}{\partial Y^2} - C_t \cdot r = 0. \tag{2.30}$$

In order to find a solution to the above stated PDE, we construct a two dimensional mesh where time steps and steps in Y_t are discretized. We use L time steps and M steps in Y to receive a mesh sketched in Figure 2.16.

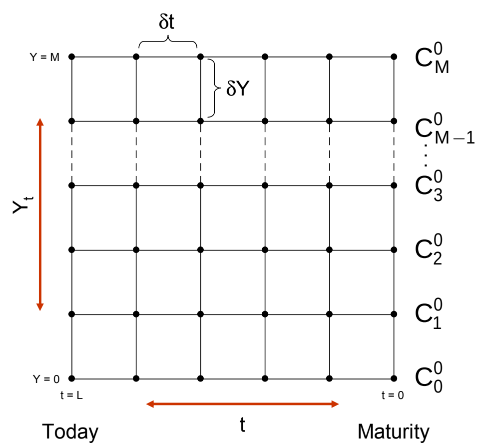

Figure 2.16: Finite differences grid with L time steps and M steps in Y.

At each node of the grid, we calculate the option price C. As the grid indicates, we start at maturity as at this point in time the payoff of the option is known, i.e. we are going backward in time. Given all option prices at maturity (i.e. $t = 0$), we use those values to calculate the values for the option prices $t = 1$ (i.e. the prior time step) satisfying the PDE in (2.30). In order to derive the missing option prices, we use the following finite differences to approximate the partial derivatives in (2.30).

$$\begin{aligned} \frac{\partial C}{\partial t} &\approx \frac{C_m^l - C_m^{l+1}}{\delta t}, \\ \frac{\partial C}{\partial Y} &\approx \frac{C_{m+1}^l - C_{m-1}^l}{2 \cdot \delta Y}, \\ \frac{\partial^2 C}{\partial Y^2} &\approx \frac{C_{m+1}^l - 2 \cdot C_m^l + C_{m-1}^l}{(\delta Y)^2}. \end{aligned} \tag{2.31}$$

With those finite differences, we receive a system of equations that allows us to calculate the option prices at the next, i.e. prior, time step.[72] We continue this roll back procedure until we have obtained the option prices at each node. The relevant option price is the one today, i.e. $t = L$, corresponding to today's value of Y. We calculate the relative pricing error for various values of δt and δY, where for all schemes $t = 0, \delta t, ..., 1$ and $Y = -1, -1 + \delta Y, ..., 5$. We start with 10 time steps and 30 steps in Y and continue by doubling both values with every new calculation, i.e. we quadruple the number of grid points. Figure 2.17 shows the relative pricing errors relative to the mesh size on a logarithmic ordinate emphasizing the exponential rate of conversion.

[72] There are several ways of deriving the option prices from the set of equations. For sense of exposition, we do not discuss the approaches and refer to Tavella and Randall (2000) for an in depth discussion of finite differences. Our calculations are based on the Crank-Nicholson method.

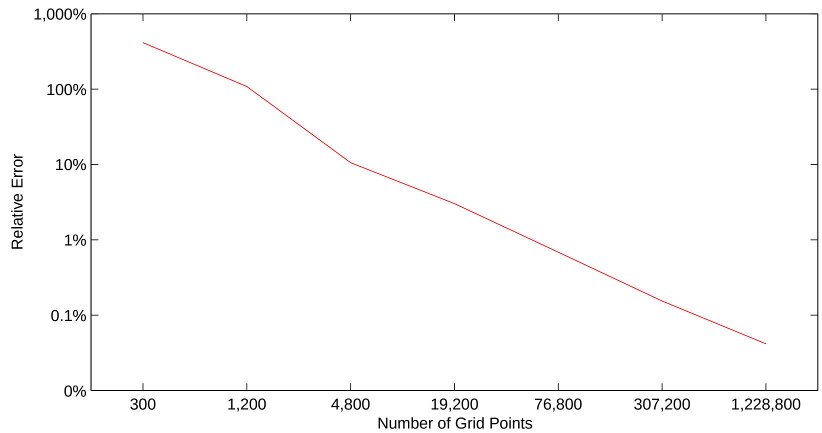

Figure 2.17: Relative pricing errors of finite differences scheme.

The relative error vanishes and the numerical option prices converge exponentially to the exact solution. In addition, one can evaluate the option price at each node allowing an easy valuation of American style options and knock-out derivatives. However, finite differences involve rather complex calculations and extensive computations. The solution for the smallest mesh was calculated after over ten minutes. The largest disadvantage of the above example is its constant mesh throughout the entire gird. This leads to numerous calculations within the grid where there is only little impact on today's option price. Thus, non constant mesh sizes could significantly increase convergence while at the same time computation efforts are considerably reduced.

2.5 Conclusion

Electricity markets throughout Europe, and beyond, have faced severe restructuring during the last decades. The transition towards a competitive market has created dramatic risk that market participants are now exposed to. In order to manage this risk, electricity as a new asset class was developed and the need for an adequate risk management led to the development of more and more derivatives products. In order to ensure a functioning risk management, the valuation of those derivatives is of utmost relevance. In the course of this development, the role of cross-border electricity markets also continues to grow in importance. As electricity as a commodity exhibits unique characteristics, those features need to be considered in an appropriate model for electricity prices. Moreover, electricity prices in cross-border markets, although akin to those in national markets, exhibit distinct features. Therefore, in addition to understanding the uniqueness of electricity as a commodity, the structure of national in contrast to cross-border electricity markets also needs to be considered. While equilibrium models are extremely important in order to understand the economic drivers of electricity prices, reduced-form models, originating from equity markets, have become the most important method of pricing derivatives contracts. Although there are many approaches for modelling national electricity prices, adequate models in cross-border markets are still scarce and constitute an important field of future research.

The Valuation of Contracts for Difference in the Nordic Market*

In this and the following chapter, we successively analyze the valuation of the most important cross-border electricity derivatives in the European market. We start with Contracts for Difference (CfDs), as shortly introduced before, and firstly introduce those contracts. Secondly, the Nordic market as the relevant one for CfDs is described. Finally, we perform an in-depth analysis of CfD prices and their risk premia before shortly summarizing our findings.

3.1 Introduction

CfDs are derivative instruments introduced in order to manage locational risk in electricity markets. In Europe, CfDs are currently listed only at

*This chapter is based on the work of Marckhoff and Wimschulte (2009).

the Scandinavian electricity exchange Nord Pool,[73] where they refer to the difference between an area price and the (unconstrained) system spot price.[74] The transmission grid in the Nordic market consists of several interconnected grid areas. Actual area prices determine the purchase cost of electricity in Nord Pool's spot market and differ from the system price at times of transmission congestion. However, the reference price for futures and forwards is the system price. Thus, for hedging locational price spreads, there are CfDs for most areas and with different delivery periods and maturities. Combined with a position in futures or forwards on the system price, CfDs are used to hedge against changes in area prices over time. A system price in combination with distinct area prices and related derivatives, available in the Nordic market, is unique and not present in any other electricity or commodity market.

In spite of the relevance of CfDs, only two studies, with limited empirical analyses, are available on CfDs in the Nordic market. Kristiansen (2004a) analyzes the pricing of season CfDs over the period November 2000 to April 2002. For most CfDs, positive risk premia, defined as the difference between average CfD prices and the average difference between area and system price during the delivery period, are identified. This is attributed to a majority of risk-averse consumers being willing to pay a risk premium in order to receive the future price spread. Conversely, contracts for the Oslo

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[73] From October 2003 to December 2005, eSpreads, CfD-like contracts on the difference between the day-ahead (system) prices for base- or peakload at the Energy Exchange Austria (EXAA) and European Energy Exchange (EEX), were listed at the EXAA, but discontinued, due to insufficient trading activity. In the UK, OTC traded CfDs do exist, but typically take the form of fixed-for-floating electricity swaps. Natural gas basis swaps at the NYMEX are the only exchange-traded commodity products comparable to CfDs. They refer to the future spread between the prices at Henry Hub and other US hubs.

[74] The system price is derived as the intersection between the supply and demand curve for the entire Nordic market (analogous to Figure 2.4) without considering any network constrains, i.e. whether delivery is physically possible.

area exhibit mostly negative risk premia, which is explained by a majority of risk-averse hydropower producers wishing to hedge their production. Kristiansen (2004b) extends the data set to season and year CfDs from November 2000 to December 2003. Although the basic findings remain unchanged, the risk premia are more dispersed.[75]

This chapter contributes to the limited number of studies on locational price spreads in electricity markets by conducting the first in-depth study of the pricing of CfDs using all CfDs listed at Nord Pool with delivery between 2001 and 2006, including contracts for five areas with monthly, quarterly, seasonal and yearly delivery periods. This totals 251 contracts and represents a much broader data set than in the two limited previous studies on CfDs. In addition, we do not restrict our analysis to average ex-post risk premia, but rather examine them on daily basis, enabling us to evaluate their development over the contract period. Further, we undertake the first investigation of the determinants of ex-post risk premia of CfDs. More specifically, we test for the dependence of risk premia on time-to-maturity and for their accordance with the Bessembinder and Lemmon (2002) model, which explains risk premia in electricity forwards with the variance and skewness of the underlying spot prices. Since CfDs can, in principle, be split into a short position in a system forward and a long position in a (non-traded) implied area forward, we also perform the analyses for these contracts.

Our results show that CfD prices contain, on average, significant risk premia and are thus not unbiased predictors of future price spreads. The risk premia exhibit significant variability in terms of sign and magnitude. They also differ substantially across areas, as they are subject to transmission congestion and hence, subject to locational price spreads to a different ex-

[75] Note that Kristiansen (2004a, 2004b) uses a risk premium definition that is different from ours. We adapt his results presented above to render them comparable to ours.

tent. Although we do not find a significant relation between risk premia and time-to-maturity for CfDs, there is a well pronounced negative relation for implied area and system forwards. This relation is in line with the theoretical results of Benth et al. (2008a). In addition, our findings indicate a negative (positive) relation between risk premia and the variance (skewness) of the underlying spot prices during the delivery period for CfDs and forwards, which supports the Bessembinder and Lemmon (2002) model. Overall, we show that existing models for the valuation of electricity forwards provide insights into the pricing and hedging of CfDs.

The remainder of this chapter is organized as follows. In Section 2, we briefly describe the spot market at Nord Pool, document the relevance of locational price spreads, both in terms of frequency and magnitude, and illustrate the design of CfDs. Section 3 discusses the pricing of CfDs and examines the determinants of their risk premia. The empirical results for each analysis are presented. The final Section 4 concludes and possible directions for further research are considered.

3.2 The Nordic Electricity Market

In this section, we introduce the Nordic electricity market as the relevant market for CfDs. After describing the spot market at Nord Pool, we discuss transmission congestion and the resulting locational price spreads. Finally, we thoroughly explain the characteristics of CfDs in the Nordic market.

3.2.1 The Spot Market at Nord Pool

The Nordic electricity market is one of the earliest and most widely liberalized electricity markets worldwide, with Norway leading the way and starting already in the early 1990s. In 1993, trading at the then solely Norwegian electricity exchange Nord Pool or its predecessor commenced and expanded across Scandinavia to Sweden (1996), Finland (1998), West Denmark (1999) and East Denmark (2000), resulting in an integrated Nordic electricity market. Currently, Nord Pool operates three different segments for electricity products: the real-time market Elbas, the day-ahead market Elspot and the derivatives market Eltermin. Each segment represents the leading electricity market in Europe in terms of traded volume. For this study, only the last two are of interest. At Elspot, which is typically referred to as a spot market, electricity for physical delivery during each single hour or hourly blocks on the subsequent day(s) is traded. The price fixing takes place in 24 individual auctions at noon on the working day prior to delivery. At Eltermin, continuously traded instruments comprise cash-settled short-term futures, medium to long-term forwards, options on futures and CfDs. The underlying of the futures and forwards is the spot price for 24-hour-delivery of electricity (baseload), calculated as the arithmetic average of the hourly spot prices, on each day of the respective delivery period.[76] Delivery periods range from one day to one year and the time until beginning of the delivery period can be up to five years. Futures are subject to daily settlement in the trading and delivery period. The settlement of forwards and CfDs takes place only during the delivery period. All financial claims in the derivatives market are guaranteed by Nord Pool, which acts as counterparty for all trades and organizes the (OTC) clearing.

Nord Pool's spot market Elspot is separated into market areas that typically reflect the grids of the national TSOs. In the period under consideration, there are at least six areas, which are shown in Figure 3.1. Norway is divided into a southern (NO1) and a central-northern area (NO2), but

[76] Futures and forwards on peakload (delivery of electricity on weekdays from 08:00 to 20:00) were introduced as recently as June 2007.

may consist of between one and six areas, depending on the TSO's considerations on the security of supply within the system. Denmark is split into the two areas West Denmark (DK1) and East Denmark (DK2) that are not linked by an interconnector. Sweden (SE) and Finland (FI) each form a separate area.

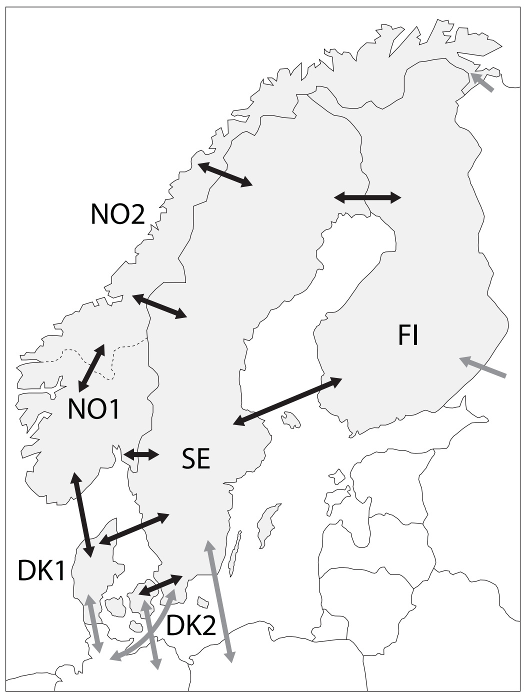


Figure 3.1: Market areas and main interconnections in the Nordic power market.

The capacity for all transmission lines between the areas is granted to Nord Pool and auctioned implicitly in the Elspot price fixing. Transmission capacities available for the next day are determined by the TSOs and published on the Nord Pool web page by 10:00 am CET, well before the

Elspot auctions at 12:00 am. In the auctions, bids for the delivery of electricity during single hours or hourly blocks in a specified delivery area are possible. All four local Nordic currencies are accepted, but converted to the primary currency prior to the price fixing and back again afterwards. Until the end of 2005, the Norwegian krone (NOK) was the primary currency, with the Euro (EUR) playing that role since then. In order to ensure consistency, we convert all spot prices into EUR using daily exchange rates published by Nord Pool. All orders for the Nordic region are combined to form one aggregate supply and demand curve for each of the 24 delivery hours, and the intersection of each pair determines the particular (unconstrained) system price. In the rare case that aggregated supply and demand curves do not intersect due to significant imbalances, bids are curtailed pro rata. Any influence of the German Kontek area on the system price calculation is limited to the available transmission capacity.[77] Bids for the two Danish areas were treated the same way until the end of 2005, but have been included in entirety since then.

If electricity flows between areas, resulting from the auctions, are within the capacity limits set by the TSOs, all area prices are equal to the system price for the specific hour throughout the entire Nordic market. However, if the electricity flows reach the available transmission capacities, i.e., congestion occurs, area prices diverge throughout the market. Currently applied methods for the calculation of locational prices in an electricity network are nodal transmission pricing, used, for example, in New York and the Pennsylvania-New Jersey-Maryland (PJM) market, and zonal transmission pricing, used, for example, in Australia, California and the Nordic

[77] The Kontek area, introduced in October 2005, comprises the Vattenfall Europe transmission grid in Eastern Germany and is connected to the Danish areas. In contrast to all other external connections to the Nordic market, the available transfer capacity is included in the implicit auction at Nord Pool's spot market.

market.[78] With nodal pricing, the locational marginal price (LMP), i.e., the price of inducing an additional MW of electricity, is calculated for each node (i.e., every point in the grid where electricity is added or removed) in the electricity network. Zonal pricing, in contrast, groups nodes that are connected by non-congested power lines into zones (or areas). Within each zone, the LMP is identical across all nodes and referred to as the zonal market clearing price (ZMCP). While nodal pricing is considered more efficient but rather complex, zonal pricing is considered simplistic but more transparent.[79]

In case of congestion in the Nordic market, separate area prices, which differ from the system price, are calculated. For this purpose, the market areas on each side of the congestion(s) are combined and new equilibrium prices are calculated, each in the same manner as before, establishing surplus area(s) with lower prices, and deficit area(s) with higher prices. The available transmission capacities are then utilized fully, by adding price-independent purchases (sales) in the surplus (deficit) area(s) and iterating the price calculations. As a result, the final equilibrium area prices are determined. Due to convex and heterogeneous supply curves among areas, the price effects of a demand increase in one area and decrease in another area do not offset. Therefore, the (volume weighted) average of the area prices is not equal to the system price and no arbitrage relationship between these prices exists. Note that all area prices can be above or below the system price, since different block bids might be accepted in

[78] Further approaches discussed in the literature are Chao-Peck pricing (Chao and Peck (1996)) and uniform pricing (Green (2007)). See Kristiansen (2004b) for a short discussion of these approaches.

[79] Although relevant in the context of CfDs in principle, a thorough analysis of transmission pricing is beyond the scope of this paper. We therefore refer to Hogan (1998) for a comprehensive discussion of transmission pricing and Bjørndal and Jørnsten (2001) for details on zonal pricing in the Nordic market.

the two calculations. The zonal pricing approach discussed above, ensures transparent price setting and electricity flows from the lower to the higher price area(s). Transmission congestion and related price differences also constitute price signals for new capacity investments.[80]

3.2.2 Transmission Congestion and Locational Price Spreads

The transfer of electricity from one area to another is driven by the considerably different generation mix and resulting cost structures in the Nordic as well as neighboring countries.[81] Electricity generation in Denmark relies mainly on coal- and gas-fired power plants (80% in 2006). Wind power also provides a substantial part. Finland uses a balanced mix of hydropower, nuclear power and conventional thermal power, but also imports substantial volumes from Russia. In Sweden, 90% of generated electricity comes from nuclear and hydropower plants in roughly equal shares. In Norway, electricity is generated almost entirely from hydropower. Since wind power is a relatively volatile supply source, hydropower output depends on the (seasonally) varying water reservoir levels, and nuclear plants often run at limited capacity in summer, the directions of electricity flows between areas vary significantly over time. As a consequence, transmission congestion and locational price differences also vary.[82]

The frequency of transmission congestion in the Nordic electricity market

[80] See Nord Pool (2006). For more details on the price fixing, see www.nordpoolspot.com. Transmission congestion within Sweden, Finland and the Danish areas is managed by the TSOs through counter-trade purchases based on bids from generators.

[81] Note that a point-of-connection tariff system is used in the Nordic market and thus there is no additional tariff for electricity transfers between areas. The design of the tariffs, however, is different across the countries. For details see Kristiansen (2004b).

[82] See Nordel (2007) for the data on electricity generation.

Area	2001	2002	2003	2004	2005	2006
Aarhus (DK1)	92.3	98.4	99.2	99.5	97.5	99.7
Copenhagen (DK2)	81.4	77.5	86.0	98.6	85.2	99.7
Oslo (NO1)	73.4	73.2	85.5	97.8	77.0	99.7
Trondheim (NO2)	73.7	73.2	85.5	98.1	77.0	99.7
Helsinki (FI)	73.7	73.2	85.5	97.8	77.0	99.7
Stockholm (SE)	73.7	73.2	85.5	97.8	77.0	99.7

Table 3.1: Frequency of differences between daily area prices and the system price.

is illustrated in Table 3.1. Differences between daily area prices for baseload and the corresponding system price occur for more than 70% of all calendar days over the period 2001 to 2006 in each area. Prices in the DK1 area even differ for almost all days from the system price, because the area's transmission connections to the NO1 and SE areas are typically not sufficient to cover all desired electricity transfers. The full inclusion of the DK1 and DK2 areas in the system price calculation consequently results in price differences in all areas almost every day in 2006 and will most likely continue. The price differences in Table 3.1 do not exhibit any persistent patterns between working and non-working days or within a year and are obviously more frequent than on an hourly basis.

In addition to the frequency of price differences, their sign and magnitude are important. Table 3.2 therefore presents descriptive statistics of the differences between area prices and the system price for baseload in absolute terms and relative to the system price (in brackets). We focus on daily baseload prices, as they are the underlying of CfDs. Panel A shows the mean difference between area and system prices for each year between 2001 and 2006, where positive values indicate higher area prices compared to the system price and vice versa. For example, the area price for Aarhus in 2005 was on average EUR 7.90 higher than the system price corresponding to 26% of the system price (as given in brackets). Panel A reveals that annual mean differences vary both between areas and within areas across time in terms of both sign and magnitude. Major reasons are swings in regional supply and demand for electricity, due to, for example, changes in water reservoir levels for hydropower or weather conditions. Mean differences are rather small for most areas, but more pronounced for the DK1 and DK2 areas, with the mean percentage difference for the DK1 area reaching a remarkable 26% in 2005. The annual standard deviations of absolute and percentage differences are small and fairly constant over time. The figures for the DK1 and DK2 areas are again substantially higher and more variable (we refrain from reporting detailed figures here). As indicated in Panel B, the frequent transmission congestion and local supply or demand shocks are also reflected in the leptokurtic distribution and extreme values of the price differences. Based on these figures, the need to manage the risk of locational price spreads in the Nordic market is evident.

As discussed above, hydropower constitutes a major share of electricity generation in the Nordic market. Because water reservoir levels are only refilled naturally via precipitation or melting water, hydropower generators determine their electricity supply based on the current hydro balance in relation to the normal situation. Therefore, variations in the current hydro balance typically become supply effective immediately.[83] Bühler and Müller-Merbach (2007b), for example, discuss the influence of cheap and flexible hydropower generation on the Nord Pool system price. They show

[83] Vehviläinen and Pyykkönen (2005) emphasize the fact that hydropower generators value current water reservoir levels based on the normal situation. See Von der Fehr et al. (2005) for a thorough analysis of the supply shock in the Nordic market between 2002 and 2003 due to abnormal low water reservoir levels.

A. Mean Absolute and Percentage Differences between Area and System Price

Area	2001		2002		2003		2004		2005		2006	
Aarhus	.59**	(.03)	-1.44	(.06)	-3.01**	(-.05)	-.12	(-.01)	7.90***	(.26)	-4.41***	(-.08)
Copenhagen	.40*	(.02)	1.67***	(.09)	.11	(.00)	-.57***	(-.02)	3.93***	(.12)	-.06	(.01)
Oslo	-.07	(.00)	-.34***	(-.02)	.42***	(.01)	.48***	(.02)	-.21***	(-.01)	.63***	(.01)
Trondheim	.31***	(.02)	-.11	(.00)	-.03	(.00)	.21***	(.01)	.06	(.00)	.38*	(.01)
Helsinki	-.31**	(-.02)	.36	(.04)	-1.39***	(-.04)	-1.24***	(-.04)	1.19***	(.04)	-.03	(.00)
Stockholm	-.29**	(-.01)	.70***	(.04)	-.20	(.00)	-.84***	(-.03)	.43***	(.01)	-.48**	(-.01)

B. Statistics for Absolute and Percentage Differences between Area and System Price 2001-2006

Area	Mean		Median		Minimum		Maximum		Std.dev.		Skewness		Kurtosis	
Aarhus	-.08	(.04)	-.02	(.00)	-93.04	(-.89)	129.69	(4.38)	10.27	(.28)	.19	(4.15)	29.14	(47.91)
Copenhagen	.91***	(.04)	.00	(.00)	-33.42	(-.57)	79.68	(2.23)	5.69	(.17)	3.27	(3.62)	29.60	(24.96)
Oslo	.15***	(.00)	.00	(.00)	-14.31	(-.26)	11.90	(.71)	1.26	(.04)	.99	(3.66)	30.14	(62.92)
Trondheim	.13**	(.01)	.00	(.00)	-17.98	(-.43)	13.85	(.82)	1.40	(.05)	.55	(4.68)	32.40	(66.85)
Helsinki	-.24*	(.00)	-.02	(.00)	-32.86	(-.54)	52.41	(1.46)	3.34	(.10)	3.35	(3.15)	69.29	(38.19)
Stockholm	-.11	(.00)	.00	(.00)	-15.18	(-.54)	11.34	(.58)	1.78	(.06)	-.43	(.70)	13.38	(20.47)

Table 3.2: Descriptive statistics for differences between daily area prices and the system price.

that an increased (decreased) availability of hydropower leads to a right (left) shift in the supply curve and therefore generally to lower (higher) system prices. The influence of hydropower on area prices depends on sufficient transmission capacities to hydropower generators. In case of congestion, the effect of hydropower is more pronounced for area prices than for the system price. For areas that are well connected to hydropower generators, the area price spread (i.e., the difference between area price and the system price) is therefore negative. If areas are not sufficiently connected to hydropower generators, cheap hydropower does not significantly dampen the area price and the area price spread is positive.

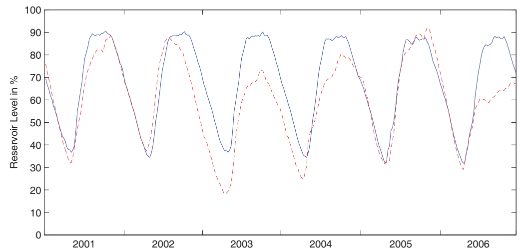
In order to test for the relation between hydropower and area price spreads, we use the difference between current water reservoir levels, measured in percent of their capacity, and their historic median as a proxy for hydropower generation capacity in the Nordic market. Figure 3.2 shows current water reservoir levels and their historic median for Norway, Finland and Sweden, where the historic median is calculated based on reservoir data starting in 1992 (Norway and Finland) and 1996 (Sweden).[84]

Water reservoir levels are, with the exception of Finland, only available on a weekly basis. To ensure consistency, we use average area price spreads for each calendar week. The difference between current and historic water reservoir levels is measured in percent of the historic median, as deviations from the median are more severe in times of low reservoir levels. We then conduct the following regression for the period 2001 and 2006:[85]

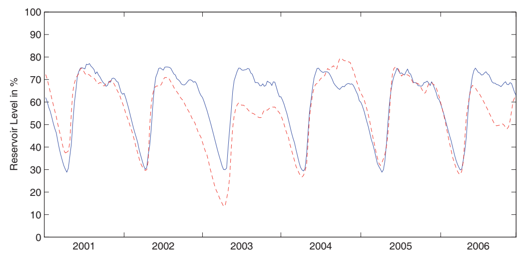
$$\bar{P}_t^A - \bar{P}_t^S = c + \beta^{No} RL_t^{No} + \beta^{Fi} RL_t^{Fi} + \beta^{Se} RL_t^{Se} + \varepsilon_t, \qquad (3.1)$$

[84] Water reservoir data is obtained from the Norwegian Water Resources and Energy Directorate (Norway), Finnish Environment Institute (Finland) and Nordel (Sweden).

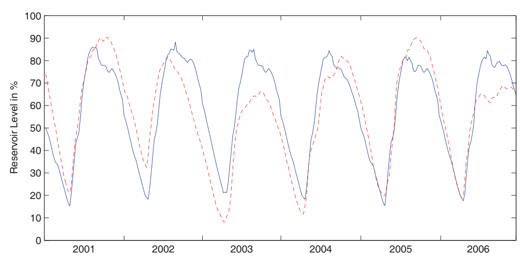
[85] All regressions are conducted using ordinary least squares.



Norway



Finland



Sweden

Figure 3.2: Water reservoir levels and their historic medians in the Nordic market.

Area	c	β^{No}	β^{Fi}	β^{Se}	Prob (F-stat.)	adj. R^2
Aarhus	3.8750***	30.0659***	8.4028	-7.7963**	.0000	.23
Copenhagen	3.2890***	23.7727***	-3.6427	-6.3625*	.0000	.23
Oslo	-.3146***	-4.5558***	.5369	1.0918***	.0000	.20
Helsinki	.9144***	12.1096***	-2.7913***	-2.2400**	.0000	.22
Stockholm	.3825*	6.4209***	-3.0463***	-1.1930*	.0000	.16

Table 3.3: Regression of area price spreads on relative water reservoir level deviations (2001-2006).

where c is a constant and ε_t is the error term. $\bar{P}_t^A$ ($\bar{P}_t^S$) is the weekly average area (system) price, and RL_t represents the relative deviation of the current water reservoir level from its historic median for Norway (No), Finland (Fi) and Sweden (Se) during week t. Table 3.3 shows the results of the above regression.

There exists a significant relation between area price spreads and relative water reservoir level deviations for all areas. In addition, the adjusted R^2 indicate good explanatory power for all regressions. The price effect of cheap hydropower is strongest in the area where it is available and generally results in lower area price spreads in this area and higher area price spreads in the other areas. This effect is observed for two reasons. First, the total amount of hydropower supply compared to electricity demand is high in the area where hydropower is available. But when aggregating the supply and demand over all areas, its impact is reduced. Thus, hydropower affects the originating area price stronger than the system price. Second, in case of congestion, cheap hydropower cannot be transferred to other areas and their prices are less affected by hydropower than the system price. This results in positive price spreads for areas without hydropower. These two effects explain the regression results in Table 3.3. Hydropower availability in Norway results in lower area price spreads in Norway but in higher ones for all other areas, as indicated by the signs of β^{No}. The hydropower availability in Sweden and Norway typically coincides. As Sweden is well connected to all areas, hydropower from Sweden dampens the area price spreads for all areas except Norway. Since Finnish hydropower only constitutes a minor share of overall capacity, its impact is limited to Finland and Sweden.

3.2.3 Contracts for Difference

In order to allow market participants to manage the risk of locational price spreads, Nord Pool introduced CfDs in November 2000.[86] Combined with a system forward covering the same delivery period, they are used to hedge against changes in area prices over time. Generally, retailers or large consumers hold long positions in CfDs in order to hedge against rising area prices, whereas generators take short positions in CfDs. The respective physical positions are traded in the spot market. The risk of two diverging area prices can be hedged with the particular CfDs, which is relevant for market participants delivering to or receiving from another area within the Nordic market. CfDs are forward contracts with reference to the difference between an area price P_t^A and the system spot price P_t^S for baseload over the delivery period. Their payoff is calculated as

[86] Instead of CfDs, area price forwards could have been introduced, but were rejected by Nord Pool so as to avoid splitting the total liquidity among several products.

$$\mathrm{CfD}(P^A, T_1, T_2) = \frac{1}{T_2 - T_1} \sum_{t=T_1}^{T_2} \left(P_t^A - P_t^S \right), \tag{3.2}$$

where T_1 and T_2 denote the start and end dates of the delivery period. Because CfD prices refer to the difference of future area and system prices, they can be positive, negative, or zero. Nord Pool currently provides CfDs for 5 areas with monthly, quarterly and yearly delivery periods.[87] Month CfDs were introduced with the April 2004 contract for all areas except the DK1 area. In 2006, Nord Pool switched from seasonal to quarterly delivery periods to meet international standards. Thus, season contracts were traded with delivery before 2006 and quarter contracts with delivery thereafter. The delivery periods for the season CfDs are January-April (Winter 1), May-September (Summer) and October-December (Winter 2). For each area, CfDs for the next two months, three quarters and year are tradable.[88] No trading is possible in the delivery period. The minimum contract size is 1 MW and prices were quoted in NOK/MWh with two decimals until the end of 2005 and subsequently in EUR/MWh. We convert all CfD prices into EUR using daily exchange rates published by Nord Pool to ensure consistency. The fulfillment of month CfDs is due as cash settlement. Quarter and year contracts, however, are not cash-settled but

[87] At Nord Pool, there are also CfDs on the difference between the German and Nordic system price, but these are not considered in this study, since they can be replicated with an appropriate position in EEX futures and Nord Pool system forwards. Also note that due to the lack of CfDs for all areas and especially the calculation methods for area prices and the system price, there is no clear arbitrage relationship for CfDs and system forwards.

[88] For Norway, there are only CfDs on the Oslo area price. For the DK2 area the first season CfD introduced was the Summer 2001 contract in contrast to the Winter 1 2001 contract for all other areas. Two additional rolling year CfDs for all areas and month CfDs for the DK1 area were introduced in November 2006 and June 2007, respectively.

rather cascaded, i.e., they are replaced by a corresponding position in CfDs with shorter delivery periods.

Our data base consists of all CfD prices from November 17, 2000 to December 31, 2006 and the corresponding spot and system forward prices. In total, there are 132 month, 20 quarter, 74 season and 25 year CfDs with delivery in that period. The open interest of all CfDs outstanding at the end of 2006 amounts to about 18,300 contracts, which constitutes 27% of open interest of the system forwards with corresponding delivery periods. All data were obtained directly from Nord Pool. For derivative products, closing prices applied for settlement are used. In case of no transactions on a certain trading day, Nord Pool employs several procedures for estimating closing prices.[89] Active exchange trading takes place only in CfDs for the two Danish areas and, similar to other markets, is concentrated in the front contracts. During the sample period, all of these contracts accounted for more than 68% of total trading volume of about 22,700 Danish month, quarter and season CfDs.[90] Given that market makers quote binding bid and ask prices for all CfDs virtually every trading day from 13:00 until the end of trading at 15:30 and OTC transactions in CfDs are regularly submitted to Nord Pool for clearing,[91] closing prices for CfDs can be regarded as realistic market prices.

In Table 3.4, the mean and standard deviation (in brackets) of CfD prices are shown. For each area, CfDs with the same delivery period (i.e., month, quarter, season, year) are aggregated, depending on their year of maturity.

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[89] See Nord Pool (2007), pp. 22-23 for details.

[90] In addition, about 5,000 Danish year CfDs were traded during the sample period. We do not include them in the above figures, since for year CfDs, only front contracts existed.

[91] Note that the OTC clearing volume was about twice the volume of all contracts traded on Nord Pool's financial market during the sample period. Detailed figures for the OTC clearing volume of CfDs are not available.

CfD prices are mostly positive and with large differences between areas. While for Oslo CfDs, about half the prices are positive, prices for Aarhus and Helsinki CfDs are positive at 72% and 84% of the time, respectively. CfD prices for the Copenhagen and Stockholm areas are almost entirely positive. The mean and standard deviation of CfD prices clearly increase for contracts with delivery in 2006, due to the full inclusion of the two Danish areas in the spot price fixing and resulting more frequent price spreads. In addition, the CfD prices for the Danish areas are typically higher and more volatile than for the other areas. This reflects the characteristics of the underlyings presented in Table 3.2. Furthermore, we find a significant increase in the rolling 10-trading day volatility of CfD prices as maturity approaches. This effect is evident for all areas and contract classes.

3.3 Pricing of CfDs: Methodology and Results

In this section, we introduce the valuation principles for CfDs. We further describe the calculation of risk-premia of CfDs and conclude this section by empirically investigating the observed market risk-premia in CfD prices.

3.3.1 Pricing of CfDs

The payoff the holder of a long position in a CfD receives during the delivery period is, as shown in (3.2), identical to receiving the area spot price and paying the system spot price each day during the delivery period. Thus, the CfD could, in principle, be replicated with a long position in an area forward and a short position in a system forward as follows

$$\mathrm{CfD}_{t,T} = F^{A}_{t,T} - F^{S}_{t,T}, \tag{3.3}$$

Contract	Delivery Period	2001		2002		2003		2004		2005		2006	
Aarhus	Month	-	-	-	-	-	-	-	-	-	-	-	-
(SYARH)	Quarter	-	-	-	-	-	-	-	-	-	-	4.48	(4.42)
	Season	.69	(1.52)	.82	(1.27)	-1.73	(4.51)	-.01	(2.38)	3.72	(2.46)	-	-
	Year	-	-	.51	(.29)	.38	(2.26)	-.39	(.50)	2.33	(.62)	5.39	(1.53)
Copenhagen	Month	-	-	-	-	-	-	.45	(.27)	3.62	(2.33)	4.39	(5.41)
(SYCPH)	Quarter	-	-	-	-	-	-	-	-	-	-	6.76	(3.22)
	Season	.26	(.19)	1.61	(.78)	.85	(.67)	.93	(.49)	2.49	(1.66)	-	-
	Year	-	-	.78	(.45)	1.48	(.49)	.99	(.40)	1.37	(.09)	5.15	(1.84)
Oslo	Month	-	-	-	-	-	-	.31	(.06)	-.19	(.22)	.67	(.81)
(SYOSL)	Quarter	-	-	-	-	-	-	-	-	-	-	.18	(.56)
	Season	-.18	(.21)	-.12	(.12)	.28	(.39)	.27	(.10)	-.02	(.31)	-	-
	Year	-	-	-.04	(.06)	-.06	(.23)	.24	(.07)	.24	(.07)	-.13	(.10)

Table 3.4: Average traded prices of Contracts for Difference by delivery period.

Contract	Delivery Period	2001	2002	2003	2004	2005	2006
Helsinki	Month	- -	- -	- -	-.40 (.15)	1.02 (.64)	.67 (.79)
(SYHEL)	Quarter	- -	- -	- -	- -	- -	.98 (.55)
	Season	.54 (.52)	.57 (.24)	.39 (.42)	.03 (.40)	.52 (.55)	- -
	Year	- -	.44 (.14)	.71 (.08)	.32 (.14)	.23 (.21)	1.03 (.37)
Stockholm	Month	- -	- -	- -	.06 (.26)	.78 (.34)	.62 (.60)
(SYSTO)	Quarter	- -	- -	- -	- -	- -	.71 (.40)
	Season	.33 (.42)	.42 (.21)	.44 (.19)	.39 (.19)	.56 (.27)	- -
	Year	- -	.28 (.16)	.52 (.07)	.47 (.12)	.35 (.08)	.74 (.17)

Table 3.4: Average traded prices of Contracts for Difference by delivery period.

where $\mathrm{CfD}_{t,T}$ refers to the CfD price on day t with maturity at time T. $F^A_{t,T}$ ($F^S_{t,T}$) denotes the area (system) forward price on day t with maturity in T. Since electricity is a flow commodity and the delivery of electricity contracts takes place during a specified delivery period, time T refers to the entire delivery period (i.e., the period between T_1 and T_2 in (3.2)). For simplicity, we use $\mathrm{CfD}_{t,T}$ instead of CfD_{t,T_1,T_2}. Although (3.3) holds theoretically, area forwards are not listed at Nord Pool's Eltermin market and thus, the simple replicating strategy is not possible. We can, however, calculate the implied area forward price for any day t as

$$F^A_{t,T} = \mathrm{CfD}_{t,T} + F^S_{t,T}. \tag{3.4}$$

The fact that CfDs are in principle replicated with a position in an area and system forward allows us to apply existing electricity forward pricing models when analyzing CfDs.

There is a large amount of research on forwards and futures on commodities and also on electricity.[92] In electricity markets, the expectations and risk preferences of market participants determine futures prices, where the futures price at time t with maturity in T, $F_{t,T}$ is split into the expected future spot price $E\left(P_T|\,\Omega\Omega_t\right)$ and a risk premium π^F_t. In this case, the futures price can be calculated as

$$F_{t,T} = E\left(P_T|\,\Omega_t\right) + \pi^F_t, \tag{3.5}$$

where Ω_t is the information set available at time t. If risk premia exist,

[92] We do not distinguish between futures and forward prices and use these terms interchangeably when introducing pricing methodologies.

futures prices are not unbiased predictors of future spot prices. The understanding of risk premia, when pricing electricity futures, is therefore important for economic agents in the electricity market. Producers rely on price forecasts, for example, for planning and budgeting purposes, while consumers use them to make their investment and consumption decisions. An assumption of unbiased futures prices would result in incorrect estimates of future spot prices and thus inefficient decisions of market participants. The risk premium π_t^F is calculated as

$$\pi_t^F = F_{t,T} - E\left(P_T \middle| \Omega_t \right). \tag{3.6}$$

For ease of exposition, $E_t\left(P_T\right)$ is used for $E\left(P_T \middle| \Omega_t \right)$ in the following. We assume that the unexpected component of $E_t\left(P_T\right)$ is white noise and uncorrelated with information at time t. Therefore, the expectations of market participants are formed rationally and errors are on average zero. Following Longstaff and Wang (2004), we thus use P_T as a proxy for $E_t\left(P_T\right)$.[93] The ex-post realized spot price is calculated as the average daily spot price during the delivery period of the futures contract and is denoted as the delivery price.[94] Applying the same approach to CfDs, we calculate the risk premium of a CfD at time t as the difference between the traded CfD price and the ex-post delivery price. The delivery price of a CfD is the average of the daily difference between the area and system prices during the delivery period. Thus, the CfD risk premium at time t with delivery in T can be calculated as

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[93] We use the ex-post risk premium based on ex-post observed delivery prices in this study. This is in contrast to the ex-ante risk premium shown in Equations 3.6 and 3.7, which is based on ex-ante expectations of market participants.

[94] We use the average delivery price as CfD prices are quoted on a per hours basis. Thus, we also need hourly prices to make CfD spot prices and their delivery prices comparable.

$$\pi_t^{\mathrm{CfD}} = \mathrm{CfD}_{t,T} - E_t\left(\mathrm{CfD}_{T,T}\right), \tag{3.7}$$

where the CfD price at time T (i.e., the ex-post delivery price $\mathrm{CfD}_{T,T}$) is used as a proxy for the expected CfD price at time t with delivery in T, $E_t\left(\mathrm{CfD}_{T,T}\right)$.

3.3.2 Risk Premia in CfD Prices

Several papers address the risk premia of electricity futures. Longstaff and Wang (2004) use day-ahead forward prices from the PJM market and find positive risk premia. Though positive on average, risk premia vary in magnitude and sign across hours. For month futures with delivery at California-Oregon-Border (COB) traded at the NYMEX, Hadsell (2006) finds significant positive risk premia for winter months and negative ones for summer months. In addition, he shows that risk premia have declined as the market matured. Botterud et al. (2002) investigate futures prices from Nord Pool between 1995 and 2001 across all traded delivery periods and also find significant positive risk premia for all contracts.[95] Bühler and Müller-Merbach (2007b) employ week and block futures from Nord Pool between 1996 and 2004. They document, on average, positive risk premia for week futures and negative risk premia for block futures. Lucia and Torró (2008) analyze week futures traded at Nord Pool between 1998 and 2005 and also find significant positive risk premia. Kristiansen (2004a, 2004b) identifies differences between the average CfD prices and the ex-post delivery prices that vary across areas and delivery periods. However, given the limited data set, he does not obtain conclusive results on risk

[95] Hadsell (2006) and Botterud et al. (2002) use risk premia definitions that are different from ours. We adapt their results to make them comparable to our findings.

premia.

We examine risk premia in CfD prices for all contracts with delivery between 2001 and 2006. As power supply and demand across areas differ in sensitivity to weather conditions, we expect risk premia to vary in sign and magnitude on a seasonal and geographical basis.[96] In order to test for risk premia in CfD prices, we calculate the risk premium according to (3.7). Table 3.5 shows the mean and standard deviation (in brackets) of ex-post CfD risk premia. Mean delivery prices are calculated as the mean daily difference between the respective area spot price and the system spot price during the delivery periods. Season contracts are not aggregated due to the different length of the delivery periods.

Table 3.5 demonstrates that CfD prices contain significant risk premia, which vary in sign and magnitude across all areas and delivery periods. Looking at the Oslo CfDs, risk premia have in most cases the opposite sign compared to all other areas.[97] The reason might be the large share of hydropower in Norway in contrast to other countries. In wet periods, Norway is a net exporter of cheap electricity from hydropower and area prices will generally be lower than the system price. This leads to an increased hedging demand from producers in Norway. Because the power generated in other Nordic countries comes mainly from thermal units, these countries are net importers in wet periods. The limited transmission capacities often

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[96] The large share of hydropower in the Nordic market makes the water reservoir level an important factor influencing electricity prices, as discussed by Bühler and Müller-Merbach (2007b) and shown in Table 3.3. Our methodological setup, however, does not allow us to clearly separate the effect of changing reservoir levels on expected spot prices and risk premia. We thus refrain from analyzing the impact of water reservoir levels on risk premia of CfDs and forwards.

[97] Further evidence of the uniqueness of the Oslo area is the fact that the price spread underlying the Oslo CfDs is negatively correlated to all other area price spreads, whereas the latter are pairwise positively correlated.

Contract		2001		2002		2003		2004		2005		2006	
Aarhus	M	-	-	-	-	-	-	-	-	-	-	-	-
(SYARH)	Q	-	-	-	-	-	-	-	-	-	-	9.05***	(6.43)
	W1	2.47***	(.20)	-1.33***	(.25)	6.02***	(7.32)	-1.17***	(1.33)	-.46***	(.93)	-	-
	S	.72*	(1.81)	-1.52***	(.78)	-2.33***	(1.27)	1.23***	(.80)	-5.56***	(1.33)	-	-
	W2	-1.37***	(.52)	14.19***	(.24)	1.72***	(1.05)	1.12***	(.48)	-5.57***	(1.38)	-	-
	Y	-	-	1.96***	(.29)	3.39***	(2.26)	-.27***	(.50)	-5.57***	(.62)	9.80***	(1.52)
Copenhagen	M	-	-	-	-	-	-	1.05***	(1.05)	-.89*	(5.45)	4.45***	(4.23)
(SYCPH)	Q	-	-	-	-	-	-	-	-	-	-	6.84***	(6.00)
	W1	-	-	-1.36***	(.52)	1.87***	(.69)	1.36***	(.24)	-2.09***	(.14)	-	-
	S	.77***	(.17)	-.59***	(.77)	-.07	(.80)	1.93***	(.52)	2.07***	(1.37)	-	-
	W2	-1.22***	(.19)	2.70***	(.26)	.72***	(.48)	1.36***	(.19)	-6.31***	(1.27)	-	-
	Y	-	-	-.89***	(.45)	1.37***	(.49)	1.56***	(.40)	-2.56***	(.09)	5.21***	(1.84)
Oslo	M	-	-	-	-	-	-	-.16***	(.50)	.02	(.28)	.05	(1.24)
(SYOSL)	Q	-	-	-	-	-	-	-	-	-	-	-.43***	(.83)
	W1	-.54***	(.06)	.05***	(.06)	-.87***	(.47)	-.22***	(.13)	.62***	(.07)	-	-
	S	-.28***	(.16)	.71***	(.16)	.56***	(.35)	-.34***	(.06)	.03	(.61)	-	-
	W2	.21***	(.10)	-.39***	(.08)	-.24***	(.16)	-.02**	(.07)	-.09***	(.14)	-	-
	Y	-	-	.30***	(.06)	-.48***	(.23)	-.24***	(.07)	.45***	(.07)	-.76***	(.10)

Table 3.5: Ex-post risk premia of Contracts for Difference.

Contract		2001		2002		2003		2004		2005		2006	
Helsinki	M	-	-	-	-	-	-	.79***	(.58)	-.18*	(1.11)	.67***	(1.86)
(SYHEL)	Q	-	-	-	-	-	-	-	-	-	-	.94***	(1.68)
	W1	1.38***	(.07)	.23***	(.18)	2.31***	(.48)	1.54***	(.39)	-.54***	(.11)	-	-
	S	1.69***	(.62)	-1.34***	(.24)	1.12***	(.51)	1.49***	(.31)	-.27**	(.61)	-	-
	W2	-.10***	(.19)	2.79***	(.10)	2.21***	(.13)	.79***	(.36)	-1.33***	(.33)	-	-
	Y	-	-	.08***	(.14)	2.10***	(.06)	1.56***	(.14)	-.96***	(.21)	1.06***	(.37)
Stockholm	M	-	-	-	-	-	-	.96***	(.72)	.36***	(.50)	1.08***	(1.44)
(SYSTO)	Q	-	-	-	-	-	-	-	-	-	-	1.12***	(.85)
	W1	1.11***	(.10)	.15***	(.25)	1.51***	(.10)	1.28***	(.15)	-.19***	(.08)	-	-
	S	1.33***	(.50)	-1.29***	(.15)	-.20***	(.26)	1.22***	(.24)	.35***	(.29)	-	-
	W2	-.22***	(.15)	.76***	(.11)	.90***	(.14)	1.23***	(.11)	.18***	(.21)	-	-
	Y	-	-	-.42***	(.16)	.72***	(.07)	1.31***	(.12)	-.08***	(.08)	1.22***	(.17)

Table 3.5: Ex-post risk premia of Contracts for Difference.

prevent the price effect of cheap hydropower to spread across all areas. This results in an opposite effect on area prices and area price spreads in all other countries, and to more hedging demand from consumers. Karakatsani and Bunn (2008) support the above argument for day-ahead forward prices in the UK. They find that different plant technical characteristics lead to changing hedging needs and thus time-varying risk premia. The CfD risk premia also vary across delivery periods, but there is no significant relation between risk premia and the length of the respective delivery period. Results in Table 3.5 further indicate that mean risk premia and their standard deviations increased in 2006, as a result of the full inclusion of the Danish areas in the system price calculation. Besides statistical significance, risk premia are also economically relevant, since, on average, they constitute a significant part of CfD prices.[98] Further analyses show that the risk premia in CfD prices exhibit seasonality. Using season CfDs, we find that the risk premia of Summer contracts are significantly different from the risk premia of Winter 1 and Winter 2 contracts, whereas the sign varies across areas.[99] The risk premia of Summer contracts are significantly lower (higher) compared to Winter contracts of Aarhus, Stockholm and Helsinki (Oslo and Copenhagen) CfDs.[100]

[98] Risk premia of implied area and system forwards (not shown here) also vary in sign and magnitude across delivery periods. Though larger than CfD risk premia in absolute terms, they are smaller on a percentage basis. In contrast to CfDs, the risk premia of implied area forwards do not differ considerably among areas.

[99] Season CfDs are used for this analysis, as their delivery periods most closely resemble seasonal patterns throughout the year. For month and quarter CfDs, the sample sizes per area are not sufficient.

[100] For our analysis of seasonality in CfD risk premia, we have regressed the risk premia of season CfDs on dummy variables for Winter 1, Summer and Winter 2. While the R^2 values are rather low (between 2% and 8%), the parameters for the summer dummy variable are significant at the 1% level for all areas. As we do not discuss those results further, we refrain from stating them in detail.

After having shown the importance of risk premia in CfD prices, we analyze the development of risk premia over time-to-maturity. Prior studies demonstrate that the risk premia of electricity futures are a negative function of time-to-maturity. Risk premia are negative for long time-to-maturities and increase (i.e., they decrease in absolute terms) with decreasing time-to-maturity. Diko et al. (2006) use OTC forward prices from three Continental electricity markets and find positive short-term, and negative long-term risk premia. They attribute this characteristic shape to price spikes in the underlying spot price process, which have a minor impact on contracts with long maturities, but become significant as time-to-maturity decreases. Since the main motivation for engaging in forward contracts is risk diversification, Benth et al. (2008a) explain the negative relation between risk premia and time-to-maturity by changing hedging pressure from market participants. They argue that the desire of market participants to hedge price risk varies with their degree of risk-aversion and planning horizon. As a consequence of long-term generation investments, electricity producers are exposed to long-term revenue uncertainty and are therefore eager to sell long-term contracts. In situations where there is a large number of producers eager to hedge their risk, while the number of consumers willing to hedge is rather small, consumers are said to have market power. This puts downward pressure on forward prices for longer time-to-maturities. Thus, producers accept forward prices below their expected spot price, resulting in negative risk premia for long-term contracts. As time-to-maturity decreases, the hedging pressure shifts towards consumers. They have a shorter hedging horizon as consumers typically hedge against possible price spikes, which are only of short-term nature. The larger number of consumers eager to hedge their risk in relation to producers shifts market power towards producers. This results in an upward pressure on forward prices for shorter time-to-maturities and consumers agree to pay a positive premium for short-term contracts. Benth et al. (2008a) base their argument on the idea that there exists a minimum (maximum) forward price producers (consumers) agree to accept (pay). The final forward price is then determined based on the market power (induced by relative hedging pressure) of market participants. If producers have full market power, they are able to charge the maximum forward price, while full market power of consumers leads to the smallest forward price possible.[101]

In order to test for a negative relation between time-to-maturity and risk premia in CfD, implied area and system forward prices, we carry out the following regression:[102]

$$\pi_t = c + \beta\ \tau_t + \varepsilon_t, \tag{3.8}$$

where c is a constant, τ_t represents the remaining time-to-maturity of the contract and ε_t is the error term. The remaining time-to-maturity is calculated as the difference in calendar days between the trading day t and the first day of the delivery period for the respective contract. The time-to-maturity of a contract observed on the Friday before the start of the delivery period on the following Monday, for example, is three. Following Shawky et al. (2003), we synchronize the end of the trading periods for CfDs, as well as the implied area and system forwards. For each calendar day until maturity, we then calculate the average risk premium over all contracts across a contract class and area. According to the forward model of Benth et al. (2008a), we expect the constant c to be positive and β to

[101] Christensen et al. (2007) analyze forward prices in West Denmark and find that the dominant producer Elsam A/S was able to influence forward prices and risk premia for certain hours between 2003 and 2006 by exerting market power.

[102] As the findings of Benth et al. (2008a) refer to forward prices and not to CfDs, we cannot constitute an expectation on the relation between time-to-maturity and risk premia of CfDs. However, as CfDs in combination with system forwards can be applied to construct area forwards, the model of Benth et al. (2008a) can be applied.

be negative for the implied area and system forwards, i.e., an increasing risk premium with decreasing time-to-maturity. Since risk premia of CfDs can, in principle, be calculated as the difference between risk premia of implied area and system forwards, the expected relation between risk premia of CfDs and their time-to-maturity is ambiguous. Table 3.6 shows the regression results for CfDs, implied area and system forwards.

CfDs generally exhibit positive risk premia for short time-to-maturities, indicated by the significant positive c. Furthermore, for most of the CfDs, there is a significant relation between risk premium and time-to-maturity. This relation, however, is not fully consistent, because the βs vary in sign and significance across areas and delivery periods. Looking at the implied area and system forward risk premia, the results conform to our expectations. The figures clearly demonstrate a significant negative relation between risk premia of quarter, season, and year implied area contracts and time-to-maturity. For system forwards, this relation is significant for all contract classes. In addition, the regression shows strong explanatory power for implied area and system forwards. Figure 3.3 demonstrates the relation between risk premia and time-to-maturity. It presents the average risk premium of quarter implied area and system forwards by time-to-maturity. Figure 3.3 reveals a non-trading effect that results from our methodology of synchronizing maturity dates of all contracts. Using this method, all contracts across a contract class and area have different non-trading days relative to their time-to-maturity, due to public holidays and weekends. The number of contracts used to calculate the average risk premium therefore diverges slightly for certain time-to-maturities, introducing some noise into the time series. For the quarter contracts in our sample, all weekends coincide in terms of time-to-maturity. For all other contract classes, the weekends do not coincide, which leads to more dispersed observations and therefore to a lower explanatory power of the regressions.

Our findings support the model by Benth et al. (2008a) that the risk premia

	Delivery		Contracts for Difference			Forwards		
	Period	N	c	β	adj. R^2	c	β	adj. R^2
System	Month	184	-	-	-	2.7491***	-.0220***	.52
	Quarter	1003	-	-	-	-4.9396***	-.0218***	.64
	Season	1002	-	-	-	-1.2821***	-.0130***	.52
	Year	1092	-	-	-	-6.2290***	-.0090***	.33
Aarhus	Month	-	-	-	-	-	-	-
(SYARH)	Quarter	276	5.0914***	.0316***	.81	9.1138***	-.0376***	.76
	Season	276	-.3570**	.0093***	.10	.7872***	-.0204***	.42
	Year	366	1.6772***	.0009*	.00	-.3209	-.0295***	.76
Copenhagen	Month	64	1.0957***	.0230**	.17	3.1111***	.0204**	.06
(SYCPH)	Quarter	276	5.0120***	.0152***	.43	9.1761***	-.0558***	.81
	Season	276	.5280***	-.0041***	.09	2.1529***	-.0370***	.44
	Year	366	1.8328***	-.0025***	.05	-1.2880***	-.0311***	.57

Table 3.6: Regression of ex-post risk premia on time-to-maturity (2001-2006).

	Delivery		Contracts for Difference			Forwards		
	Period	N	c	β	adj. R^2	c	β	adj. R^2
Oslo	Month	64	-.0392	.0017	.02	1.7950***	.0076	.01
(SYOSL)	Quarter	276	-.0152	-.0034***	.73	4.0477***	-.0731***	.88
	Season	276	.0328***	-.0006***	.13	1.1497***	-.0302***	.29
	Year	366	.0765***	-.0002***	.05	-2.6427***	-.0290***	.44
Helsinki	Month	64	.3833***	-.0006	-.01	2.2282***	.0048	-.01
(SYHEL)	Quarter	276	.6769***	.0023***	.16	4.8439***	-.0683***	.87
	Season	276	.6217***	.0016***	.04	1.7427***	-.0281***	.32
	Year	366	.5705***	.0001	.00	-1.8675***	-.0292***	.44
Stockholm	Month	64	.7947***	-.0015	.00	2.5720***	.0070	.00
(SYSTO)	Quarter	276	1.1390***	-.0001	.00	5.2628***	-.0703***	.88
	Season	276	.4865***	.0003*	.01	1.6080***	-.0293***	.30
	Year	366	.5421***	-.0003***	.08	-2.0974***	-.0292***	.43

Table 3.6: Regression of ex-post risk premia on time-to-maturity (2001-2006).

of electricity forwards are a negative function of time-to-maturity. While the βs are, with the exception of month implied area contracts, always significantly negative, the signs of the constant c are negative for year contracts and positive for all other delivery periods. As the sign of the constant c represents the overall level of relative hedging pressure, the negative c for year contracts indicates that, for this time horizon, the hedging pressure from generators is always larger than from consumers. This further supports the model.

3.3.3 Determinants of Risk Premia

Besides analyzing the development of risk premia in CfD prices with changing time-to-maturity, a thorough understanding of the determinants of risk premia is essential for risk management. Bessembinder and Lemmon (2002) develop a static equilibrium model for electricity forward prices. Their model considers electricity producers and retailers who face demand uncertainty and decide whether to trade in the wholesale spot or forward market. In equilibrium, the forward price, as discussed above, is determined as

$$F_{t,T} = E\left(P_T\right) + \alpha Var(P_T) + \gamma Skew(P_T), \tag{3.9}$$

where $Var(P_T)$ and $Skew(P_T)$ indicate the variance and unstandardized skewness of the spot price at maturity P_T. Longstaff and Wang (2004) test this implication for PJM hourly day-ahead prices (i.e., one day forwards) between 2000 and 2002 and find support for the model. However, for the same contracts over an extended period from 2000 to 2007, Ullrich (2007) shows that both implications hold only for relatively low spot prices. For day-ahead contracts at the NYISO, Hadsell and Shawky (2006) detect a

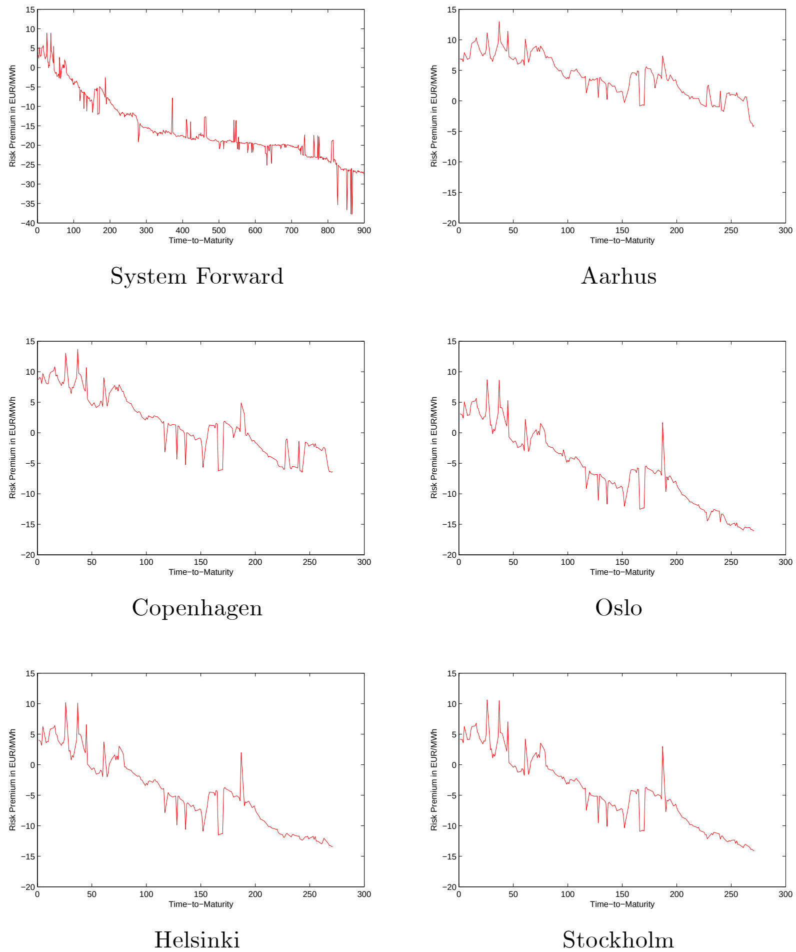

Figure 3.3: Ex-post risk premia of system and implied area quarter forwards by time-to-maturity.

strong dependence of risk premia on the volatility of real-time prices for most areas. Furió and Meneu (2008) use OTC forwards with one and two month to delivery from the Spanish electricity market and confirm both implications of the Bessembinder and Lemmon model. Lucia and Torró (2008) investigate week futures prices from Nord Pool between 1998 and 2005 and confirm the Bessembinder and Lemmon model for the subperiod 1998-2002. Between 2003 and 2005, however, they find no significant relation between risk premia and the variance and skewness of spot prices. Christensen et al. (2007) analyze Nord Pool and OTC forward prices for delivery in West Denmark between 2002 and 2006. While there exists a significant negative relation between risk premia and spot price variance, a positive relation between risk premia and spot price skewness is not supported by the data.

We test the implications of the Bessembinder and Lemmon (2002) model for CfDs, as well as implied area forwards. For the risk premia of implied area forwards, as defined in (3.6), we conduct, analogous to Longstaff and Wang (2004), the following regression[103]

$$\pi_i^F = c + \alpha Var(P^A) + \gamma Skew(P^A) + \varepsilon_i. \tag{3.10}$$

For the regression of CfD risk premia on the variance and skewness of the underlying spot price spread, the regression set up is modified. Assuming that the model holds for implied area and system forwards, we replace $F_{t,T}^A$ and $F_{t,T}^S$ in (3.3), using (3.9), and solve for the CfD risk premium. Recalling that the CfD risk premium in terms of implied area and system

[103] We also regress the risk premia of month system forwards on the variance and unstandardized skewness of spot prices. For other delivery periods, the sample size is not sufficient.

prices is defined as $\pi_t^{\mathrm{CfD}} = \left(F_{t,T}^A - F_{t,T}^S\right) - E_t\left(P_T^A - P_T^S\right)$, the following regression is carried out

$$\pi_i^{\mathrm{CfD}} = c + \alpha_1 Var(P^A) + \alpha_2 Var(P^S) + \gamma_1 Skew(P^A) + \gamma_2 Skew(P^S) + \epsilon_i. \tag{3.11}$$

In both regressions, c is a constant and ε_i is the error term. The variance and unstandardized skewness of the area (system) spot price P^A (P^S) are calculated based on daily closing prices during the delivery period of each contract.[104] The regressions are conducted for each contract class (i.e., month, quarter, season, year contracts) separately, but within a certain class, contracts over all areas are aggregated. The aim of this aggregation is to obtain a larger sample size and more robust results.

We expect the variance of the area spot prices to have a negative effect, and the skewness to have a positive effect on the risk premia of implied area forwards and CfDs to be in line with Bessembinder and Lemmon (2002). As CfDs imply short positions in system forwards, however, we expect the variance of the system spot price to have a positive impact and the skewness to have a negative one on CfD risk premia. Table 3.7 shows the regression results for each contract class.[105]

For season and year implied area forwards (Panel A) and CfDs (Panel B), all coefficients are significant and their signs are in line with the model.

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[104] The variance is calculated as $\frac{1}{n}\sum_{i=1}^{n}\left(P_i - \overline{P}\right)^2$, where n is the number of days during the delivery period and P_i is the daily spot price. $\overline{P}$ is the mean spot price during the delivery period. Analogous, the unstandardized skewness is calculated as $\frac{1}{n}\sum_{i=1}^{n}\left(P_i - \overline{P}\right)^3$.

[105] For quarter CfDs and implied area forwards the sample size is not sufficient. For the same reason, only month system forwards are considered in our analysis.

A. Implied Area Forwards

Contract	N	c	α	γ	Prob(F-stat.)	adj. R^2
Month	132	3.0488***	-.0106	-.0002	.5770	.00
Season	74	2.5758***	-.0892***	.0011***	.0000	.73
Year	25	-.1239	-.0712***	.0008**	.0019	.38

B. Contracts for Difference

Contract	N	c	α_1	α_2	γ_1	γ_2	Prob(F-stat.)	adj. R^2
Month	132	.3081	.0358**	-.0108	-.0018***	-.0008***	.0000	.38
Season	74	.7594***	-.0563***	.0555***	.0004***	-.0004**	.0000	.77
Year	25	1.0871**	-.1324***	.1375***	.0027***	-.0029***	.0000	.69

Table 3.7: Regression of ex-post risk premia on variance and skewness of spot prices (2001-2006).

In addition, the adjusted R^2 shows strong explanatory power. The results for month implied area forwards and CfDs are not significant and do not support the model respectively. A reason might be the large dispersion of the underlying spot price variance and skewness among areas and periods, due to the impact of short-term price shocks on month contracts. This dispersion is not distinct for contracts with longer delivery periods as they are less affected by short-term shocks. For month system forwards (not shown here), the results are in line with the model, with the α (γ) being significantly negative (positive). Overall, our findings for CfDs, as well as implied area and system forwards, support the Bessembinder and Lemmon (2002) model. Further, the importance of hydropower for electricity prices in the Nordic market allows us to relate the availability of hydropower to the variance and skewness of area prices and their spreads. Due to frequent transmission congestion, hydropower affects area prices and the system price to a different extend and consequently influences the distribution of area price spreads. The results in Table 3.7 thus indicate a relation between the availability of hydropower and the risk premia of CfDs and implied area forwards.

3.4 Summary and Outlook

Contracts for Difference (CfDs) are financial products for managing locational price spreads and are the only cross-border derivatives traded in the Nordic market. In this chapter, we analyze the pricing of CfDs at the Nordic electricity exchange Nord Pool over the period 2000 to 2006. We find a significant impact of hydropower generation on area price spreads as the underlying of CfDs. Further, we test CfD prices for risk premia, calculated as the difference between a CfD price and the ex-post realized delivery price. The results document significant average risk premia with substantial variability in terms of sign and magnitude. Since market areas are affected by transmission congestion to a different extent, the risk premia in CfD prices also differ substantially across areas. In addition, we examine the development of risk premia with decreasing time-to-maturity. While the results for CfDs are inconclusive, we find a significant negative relation for implied area and system forwards, the two constituents of CfDs. This provides empirical support for the theoretical results of Benth et al. (2008a). In order to analyze determinants of risk premia, we investigate the implications of the electricity forward pricing model by Bessembinder and Lemmon (2002), which predicts a negative (positive) relationship between risk premia and the variance (skewness) of spot prices. Our findings support these model implications for CfDs as well as implied area and system forwards. They also indicate the influence of hydropower availability on prices and risk premia in the Nordic market. Overall, we show that existing models for the valuation of electricity forwards provide market participants with insights into the pricing and hedging of CfDs.

Future research could usefully extend our analyses by further exploring the dynamics of risk premia in CfD prices and especially their determinants. Results in Tables 3.6 and 3.7 indicate that in general, the empirical findings for month CfDs and month area forwards are not in line with the discussed models. This is in contrast to our results for quarter, season and year contracts and indicates that risk premia of month CfDs are influenced by other factors. The analysis of potential short-term factors of risk premia on CfDs is an interesting field for further research. In addition, investigations of the pricing of CfDs with reduced-form models, for example, based on the regime-switching approach for electricity spot prices of Haldrup and Nielsen (2006b), or equilibrium models, such as an extension of the dynamic forward pricing model of Bühler and Müller-Merbach (2007b), seem promising. Ideally, future work would be able to include the CfD prices from Nord Pool's OTC clearing.

Chapter 4

Jump Risk Premia in Short-Term Electricity Spread Options

In this chapter, we analyze Physical Transmission Rights (PTRs). After shortly introducing those contracts, we describe the German-Dutch electricity market, as the PTRs discussed in this chapter refer to the German-Dutch cross-border market. Following the discussion of the relevant market and product, we introduce our model in order to value PTRs and estimate risk premia. We then estimate the empirical as well as the risk-neutral parameters of our model and discuss the results. Before concluding this chapter, we perform an in depth analysis of the residuals of our estimation. This offers further insights in the valuation of PTRs and shows promising areas for further research.

4.1 Introduction

The second derivative instrument, besides CfDs, currently applied in European cross-border electricity markets are Physical Transmission Rights (PTRs). These are option contracts that allow access to cross-border transmission lines for a specific period of time. Therefore, PTRs offer the opportunity of buying electricity in a specific region A and selling it in another region B. From a financial standpoint, the payoff of this transaction is equal to the difference between both electricity prices. For this reason, PTRs can be seen as exchange options where the electricity price in region P^B is exchanged for the electricity price in region P^A. Thus, the payoff of a PTR can be stated as

$$PTR = \left(P^B - P^A\right)^+ . \tag{4.1}$$

Although the payoff of a PTR can be considered as the payoff of an exchange option, modelling the spread directly as discussed above might also be applicable. Therefore, we can apply standard models for the valuation of electricity derivatives discussed in Chapter 1.

Albeit modelling the spread between electricity prices directly allows us to refer to standard models for pricing electricity derivatives, the unique characteristics of spread processes need to be incorporated. While Seifert and Uhrig-Homburg (2007) show that spikes in the German electricity market generally last two to three days until prices revert back to their long-term mean-reversion level, the duration of spikes in the spread between electricity prices is usually remarkably shorter. Jumps in the day-ahead spread between Dutch and German electricity prices, for example, do not last longer than one day. In order to match these specific spread characteristics, we develop a model based on Simonsen et al. (2004) that produces these pronounced spikes. We separate the diffusion part from observed prices and model the spikes as a normally distributed component that is occasionally added to the underlying diffusion process.[106] The jump times are Bernoulli distributed in order to model that jumps occur instantaneously and then disappear after one day. The separation of the underlying spread into a diffusion and a spike component further allows us to include only de-spiked prices into the valuation of derivatives. Since spikes only last one day and their occurrence has absolutely no impact on the overall price level, using observed spreads for the valuation of derivatives could lead to large distortions.

In this chapter, we use the aforementioned model for the valuation of hourly day-ahead PTR options on the German-Dutch interconnector. We include all hourly PTRs between January 1, 2001 and December 31, 2008, i.e. 2,922 observations for each hour of the day. Our dataset further includes all corresponding hourly day-ahead electricity prices in Germany and the Netherlands. As PTRs and electricity is auctioned explicitly for each hour of the day and each hour has its unique characteristics, we analyze all 24 hours separately. In order to estimate the empirical (or physical) and risk-neutral parameters of our model, we use the method of Markov Chain Monte Carlo (MCMC). MCMC is widely used for estimating equity models and is applied by Eraker et al. (2003) as well as Eraker (2004) and more recently by Rodrigues and Schlag (2009). This efficient and robust estimation procedure does not only allow for the simultaneous estimation of all parameters, but also the estimation of explicit vectors for jump times and jump sizes. Using the latter, we can easily separate the de-spiked price process from observed market prices.

This chapter contributes to the current research in various ways. First, to the best of our knowledge, we are the first to comprehensively analyze day-

[106] Throughout this chapter we refer to the diffusion component as the de-spiked process.

ahead PTR options that are currently the most widely used instruments for managing cross-border electricity flows in the German electricity market.[107] Second, we develop a model capable of incorporating the unique features of hourly electricity spreads. Third, we estimate our model and analyze the market price of jump risk inherent in day-ahead PTR options. Due to their extremely short time to maturity, jump risk can be considered as the main driver of these contracts. Empirical evidence indicates that our model describes an adequate approach for the valuation of hourly PTR options especially during calm hours. Further, our results show that investors are willing to pay a premium for hourly PTR options for turbulent hours of the day, i.e. hours 8 to 22. This price premium can be explained by increased hedging demand of investors and emphasizes the importance of these contracts and the need for adequate risk managing tools in cross-border electricity markets. Finally, this paper contributes to the existing literature as it is the first analysis of electricity prices that includes option price data. Previous research of electricity prices, as for example discussed in chapter 2.4, relies solely on spot price data. Using PTR option prices with hourly and monthly delivery period, we find significant seasonality in the residuals of hourly and monthly PTRs. A seasonal trend is evident for both contracts, i.e. hourly and monthly. While a systematic connection between the seasonality in residuals and the number of observed spikes, however, is ambiguous for hourly contracts, this relation is highly significant for monthly PTRs.

In the course of this chapter, we first discuss the national and cross-border electricity markets of Germany and the Netherlands. We then introduce our model for the valuation of PTR options and explain the MCMC estimation of the empirical and risk-neutral parameters. Finally, we discuss the results

[107] With the exception of the German and western Denmark interconnector, all German cross-border connections currently use explicit auctions of PTRs to allocate day-ahead cross-border capacity.

of our analysis before we shortly sum up our findings.

4.2 German-Dutch Electricity Market

This section illustrates the German as well as the Dutch electricity markets. It further describes the fundamentals of the cross-border market for both countries. These fundamentals are necessary in order to fully understand the functioning of the later discussed PTR options.

4.2.1 National Electricity Markets

The German and Dutch exchange based electricity markets are very alike. In Germany, electricity is traded on the European Energy Exchange (EEX). The spot market mainly consists of a day-ahead market where every working day, electricity is auctioned for each of the 24 hours of the next day (or days in case of holidays or weekends). The overall trading volume in 2007 in the spot market at EEX was 124 TWh, compared to a total electricity consumption in Germany of 556 TWh. In the Netherlands, electricity is traded on the Amsterdam Power Exchange (APX). Here, the spot market is also a day-ahead market where every day electricity is auctioned analogous to the EEX.[108] The spot market trading volume at APX in 2007 was about 21 TWh compared to a total consumption in the Netherlands of 117 TWh.[109]

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[108] At EEX and APX, there also exists an intraday market since September 2006, where electricity is traded continuously for certain time blocks.

[109] Exchange based information is available at http://www.eex.de and also at http://www.apxgroup.com respectively. Consumption figures are obtained from http://www.ucte.org.

Country	Coal	Natural Gas	Oil	Nuclear	Hydro Power	Other Renew.	Other	Total
Germany	299.8	84.1	11.3	140.5	28.5	72.9	0.1	637.1
	(47.1)	(13.2)	(1.8)	(22.1)	(4.5)	(11.4)	(0.0)	(100.0)
Netherlands	24.9	62.6	2.2	4.2	0.1	9.0	0.2	103.2
	(24.1)	(60.6)	(2.1)	(4.1)	(0.1)	(8.8)	(0.2)	(100.0)

Table 4.1: Gross electricity generation in Germany and the Netherlands in 2007 in TWh. Relative values are given in brackets in percent of total power generation. Source: Eurostat.

Although the market set up in both countries is comparable, their physical electricity markets demonstrate fundamental differences in terms of their power generation mix. These discrepancies are extremely relevant as they economically determine the overall electricity price level and therefore the sign and level of the price spread between these countries. The Netherlands, as one of the largest natural gas producers in the world, generates a major share of electricity from natural gas power plants.[110] Other fuels play only a minor role in the generation of electricity. In Germany, coal fired and nuclear power plants are the most important source of electricity generation, while natural gas and renewables also contribute significantly to the power generation mix. Table 4.1 provides details on the gross power generation in both countries in 2007.

The power generation mix shown in Table 4.1 has a significant effect on the spread between German and Dutch electricity prices. Power plants are

[110] In 2007, the Netherlands produced about 76.3 billion cubic meters of natural gas which corresponds to almost 39% of the European Union's natural gas production. See http://www.cia.gov for details.

generally stacked according to their marginal cost of power generation, where the power plant with the lowest marginal cost is used first.[111] In Germany, baseload demand is generally covered by renewables (without pumped storages), lignite and nuclear power. Natural gas and hard coal power plants are added as demand increases. Peakload is usually covered by gas and oil fired plants or pumped storage facilities. The Netherlands cannot rely on relatively cheap nuclear power or lignite and need to refer to more expensive hard coal and natural gas power plants for base and medium load. Therefore, the usage of efficient powerplants to cover peak demand in the Netherlands is limited and Dutch day-ahead electricity prices are in general higher and more erratic than prices in Germany.

4.2.2 German-Dutch Cross-Border Market

The German and Dutch power grids are currently connected via three high voltage (380kV) cross-border cables. In Germany, the two southern cables are operated by the transmission system operator (TSO) RWE TSO and the northern one by E.ON Netz. In the Netherlands, TenneT is the sole TSO operating all interconnectors. The capacity of the cross-border connections is auctioned explicitly by Auction BV, a 100% subsidiary of TenneT. The capacity is auctioned for each direction separately via PTR options.[112]

There are three different types of PTR options auctioned for the German-

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[111] This stacking is called merit order and is also applicable within a power plant concerning different generators. In addition to marginal costs, other factors, e.g. lead time and minimum usage time, need to be considered.

[112] The connections from RWE TSO and E.ON Netz are auctioned separately. Since the capacity of the RWE TSO cables is by far larger, we only refer to these interconnectors and the respective PTRs throughout this paper.

Dutch interconnector. They differ in the length of the delivery period and are hourly, monthly and yearly PTRs. All PTRs have a volume of 1MW. Hourly PTRs are auctioned day-ahead for every single hour of the following day. Monthly PTRs are auctioned on the 10th working day of the month preceding the delivery month. Yearly PTRs are auctioned on the first working day after September 27th in the year preceding the year of delivery. In case of remaining capacity from the first auction of yearly PTRs, a second auction is held on the first working day after the 27th of November. In general, about one third of the entire available capacity is reserved for each of the three contract types. Investors bid price/volume combinations where several bids per investor are allowed. In case the requested volume is lower than the available capacity, the PTR price is zero. Otherwise, all investors pay the price of the lowest successful bid. If there is more volume requested at the lowest successful bid, the allocation for those bids is partitioned relative to the requested volume.

Although monthly and yearly PTRs are auctioned separately, they actually consist of a portfolio of hourly PTRs for each hour of the respective delivery period. Therefore, the owner of such a portfolio has the right to exercise each hourly PTR separately.[113] In case of monthly or yearly contracts, investors need to nominate the PTR for each hour three days prior to execution, i.e. they must state whether they use their PTR or not. Should investors nominate the usage of their PTR, they are obliged to induce the corresponding amount of electricity into the grid. In case PTRs are returned, the capacity is available for the hourly auction and the owner is refunded with the proceeds of the auction. If an investor does not nominate the PTR at all, the capacity is returned to the hourly auction and the owner is not refunded. This approach is called the use it or lose it principle. Concerning the fulfillment of the PTRs, these contracts are offered firm,

[113] Therefore, an investor buying a monthly PTR for January actually receives 744 ($31 \cdot 24$) hourly contracts.

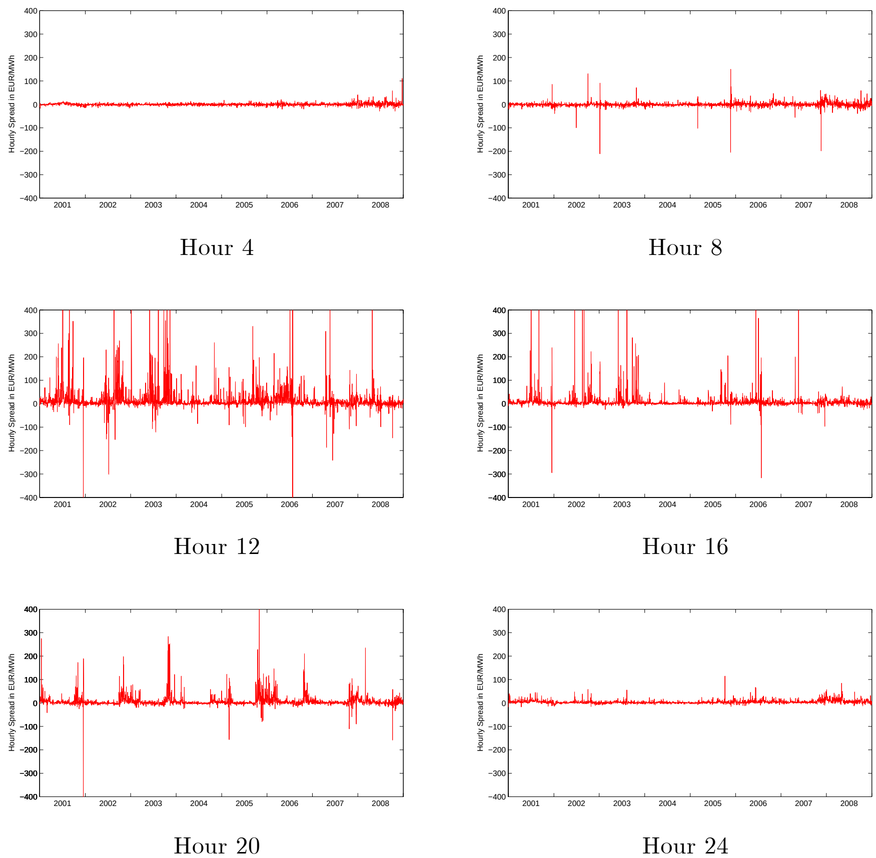


Figure 4.1: Hourly price spreads between German and Dutch day-ahead prices.

i.e. the TSO has no right to curtail the granted capacity of a PTR holder. However, there are two exceptions to the rule. In case of power system safety requirements or force majeure, the TSO might revoke the right of inducing electricity into the grid. In the first case, the TSO is required to compensate for the losses and has to pay the holder of the PTR 110% of the initially paid price. In case of force majeure, the TSO only refunds 100% of the paid PTR price. Out of the 2,922 days between 2001 and 2008, only for six days there was no PTR price available.

As all three types of PTR options basically refer to hourly contracts, the underlying of the PTR is the hourly spread between German and Dutch day-ahead electricity prices. Figure 4.1 shows this spread between 2001 and 2008.

Hourly spreads between Germany and the Netherlands vary significantly across different hours of the day. While spreads usually fluctuate mildly with occasional jumps during off peak hours, spreads are highly erratic during peak hours. Since the merit order leads to a concave marginal cost function, price spikes are more likely when general demand is already high as for the peak hours during the day. Furthermore, price spreads and spikes are mostly positive, i.e. Dutch prices are usually higher than German ones. This is in line with the power generation mix of the Netherlands compared to Germany discussed above. Table 4.2 provides detailed descriptive statistics for the German and Dutch hourly day-ahead spread, where a positive spread refers to higher prices in the Netherlands and vice versa.

Figures in Table 4.2 confirm the erratic behavior of the electricity price spread and shows a large dispersion of the descriptive statistics across different hours. The minimum and maximum values indicate the relevance and idiosyncratic occurrence of jumps in national electricity prices. Also, the skewness and kurtosis shows significant values for all hours, although during peak hours these figures are even more severe. The varying charac-

Hour	Mean	Median	Minimum	Maximum	Std.dev.	Skewness	Kurtosis
1	1.93	1.25	-28.31	137.02	6.55	4.13	66.69
2	1.05	0.50	-21.47	47.69	5.81	1.68	9.56
3	0.65	0.05	-27.00	110.70	6.08	3.15	43.49
4	0.18	-0.14	-28.91	110.61	5.83	3.22	50.74
5	-0.52	-0.56	-31.18	110.46	5.79	2.78	50.94
6	-0.92	-1.02	-43.11	49.31	5.52	0.53	9.07
7	-0.16	-0.73	-35.24	61.62	6.91	1.32	8.61
8	-0.20	-0.64	-210.92	150.08	11.9	-4.01	118.14
9	3.55	1.07	-256.84	467.42	23.32	6.36	131.26
10	13.60	3.24	-301.67	1,945.94	62.78	16.69	434.86
11	17.11	4.24	-546.78	1,544.92	67.31	10.77	198.79
12	19.56	4.98	-1,640.15	1,943.90	87.09	6.03	178.58
13	12.08	6.69	-273.58	1,752.34	58.12	16.58	409.82
14	16.09	4.04	-315.05	1,751.58	65.54	13.25	279.94
15	13.04	3.57	-430.09	1,524.92	55.78	11.98	240.40
16	11.12	2.95	-317.34	1,754.91	67.56	16.92	366.55
17	10.30	2.78	-564.99	1,736.99	59.37	17.64	441.38
18	24.82	3.28	-412.31	1,952.97	102.13	8.77	111.75
19	10.59	2.03	-2,186.62	742.92	69.19	-10.98	407.37
20	7.18	2.00	-407.21	450.04	26.50	3.75	82.93
21	5.25	1.90	-178.53	376.49	18.60	8.38	137.50
22	3.37	1.42	-26.08	468.56	12.65	18.55	633.82
23	2.13	0.92	-29.24	74.43	7.93	2.60	15.08
24	4.55	2.82	-17.73	114.98	7.98	3.22	23.58

Table 4.2: Descriptive statistics of German-Dutch day-ahead spreads between 2001 and 2008.

teristics of electricity price spreads across the day motivates the distinct modelling of each hour. Further, the mostly positive results for the mean and skewness in addition to the larger maximum compared to the minimum values (in absolute terms) confirm the generally higher and more erratic electricity prices in the Netherlands.

As stated in (4.1), the PTR is an option on the spread between German and Dutch hourly day-ahead price. Since hourly PTRs are auctioned with only one day to maturity, we expect their prices to closely reflect the price spreads in Table 4.2. The descriptive statistics of hourly PTRs as well as the average volume auctioned (in MW) are shown in Table 4.3.

Table 4.3 shows that in general PTR prices reflect the underlying price spread. For peak hours, mean PTR prices as well as their standard deviations are higher than for off-peak hours. Further, skewness and kurtosis are extremely high for all hours but the greatest values are observed for off-peak hours. Traded volumes indicate relatively large amounts of available capacity, considering that these values are given per MWh and that hourly PTRs only constitute about one third of overall capacity auctioned. Further, the figures show lower capacities for peak hours compared to off-peak hours. One reason might be the lower levels of returned capacity from year and month auctions, i.e. more PTR holders exercise their long-term options rather than selling it in the hourly auction. Another reason could be the delivery of less electricity from the Netherlands to Germany, compared to off-peak hours, which also leads to lower capacities for delivery from Germany to the Netherlands.[114]

Despite overall similarities, there are significant differences between PTR prices and electricity price spreads, especially when considering the ex-

[114] Since opposed currents cancel out, scheduled electricity flows from the Netherlands to Germany increase the available capacity for the opposite direction.

Hour	Mean	Median	Minimum	Maximum	Std.dev.	Skewness	Kurtosis	Avg. MW
1	0.99	0.22	0.00	25.00	2.07	4.36	26.58	777.78
2	0.57	0.10	0.00	25.01	1.51	6.21	55.57	782.61
3	0.50	0.06	0.00	25.01	1.48	7.09	71.20	783.24
4	0.44	0.06	0.00	20.01	1.29	7.24	73.09	789.76
5	0.39	0.05	0.00	20.01	1.20	7.73	84.87	791.68
6	0.40	0.05	0.00	20.01	1.28	7.88	83.62	792.45
7	0.86	0.09	0.00	350.00	8.88	34.52	1,267.75	762.00
8	1.33	0.25	0.00	350.00	9.09	32.15	1,147.80	693.83
9	3.90	1.00	0.00	350.00	13.27	13.76	278.31	661.55
10	9.56	2.25	0.00	399.00	24.74	6.98	68.64	642.18
11	11.91	3.11	0.00	399.00	27.47	5.95	50.41	636.36
12	14.16	3.74	0.00	579.12	32.97	6.89	74.61	635.48

Table 4.3: Descriptive statistics of German-Dutch PTR prices between 2001 and 2008.

tremely short time to maturity of each PTR. The reason is that price spikes are very hard to predict even with only one day to maturity. Positive (negative) price spikes occur in case the Dutch day-ahead electricity price jumps up (down) compared to the German one or vice versa. Jumps in national electricity prices are idiosyncratic and occur in case of a sudden drop in supply or an unpredicted increase in demand, where the first is usually more common. Figure 4.2 shows the relation between traded PTR prices and the resulting payoffs, i.e. the maximum of the underlying spread and zero, between 2001 and 2008. A negative value indicates PTR prices above the corresponding payoff. It is evident that there is a large dispersion between paid PTR prices and resulting payoffs. PTR prices considerably above their payoff as well as payoffs significantly in excess of PTR prices are both frequently observed across all hours. This confirms the unpredictability of price spikes.

In addition to unforeseeable price spikes, the PTR auction process further induces a great amount of uncertainty to investors. As PTRs are physical contracts that securitize the right to deliver electricity via an interconnector, the exercise of these options requires a specific period of lead time.[115] In order to profit from a purchased PTR option, assuming that no prior delivery agreement exists, the investor needs to buy electricity in the German day-ahead market and sell the same amount in the Dutch day-ahead auction. However, while the PTRs are auctioned at 9am the day before maturity, the auctions for German and Dutch day-ahead electricity close at noon and 11am respectively. Auction results are submitted 15 minutes later for German day-ahead electricity and 30 minutes later otherwise. Since there is no obligation to use the purchased PTR, the holder can announce its usage until 2pm. In case the investor bought electricity in the

[115] Lead time is required since an instantaneous delivery of electricity is hardly feasable. Further, demand and supply have to be balanced at all time so that the TSO needs to schedule any changes in the supply and demand.

Dutch or German day-ahead auction and the price spread does not allow for a profitable usage of the PTR option, positions need to be closed via the intraday market in order to prevent losses. Since the price at which the positions are closed is also unpredictable, this schedule of auctioning the PTR options offers a great amount of uncertainty to investors.

4.3 Model

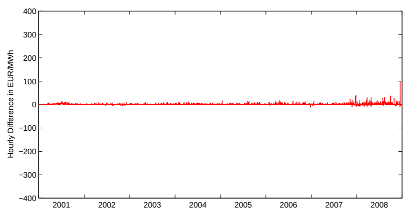
In this section, we lay out the fundamental model for the valuation of PTR options. We start with describing the underlying process of the respective electricity prices spread and then derive a closed-form solution for the PTR option price.

4.3.1 Underlying Processes

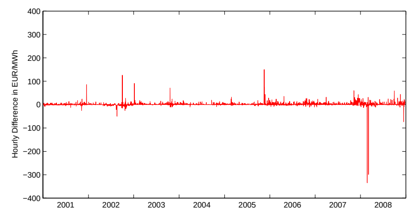
In order to model the hourly spread between German and Dutch electricity prices, we decompose the price spread at time t, P_t, into a diffusion component S_t and a jump component J_t. Further, we assume no significant seasonality in price spreads as national electricity prices in Germany and the Netherlands are expected to follow identical seasonal trends based on intra-week and annual demand patterns. The price spread P_t can be written as

$$P_t = S_t + J_t. \tag{4.2}$$

S_t follows an Ornstein-Uhlenbeck process since the price spread is subject to mild variations in the short-term but reverts back to the mean-reversion level in the long run. J_t mirrors the fact that the price spread is subject



Hour 4



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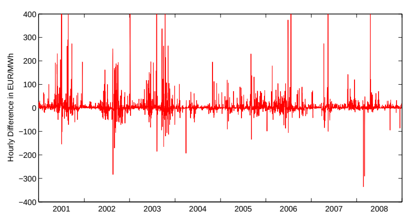

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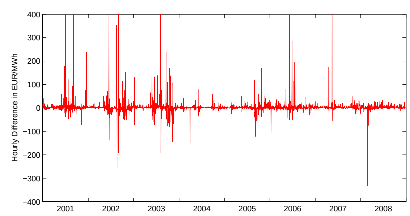

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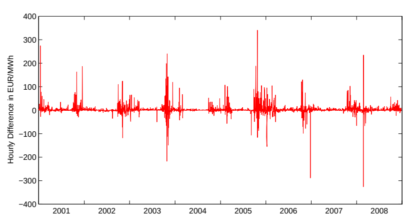

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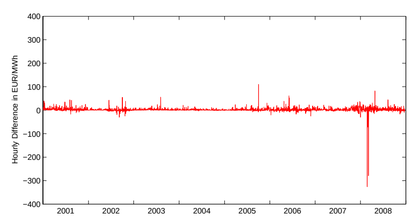


Hour 24

Figure 4.2: Difference between PTR prices and resulting payoffs.

to occasional jumps, which in general only last for one day. Formally, we have

$$
\begin{aligned}
dS_t &= \kappa \left(\nu - S_t\right) dt + \sigma_D dW_t, \\
J_t &= \left\{ \begin{array}{ll} N_t, & \text{with probability } p_J \\ 0, & \text{with probability } (1 - p_J) \end{array} \right. ,
\end{aligned}
\tag{4.3}
$$

with

$$
t \in t_1, ..., t_2,
$$

where $N_t \sim \mathcal{N}\left(\mu_J, \sigma_J^2\right)$ and t_1 (t_2) is the beginning (end) of day t. Using this set up, we are able to model price spikes in the underlying process, i.e. extreme jumps that only last for one day. Further, since we separate the jump and diffusion component, the mean-reversion speed κ is not biased by jumps in the underlying and we therefore receive more realistic values when estimating κ. In addition, we are able to use the current de-spiked value S_t when calculating the PTR option price. This is reasonable as the occurrence of a spike should have no influence on the PTR price. Following Mikosch (1999) and Cont and Tankov (2003) we get for the process of P_t

$$
P_t = S_0 e^{-\kappa t} + \nu \left(1 - e^{-\kappa t}\right) + \sigma_D e^{-\kappa t} \int_0^t e^{\kappa s} dW_s + J_t. \tag{4.4}
$$

The mean and variance of P_t are

$$\begin{aligned} E\left[P_t\right] &= S_0 e^{-\kappa t} + \nu\left(1 - e^{-\kappa t}\right) + p_J \mu_J, \\ Var\left[P_t\right] &= \left(1 - e^{-2\kappa t}\right)\frac{\sigma_D^2}{2\kappa} + \varsigma_J, \end{aligned} \tag{4.5}$$

where $\varsigma_J = \sigma_D^2 p_J + \mu_J^2 p_J\left(1 - p_J\right)$ is the variance of the jump component J_t. Note that the variance of the jump component is large compared to that of the diffusion component. In order to obtain the SDE under the risk-neutral measure $\mathcal{Q}$, we use the Girsanov theorem.[116] In our model, we only consider the effect of the measure change on the jump component. Due to the short time to maturity of only one day, we neglect the impact of the market price of diffusion risk. Therefore, we receive more pronounced results for our estimation of the jump risk premia and all diffusion parameters remain constant, i.e. $\kappa_J^Q = \kappa_J$, $\nu^Q = \nu$ and $\sigma_J^Q = \sigma_J$.[117] Under the risk-neutral measure the jump intensity as well as the mean jump size change whereas the variance of the jump size remains constant, i.e. $\sigma_J^Q = \sigma_J$. Thus, we can characterize S_t and J_t under the risk-neutral measure as

$$\begin{aligned} dS_t &= \kappa\left(\nu - S_t\right)dt + \sigma_D dW_t^Q, \\ J_t^Q &= \begin{cases} N_t^Q, & \text{with probability } p_J^Q \\ 0, & \text{with probability } \left(1 - p_J^Q\right) \end{cases}. \end{aligned} \tag{4.6}$$

Under the risk-neutral measure, the jump probability is p_J^Q and $N^Q \sim$

---

[116] See Benth et al. (2008b) for a detailed discussion of the measure change for jump processes.

[117] We use the super index $\mathcal{Q}$ to indicate the risk neutral measure. For the empirical measure $\mathcal{P}$, we omit the super index.

$\mathcal{N}\left(\mu^Q, \sigma_J^2\right)$. The corresponding process for P_t follows as

$$P_t = S_0 e^{-\kappa t} + \nu\left(1 - e^{-\kappa t}\right) + \sigma_D e^{-\kappa t} \int_0^t e^{\kappa s} dW_s^Q + J_t^Q. \tag{4.7}$$

4.3.2 Derivation of Call-Price

We derive the price of a PTR option based on the approach of Merton (1976). When conditioning the spread on the occurrence of a jump at time t, the spread is normally distributed. Given a conditional normal distribution, the option price can be easily derived as the discounted expected value under the risk-neutral measure. Based on the risk-neutral process for P_t from (4.7), we can write the price of a PTR option as

$$PTR = e^{-rt} \sum_{n=0}^{1} \Pr\left(n\ jumps\right) E^Q\left[\left.(P_t)^+\right| n\ jumps\right]. \tag{4.8}$$

Due to the set up of our model, the number of jumps until maturity is of no relevance for the valuation of the PTR option. This is based on the idea that spikes only last one day and vanish without any influence on the general price level. The only relevant jump time for the valuation of a PTR option is at maturity. Therefore, we only need to distinguish two scenarios. Either there is a jump in the underlying spread at maturity or there is no jump. Considering the probability of a jump under the risk-neutral measure, i.e. p_J^Q, we can write the value of the PTR option at time zero and maturity at time t as

$$PTR = \sum_{n=0}^{1} \left(p_J^Q\right)^n \left(1 - p_J^Q\right)^{1-n} \left\{ \left[S_0 e^{-\kappa t} + \nu \left(1 - e^{-\kappa t}\right) + n\mu_J^Q\right] \Phi\left(d\right) \right.$$
$$\left. + \sqrt{\frac{\sigma_D^2}{2\kappa}\left(1 - e^{-2\kappa t}\right) + n\sigma_J^2} \frac{e^{-\frac{1}{2}d^2}}{\sqrt{2\pi}} \right\} e^{-rt}, \tag{4.9}$$

where $\Phi(\cdot)$ is the cumulative normal distribution function and d is defined as

$$d = \frac{S_0 e^{-\kappa t} + \nu \left(1 - e^{-\kappa t}\right) + n\mu^Q}{\sqrt{\frac{\sigma_D^2}{2\kappa}\left(1 - e^{-2\kappa t}\right) + n\sigma_J^2}}.$$

(4.9) shows that if a jump occurs, the mean and variance of the spread are adjusted by the mean and variance of the jump component. In case of no jump, only the diffusion mean and variance enter the expected spot price. Since only two scenarios need to be considered, i.e. jump and no jump, the formula is kept easily tractable.

4.4 MCMC Estimation

This section discusses the estimation of our model parameters. We start by describing the discrete model required for the estimation. Following the discretization, we elaborate on the estimation methodology for the empirical as well as risk-neutral parameters.

4.4.1 Discrete Process

In order to estimate the empirical and risk-neutral parameters, we first need to discretize the process P_t assuming that $\Delta t = 1$. Special interest in the course of the discretization receives the jump component J_t. We decompose the jump component into the product of a jump time variable q_t and a jump size variable N_t. While q_t is Bernoulli distributed to indicate whether a jump occurs at time t, N_t is normally distributed to determine the jump size. This setup allows us to model positive and negative jumps in the underlying spread.

Given the process for P_t under the empirical measure $\mathcal{P}$ from (4.4), we can write the discrete process of P_t as

$$P_{t+1} \left| S_t \right. = S_t e^{-\kappa} + \nu \left(1 - e^{-\kappa t}\right) + \sqrt{1 - e^{-2\kappa}} \varepsilon_{t+1} + N_{t+1} q_{t+1}, \tag{4.10}$$

where $\varepsilon_{t+1} \sim \mathcal{N}\left(0, \frac{\sigma_D^2}{2\kappa}\right)$. For simplicity we assume $P_0 = S_0 = 0$. Since a spike today has virtually no impact on tomorrow's price, P_t is conditioned on today's de-spiked price S_t. Given the discrete process of P_t, we can formulate the full information likelihood of $P = \{P_t\}_{t=1}^T$ as

$$p\left(P \left| \Theta, q, N \right.\right) = \prod_{t=0}^{T-1} p\left(P_{t+1} \left| S_t, q_{t+1}, N_{t+1}, \Theta \right.\right), \tag{4.11}$$

where

$$p\left(P_{t+1}\left|S_t, q_{t+1}, N_{t+1}, \Theta\right.\right)$$
$$\propto \exp\left(-\frac{1}{2}\frac{\left(P_{t+1} - S_t e^{-\kappa} - \nu\left(1 - e^{-\kappa t}\right) - N_{t+1}q_{t+1}\right)^2}{\left(1 - e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}\right). \tag{4.12}$$

The full information likelihood is an essential part in the parameter estimation. The likelihood for the vector of observed price spreads P is simply the product of the likelihood functions of all of its elements P_t, for all $t = 1, ..., T$. Normality of P_t is guaranteed since it is conditioned on the occurrence of a jump. The risk-neutral discrete process of P_t can be derived identically as

$$P_{t+1}\left|S_t\right. = S_t e^{-\kappa t} + \nu\left(1 - e^{-\kappa t}\right) + \sqrt{1 - e^{-2\kappa}}\varepsilon_{t+1}^Q + N_{t+1}^Q q_{t+1}^Q. \tag{4.13}$$

The full information likelihood for the risk-neutral process of P_t follows analogous to (4.11).

4.4.2 Empirical Parameters

In order to estimate the parameters of the discretized process, we use the Markov Chain Monte Carlo (MCMC) method. Via MCMC, we generate random samples from the joint posterior distribution, $p\left(\Theta, X\left|P\right.\right)$, of parameters Θ and latent state variables X. The set of parameters includes all relevant parameters to be estimated, i.e. $\Theta = \left\{\kappa, \sigma_D^2, \nu, \mu_J, \sigma_J^2, p_J\right\}$, whereas the latent state variables include jump times and jump sizes, i.e. $X = \{q, N\}$. Since the joint posterior distribution is in general not known, we apply the Clifford-Hemmersley theorem that allows us to draw from the complete conditional distributions, i.e. $p\left(\Theta\,|P,X\right)$ and $p\left(X\,|P,\Theta\right)$, instead. If drawing from the complete conditional distribution is still not feasible, the Clifford-Hemmersley theorem can be reapplied until each parameter, conditional on observed prices, latent state variables and all other parameters, is drawn separately. The same holds for the conditional distribution of the state variables. Drawing all parameters iteratively, we receive a Markov Chain that eventually converges to the target posterior distribution. We finally receive the estimated parameter values as the arithmetic mean of all non discarded Monte Carlo draws. We discard the first drawings of our estimation to let the Markov chain come close to its stationary distribution and therefore, to receive more robust results.[118]

MCMC is based essentially on the theory of Bayes. Using Bayes rule, we are able to state the posterior distribution as the factor of the likelihood (see (4.11)), the distribution of the state variables, $p\left(X\,|\Theta\right)$, and the prior distribution of the parameter, $p\left(\Theta\right)$. As we only require a proportionality relation, we can express the posterior distribution as

$$p\left(\Theta,X\,|P\right)\propto p\left(P\,|\Theta,X\right)p\left(X\,|\Theta\right)p\left(\Theta\right), \tag{4.14}$$

In case the above stated product of distributions can directly be drawn from, we can use the so called Gibbs algorithm in order to draw a new sample. By choosing appropriate prior distributions for the parameters, we mainly refer to Gibbs sampling in this chapter. If the posterior distribution cannot directly be sampled from, we use a Metropolis-Hastings step. Here, a new sample is drawn from a proposed distribution. This sample

[118] See Gamerman and Lopes (2006) for a textbook treatment of MCMC methods and Bayesian theory. Johannes and Polson (2003) thouroughly discuss MCMC methods and give various examples for financial models.

candidate is then accepted as a drawing from the posterior distribution according to a given acceptance criterion. In case of the Metropolis-Hastings algorithm, the conditional posterior only needs to be evaluated numerically. We use the Metropolis-Hastings method when drawing samples for the mean-reversion speed κ.

For our estimation, we start with the jump times q_t. In each iteration step, we generate a vector $q \in \{q_1, ..., q_T\}$ with length equal to the number of observed market prices. Each element is Bernoulli distributed indicating if a jump occurred at time t or not, i.e. $q_i \in \{0, 1\}$, for all $i = 1, ..., T$. The conditional probability of a jump in the next time step, i.e. $q_{t+1} = 1$, has the following distribution[119]

$$q_{t+1} | \, \Theta, P_{t+1}, S_t, N_{t+1} \sim \mathcal{B}ernoulli\left(\varphi_{t+1}\right). \tag{4.15}$$

φ_{t+1} is the Bernoulli probability of a jump in the next time step. With the vector of jump times, we draw a new jump probability p_J via Gibbs. We assume that p_J has a beta prior distribution, i.e. $p_J \sim \mathcal{B}eta\left(\alpha_{p_J}, \beta_{p_J}\right)$. A beta prior ensures that the jump probability is bound between zero and one. Further, the beta distribution is conjugate to the Bernoulli likelihood.[120] The posterior distribution of p_J then follows as

$$p\left(p_J \left| q\right.\right) \propto p\left(q \left| p_J\right.\right) p\left(p_J\right). \tag{4.16}$$

[119] We refer to Appendix B for details on the posterior distributions.

[120] A prior distribution is called conjugate to a likelihood function if the resulting posterior distribution is from the same family as the prior, i.e. the product of Bernoulli likelihood and a beta prior is also beta distributed with altered parameters. With the exception of the mean reversion speed, all prior distributions in this paper are conjugate.

In case we omit indices of observed prices, jump times or jump sizes, we refer to the entire vector. Indices, in contrast, refer to a specific element of the respective vector. The latter is used when drawing the vector of jump times and jump sizes since each element is drawn individually. The first is applied for the estimation of the parameters.

When drawing the vector of jump sizes we proceed analogous to jump times such that we need to generate a vector of jump sizes $N \in \{N_1, ..., N_T\}$, where each element is normally distributed with mean μ_J, and variance σ_J^2. The posterior distribution of each element can therefore be stated as

$$\begin{aligned} & p\left(N_{t+1} \middle| \Theta, q_{t+1}, P_{t+1}, S_t\right) \\ \propto & p\left(P_{t+1} \middle| S_t, N_{t+1}, q_{t+1}, \Theta\right) p\left(N_{t+1} \middle| \Theta\right). \end{aligned} \tag{4.17}$$

After having drawn jump times, jump sizes and the jump probability, we continue by successively drawing all remaining parameters. The mean jump size μ_J has the following posterior distribution, assuming a normal prior, i.e. $\mu_J \sim \mathcal{N}\left(m_J, s_J^2\right)$

$$p\left(\mu_J \middle| N, q, \Theta_{-\mu_J}\right) \propto p\left(N \middle| \Theta\right) p\left(\mu_J\right). \tag{4.18}$$

$\Theta_{-\mu_J}$ refers to the vector of parameters without μ_J. For the variance of the jump size σ_J^2 we assume an inverse gamma prior distribution, i.e. $\sigma_J^2 \sim \mathcal{IG}\left(\alpha_{\sigma_J}, \beta_{\sigma_J}\right)$, to insure positivity of the variance. Thus, the posterior follows as

$$p\left(\sigma_J^2 \middle| N, q, \Theta_{-\sigma_J^2}\right) \propto p\left(N \middle| \Theta\right) p\left(\sigma_J^2\right). \tag{4.19}$$

Afterwards, we draw a sample for the mean-reversion speed κ as well as the variance of the diffusion process σ_D^2. For κ we assume a normal prior, i.e. $\kappa \sim \mathcal{N}\left(m_\kappa, s_\kappa^2\right)$ and therefore the posterior of κ is

$$p\left(\kappa \mid N, q, \Theta_{-\kappa}, P, S\right) \propto p\left(P \mid S, N, q, \Theta\right) p\left(\kappa\right), \tag{4.20}$$

where $S = \{S_t\}_{t=1}^{T}$. Since a direct draw from the above mentioned product of the likelihood and the prior of κ is not feasible, we use the Metropolis-Hastings approach in order to sample κ. Here, we only need to evaluate $\pi\left(\kappa_{g+1}\right)/\pi\left(\kappa_g\right)$, where κ_g is the g^{th} sample drawing of κ and $\pi\left(\kappa_g\right)$ is the posterior distribution of κ_g. Using the random walk Metropolis-Hastings algorithm, we draw a proposed κ_{g+1} as $\kappa_{g+1} = \kappa_g + \varepsilon_\kappa$, where $\varepsilon_\kappa \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$. Afterwards, we have to decide whether to accept κ_{g+1} as a potential sample drawing for the above stated posterior distribution (from which we could not draw directly). In case we do not accept κ_{g+1} we set $\kappa_{g+1} = \kappa_g$, where the probability α of the acceptance of κ_{g+1} is calculated as[121]

$$\alpha\left(\kappa_g, \kappa_{g+1}\right) = \min\left(\frac{\pi\left(\kappa_{g+1}\right) q\left(\kappa_g \mid \kappa_{g+1}\right)}{\pi\left(\kappa_g\right) q\left(\kappa_{g+1} \mid \kappa_g\right)}, 1\right). \tag{4.21}$$

Afterwards we need to draw a new σ_D^2 from its posterior distribution, where we assume, analog to the variance of the jump size, an inverse gamma prior distribution of σ_D^2, i.e. $\sigma_D^2 \sim \mathcal{IG}\left(\alpha_D, \beta_D\right)$ .Thus, the posterior of σ_D^2 is

$$p\left(\sigma_D^2 \mid N, q, \Theta_{-\sigma_D^2}, P\right) \propto p\left(P \mid S, N, q, \Theta\right) p\left(\sigma_D^2\right). \tag{4.22}$$

[121] $q\left(\kappa_{g+1} \mid \kappa_g\right)$ is the proposal density of κ_{g+1}. In case of symetrical proposal densities, these cancel out simplifying the acceptance criterion to the fraction of posterior distributions.

Finally, we need to draw a new sample for the mean-reversion level ν. Assuming that $\nu \sim \mathcal{N}\left(m_\nu, s_\nu^2\right)$, the conditional posterior follows as

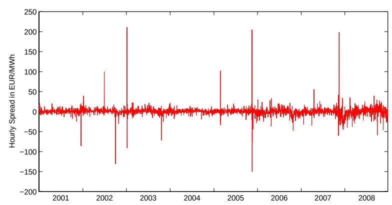
$$p\left(\nu \middle| N, q, \Theta_{-\nu}, P\right) \propto p\left(P \middle| S, N, q, \Theta\right) p\left(\nu\right). \tag{4.23}$$

Repeating the drawing of the above mentioned state variables and parameters, the distribution of the resulting Markov Chains will eventually converge to their target posterior distributions. In order to find starting values for the estimation procedure, we randomly draw parameter values from their prior distributions. The starting vector of jump sizes is constructed from observed market data. We discard the first 10,000 iterations and use additional 10,000 drawings as a basis for our parameter estimation. Appropriate choices of prior distributions and their parameters (called hyperparameters) improve fast convergence and are generally used to induce exogenous information into the estimation procedure. However, in our estimation, the results of the parameter estimates are quite insensitive towards changes in hyperparameters.[122]

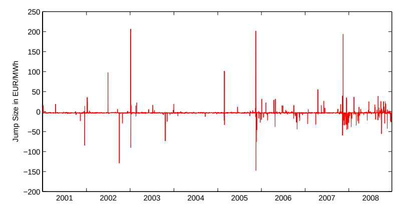
Besides receiving point estimates for parameter values, MCMC additionally allows us to separate the jump and the diffusion part of the underlying spread. Since we receive a vector of jump times and a vector of jump sizes in each iteration step, we can calculate the diffusion part of the underlying at time t, i.e. S_t, as the difference between the observed spread and the product of jump time and jump size at time t. After having finished the parameter estimation, we receive the final value for S_t as the arithmetic

[122] The mean jump size as well as the mean-reversion level are normally distributed with mean 0.0 and variance 2.25. The mean reversion speed is also normally distributed but with a mean of 0.3 and a variance of 0.01. The variances of the jump size and the diffusion variance are inverse-gamma distributed with an alpha of 10.0 and a beta of 2.0. The jump probability is beta distributed with an alpha and beta equal to 2.0.

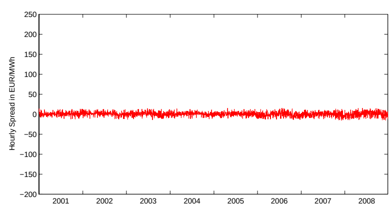
mean of non discarded iterations of S_t for all $t = 1, ..., T$. The calculation of the final jump times and jump sizes follows analogous to the vector S. Figure 4.3 shows the underlying spread, the de-spiked process as well as the jump times and jump sizes exemplarily for the eighth hour.



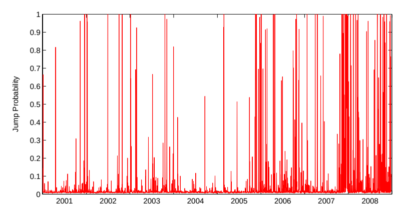
Underlying Spread



Jump Size



De-Spiked Process



Jump Probability

Figure 4.3: Estimated de-spiked process for hour 8.

4.4.3 Risk-neutral Parameters

In order to estimate the risk-neutral parameters, we need to include option prices into our estimation, since spot prices do not contain information about market prices of risk. For the observed PTR prices C, we assume

$$C_t = PTR_t \left(\Theta^Q, \psi_t\right) + \varepsilon_t^c, \tag{4.24}$$

where $\varepsilon_t^c \sim \mathcal{N}\left(0, \sigma_c^2\right)$. $PTR_t\left(\Theta^Q, \psi_t\right)$ is the model price of C_t, given in (4.9), as a function of the parameters Θ^Q and additional factors ψ_t, i.e. the current value of the de-spiked price S_t, interest rate and time to maturity.[123]

(4.24) states that PTR prices are observed with an error. This assumption is made in order to avoid a singularity problem. Thus, the observed option prices are normally distributed around their theoretical value, i.e. $C_t \sim \mathcal{N}\left(PTR_t\left(\Theta^Q, \psi_t\right), \sigma_c^2\right)$. Given the empirical parameters, we now only need to estimate those parameters that change when changing to the risk-neutral measure. The set of parameters therefore is $\Theta^Q = \left\{\kappa, \sigma_D^2, \nu, \mu_J^Q, \sigma_J^2, p_J^Q, \sigma_c^2\right\}$. Since we do not want to induce any information on how the risk-neutral parameters change when changing to the risk-neutral measure $\mathcal{Q}$, we use the same prior distributions and hyperparameters as before. The full information likelihood of C follows as

$$p\left(C\left|\Theta^Q, S, \sigma_c^2\right.\right) = \prod_{t=1}^{T} p\left(C_t\left|\Theta^Q, S_t, \sigma_c^2\right.\right). \tag{4.25}$$

We point out that the conditional distribution of C_t does not depend on the jump times and jump sizes. Therefore, PTR prices contain no information on jump times and jump sizes. This does of course not mean that jump times and jump sizes are identical across measures. But in order to generate the vectors q and N, we refer to the same posterior distributions as for the empirical estimation given in (4.15) and (4.17) and we refrain from stating them again.

[123] Throughout this paper, we assume a risk free rate of interest of 2% p.a. As we only use PTR options with one day to maturity, the interest rate has virtually no effect on our results.

The first parameter whose conditional distribution changes is the jump probability p_J^Q. Given the beta prior distribution stated above, the posterior for the jump probability follows as

$$p\left(p_J^Q \left| q^Q, C\right.\right) \propto p\left(q^Q \left| p_J^Q\right.\right) p\left(C_t \left| \Theta^Q, S_t, \sigma_c^2\right.\right) p\left(p_J^Q\right). \tag{4.26}$$

Since we cannot draw from this distribution directly, we apply Metropolis-Hastings algorithm with a $\mathcal{B}eta$ proposal density. For the mean jump size μ_J^Q the posterior is

$$\begin{aligned} & p\left(\mu_J \left| N^Q, q^Q, \Theta^Q_{-\mu_J^Q}, C\right.\right) \\ \propto & p\left(N^Q \left| \Theta^Q\right.\right) p\left(C_t \left| \Theta^Q, S_t, \sigma_c^2\right.\right) p\left(\mu_J^Q\right). \end{aligned} \tag{4.27}$$

As we can also not draw from this distribution directly, we use again a Metropolis-Hastings algorithm with a normal proposal density.[124] Finally, we need to draw the variance of the error term from the observed option prices where we assume, in line with before estimated variances, an inverse gamma prior, i.e. $\sigma_c^2 \sim \mathcal{IG}\left(\alpha_c, \beta_c\right)$, with $\alpha_c = 10$ and $\beta_c = 2$. The posterior distribution is[125]

[124] For the proposal density of the jump probability we use the parameters $\alpha = 2, \beta = 2$. The normal proposal density for the mean jump size has mean 0 and variance 100. Both proposal densities are symmetric wich simplifies the acceptance criteria in the Metropolis-Hastings step.

[125] We refer to Appendix B for details on the posterior distribution of σ_c^2.

$$p\left(\sigma_c^2 \middle| N^Q, q^Q, \Theta^Q_{-\sigma_c^2}, C, P\right) \propto p\left(C \middle| P, S, N^Q, q^Q, \Theta^Q\right) p\left(\sigma_c^2\right), \quad (4.28)$$

from which we can sample directly. Thus, we apply the Gibbs algorithm. As the rest of the parameters is identical across measures, we refrain form drawing them again and use their values estimated before.

4.5 Parameter Estimates

In Table 4.4, the estimation results for the empirical as well as risk-neutral parameters are given for all 24 hours. All parameters shown are those discussed in the prior section and given in absolute values based on daily observations. As mentioned above, the presented values are calculated as the arithmetic mean of the 10,000 non discarded Monte Carlo iterations. Further, below each parameter, the standard error of the estimation is given in brackets. The low standard errors in relation to the parameter values confirm a fast convergence of our MCMC algorithm.

For the empirical parameters, our initial expectations for the underlying spread are confirmed by the parameter estimates. First of all, figures in Table 4.4 show a relatively large mean-reversion speed κ compared to values observed in national electricity markets. This confirms the observed oscillatory behavior of the diffusion process as presented in Figure 4.3.[126] Second, the estimation results of the remaining parameters support the time-series properties of the spread. The values of the mean-reversion levels are in line

[126] Note that the mean-reversion speed only refers to the diffusion component. Therefore, the spikes in the underlying spread do not affect the estimated values for κ.

	Empirical Parameters						Risk-Neutral Parameters		
Hour	κ	σ_D^2	ν	μ_J	σ_J^2	p_J	μ_J^Q	p_J^Q	σ_c^2
1	1.4486	56.19	1.3127	4.5063	204.88	0.1053	-1.6473	0.0659	4.49
	(0.0006)	(0.0341)	(0.0012)	(0.0079)	(0.2473)	(0.0001)	(0.0069)	(0.0001)	(0.0012)
2	1.2784	36.88	0.4322	3.4572	106.29	0.1650	-2.6264	0.0920	3.31
	(0.0005)	(0.0219)	(0.0011)	(0.0059)	(0.1041)	(0.0002)	(0.0028)	(0.0002)	(0.0007)
3	1.2955	41.59	0.1346	3.3801	145.93	0.1312	-2.5935	0.0702	3.78
	(0.0006)	(0.0224)	(0.0011)	(0.0071)	(0.1642)	(0.0002)	(0.0023)	(0.0002)	(0.0007)
4	1.3654	53.08	-0.0461	2.2074	197.75	0.0732	-2.9084	0.0317	3.84
	(0.0006)	(0.0264)	(0.0011)	(0.0089)	(0.3141)	(0.0001)	(0.0073)	(0.0002)	(0.0006)
5	1.5132	63.37	-0.5918	0.8022	203.53	0.0674	-2.8833	0.0267	4.39
	(0.0006)	(0.0348)	(0.0011)	(0.0094)	(0.3263)	(0.0001)	(0.0006)	(0.0009)	(0.0009)
6	1.6314	63.44	-1.0007	0.7735	131.93	0.0974	-3.1499	0.0491	4.82
	(0.0006)	(0.0364)	(0.0011)	(0.0075)	(0.1921)	(0.0002)	(0.0043)	(0.0002)	(0.0006)
7	1.5528	75.97	-0.6849	3.2404	165.80	0.1395	1.9713	0.1166	81.97
	(0.0007)	(0.0449)	(0.0013)	(0.0073)	(0.1753)	(0.0002)	(0.0064)	(0.0001)	(0.0213)
8	1.4206	113.12	-0.4416	0.8069	1,622.60	0.0566	1.2225	0.0418	85.81
	(0.0006)	(0.0552)	(0.0016)	(0.0134)	(2.2065)	(0.0001)	(0.0054)	(0.0002)	(0.0220)

Table 4.4: Estimated empirical and risk-neutral parameters.

	Empirical Parameters						Risk-Neutral Parameters		
Hour	κ	σ_D^2	ν	μ_J	σ_J^2	p_J	μ_J^Q	p_J^Q	σ_c^2
9	1.3511	128.56	1.3972	2.4266	5,156.84	0.0895	3.0785	0.0800	174.27
	(0.0006)	(0.0666)	(0.0018)	(0.0142)	(5.3434)	(0.0001)	(0.0036)	(0.0002)	(0.0453)
10	1.7476	335.33	3.9393	3.9461	12,936.33	0.1290	4.7790	0.1331	606.56
	(0.0007)	(0.1894)	(0.0024)	(0.0148)	(10.6475)	(0.0001)	(0.0133)	(0.0001)	(0.1576)
11	1.8726	638.14	5.8196	2.7146	23,143.02	0.1161	4.2312	0.1235	746.88
	(0.0007)	(0.3761)	(0.0032)	(0.0149)	(19.8108)	(0.0001)	(0.0153)	(0.0001)	(0.1969)
12	1.8348	810.10	6.2835	2.4282	32,081.58	0.1383	3.8410	0.1395	1,075.12
	(0.0006)	(0.4475)	(0.0035)	(0.0149)	(25.0247)	(0.0001)	(0.0160)	(0.0001)	(0.2786)
13	1.8551	403.06	4.6101	2.4941	15,852.15	0.0916	4.2589	0.1021	55.42
	(0.0006)	(0.2257)	(0.0025)	(0.0148)	(15.1873)	(0.0001)	(0.0149)	(0.0001)	(0.1464)
14	1.9521	459.85	5.0874	2.9444	20,728.57	0.1083	4.3394	0.1155	688.53
	(0.0008)	(0.2517)	(0.0025)	(0.0149)	(17.3976)	(0.0001)	(0.0145)	(0.0001)	(0.1803)
15	1.9533	347.13	4.4949	2.2971	21,650.97	0.0975	3.7397	0.1002	538.46
	(0.0007)	(0.1758)	(0.0022)	(0.0148)	(19.0381)	(0.0001)	(0.0156)	(0.0001)	(0.1392)
16	1.8549	268.06	3.8224	1.0518	38,264.94	0.0632	2.9409	0.0604	417.05
	(0.0007)	(0.1364)	(0.0019)	(0.0150)	(40.3128)	(0.0001)	(0.0094)	(0.0001)	(0.0225)

Table 4.4: Estimated empirical and risk-neutral parameters.

	Empirical Parameters						Risk-Neutral Parameters		
Hour	κ	σ_D^2	ν	μ_J	σ_J^2	p_J	μ_J^Q	p_J^Q	σ_c^2
17	1.8386	220.12	3.4703	1.7472	20,305.41	0.0744	3.5207	0.0786	433.57
	(0.0006)	(0.1033)	(0.0018)	(0.0149)	(20.3079)	(0.0001)	(0.0163)	(0.0001)	(0.1124)
18	1.8040	262.60	3.6699	2.6711	46,001.99	0.1479	3.8846	0.1473	2,345.89
	(0.0007)	(0.1501)	(0.0021)	(0.0149)	(31.7184)	(0.0001)	(0.0159)	(0.0001)	(0.6180)
19	1.4990	188.97	2.5456	3.0567	18,822.19	0.1287	4.0312	0.1505	953.62
	(0.0006)	(0.1027)	(0.0021)	(0.0148)	(14.7670)	(0.0001)	(0.0161)	(0.0001)	(0.2471)
20	1.4863	166.56	2.4207	5.5006	4,909.00	0.1212	5.4211	0.1233	307.91
	(0.0006)	(0.0992)	(0.0020)	(0.0147)	(4.4557)	(0.0001)	(0.0123)	(0.0001)	(0.0800)
21	1.4791	159.85	2.5815	4.6026	3,282.18	0.0797	4.8525	0.0792	110.80
	(0.0006)	(0.0901)	(0.0019)	(0.0147)	(3.5864)	(0.0001)	(0.0135)	(0.0001)	(0.0292)
22	1.4997	118.16	2.1252	3.1520	2,318.22	0.0454	3.9104	0.0437	106.27
	(0.0006)	(0.0600)	(0.0016)	(0.0147)	(3.6696)	(0.0001)	(0.0146)	(0.0000)	(0.0279)
23	1.2860	53.22	0.8971	6.5293	217.47	0.1528	5.5574	0.1427	96.38
	(0.0006)	(0.0339)	(0.0013)	(0.0075)	(0.1950)	(0.0001)	(0.0094)	(0.0001)	(0.0225)
24	2.0768	182.32	3.5926	8.6124	305.00	0.0620	7.5670	0.0662	91.64
	(0.0008)	(0.1011)	(0.0016)	(0.0126)	(0.5484)	(0.0001)	(0.0021)	(0.0001)	(0.0241)

Table 4.4: Estimated empirical and risk-neutral parameters.

with the median price spreads in Table 4.2 in terms of sign as well as level. While hours 4 to 8 have negative mean-reversion levels, those for the other hours are all positive and generally higher for peak hours. The mean jump sizes and jump probabilities show no clear pattern across different hours. However, the variance of the jump sizes vary significantly during the day. For morning hours and late at night, the variance of the jump sizes are rather moderate. During peak hours, in contrast, jump variances increase extremely reaching their peak for hour 18 with a variance of over 46,000. Comparing the figures with the time-series results in Table 4.2 as well as the trajectories in Figure 4.1, these values are in line with intuition. With minimum and maximum price ranges of over 3,500 and kurtoses of up to 600, these jump size variances are also in line with observed prices. Overall, the empirical parameter estimates in Table 4.4 confirm the erratic and extremely spiky behavior of daily price spreads from Table 4.2 and Figure 4.1 and supports our approach of modelling each hour separately.

In order to clarify the results in Table 4.4 and to analyze the results for the risk-neutral parameters, we compare the densities of the prices spreads for the empirical parameters with those of the risk-neutral ones. Figure 4.4 shows the difference between empirical and risk-neutral densities of the underlying spread. A positive value refers to a higher probability under the risk-neutral measure.

Figure 4.4 confirms the results of Table 4.4. For hours 4 and 24, negative (positive) outcomes of the spread become more (less) likely under the risk-neutral measure. This shift results in relatively smaller PTR option prices, since smaller price spreads also mean lower option payoffs. Although less pronounced for hour 24, note the different scale for hours 12 to 24 compared to hours 4 and 8, the left shift of the distribution is still significant. For the remaining hours, the differences between the empirical and risk-neutral densities are also clearly visible, although only minimal for hour 12, since the densities are dominated by the extreme jump size variance. However,

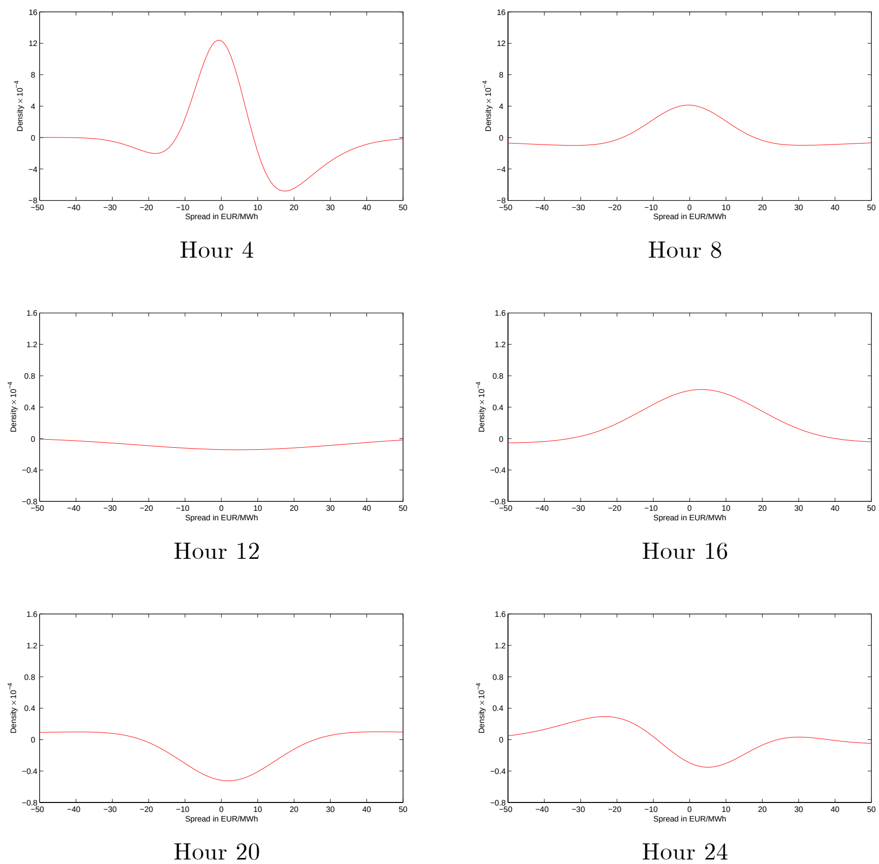


Figure 4.4: Difference between empirical and risk-neutral densities.

in contrast to hours 4 and 24, the difference between empirical and risk-neutral densities is symmetric, i.e. there is no shift in the probability of positive and negative outcomes. Thus, there is only a slight or even non risk adjustment for PTR options during those hours resulting in relatively expensive PTR prices.

In order to explain our results, two aspects are of relevance. First of all, PTR options are physically settled in contrast to the widely used financially settled options. In case of cross-border supply agreements, investors might need to purchase PTR options at short notice. Since the fulfillment of such an agreement is generally of utmost priority, investors might be willing to pay a premium for PTR options. As the PTR auction is held before the day-ahead auction in the Dutch electricity market, investors are in general not able to purchase any electricity in the Dutch market at all or at the desired price in order to fulfill a potential supply agreement. Thus, the auction set up increases the hedging demand of market participants. This demand to hedge the delivery risk is amplified by the lower amount of available capacity during turbulent hours, inducing an insurance premium in PTR options. The other reason might be the usage of PTR options as purely speculative contracts. As PTR prices are generally small with regards to the extreme price spikes inherent in the underlying price process, PTRs can be thought of as a bet on the occurrence of a jump. As the forecast of jumps in the underlying spread, even with only one day to maturity, is extremely difficult as shown in Figure 4.2, investors are willing to pay a premium for PTRs in order to benefit from occasional but highly profitable jumps.

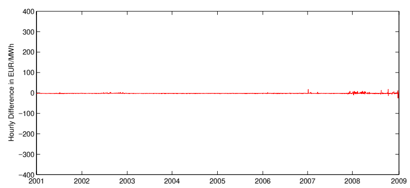
Besides the dichotomy in the behavior of market participants investing in PTR options, the pricing performance of our model also differs depending on the volatility in the market. The model fit is indicated by the variance of the residuals between market and model prices, i.e. σ_c^2. While σ_c^2 is rather low for calm hours with only moderate jump size variances, it significantly increases for turbulent hours. Our model therefore describes an adequate approach for the valuation of hourly PTR options during calm hours while in times of extreme volatility its pricing performance decreases.

4.6 Analysis of Residuals

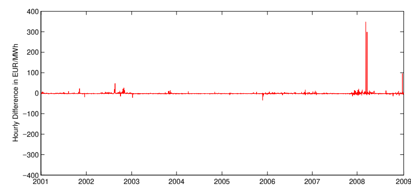
After having applied our model for the valuation of hourly PTRs, we now proceed with an analysis of the residuals of our estimation, i.e. the difference between the market and model prices. This analysis helps to gain further insights in the valuation of PTRs and thus leads to a better understanding of these contracts. Further, the residuals might indicate promising extensions to our current model. In order to gain insights into the valuation of PTRs, we proceed in two steps. First, we use the residuals of the hourly PTRs. Due to the discrepancies of different hours during a day, as discussed above, we analyze each of the 24 hours separately. In a second step, we use the above estimated hourly parameters for the valuation of monthly PTR contracts. These parameters are applicable as monthly PTRs are just a portfolio of the corresponding hourly contracts. Then, we also analyze the corresponding residuals between market and model prices of monthly PTRs. Thus, we are able to identify differences in the valuation of hourly and monthly PTR contracts.

4.6.1 Residuals of Hourly PTRs

The residuals of our hourly PTRs are the difference between the estimated model price and the corresponding observed market price. Therefore, the resulting residuals can be regarded as estimation errors of our model. As shown in Figure 4.1, the underlying spread varies extremely across hours as well as over the period between 2001 and 2008. Based on the estimated parameters in Table 4.4, especially the σ_c^2, we expect the residuals to be also highly erratic. Figure 4.5 shows the residuals for our estimated model between 2001 and 2008. Residuals are calculated such that a positive difference between model and market prices refers to a higher market value compared to the model price and vice versa.



Hour 4



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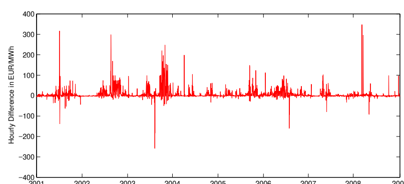
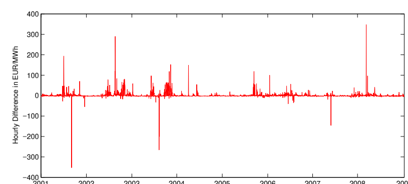
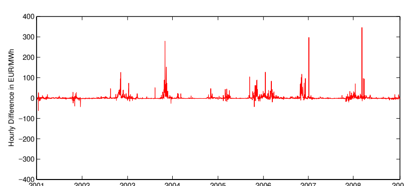


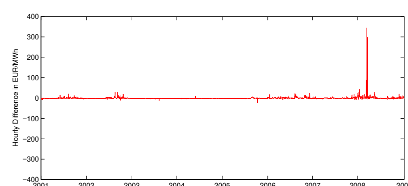
Hour 12



Hour 16



Hour 20



Hour 24

Figure 4.5: Hourly difference between market and model PTR prices.

Figure 4.5 shows the erratic behavior of the residuals. The prominent spiky trajectory of the residuals is especially evident for turbulent hours during the day. The resulting residuals indicate that our model is not able to capture the extreme spikes in PTR option prices adequately leaving room for improvement of our model. In order to identify shortcomings of our model, we need to refer to the characteristics of the underlying spread in Figure 4.1 as any model extension needs to be founded on the underlying prices to be modelled. Figure 4.1 shows that, especially during turbulent hours, observed spikes are not distributed evenly across the relevant time span. Based on the given spreads, a more flexible jump pattern could improve our model performance.

In order to investigate a systematic behavior in the underlying spread that might indicate potential extensions to our model, we first analyze if the residuals are subject to a seasonal behavior. Second, we build on the seasonal pattern and investigate if the seasonal pattern of the spread is closely connected to the number of observed spikes in the underlying. In case of a positive relation between the residuals and the number of spikes, adopting a more flexible jump pattern could lead to more realistic PTR model prices as market PTR prices are systematically higher than model prices in case of a higher number of jumps and vice versa. For our analysis, we use a weekly seasonality in form of a sinusoidal function with a period of seven days and phase zero.[127] Therefore, in order to test for seasonality, we perform the following regression

$$C_t - PTR_t = \gamma_1 + \gamma_2 \cdot \sin\left(2\pi \cdot \frac{t}{7}\right) + \varepsilon_t, \tag{4.29}$$

where analogous to (4.24), C_t (PTR_t) refers to the market (model) price

[127] We have also used different frequencies (i.e. monthly and annually) and phases as well as a piecewise constant seasonality function. However, the function we apply best describes the seasonal pattern of the residuals.

of the PTR contract. γ_1 and γ_2 are constant parameters, t is a variable corresponding to the respective day of the week and ε_t is the *iid* normally distributed residual of the regression.

Beside testing for weekly seasonality in the residuals of hourly PTRs, we are interested in a relation between residuals and observed spikes in the underlying spread. In our analysis we follow, amongst other, Kluge (2006) and classify a price movement as a jump (or spike) depending on the number of standard deviations this price deviates from the mean value. Any value that deviates more than four standard deviations from the average value is in our analysis considered to be a spike.[128] The results of regression (4.29) as well as the number of observed spikes in the underlying spread per weekday and the correlation (ρ) between the seasonality of the residuals and the spikes are shown in Table 4.5 for each hour separately.

Table 4.5 shows that the residuals are on average negative during calm hours, whereas during turbulent hours, they are generally positive as indicated by the sign of γ_1. This demonstrates that the model underestimates market prices when the underlying spread is highly erratic and overestimates PTR prices in calm hours. Besides significant average residuals, we also find evidence for seasonality in residuals for hours 9 to 18. Moreover, for those hours, the number of observed spikes during the beginning of the week is rather high, while at the end of the week, almost no spikes occur. During the rest of the day, the occurrence of spikes is more dispersed and several spikes are observed at the weekend. This effect is mainly due to the lower standard deviation during calm hours and therefore a lower threshold for prices to be considered a spike.

[128] The choice of the number of standard deviation is somewhat heuristic. While three is often used, we use a higher value in order to include only the extreme jump in our analysis. However, the results do not change considerably when choosing any other value between three and four.

	Regression Results		Number of Spikes							Korrel
Hour	γ_1	γ_2	Mon	Tue	Wed	Thu	Fri	Sat	Sun	ρ
1	-1.7604***	0.0415	3	4	2	2	3	2	7	0.21
2	-2.0510***	-0.1053***	6	2	1	2	2	2	9	-0.20
3	-2.0620***	-0.0786**	3	2	1	2	1	2	5	-0.21
4	-2.0613***	-0.0157	4	2	0	1	0	2	4	-0.40
5	-2.0140***	-0.0015	1	2	2	1	1	2	2	-0.26
6	-1.9510***	0.0110	2	1	0	1	0	2	5	0.07
7	-1.6227***	-0.0270	1	1	0	1	3	3	10	0.25
8	-1.2257***	0.0317	2	0	4	2	1	0	1	0.23

Table 4.5: Seasonality in residuals for hourly PTRs and number of spikes per weekday.

	Regression Results		Number of Spikes							Korrel
Hour	γ_1	γ_2	Mon	Tue	Wed	Thu	Fri	Sat	Sun	ρ
9	0.6057***	0.9278***	3	4	9	7	3	0	0	0.27
10	4.2203***	2.5201***	4	6	5	3	4	1	0	0.53
11	5.8016***	3.0037***	5	8	5	4	5	1	1	0.57
12	7.2046***	3.8356***	5	7	7	3	2	1	0	0.75
13	4.5301***	2.3977***	4	6	6	1	4	0	0	0.60
14	4.6776***	2.6226***	10	9	8	2	1	0	0	0.88
15	3.3336***	1.8399***	8	9	7	6	1	1	0	0.79
16	1.8990***	1.3425***	4	8	3	1	2	1	0	0.74

Table 4.5: Seasonality in residuals for hourly PTRs and number of spikes per weekday.

	Regression Results		Number of Spikes							Korrel
Hour	γ_1	γ_2	Mon	Tue	Wed	Thu	Fri	Sat	Sun	ρ
17	2.4213***	1.3472***	9	6	5	2	2	0	0	0.79
18	7.5243***	3.9438***	9	8	9	10	3	3	0	0.51
19	4.9604***	1.2268*	7	2	2	6	1	5	1	0.07
20	2.2455***	-0.0409	5	3	3	7	1	7	7	0.12
21	0.6391***	-0.6228**	3	2	3	6	3	2	6	0.19
22	-0.3861**	-0.3496***	2	2	1	1	0	2	13	-0.10
23	-0.7530***	-0.3444	2	5	1	3	1	5	17	-0.03
24	-0.6831***	0.0650***	9	5	4	3	8	4	4	0.14

Table 4.5: Seasonality in residuals for hourly PTRs and number of spikes per weekday.

Results in Table 4.5 show an ambiguous relation between residuals and observed spikes. Concerning our model, the results indicate that for turbulent hours, an extension towards non-constant, i.e. seasonal, jump intensities might offer improved modelling results. However, this does not hold for calm hours where the relation between residuals and observed spikes is not significant or even negative. Our results indicate that for different hours of the day, not only different parameters need to be estimated, but also, different models should be applied.

4.6.2 Residuals of Monthly PTRs

After having analyzed the residuals of hourly PTRs, we now investigate residuals from monthly PTRs. In the course of our analysis, we first need to derive model prices for monthly PTRs and then compare these prices with the observed market prices to receive the residuals. For our analysis, we use all monthly PTR contracts between February 2001 and December 2008, i.e. 95 PTR prices.[129] Monthly PTRs are just portfolios of the corresponding hourly PTRs. Further, when buying a monthly PTR, its holder needs to decide for every single hour separately, whether to exercise the PTR option or not. In order to value monthly PTRs, we simply use the estimated parameters in Table 4.4 and calculate the arithmetic mean of all hourly PTRs for the respective month.[130] So the March PTR price, for example,

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[129] Although the January 2001 PTR price is also available, we do not have information on the underlying spread at the time of the PTR auction in December 2000. Thus, we are not able of deriving a model price for the January 2001 contract and do not include it in our analysis.

[130] We calculate the arithmetic mean of all hourly contracts as monthly PTRs are priced on a per hour basis. Thus, we need an hourly price in order to make model and market prices comparable.

is just a portfolio of 744 hourly PTRs.[131]

Following the calculation of the monthly PTR model prices, the residuals are then derived as the difference between observed market and model prices. Figure 4.6 shows the residuals for all monthly PTRs between February 2001 and December 2008. Again, a positive residual refers to a market price above the corresponding model price and vice versa.

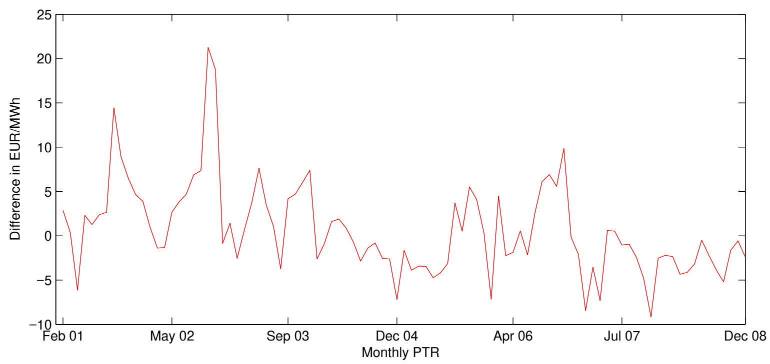

Figure 4.6: Difference between market and model prices for all monthly PTRs between 2001 and 2008.

Figure 4.6 shows that the residuals fluctuate around zero and are slightly positively skewed.[132] Especially, extremely high market prices are not adequately matched by our model. In addition to its fluctuation, the trajectory of the residuals indicate an annual seasonality. In order to further investigate the seasonal trend, we perform a regression of the residuals on a

[131] For simplicity reasons, we calculate the respective time-to-maturity in days and not hours. Thus, all 24 hourly PTRs for a given day have the same time to maturity.

[132] We refrain from stating detailed descriptive statistics as these do not offer any additional insight for our analysis.

sinusoidal function with a period of 12 month. Analogous to our analysis for residuals of hourly PTRs, we perform the following regression for monthly PTRs[133]

$$C_t - PTR_t = \gamma_1 + \gamma_2 \cdot \sin\left(2\pi \cdot \frac{t}{12}\right) + \varepsilon_t. \tag{4.30}$$

Here, C_t (PTR_t) refers to the market (model) price of the monthly PTR contract. Again, γ_1 and γ_2 are constant parameters, t is a variable corresponding to the respective month and ε_t is the *iid* normally distributed residual of the regression.

In addition to our analysis of the seasonality in residuals of monthly PTR, we also investigate the relation to spikes in the underlying spread. Therefore, we count the number of observed spikes during each calendar month are calculate the correlation between the seasonality function and the underlying spikes. Table 4.6 provides information on the results of regression (4.30) as well as the number of spikes per month.

Figures in Table 4.6 are in line with our expectation from observed residuals in Figure 4.6. Residuals are on average positive, as indicated by the significant positive γ_1. Further, residuals show a highly significant seasonality with lower values during the first half of the year and peaks during the fall. This seasonal behavior is corresponding to the number of observed spikes during each month also shown in Table 4.6. While during spring, spikes occur less frequently, their appearance peaks during August and October. Consequently, the corresponding correlation between the seasonality of the residuals and the number of observed spikes is 0.88.

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[133] For our regression, we use monthly values. Thus, the period of our sinusoidal function is 12.

Regression Results	Number of Spikes					
γ_1	Jan	Feb	Mar	Apr	May	Jun
1.5349***	53	9	21	10	37	31
γ_2	Jul	Aug	Sep	Oct	Nov	Dec
-2.5267***	49	84	65	77	69	67

Table 4.6: Seasonality in residuals for monthly PTRs and number of spikes per calendar month.

In order to emphasize the strong relation between the seasonality in residuals and the number of observed spikes, Figure 4.7 shows both trajectories, i.e. seasonality and frequency of jumps, for each calendar month. The left y-axis shows the number of observed spikes and is depicted by the red, erratic line. The right y-axis displays the seasonal trend of the residuals and is shown by the blue, smooth line.

While for hourly contracts, the relation between spikes and the over- or undervaluation of PTRs was ambiguous, the correlation for monthly PTRs is evident. In months with more frequent spikes, market participants are willing to pay a higher price for PTRs compared to months when spikes are rare. One reason for the clear relation for monthly PTRs might be that spikes are extremely hard to predict on a daily basis. However, on a monthly basis, the number of spikes is more stable and therefore easier to foresee. Thus, in order to adequately value monthly PTRs, the extension

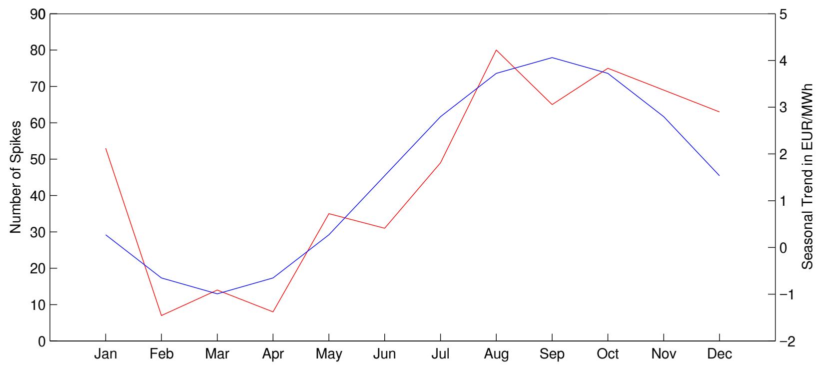

Figure 4.7: Number of spikes (bumpy, left axis) and seasonal trend in the difference between market and model prices (smooth, right axis) per calendar month.

of our model towards non-constant, seasonal jump intensities seems highly promising.

4.7 Conclusion

In this chapter, we analyze hourly Physical Transmission Right (PTR) prices for the German-Dutch interconnector between 2001 and 2008. We model this price spread directly considering the unique features of the underlying, especially the extremely short-term price spikes. Due to the diverse characteristics of price spreads across hours, modelling each hour separately is essential. We find that investors are willing to pay a premium for hourly PTR options for turbulent hours of the day, i.e. hour 8 to 22. This price premium can be explained by increased hedging demand or a speculation premium from investors in the German-Dutch cross-border electricity market. The extensive demand for PTRs emphasizes the importance of these contracts and the need for adequate risk management tools in cross-border electricity markets.

Further, we find evidence of seasonality in residuals of hourly as well as monthly PTR contracts. Moreover, this seasonality is correlated to the occurrence of spikes in the underlying spread. This indicates that in times of higher frequencies of spikes, our model underestimates the market prices while during times of lesser spikes, market prices are generally lower than model prices. For month contracts, this relation is highly significant. For hourly PTRs, the relation between seasonality in residuals and spikes in the underlying is ambiguous. While it is significant during turbulent hours, the relation is not evident for calm hours. Our results indicate that the introduction of non-constant seasonal jump intensities poses a promising extension in order to improve model performance. This especially holds for month contracts as it is easier to estimate the occurrence of spikes on a monthly than on a daily basis. While Seifert and Uhrig-Homburg (2007) is the only paper to discuss non-constant jump intensities in electricity markets, we are the first, to the best of our knowledge, to include option prices in our analysis. Furthermore, as jumps in the underlying spread are based on idiosyncratic jumps in one of the national electricity prices, our results offer not only insights for cross-border derivatives, but also for the valuation of derivatives in national electricity markets as well.

Future research could certainly analyze variations in the parameters of our model over time, which is, however, rather tedious and implies the risk of receiving unstable estimates. Moreover, the adoption of a time-varying jump-intensity, as used by Seifert and Uhrig-Homburg (2007), could improve the pricing performance and shed more light on the behavior of jumps in this market. In addition, applying the currently famous regime-switching models could also be promising for modelling price spreads. These models have proven to adequately mirror electricity prices and are, amongst others, used by Haldrup and Nielsen (2006b). Finally, testing our model for other underlyings, such as cross-commodity spreads, seems an interesting field of research.

Chapter 5

Conclusion

The ongoing liberalization in European electricity markets over the past decades has led to an increased competition in national markets and to a continuous rise in cross-border electricity flows. The congestion of electricity transmission lines and resulting spreads between the prices of connected grid zones are an important risk faced by participants in the electricity markets, especially with respect to cross-border trading. Although cross-border transmission capacities are constantly expanded and more efficient congestion management methods are applied, congestion and therefore locational price spreads will continue to appear frequently in the whole of Europe for the foreseeable future. Consequently, the management of locational price spreads remains important and a thorough understanding of the pricing of derivative products and therefore a further analysis of risk premia is of utmost interest to market participants as well as outside speculators.

This thesis discusses the valuation of the most important cross-border derivatives in the European electricity market. It analyzes the drivers of risk premia and their impact on the valuation of these contracts. While Chapter 3 discusses the pricing of Contracts for Difference (CfDs), Chapter 4 examines the valuation of Physical Transmission Rights (PTRs) with delivery of one hour and an entire calendar month respectively.

In Chapter 3 we analyze the ex-post risk premia of CfDs traded at Nord Pool. It is shown that CfDs contain significant risk premia that substantially vary in both sign and magnitude across market areas. We then investigate the development of these risk premia over time-to-maturity and identify their main economic drivers. Results show a strong coherency between ex-post risk premia and time-to-maturity. Although not significant for CfDs, this relation is highly significant for implied area and system forwards, the two constituents of CfDs. In addition, Chapter 3 identifies a strong relationship between risk premia and the variance and skewness of the underlying spot prices and also finds a significant impact of hydropower on spot prices and risk premia in the Nordic market. Thus, Chapter 3 confirms the two prominent models of Bessembinder and Lemmon (2002) as well as Benth et al. (2008a) for cross-border electricity markets. Future research could especially investigate influencing factors of short-term risk premia, e.g. for month contracts in contrast to season or year contracts. In addition, investigations of the pricing of CfDs with reduced-form models, for example, based on the regime-switching approach for electricity spot prices, or equilibrium models, seem promising. Ideally, future work would be able to include the CfD prices from Nord Pool's OTC clearing.

Chapter 4 discusses the valuation of hourly PTRs for the German-Dutch interconnector. We propose a spike-diffusion model and estimate its physical and risk-neutral parameters. Using those parameters, we compare the empirical and risk-neutral densities for the underlying price spreads. Our results show first of all that the spike-diffusion model adequately describes the underlying PTR prices especially during calm hours. Second, the estimated parameters show that during calm hours PTRs are traded at a discount, whereas market participants are willing to pay a premium for PTRs during turbulent hours. The premium implicit in those PTRs can be explained by either increased hedging demand or speculation of market participants. Furthermore, we find evidence for seasonality in the residuals of hourly and monthly PTR option prices. For monthly PTRs and hourly PTRs during turbulent hours, this seasonality is strongly related to jumps in the underlying spread. This result, in contrast to prior work, is the first based on not only spot but also option prices and offers further insights in the valuation of derivatives in cross-border as well as national electricity markets. Future research could analyze variations in the parameters of our model over time especially the adoption of a time-varying jump-intensity. In addition, applying the currently famous regime-switching models could also be promising for modelling price spreads. Finally, our model could be test for other underlyings, especially further cross-commodity spreads.

As any analysis relies on certain assumption, so do the analyses in Chapters 3 and 4 of this thesis. In Chapter 3, we assume that all CfDs are liquidly traded. Although the trading volume of CfDs is constantly increasing, liquidity compared to equity markets is still poor. Moreover, in Chapters 3 and 4, we do not discuss transaction costs for trading CfDs and PTRs. Finally, we assume that no market member is in the position to exert market power. In the market for CfDs with the rather low liquidity, a major market participants may well be able to influence prices. In the German-Dutch electricity market for PTRs, where contracts are auctioned, the auction mechanism might also lead to market imperfections. As the results of this thesis heavily rely on all those assumptions, an analysis on how a relaxation of one or more of them might influence results seems highly interesting and is left for future research.

Appendix

A Itô Lemma for Jump Processes

The Itô Lemma for jump-diffusion processes is extensively discussed in the literature. The following description is based on Cont and Tankov (2003) proposition 8.14.

Let X be a diffusion process with jumps, defined as the sum of a drift term, a Wiener stochastic integral and a compound Poisson process as

$$X_t = X_0 + \int_0^t b_s ds + \int_0^t \sigma_s dW_s + \sum_{i=1}^{N_t} \Delta X_i,$$

where b_t and σ_t are continuous nonanticipating processes with

$$E\left[\int_0^T \sigma_t^2 dt\right] < \infty.$$

Then, for any $C^{1,2}$ function $f : [0,T] \times \mathbb{R} \to \mathbb{R}$, the process $Y_t = f(t, X_t)$

can in differential notation be represented as

$$\begin{aligned} dY_t &= \frac{\partial f}{\partial t}(t, X_t)\,dt + b_t \frac{\partial f}{\partial x}(t, X_t)\,dt + \frac{\sigma_t^2}{2}\frac{\partial^2 f}{\partial x^2}(t, X_t)\,dt \\ &\quad + \frac{\partial f}{\partial x}(t, X_t)\,\sigma_t dW_t + \left[f\left(X_{t-} + \Delta X_t\right) - f\left(X_{t-}\right)\right]. \end{aligned}$$

B Posterior Densities for the Estimated Parameters of Hourly PTRs

The Bernoulli probability of a jump is needed in order to generate a vector of jump. Each probability is calculated as

$$\begin{aligned} \varphi_{t+1} &= \Pr\left(q_{t+1} = 1 \middle| \Theta, P_{t+1}, S_t, N_{t+1}\right) \\ &= \left(1 + \frac{1 - p_J}{p_J}\exp\left(-\frac{2N_{t+1}\left(P_{t+1} - S_t e^{-\kappa} - \nu\left(1 - e^{-\kappa t}\right)\right) - N_{t+1}^2}{2\left(1 - e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}\right)\right)^{-1}. \end{aligned}$$

The full posterior distribution for the jump probability is

$$\begin{aligned} p\left(p_J \middle| q\right) &\propto p\left(q \middle| p_J\right) p\left(p_J\right) \\ &\propto p_J^{\sum_{t=1}^T q_t}\left(1 - p_J\right)^{T - \sum_{t=1}^T q_t} p_J^{\alpha_J - 1}\left(1 - p_J\right)^{\beta_J - 1} \\ &\propto p_J^{\alpha_J + \sum_{t=1}^T q_t - 1}\left(1 - p_J\right)^{\beta_J + T - \sum_{t=1}^T q_t - 1}. \end{aligned}$$

The full posterior distribution for the jump size is

$$\begin{aligned}
& p\left(N_{t+1}\middle|\,\Theta, q_{t+1}, P_{t+1}, S_t\right) \\
\propto\ & p\left(P_{t+1}\middle|\,S_t, N_{t+1}, q_{t+1}, \Theta\right) p\left(N_{t+1}\,|\Theta\right) \\
\propto\ & \exp\left(-\frac{1}{2}\frac{\left(N_{t+1}-\frac{\sigma_J^2 q_{t+1}\left(P_{t+1}-S_t e^{-\kappa}-\nu\left(1-e^{-\kappa t}\right)\right)+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}\mu_J}{\sigma_J^2 q_{t+1}^2+(1-e^{-2\kappa})\frac{\sigma_D^2}{2\kappa}}\right)^2}{\frac{\sigma_J^2(1-e^{-2\kappa})\frac{\sigma_D^2}{2\kappa}}{\sigma_J^2 q_{t+1}^2+(1-e^{-2\kappa})\frac{\sigma_D^2}{2\kappa}}}\right).
\end{aligned}$$

The full posterior distribution for the mean jump size is

$$\begin{aligned}
p\left(\mu_J\middle|\,N, q, \Theta_{-\mu_J}, P\right) &\propto p\left(N\middle|\,\Theta\right) p\left(\mu_J\right) \\
&\propto \exp\left(-\frac{1}{2}\frac{\left(\mu_J-\frac{s_\mu^2\sum_{t=0}^{T-1}N_{t+1}+\sigma_J^2 m_\mu}{Ts_\mu^2+\sigma_J^2}\right)^2}{\frac{\sigma_J^2 s_\mu^2}{Ts_\mu^2+\sigma_J^2}}\right).
\end{aligned}$$

The full posterior distribution for the variance of the jump size is

$$\begin{aligned}
p\left(\sigma_J^2\middle|\,N, q, \Theta_{-\sigma_J^2}, P\right) &\propto p\left(N\middle|\,\Theta\right) p\left(\sigma_J^2\right) \\
&\propto \left(\frac{1}{\sigma_J^2}\right)^{\widetilde{\alpha}_J+1}\exp\left(-\frac{\widetilde{\beta}_J}{\sigma_J^2}\right)\frac{\left(\widetilde{\beta}_J\right)^{\widetilde{\alpha}_J}}{\Gamma\left(\widetilde{\alpha}_J\right)}.
\end{aligned}$$

Therefore, the posterior distribution of σ_J^2 is also inverse gamma with parameters $\widetilde{\alpha}_J$ and $\widetilde{\beta}_J$, which follow from the prior parameters as

$$\begin{aligned}\widetilde{\alpha}_J &= \frac{1}{2}T + \alpha_J, \\ \widetilde{\beta}_J &= \frac{1}{2}\sum\nolimits_{t=0}^{T-1}\left(N_{t+1} - \mu_J\right)^2 + \beta_J.\end{aligned}$$

The full posterior distribution for the variance is

$$\begin{aligned}p\left(\sigma_D^2 \middle| N, q, \Theta_{-\sigma_D^2}, P\right) &\propto p\left(P \middle| S, N, q, \Theta\right) p\left(\sigma_D^2\right) \\ &\propto \left(\frac{1}{\sigma_D^2}\right)^{\widetilde{\alpha}_D+1} \exp\left(-\frac{\widetilde{\beta}_D}{\sigma_D^2}\right) \frac{\left(\widetilde{\beta}_D\right)^{\widetilde{\alpha}_D}}{\Gamma\left(\widetilde{\alpha}_D\right)}.\end{aligned}$$

Therefore, the posterior distribution of σ_D^2 is also inverse gamma with parameters $\widetilde{\alpha}_D$ and $\widetilde{\beta}_D$, which follow from the prior parameters as

$$\begin{aligned}\widetilde{\alpha}_D &= \frac{1}{2}T + \alpha_D, \\ \widetilde{\beta}_D &= \frac{\kappa}{\left(1 - e^{-2\kappa}\right)}\sum\nolimits_{t=0}^{T-1}\left(P_{t+1} - S_t e^{-\kappa} - N_{t+1}q_{t+1}\right)^2 + \beta_D.\end{aligned}$$

The full posterior distribution for the mean-reversion level is

$$\begin{aligned}& p\left(\nu \middle| N, q, \Theta_{-\nu}, P\right) \\ \propto\ & p\left(P \middle| S, N, q, \Theta\right) p\left(\nu\right) \\ \propto\ & \exp\left(-\frac{1}{2}\frac{\left(\nu - \frac{\left(1-e^{-\kappa}\right)s_\nu^2\sum_{t=0}^{T-1}\left(P_{t+1}-S_t e^{-\kappa}-N_{t+1}q_{t+1}\right)+m_\nu\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}{\left(1-e^{-\kappa}\right)^2 s_\nu^2 T+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}\right)^2}{\frac{\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}s_\nu^2}{\left(1-e^{-\kappa}\right)^2 s_\nu^2 T+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}}\right).\end{aligned}$$

The full posterior distribution for the variance of observed PTR prices is

$$\begin{aligned} p\left(\sigma_c^2 \middle| N^Q, q^Q, \Theta^Q_{-\sigma_c^2}, C, P\right) &\propto p\left(C \middle| P, S, N^Q, q^Q, \Theta^Q\right) p\left(\sigma_c^2\right) \\ &\propto \left(\frac{1}{\sigma_c^2}\right)^{\widetilde{\alpha}_c+1} \exp\left(-\frac{\widetilde{\beta}_c}{\sigma_c^2}\right) \frac{\left(\widetilde{\beta}_c\right)^{\widetilde{\alpha}_c}}{\Gamma\left(\widetilde{\alpha}_c\right)}. \end{aligned}$$

Therefore, the posterior distribution of σ_c^2 is also inverse gamma with parameters $\widetilde{\alpha}_c$ and $\widetilde{\beta}_D$, which follow from the prior parameters as

$$\begin{aligned} \widetilde{\alpha}_c &= \frac{1}{2}T + \alpha_c, \\ \widetilde{\beta}_c &= \frac{1}{2}\sum_{t=0}^{T}\left(C_{t+1} - PTR^{Nq}_{t+1}\right)^2 + \beta_c. \end{aligned}$$

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