Non-negativity Constraints in Synchronization Economics

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Introduction :

In the preceding paper entitled "The Validity of the Generalized Reproduction Scheme,"[1] I had, in comparison with my scheme, reviewed Piero Sraffa's theory and I pointed out that the main defect of his theory laid deeply rooted in the assumption that "the wage is paid *post factum* as a share of the annual product, thus abandoning the classical economists' idea of a wage 'advanced' from capital,"[3] just contrary to what Karl Marx also assumed in his reproduction scheme in "Capital."

As for the concept of "advance," Joseph A. Schumpeter described the historical developments of economic thoughts as dividing the basic methodological ideas of economic analyses into two groups such as advance economics as seen in the physiocrat notion, "substantially the old idea of Quesnay,"[4] and successors of this concept like Karl Marx, Oskar Lange etc. on the one hand, and synchronization economics on the other. He noticed that there existed these two major theoretical cur-

1) Yuichi Shinzawa, "The Validity of the Generalized Reproduction Scheme—In Comparison with the Theories of P. Sraffa and of M. Kalecki—," *THE WASEDA COMMERCIAL REVIEW*, No. 277, Jun. 1979, pp. 91–119.

2) Piero Sraffa, *Production of Commodities by Means of Commodities*, (London: Cambridge University Press, 1960).

3) Sraffa, P., *ibid.*, p. 10.

4) Joseph A. Schumpeter, *History of Economic Analysis*, (New York: Oxford University Press, 1954), p. 564.

rents of economic thoughts such as when he said that "I cannot emphasize too strongly that this (the notion of advance) was a particular way of *interpreting* economic process and not at all directly suggested by the practice of life: in practice, the employer 'hires' the workman —or he may be said to 'buy' the latter's services—but he does not advance anything to him. Moreover, this interpretation means more than recognition of the trivial facts that whatever is being consumed must have been produced before; or that society at any moment always lives on the past and works for the future ; or, finally, that initial stocks are always among the date we must start from....... If 'capitalists' actually advance labor's real income and if this is to mean more than a monetary arrangement, discounts and 'abstinence' will have to be admitted among the essentials of the economic process whether we like them or not; that is, no analysis of production and consumption will be complete that does not take account of them in one way or another. This is important enough to justify a distinctive label for all analytic patterns that do work with the notion under discussion. We may call them *advance economics* and distinguish them from *synchronization economics,* that is, all analytic patterns that do not in a stationary process assign any fundamental role to the fact that what society lives on at any given moment is the result of past production, on the ground that, once a stationary process has been established, the flow of consumers' goods and the flow of productive services are synchronized so that the process works *as if* society did live on current production."[5)]

This quotation surely enough tells us what the notion of advance means. However, he did not try to find out or to construct a theory in order to synthesize the two currents of economic thoughts, by only saying that "for that matter, Walras' system proves sufficiently that these facts do not force us to make them the pivots of our analysis. But if we do make them the pivots, then a number of consequences suggest themselves that are not avoided simply by refusing to recog-

5) Schumpeter, J. A., *ibid.,* pp. 564–565.

nize them."[6]

The advent of Sraffa's theory in 1960 added one more theoretical notion to the analysis of economics; that is, the concept of the wage paid *post factum*. We may call it *post factum economics*. Nowadays, we can find a lot of manuscripts of the Sraffa type based especially on the theorem of Frobenius without paying much attention to the decisive difference laying between the assumption of Marx's economic model (or the Quesnay's) and that of Sraffa's type. Even if we can recognize the mathematical elegancy in solving of certain economic models (or systems), such kind of elegancy has no meaning without making the efforts to find out some of the persuasive expressions such as a bridge to combine the different theories as I have done in my explanation of the generalized reproduction scheme. I dare say once again that the Marxian model belongs to *advance economics*, and that the Sraffayan system belongs to *post factum economics*. Therefore a man who wishes to prove mathematically the validity of Marx's theory with the Sraffayan system has to first solve the differences in the assumptions of both theories in order to apply some mathematical treatment in Sraffa's theory to it.

The *post factum* model of the Sraffa type can not clarify the importance of the individual organic composition of capital, rate of profit and standard ratio in each individual sector upon which Marx placed emphasis.[7] I proved that the standard ratio (R_i^s) in the i th sector is not necessarily the same as those of the other sectors. Instead it depends on the following three parameters: the organic composition of capital (α_i), the capitalists' propensity to consume (γ_i) and the rate of surplus value (β_i) in each sector, where

$$R_i^s = \frac{1+\gamma_i\beta_i}{\alpha_i} \tag{1}$$

Furthermore, I could succeed in combining my generalized scheme

6) Schumpeter, J. A., *ibid.*, p. 564.

7) *cf.* Karl Marx, *Capital*, Vol. III, Part I, II and III.

with Sraffa's system which I had shown to be a special case and have included it in my scheme. In this present article, starting with the problem that the standard ratio R_i^s will vary as $R_i^{s\prime}$ in accordance with changes in the parameters α_i, β_i and γ_i into α_i', β_i' and γ_i', where

$$R_i^{s\prime}=\frac{1+\gamma_i'\beta_i'}{\alpha_i'} \tag{2}$$

I will show a condition due to the changes in the volume of output in Section 1. After illustrating some simple examples in Section 2, I will refer to non-negativity constraints in consumption in synchronization economics in Section 3, and non-negativity constraints of total output in Section 4, which are required as long as we treat only the realm of commodities (and services) flows in synchronization economics.

1. Changes in the Volume of Output in reference to Changes in the Parameters in Iteration Processes:

This section will treat changes in the elements of the generalized reproduction scheme with iteration processes, in reference to changes in the parameters α_i, β_i and γ_i. Changes in these three parameters of α_i, β_i and γ_i into α_i', β_i' and γ_i' respectively induce not only a change in the standard ratio R_i^s into $R_i^{s\prime}$, but also a change in the volume of commodity Q_i into Q_i'. For the sake of simplicity, we will assume that there is no divergence between a relative value μ_i and a relative price ν_i and, that the technological level of production is constant during the period in consideration. An equation of the i th sector in the generalized reproduction scheme of a certain period as I had proved is as follows:

$$(a_{1i}\nu_1+a_{2i}\nu_2+\cdots\cdots+a_{ni}\nu_n)+\frac{1+\gamma_i\beta_i}{\alpha_i+1+\gamma_i\beta_i}\mu_i=\nu_i \tag{3}$$

Multiplying both sides of this equation by the volume of commodity Q_i produced in the i th sector, we will get the following equation:

$$(a_{1i}\nu_1+a_{2i}\nu_2+\cdots\cdots+a_{ni}\nu_n)Q_i+\frac{1+\gamma_i\beta_i}{\alpha_i+1+\gamma_i\beta_i}\mu_iQ_i=\nu_iQ_i \tag{4}$$

As mentioned above, if the three parameters α_i, β_i and γ_i change into α_i', β_i' and γ_i', the volume of commodity Q_i is changed into Q_i', and the equation (5) will be as follows:

$$Z_i(a_{1i}\nu_1+a_{2i}\nu_2+\cdots\cdots+a_{ni}\nu_n)Q_i'+\frac{1+\gamma_i'\beta_i'}{\alpha_i'+1+\gamma_i'\beta_i'}\mu_iQ_i'=\nu_iQ_i' \qquad (5)$$

where

$$Z_i=\frac{\alpha_i'(\alpha_i+1+\gamma_i\beta_i)}{\alpha_i(\alpha_i'+1+\gamma_i'\beta_i')} \qquad (6)$$

which is the weight given to each original element a_{ji}. Equating each new element to a_{ji}' after the changes in the parameters, the equation (5) is rewritten as

$$(a_{1i}'\nu_1+a_{2i}'\nu_2+\cdots\cdots+a_{ni}'\nu_n)Q_i'+\frac{1+\gamma_i'\beta_i'}{\alpha_i'+1+\gamma_i'\beta_i'}\mu_iQ_i'=\nu_iQ_i' \qquad (7)$$

Even if we assume the constancy in the technological level of production, original elements a_{1i}, a_{2i}, $\cdots\cdots$, a_{ni} are superficially changed into a_{1i}', a_{2i}', $\cdots\cdots$, a_{ni}' with the change in volume of commodity. The reader may feel that changes in the elements present changes in coefficients of the technological level, and therefore an assumption on constancy in the technological level would be wrong. I wish to give notice that such kind of reasoning is too short sighted. Because the constancy in the technological level means that the relationship between the so-called coefficients of technology, which is the ratio (a_{ji}) of commodities (Q_{ji}) used up for producing a certain commodity (Q_i), and the coefficient of total output measured by the total value of labour power or total wages paid for workers as a unity, can be given as $\alpha_i+1+\gamma_i\beta_i$. When we therefore assume the constancy of the technological level, we have the following relational equation:

$$(\alpha_i+1+\gamma_i\beta_i)a_{ji}\nu_j=(\alpha_i+1+\gamma_i'\beta_i')a_{ji}'\nu_j \qquad (8)$$

$$a_{ji}'=\frac{(\alpha_i+1+\gamma_i\beta_i)a_{ji}}{\alpha_i+1+\gamma_i'\beta_i'} \qquad (9)$$

Summing up the left hand side of the equation (3) and substituting the result into the equation (5), we can easily get equation (6).

Changes in the generalized reproduction scheme are simply explained by expressing the first term at the left hand side of equation (3) by the concept of the organic composition of capital α_i, as

$$a_{1i}\nu_1 + a_{2i}\nu_2 + \cdots\cdots + a_{ni}\nu_n = \frac{\alpha_i \mu_i}{\alpha_i + 1 + \gamma_i \beta_i} \tag{10}$$

where equation (4) is

$$\frac{\alpha_i + 1 + \gamma_i \beta_i}{\alpha_i + 1 + \gamma_i \beta_i} \mu_i Q_i = \nu_i Q_i \tag{11}$$

and equation (7) is

$$\frac{\alpha_i + 1 + \gamma_i' \beta_i'}{\alpha_i + 1 + \gamma_i' \beta_i'} \mu_i Q_i' = \nu_i Q_i' \tag{12}$$

If we do not make any assumptions to the changes in the technological level, the organic composition of capital has the possibility to change, so that equations (8) and (9) will be

$$\frac{(\alpha_i + 1 + \gamma_i \beta_i)}{\alpha_i} a_{ji} \nu_j = \frac{(\alpha_i' + 1 + \gamma_i' \beta_i')}{\alpha_i'} a_{ji}' \nu_j \tag{13}$$

or

$$a_{ji}' \nu_j = \frac{\alpha_i'(\alpha_i + 1 + \gamma_i \beta_i)}{\alpha_i(\alpha_i' + 1 + \gamma_i' \beta_i')} a_{ji} \mu_i = Z_i a_{ji} \mu_i \tag{14}$$

As seen in equation (4) the initial condition is

$$\left(\frac{\alpha_i}{\alpha_i + 1 + \gamma_i \beta_i} + \frac{1}{\alpha_i + 1 + \gamma_i \beta_i} + \frac{\gamma_i \beta_i}{\alpha_i + 1 + \gamma_i \beta_i}\right) \mu_i Q_i = \nu_i Q_i \tag{15}$$

However, in case of a change in the parameters from α_i, β_i and or γ_i into α_i', β_i' and or γ_i', the following iteration processes will occur in the scheme.

Suppose that capitalists change their propensity to consume from γ_i to γ_i' and or their rate of surplus value from β_i to β_i' and or there is a change in the technological level, namely, in the organic composition of capital, the ratio of capitalists' propensity to consume for total products will vary as follows:

$$\frac{(\gamma_i'-\gamma_i)\beta_i}{\alpha_i+1+\gamma_i\beta_i}\gtreqless 0 \qquad \text{for} \qquad \gamma_i'\gtreqless\gamma_i \tag{16}$$

Therefore the relations parenthesized on the left hand of equation (15) in the first step have to be rewritten as

$$\frac{\alpha_i}{\alpha_i+1+\gamma_i\beta_i}+\frac{1}{\alpha_i+1+\gamma_i\beta_i}+\frac{\gamma_i\beta_i}{\alpha_i+1+\gamma_i\beta_i}+\frac{(\gamma_i'-\gamma_i)\beta_i}{\alpha_i+1+\gamma_i\beta_i} \tag{17}$$

and $\frac{\gamma_i-\gamma_i'}{\alpha_i+1+\gamma_i\beta_i}$ is either invested or consumed in terms of $\gamma_i\lesseqgtr\gamma_i'$ at the second step. We then have to add the result calculated to the sum of the first step

$$+\frac{\alpha_i'(\gamma_i-\gamma_i')\beta_i}{(\alpha_i'+1)(\alpha_i+1+\gamma_i\beta_i)}+\frac{(\gamma_i-\gamma_i')\beta_i}{(\alpha_i'+1)(\alpha_i+1+\gamma_i\beta_i)}$$
$$+\frac{(\gamma_i-\gamma_i')\beta_i\beta_i'\gamma_i'}{(\alpha_i'+1)(\alpha_i+1+\gamma_i\beta_i)} \tag{18}$$

at the second step.

Through the same process we subsequently have to add

$$+\frac{\alpha_i'(\gamma_i-\gamma_i')\beta_i\beta_i'(1-\gamma_i')}{(\alpha_i'+1)^2(\alpha_i+1+\gamma_i\beta_i)}+\frac{(\gamma_i-\gamma_i')\beta_i\beta_i'(1-\gamma_i')}{(\alpha_i'+1)^2(\alpha_i+1+\gamma_i\beta_i)}$$
$$+\frac{(\gamma_i-\gamma_i')\beta_i\beta_i'^2(1-\gamma_i')\gamma_i'}{(\alpha_i'+1)^2(\alpha_i+1+\gamma_i\beta_i)} \tag{19}$$

at the third step

$$+\frac{\alpha_i'(\gamma_i-\gamma_i')\beta_i\beta_i'^2(1-\gamma_i')^2}{(\alpha_i'+1)^3(\alpha_i+1+\gamma_i\beta_i)}+\frac{(\gamma_i-\gamma_i')\beta_i\beta_i'^2(1-\gamma_i')^2}{(\alpha_i'+1)^3(\alpha_i+1+\gamma_i\beta_i)}$$
$$+\frac{(\gamma_i-\gamma_i')\beta_i\beta_i'^3(1-\gamma_i')^2\gamma_i'}{(\alpha_i'+1)^3(\alpha_i+1+\gamma_i\beta_i)} \tag{20}$$

at the fourth step

....

and so on.

The final result of summing up all the items is

$$1+\frac{(\gamma_i-\gamma_i')\beta_i}{(\alpha_i+1+\gamma_i\beta_i)}\frac{\beta_i'}{(\alpha_i'+1+\gamma_i'\beta_i'-\beta_i')} \tag{21}$$

so that

$$Q_i'=\left\{1+\frac{(\gamma_i-\gamma_i')\beta_i}{(\alpha_i+1+\gamma_i\beta_i)}\frac{\beta_i'}{(\alpha_i'+1+\gamma_i'\beta_i'-\beta_i')}\right\}Q_i \qquad (22)$$

Equation (22) is a condition which combines the change in the volume of products with each change in the parameters. A change in the capitalists' propensity to consume plays a decisive role in this equation, because no change in the volume of products arises if γ_i is equal to γ_i'. In the case where there is a change in the parameters, this will be merely seen as a necessary condition for changing the volume of products. By illustrating the four examples in the next section, I will show that this equation is a necessary condition but at the same time it is not a sufficient condition.

2. Some Illustrations Using Three Simple Equation Systems:

This section will treat four kinds of schemes consisting of three sectors as in the case of the changing parameters α_i, β_i and γ_i, so as to give aid in visualizing the relations mentioned in the preceding section.

Case I:

As a basis for this analysis, an extended reproduction scheme[8] in the absolute dimension is illustrated for the sake of convenience in TABLE I-1, I-2, I-3 and I-4 in accordance with the classified items respectively Value, Volume, Coefficients of Technology and Prices. Thus the absolute values are assumed to be given in this table. If we were to treat this kind of analysis in the relative dimension where all the figures in each row are divided only by the absolute price of a certain commodity selected as a standard commodity (in the present examples the absolute price of products in the 3rd sector is chosen), measuring others in TABLE I-1 there will be no essential

8) Figures illustrated in TABLE I were used in my report given to the Japan Society of Monetary Economics in May, 1978.

TABLE I-1

			Invariable Capital				Variable Capital	Capitalists' Consumption	Total Output	Organic Composition of Capital	The Rate of Surplus Value	Capitalists' Propensity to Consume
			Individual Sector			Total						
			A_{1i}	A_{2i}	A_{3i}	C_i	V_i	Mk_i	A_i	α_i	β_i	γ_i
Value	Sectors	1	100	40	20	160	40	20	220	4	1	0.5
		2	96	39	15	150	50	25	225	3	1	0.5
		3	24	30	26	80	40	20	140	2	1	0.5
	Total		220	109	61	390	130	65	585	3	1	0.5
	Consumption		0	116	79		195.					

TABLE I-2

			Input Individual Sector			Total Output
			Q_{1i}	Q_{2i}	Q_{3i}	Q_i
Volume	Sectors	1	20.	10.	10.	44.
		2	19.2	9.75	7.5	56.25
		3	4.8	7.5	13.	70.
	Total		44.	27.25	30.5	

TABLE I-3

		Coefficients of Technology		
		a_{1i}	a_{2i}	a_{3i}
Sectors	1	$0.\dot{4}\dot{5}$	$0.2\dot{2}\dot{7}$	$0.2\dot{2}\dot{7}$
	2	$0.341\dot{3}$	$0.17\dot{3}$	$0.1\dot{3}$
	3	0.06857	0.10714	0.18571

TABLE I-4

		Value		Price	
		Absolute	Relative	Absolute	Relative
		υa_i	μ_i	ρ_i	ν_i
Sectors	1	5	2.5	5	2.5
	2	4	2.	4	2.
	3	2	1.	2	1.

difference between the two dimensions. But actually speaking, we can not measure the absolute value of both the value of labour power and the utility. Were we to assume that the relative value μ_i is equal to the relative price ν_i, if given the coefficients of technology a_{ij} and the three kinds of parameters such as α_i, β_i and γ_i, we could then easily solve for relative prices or relative values by using the following equational system which I have followed from the generalized reproduction scheme.[9]

$$\boldsymbol{\mu}_n = \boldsymbol{\nu}_n = [\mathbf{I} - \mathbf{A}'_{nn} - \mathbf{R}_{nn}]^{-1}\mathbf{g}_n \tag{23}$$

where

$$\boldsymbol{\nu}_n = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}, \quad \boldsymbol{\mu}_n = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A}'_{nn} = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}, \quad \mathbf{g}_n = \begin{bmatrix} a_{31} \\ a_{32} \end{bmatrix}, \quad \mathbf{R}_{nn} = \begin{bmatrix} \dfrac{1+\gamma_1\beta_1}{\alpha_1+1+\gamma_1\beta_1} & \dfrac{1+\gamma_2\beta_2}{\alpha_2+1+\gamma_2\beta_2} \end{bmatrix} \tag{24}$$

$$\begin{aligned}
&0.\dot{4}\dot{5}\nu_1 + 0.2\dot{2}\dot{7}\nu_2 + 0.2\dot{2}\dot{7} + 0.\dot{2}\dot{7}\mu_1 = \nu_1 \\
&0.341\dot{3}\nu_1 + 0.17\dot{3}\nu_2 + 0.\dot{1}\dot{3} + 0.\dot{3}\mu_2 = \nu_2 \\
&(1 - 0.\dot{4}\dot{5} - 0.\dot{2}\dot{7})\mu_1 - 0.2\dot{2}\dot{7}\mu_2 = 0.2\dot{2}\dot{7} \\
&-0.341\dot{3}\mu_1 + (1 - 0.17\dot{3} - 0.\dot{3})\mu_2 = 0.\dot{1}\dot{3}
\end{aligned} \tag{25}$$

$$\begin{bmatrix} 0.\dot{2}\dot{7} & -0.2\dot{2}\dot{7} \\ -0.341\dot{3} & 0.49\dot{3} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0.2\dot{2}\dot{7} \\ 0.\dot{1}\dot{3} \end{bmatrix}$$

The solution to this simultaneous equation is

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \frac{1}{0.0569\dot{6}} \begin{bmatrix} 0.14\dot{2}\dot{4} \\ 0.11\dot{3}\dot{9} \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2 \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{26}$$

We have tested all the relative prices in TABLE II, III, IV and V successively by this procedure and the final results are shown in these tables.

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9) Shinzawa, Yuichi, "An Essay on Solving the So-called Transformation Problem," *WBES*, No. 14, 1978.

Case II:

In this scheme, capitalists' rate of surplus value β_i' is assumed to be zero: the volume of capitalists' consumption M_{ki} is also zero. This scheme may be thought as that kind of socialist society in which there is no exploitation of surplus value and workers can receive a part of the income (V_i) as wages and another part of the income M_i as a kind of property income which comes from workers' offering their property to society as social fund. Therefore γ_i' in this scheme can be thought as workers' additional rate of consumption. Accordingly each item in column V_i consists of a wage and additional volume of consumption and each item M_{ki} is zero. Even if this scheme may be called either peoples' capitalist society or socialist society apart from serious arguments on an ideological problem, it is absolutely necessary for any society to invest a part of the surplus value into the reproduction process in order to develop the economic society so as to raise the standard living of its people. Thus columns A_{ij}, C_i and A_i are the same as those in Case I in an extended reproduction scheme if we include the part of workers' additional consumption in the variable capital, the organic composition of capital changes from α_i s into α_i' s, and all the figures of coefficients of technology superficially changes from TABLE I–4 into II–4. But there are no substantial changes in technology so that relative values and prices are quite the same as those of TABLE I.

Case III:

This scheme is a typical example of a simple reproduction scheme which is an extreme case. Namely, capitalists spend all surplus value which they can get, for consumption without any intention to invest. The volumes of total output and variable capital decrease but capitalists' consumption increases in comparison with those of TABLE I and the organic composition of capital as a whole changes from 3 to 3.02358.

TABLE II-1

	Invariable Capital				Variable Capital	Capitalists' Consumption	Total Output	Organic Composition of Capital	The Rate of Surplus Value	Workers' Additional Rate of Consumption
	Individual Sector			Total						
	$A_{1}i$	$A_{2}i$	$A_{3}i$	Ci	Vi	Mki	Ai	$\alpha i'$	$\beta i'$	$\gamma i'$
Value Sectors 1	100	40	20	160	40+20	0	220	2.$\dot{6}$	0	0.5
Value Sectors 2	96	39	15	150	50+25	0	225	2.	0	0.5
Value Sectors 3	24	30	26	80	40+20	0	140	1.$\dot{3}$	0	0.5
Total	220	109	61	390	195	0	585	2.	0	
Consumption	0	116	79		195					

TABLE III-1

			Invariable Capital				Variable Capital	Capitalists' Consumption	Total Output	Organic Composition of Capital	The Rate of Surplus Value	Capitalists' Propensity to Consume
			Individual Sector			Total						
			A_{1i}	A_{2i}	A_{3i}	C_i	V_i	Mk_i	A_i	α_i	β_i	γ_i
Value	Sectors	1	90.	36.	18.	144.	36.	36.	216.	4.	1	1
		2	84.	34.125	13.125	131.25	43.75	43.75	218.75	3.	1	1
		3	20.	25.	$21.\dot{6}$	$66.\dot{6}$	$33.\dot{3}$	$33.\dot{3}$	$133.\dot{3}$	2.	1	1
	Total		194.	95.125	$52.791\dot{6}$	$341.91\dot{6}$	$113.08\dot{3}$	$113.08\dot{3}$	$568.08\dot{3}$	3.02358	1	1
	Consumption		22.	123.625	$80.541\dot{6}$		$226.1\dot{6}$					

TABLE III-2

			Input Individual Sector			Total Output
			Q_{1i}	Q_{2i}	Q_{3i}	Q_i
Volume	Sectors	1	18.	9.	9.	43.2
		2	16.8	8.53125	6.5625	54.6875
		3	4.	6.25	$10.8\dot{3}$	$66.\dot{6}$
	Total		38.8	23.78125	26.39583	
			4.4	30.90625	40.27084	

TABLE III-3

		Coefficients of Technology		
		a_{1i}	a_{2i}	a_{3i}
Sectors	1	$0.41\dot{6}$	$0.208\dot{3}$	$0.208\dot{3}$
	2	0.3072	0.156	0.12
	3	0.06	0.09375	0.1625

TABLE IV-1

			Invariable Capital				Variable Capital	Capitalists' Consumption	Total Output	Organic Composition of Capital	The Rate of Surplus Value	Capitalists' Propensity to Consume
			Individual Sector			Total						
			A_{1i}	A_{2i}	A_{3i}	C_i	V_i	Mk_i	A_i	α_i	β_i	γ_i
Value	Sectors	1	112.5	45.	22.5	180.	45.	0	225.	4.	1	0
		2	112.0	45.5	17.5	175.	58.$\dot{3}$	0	233.$\dot{3}$	3.	1	0
		3	30.	37.5	32.5	100.	50.	0	150.	2.	1	0
	Total		254.5	128.	72.5	455.	153.$\dot{3}$	0	608.$\dot{3}$	2.96739	1	0
	Consumption		-29.5	105.$\dot{3}$	77.5		153.$\dot{3}$					

TABLE IV-2

			Input Individual Sector			Total Output
			Q_{1i}	Q_{2i}	Q_{3i}	Q_i
Volume	Sectors	1	22.5	11.25	11.25	45.
		2	22.4	11.375	8.75	58.$\dot{3}$
		3	6.	9.375	16.25	75.
	Total		50.9	32.	36.25	
			-5.9	26.$\dot{3}$	38.75	

TABLE IV-3

		Coefficients of Technology		
		a_{1i}	a_{2i}	a_{3i}
Sectors	1	0.5	0.25	0.25
	2	0.384	0.195	0.15
	3	0.08	0.125	0.21$\dot{6}$

Case IV:

This scheme is also an extreme case, in which capitalists spend all surplus value they get for investment. But this table can not insist on its existence just as it can be seen, because of the simple constraint that we can not sell more products than the total volume of products we produce, as you can notice this fact at the bottom of the first column of TABLE IV-1 and 2.

In input-output analysis, the total sum of column vectors of value which are input in each sector, as in the i th sector, has to be less than or equal to the total sum which is the output in the row vectors of value which are input from other sectors including self-consumption in the i th sector. The main defect of TABLE IV lies in the fact that the 1st sector violates this requirement. Then, can not we solve for this kind of problem? In next section, I will treat this problem more mathematically to find some clues in the solving of this type problem.

3. Non-negativity Constraints in Consumption:

As mentioned in the previous article "The Meaning of the Rate of Profit r,"[10] an equation of the i th sector in the input phase of relative dimension is

$$(a_{1i}\nu_1 + a_{2i}\nu_2 + \cdots\cdots + a_{ni}\nu_n)Q_i + \frac{1+\gamma_i\beta_i}{\alpha_i+1+\gamma_i\beta_i}\mu_i Q_i = \nu_i Q_i \qquad (4)$$

after changing parameters from α_i, β_i and γ_i into α_i', β_i' and γ_i', our system consists of the following phases as seen in the equation systems (27) and (28).

Input phase

$$Z_1(a_{11}\nu_1 + a_{21}\nu_2 + \cdots\cdots + a_{n1}\nu_n)Q_1' + \frac{1+\gamma_1'\beta_1'}{\alpha_1'+1+\gamma_1'\beta_1'}\mu_1 Q_1' = \nu_1 Q_1'$$

10) Shinzawa, Y., "The Validity of the Generalized Reproduction Scheme," *op. cit.*, pp. 109–113.

$$Z_2(a_{12}\nu_1 + a_{22}\nu_2 + \cdots\cdots + a_{n2}\nu_n)Q_2' + \frac{1+\gamma_2'\beta_2'}{\alpha_2'+1+\gamma_2'\beta_2'}\mu_2 Q_2' = \nu_2 Q_2'$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$Z_n(a_{1n}\nu_1 + a_{2n}\nu_2 + \cdots\cdots + a_{nn}\nu_n)Q_n' + \frac{1+\gamma_n'\beta_n'}{\alpha_n'+1+\gamma_n'\beta_n'}\mu_n Q_n' = \nu_n Q_n' \tag{27}$$

Output phase

$$Z_1 a_{11}\nu_1 Q_1' + Z_2 a_{12}\nu_1 Q_2' + \cdots\cdots + Z_n a_{1n}\nu_1 Q_n' + R_{c1} = \nu_1 Q_1'$$

$$Z_1 a_{21}\nu_2 Q_1' + Z_2 a_{22}\nu_2 Q_2' + \cdots\cdots + Z_n a_{2n}\nu_2 Q_n' + R_{c2} = \nu_2 Q_2'$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$Z_1 a_{n1}\nu_n Q_1' + Z_2 a_{n2}\nu_n Q_2' + \cdots\cdots + Z_n a_{nn}\nu_n Q_n' + R_{cn} = \nu_n Q_n' \tag{28}$$

where

$$Z_i = \frac{\alpha_i'(\alpha_i + 1 + \gamma_i\beta_i)}{\alpha_i(\alpha_i' + 1 + \gamma_i'\beta_i')} \tag{6}$$

R_{ci} is a value of the difference between input phase and output phase, which is ordinarily positive and thought to be a part that is consumed, but its sign (positive or negative) depends on the situation. We can not qualify anything to the sign of R_{ci} but we assume that the relative values (μ_i) are equal to the relative prices (ν_i) respectively and ν_n is equal to a unity. In each sector, if we subtract the equation of output phase from that of input phase respectively, we obtain the following equations:

$$\left\{(a_{21}\nu_2 + \cdots\cdots + a_{n1}\nu_n)Z_1 + \frac{1+\gamma_1'\beta_1'}{\alpha_1'+1+\gamma_1'\beta_1'}\nu_1\right\}Q_1' - a_{12}\nu_1 Z_2 Q_2' - \cdots\cdots$$
$$- a_{1n}\nu_1 Z_n Q_n' = R_{c1}$$

$$- a_{21}\nu_2 Z_1 Q_1' + \left\{(a_{12}\nu_1 + \cdots\cdots + a_{n2}\nu_n)Z_2 + \frac{1+\gamma_2'\beta_2'}{\alpha_2'+1+\gamma_2'\beta_2'}\nu_2\right\}Q_2' - \cdots\cdots$$
$$- a_{2n}\nu_2 Z_n Q_n' = R_{c2}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$- a_{n1}\nu_n Z_1 Q_1' - a_{n2}\nu_n Z_2 Q_2' + \cdots\cdots$$
$$+ \left\{(a_{1n}\nu_1 + \cdots\cdots + a_{n-1\,n}\nu_{n-1}) + \frac{1+\gamma_n'\beta_n'}{\alpha_n'+1+\gamma_n'\beta_n'}\nu_n\right\}Q_n' = R_{cn} \tag{29}$$

Summing up all left sides and right sides of the equation system (29), we obtain

$$R_{c1}+R_{c2}+\cdots\cdots+R_{cn}$$
$$=\frac{(1+\gamma_1'\beta_1')\nu_1Q_1'}{\alpha_1'+1+\gamma_1'\beta_1'}+\frac{(1+\gamma_2'\beta_2')\nu_2Q_2'}{\alpha_2'+1+\gamma_2'\beta_2'}+\cdots\cdots+\frac{(1+\gamma_n'\beta_n')\nu_nQ_n'}{\alpha_n'+1+\gamma_n'\beta_n'} \tag{30}$$

For the sake of simplicity, redefining diagonal elements and other elements such as

$$d_{ii}=\Big\{(a_{1i}\nu_1+a_{2i}\nu_2+\cdots\cdots+a_{i-1i}\nu_{i-1}+a_{i+1i}\nu_{i+1}+\cdots\cdots$$
$$+a_{ni}\nu_n)Z_i+\frac{(1+\gamma_i'\beta_i')\nu_i}{\alpha_i'+1+\gamma_i'\beta_i'}\Big\} \tag{31}$$

and

$$d_{ij}=a_{ij}\nu_iZ_j$$

the equation system (29) becomes

$$\begin{aligned} d_{11}Q_1'-d_{12}Q_2'-\cdots\cdots-d_{1n}Q_n'&=R_{c1}\\ -d_{21}Q_1'+d_{22}Q_2'-\cdots\cdots-d_{2n}Q_n'&=R_{c2}\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ -d_{n1}Q_1'-d_{n2}Q_2'-\cdots\cdots+d_{nn}Q_n'&=R_{cn} \end{aligned} \tag{32}$$

In this system, R_{ci} is not necessarily guaranteed to be positive as shown in Case IV. When several R_{ci} s of certain sectors show up as negative after making a calculation of iteration process as explained in Section 1, we should change (increase or decrease) some of the volumes of the products in whole sectors in order to change these negative R_{ci} s into positive R_{ci}^* such as:

$$\begin{aligned} d_{11}(Q_1'+\Delta Q_1)-d_{12}(Q_2'+\Delta Q_2)-\cdots\cdots-d_{1n}(Q_n'+\Delta Q_n)&=R_{c1}^*\geqq 0\\ -d_{21}(Q_1'+\Delta Q_1)+d_{22}(Q_2'+\Delta Q_2)-\cdots\cdots-d_{2n}(Q_n'+\Delta Q_n)&=R_{c2}^*\geqq 0\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ -d_{n1}(Q_1'+\Delta Q_1)-d_{n2}(Q_2'+\Delta Q_2)-\cdots\cdots+d_{nn}(Q_n'+\Delta Q_n)&=R_{cn}^*\geqq 0 \end{aligned} \tag{33}$$

or using the equation system (32)

$$R_{c1}+d_{11}\Delta Q_1-d_{12}\Delta Q_2-\cdots\cdots-d_{1n}\Delta Q_n=R_{c1}^*\geqq 0$$
$$R_{c2}-d_{21}\Delta Q_1+d_{22}\Delta Q_2-\cdots\cdots-d_{2n}\Delta Q_n=R_{c2}^*\geqq 0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$R_{cl} - d_{l1}\Delta Q_1 - d_{l2}\Delta Q_2 - \cdots\cdots + d_{ll}\Delta Q_l - \cdots\cdots - d_{ln}\Delta Q_n = R_{cl}^* \geqq 0$$
$$R_{cm} - d_{m1}\Delta Q_1 - d_{m2}\Delta Q_2 - \cdots\cdots + d_{mm}\Delta Q_m - \cdots\cdots - d_{mn}\Delta Q_n = R_{cm}^* \geqq 0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$R_{cn} - d_{n1}\Delta Q_1 - d_{n2}\Delta Q_2 - \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots + d_{nn}\Delta Q_n = R_{cn}^* \geqq 0 \tag{34}$$

In this equation system, all R_{ci}^*s are more than or at least equal to zero, and we assume that $R_{c1}, \cdots\cdots, R_{cl}$ are negative and $R_{cm}, \cdots\cdots, R_{cn}$ are positive. Under these assumptions negative $R_{c1}, \cdots\cdots, R_{cl}$ should be changed to positive $R_{c1}^*, \cdots\cdots, R_{cl}^*$ which are more than or at least equal to zero, by increasing $\Delta Q_1, \cdots\cdots, \Delta Q_l$ on the one hand and decreasing $\Delta Q_m, \cdots\cdots \Delta Q_n$ on the other hand, keeping $R_{cm}^*, \cdots\cdots, R_{cn}^*$ more than or at least equal to zero.

As in the equation (30), after changing ΔQ_i

$$R_{c1}^* + R_{c2}^* + \cdots\cdots + R_{cn}^*$$
$$= \frac{(1+\gamma_1'\beta_1')\nu_1}{\alpha_1'+1+\gamma_1'\beta_1'}(Q_1'+\Delta Q_1) + \frac{(1+\gamma_2'\beta_2')\nu_2}{\alpha_2'+1+\gamma_2'\beta_2'}(Q_2'+\Delta Q_2)$$
$$+ \cdots\cdots + \frac{(1+\gamma_n'\beta_n')\nu_n}{\alpha_n'+1+\gamma_n'\beta_n'}(Q_n'+\Delta Q_n) \tag{35}$$

if the total volume of consumption can not be changed,

$$R_{c1} + R_{c2} + \cdots\cdots + R_{cn} = R_{c1}^* + R_{c2}^* + \cdots\cdots + R_{cn}^* \tag{36}$$

so that

$$(R_{c1}^* - R_{c1}) + (R_{c2}^* - R_{c2}) + \cdots\cdots + (R_{cn}^* - R_{cn})$$
$$= \frac{(1+\gamma_1'\beta_1')\nu_1}{\alpha_1'+1+\gamma_1'\beta_1'}\Delta Q_1 + \frac{(1+\gamma_2'\beta_2')\nu_2}{\alpha_2'+1+\gamma_2'\beta_2'}\Delta Q_2 + \cdots\cdots$$
$$+ \frac{(1+\gamma_n'\beta_n')\nu_n}{\alpha_n'+1+\gamma_n'\beta_n'}\Delta Q_n$$
$$= R_1'\nu_1\Delta Q_1 + R_2'\nu_2\Delta Q_2 + \cdots\cdots + R_l'\nu_l\Delta Q_l + R_m'\nu_m\Delta Q_m + \cdots\cdots$$
$$+ R_n'\nu_n\Delta Q_n = 0 \tag{37}$$

where

$$R_i' = \frac{1+\gamma_i'\beta_i'}{\alpha_i'+1+\gamma_i'\beta_i'} \tag{38}$$

From the equations (34) and (36), the equation (37) can be rewritten as

$$(R_{c1}^*-R_{c1})+(R_{c2}^*-R_{c2})+\cdots\cdots+(R_{cn}^*-R_{cn})$$
$$=(d_{11}-d_{21}-\cdots\cdots-d_{n1})\Delta Q_1$$
$$+(-d_{12}+d_{22}-\cdots\cdots-d_{n2})\Delta Q_2+\cdots\cdots$$
$$+(-d_{1n}-d_{2n}-\cdots\cdots+d_{nn})\Delta Q_n=0 \qquad (39)$$

Now, $R_{c1},\cdots\cdots,R_{cl}$ are assumed to be negative, so that we have to increase $Q_1',\cdots\cdots,Q_l'$ such as

$$\Delta Q_1 \geqq \frac{|R_{c1}|}{\nu_1}>0,\cdots\cdots,\quad \Delta Q_l \geqq \frac{|R_{cl}|}{\nu_l}>0 \quad \text{at least.}^{11)} \qquad (40)$$

In the equation (37), if $\Delta Q_1,\cdots\cdots,\Delta Q_l$ are determined as positive at the minimal level,

$$H=R_1'\nu_1\Delta Q_1+R_2'\nu_2\Delta Q_2+\cdots\cdots+R_l'\nu_l\Delta Q_l>0 \qquad (41)$$

is known, but other elements, namely,

$$R_m'\nu_m\Delta Q_m+\cdots\cdots+R_n'\nu_n\Delta Q_n<0 \qquad (42)$$

can not be determined because $\Delta Q_m',\cdots\cdots,\Delta Q_n'$ are left unknown.

From the equation (37) and (41)

$$\Delta Q_m=\frac{1}{R_m'\nu_m}\left\{-H-(R_{m+1}'\nu_{m+1}\Delta Q_{m+1}+\cdots\cdots+R_n'\nu_n\Delta Q_n)\right\}$$

$$\Delta Q_{m+1}=\frac{1}{R_{m+1}'\nu_{m+1}}\left\{-H-(R_m'\nu_m\Delta Q_m+\cdots\cdots+R_n'\nu_n\Delta Q_n)\right\}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\Delta Q_n=\frac{1}{R_n'\nu_n}\left\{-H-(R_m'\nu_m\Delta Q_m+\cdots\cdots+R_{n-1}'\nu_{n-1}\Delta Q_{n-1})\right\} \qquad (43)$$

Taking one equation of the ith sector ($i=1,\cdots\cdots,l$) in output phase (28),

$$\nu_iQ_i'=Z_1a_{i1}\nu_iQ_1'+Z_2a_{i2}\nu_iQ_2'+\cdots\cdots+Z_la_{il}\nu_iQ_l'+Z_ma_{im}\nu_iQ_m'$$
$$+\cdots\cdots+Z_na_{in}\nu_iQ_n'+R_{ci} \qquad (44)$$

By an assumption R_{ci} as being negative, we have to change R_{ci} to R_{ci}^* to be up to zero at least. Namely the equation (44) will be

11) Notice that as $|R_{ci}|$ belongs to the absolute dimension, we have to divide it by an absolute price p_i to get an increment $\Delta Q_i'$.

$$\nu_i(Q_i'+\Delta Q_i)=Z_1a_{i1}\nu_i(Q_1'+\Delta Q_1)+Z_2a_{i2}\nu_i(Q_2'+\Delta Q_2)+\cdots\cdots$$
$$+Z_ia_{ii}\nu_i(Q_i'+\Delta Q_i)+\cdots\cdots+Z_la_{il}\nu_i(Q_l'+\Delta Q_l)$$
$$+Z_ma_{im}\nu_i(Q_m'+\Delta Q_m)+\cdots\cdots+Z_na_{in}\nu_i(Q_n'+\Delta Q_n) \tag{45}$$

If

$$H_i=\nu_i(Q_i'+\Delta Q_i)-\sum_{j=1}^{l}Z_ja_{ij}\nu_i(Q_j'+\Delta Q_j) \tag{46}$$

equation (45) is rewritten as:

$$H_i=Z_ma_{im}\nu_i(Q_m'+\Delta Q_m)+\cdots\cdots+Z_na_{in}\nu_i(Q_n'+\Delta Q_n) \tag{47}$$

Taking one element, say, ΔQ_n from the equation system (43) and substitute it into (47), we obtain

$$D_i=e_{im}\Delta Q_m+e_{im+1}\Delta Q_{m+1}+\cdots\cdots+e_{in-1}\Delta Q_{n-1} \tag{48}$$

where

$$D_i=H_i-\sum_{j=m}^{n}Z_ja_{ij}\nu_iQ_j'+\frac{Z_na_{in}\nu_i}{R_n'\nu_n}H \tag{49}$$

and

$$e_{ij}=Z_ja_{ij}\nu_i-\frac{Z_na_{in}\nu_iR_j'\nu_j}{R_n'\nu_n} \tag{50}$$

Therefore

$$\begin{aligned} D_1&=e_{1m}\Delta Q_m+e_{1m+1}\Delta Q_{m+1}+\cdots\cdots+e_{1n-1}\Delta Q_{n-1}\\ D_2&=e_{2m}\Delta Q_m+e_{2m+1}\Delta Q_{m+1}+\cdots\cdots+e_{2n-1}\Delta Q_{n-1}\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ D_l&=e_{lm}\Delta Q_m+e_{lm+1}\Delta Q_{m+1}+\cdots\cdots+e_{ln-1}\Delta Q_{n-1} \end{aligned} \tag{51}$$

When l is equal to $n-m$, we can uniquely solve $(l-1)$ kinds of simultaneous equation systems, each of which is based on a certain sector chosen as the n th sector. Generally speaking, we have the following three cases:

Case 1 $\quad l>n-m$ (52)

Case 2 $\quad l=n-m$ (53)

Case 3 $\quad l<n-m$ (54)

This kind of identification problem often arises in solving simultaneous equation system. Therefore in Case 1 we have to select $k=n-m$ equations where the combination of selecting k equations is ${}_lC_{n-m}$,

TABLE V-1

			Invariable Capital				Variable Capital	Capitalists' Consumption	Total Output	Organic Composition of Capital	The Rate of Surplus Value	Capitalists' Propensity to Consume
			Individual Sector			Total						
			A_{1i}	A_{2i}	A_{3i}	C_i	V_i	Mk_i	A_i	α_i'	β_i'	γ_i'
Value	Sectors	1	127.25	50.9	25.24	203.6	50.9	0	254.5	4.	1	0
		2	$95.69\dot{4}\dot{5}$	$38.875\dot{9}\dot{0}$	$14.952\dot{2}\dot{7}$	$149.52\dot{2}\dot{7}$	$49.84\dot{0}\dot{9}$	0	$199.\dot{3}\dot{6}$	3.	1	0
		3	$31.55\dot{5}\dot{4}$	39.44432	34.18508	$105.1\dot{8}\dot{4}$	$52.59\dot{2}\dot{4}$	0	$157.77\dot{7}\dot{2}$	2.	1	0
	Total		254.5	129.22023	74.37735	$458.30\dot{7}\dot{5}$	$153.\dot{3}$	0	611.6409	2.98892	1	0
	Consumption		0.	70.14341	83.39924		$153.\dot{3}$					

TABLE V-2

			Input			Total Output
			Individual Sector			
			Q_{1i}	Q_{2i}	Q_{3i}	Q_i
Volume	Sectors	1	25.45	12.725	12.725	50.9
		2	$19.318\dot{9}\dot{0}$	9.71898	7.47614	$49.84\dot{0}\dot{9}$
		3	$6.311\dot{0}\dot{9}$	9.86108	17.09254	$78.888\dot{6}\dot{3}$
	Total		50.9	32.30506	37.18867	
			0.	17.53585	41.69996	

TABLE V-3

		Coefficients of Technology		
		a_{1i}	a_{2i}	a_{3i}
Sectors	1	0.5	0.25	0.25
	2	0.384	0.195	0.145
	3	0.08	0.125	$0.21\dot{6}$

TABLE V-4

		Value		Price	
		Absolute	Relative	Absolute	Relative
		υa_i	μ_i	ρ_i	ν_i
Sectors	1	5	2.5	5	2.5
	2	4	2.	4	2.
	3	2	1.	2	1.

from $D_1, \cdots\cdots, D_l$, and in Case 3, we have to select $k'=l$ unknown variables from $\varDelta Q_m, \cdots\cdots, \varDelta Q_{n-1}$ where the combination of selecting k' unknown variables is ${}_{n-m}C_l$.

As to Case 2, I have mentioned above.

All solutions described above do not contradict any premises and assumptions which we have placed. When we search for optimality in the reproduction process with a certain necessity of economic policy, for instance, by putting constraints on the output such that R_i^* is more than a certain level, our model can be built on a linear inequality model using the techniques of linear programming. In this section I will not proceed to discuss this problem furthermore.

As the result of these procedures, Case IV can be uniquely solved because l is equal to one, $n-m$ is also equal to one, and the contents of TABLE IV are changed into TABLE V.

4. Non-negativity Constraints of Total Output

In order to firmly prove that my reasoning is never absurd, I will take one more example of the five different spheres of production from Volume III of "Capital," which I used in solving the transformation problem.[12] The figures of the elements in a matrix of invariable capital in TABLE VI were arbitrarily selected after due consideration of the condition of Simon-Hawkins. Purposely to use the non-negativity constraints of consumption, I will treat an extreme case by assuming that capitalists do not spend their surplus value for consumption but invest all value they exploit in the reproduction process. For the sake of simplicity the technological level is assumed to be constant. Therefore the organic composition of capital does not change. And workers are assumed to be able to transfer freely among sectors. TABLE VII is the result of calculation of the first step. As I noticed

12) Karl Marx, *Capital,* (Moscow: Progress Publishers, 1956), Vol. III, p. 155 and Yuichi Shinzawa, "An Essay on Solving the So-called Transformation Problem," *op. cit.*, footnote 9).

in the preceding section, we obtain different kinds of solutions of simultaneous equations from different non-negative constraints of consumption. Are all solutions valid *in economics?* Merely the non-negativity constraints of consumption are solely not always sufficient. All solutions ΔQ_i s have to satisfy the following conditions such as

$$\begin{aligned} &Q_1' + \Delta Q_1 \geqq 0 \\ &Q_2' + \Delta Q_2 \geqq 0 \\ &\cdots\cdots\cdots\cdots \\ &Q_i' + \Delta Q_i \geqq 0 \\ &\cdots\cdots\cdots\cdots \\ &Q_n' + \Delta Q_n \geqq 0 \end{aligned} \tag{55}$$

I call this condition, $Q_i + \Delta Q_i \geqq 0$, non-negativity constraints of total output (the second constraints). Only when both constraints of non-negativity in consumption and total output are satisfied, we can have a region of feasible solutions, in case of not taking the realm of finance into consideration. From the view point of non-negativity constraints of consumption our example does not clearly satisfy them. It is necessary to increase ΔQ_1 and ΔQ_2 in order to satisfy these constraints by which R_i^* is equal to at least zero.

$$\begin{aligned} \Delta Q_1 &\geqq \frac{|R_{c1}|}{p_1} = 2.69920 \\ \Delta Q_2 &\geqq \frac{|R_{c3}|}{p_3} = 0.44692 \end{aligned} \tag{40'}$$

In this case solutions are entirely the same regardless of having chosen a sector as a basis of calculation, so that if we choose the second sector as a basis, our equation system is as follows:

$$H = R_1'\nu_1 \Delta Q_1 + R_3'\nu_3 \Delta Q_3 = 2.37840 \tag{41'}$$

$$\begin{aligned} H_1 &= \nu_1(Q_1' + \Delta Q_1) - \{Z_1 a_{11} \nu_1 (Q_1' + \Delta Q_1) + Z_3 a_{13} \nu_1 (Q_3' + \Delta Q_3)\} \\ &= 35.82760 \\ H_3 &= \nu_3(Q_3' + \Delta Q_3) - \{Z_1 a_{31} \nu_3 (Q_1' + \Delta Q_1) + Z_3 a_{33} \nu_3 (Q_3' + \Delta Q_3)\} \\ &= 39.20280 \end{aligned} \tag{46'}$$

$$D_1 = H_1 - (Z_2 a_{12} \nu_1 Q_2' + Z_4 a_{14} \nu_1 Q_4' + Z_5 a_{15} \nu_1 Q_5') + \frac{Z_2 a_{12} \nu_1}{R_2' \nu_2} H$$

$$=0.23981 \tag{49'}$$

$$D_3 = H_3 - (Z_2 a_{32} \nu_3 Q_2' + Z_4 a_{34} \nu_3 Q_4' + Z_5 a_{35} \nu_3 Q_5') + \frac{Z_2 a_{32} \nu_3}{R_2' \nu_2} H$$

$$= -1.19270$$

$$e_{ij} = Z_j a_{ij} \nu_i - \frac{Z_n a_{in} \nu_i R_j' \nu_j}{R_n' \nu_n} \tag{50}$$

$$e_{14} = -0.06, \qquad e_{15} = 0.14$$

$$e_{34} = 1.92, \qquad e_{35} = 0.03\dot{6}$$

From the following equations

$$e_{14} \Delta Q_4 + e_{15} \Delta Q_5 = D_1$$

$$e_{34} \Delta Q_4 + e_{35} \Delta Q_5 = D_3$$

$$\begin{bmatrix} -0.06 & 0.14 \\ 1.92 & 0.03\dot{6} \end{bmatrix} \begin{bmatrix} \Delta Q_4 \\ \Delta Q_5 \end{bmatrix} = \begin{bmatrix} 0.23981 \\ -1.19270 \end{bmatrix} \tag{51'}$$

finally we obtain

$$\Delta Q_4 = -0.64860$$

$$\Delta Q_5 = 1.43501$$

From equation (43)

$$\Delta Q_2 = \frac{1}{R_2' \nu_2} \{ -H - (R_4' \nu_4 \Delta Q_4 + R_5' \nu_5 \Delta Q_5) \} \tag{43'}$$

we then get

$$\Delta Q_2 = -2.74797$$

By using these results

$$\begin{aligned} Q_1' + \Delta Q_1 &= 20.\dot{4}\dot{5} \quad + 2.69920 = 23.15375 > 0 \\ Q_2' + \Delta Q_2 &= 24.28571 - 2.74798 = 21.53774 > 0 \\ Q_3' + \Delta Q_3 &= 13.\dot{3} \quad + 0.44692 = 13.78025 > 0 \\ Q_4' + \Delta Q_4 &= 13.60294 - 0.64860 = 12.95434 > 0 \\ Q_5' + \Delta Q_5 &= 51.31579 + 1.43502 = 52.75081 > 0 \end{aligned} \tag{55'}$$

The final figures are illustrated in TABLE VIII.

Starting from TABLE VII, I will show one extreme example in which an increase in products (ΔQ_3) by 3 units produced in the 3rd sector involves an extreme decrease in products in the 2nd sector, as seen in TABLE VIII. By the non-negativity constraints of consumption,

TABLE VI

	A_{1i}	A_{2i}	A_{3i}	A_{4i}	A_{5i}	C_i	V_i	A_i	α_i	β_i	γ_i
I	30.9375	22.5	28.125	6.75	1.6875	90.	22.5	112.5	4.	1	0
II	40.07143	18.21429	17.	1.21429	8.5	85.	36.42857	121.42857	$2.\dot{3}$	1	0
III	20.	$1.\dot{3}$	$26.\dot{6}$	$30.\dot{6}$	$1.\dot{3}$	80.	$53.\dot{3}$	$133.\dot{3}$	1.5	1	0
IV	16.32353	4.35294	59.85294	10.88235	1.08824	92.5	16.32353	108.82353	$5.\dot{6}$	1	0
V	20.01316	30.27632	6.15789	37.46053	3.59211	97.5	5.13158	102.63158	19.0	1	0
Total	127.34562	76.67688	137.80250	86.97383	16.20117	445.		578.71701	3.32792	1	0
Consumption	−14.84562	44.75170	−4.46917	21.84970	86.43041		133.71701				

TABLE VII

	A_{1i}	A_{2i}	A_{3i}	A_{4i}	A_{5i}	C_i	V_i	A_i	Q'_i	ΔQ_i	$Q'_i+\Delta Q_i$
I	35.02004	25.46912	31.83640	7.64074	1.91018	101.87629	25.46912	127.34562	$20.\dot{4}\dot{5}$	2.69920	23.15375
II	35.53727	16.15330	15.07642	1.07689	7.53821	75.38208	32.30660	107.68868	24.28571	−2.74798	21.53774
III	20.67038	1.37803	27.56050	31.69458	1.37803	82.68150	55.12100	137.80250	$13.\dot{3}$	0.44692	13.78025
IV	15.54520	4.14539	56.99908	10.36347	1.36347	88.08949	15.54520	103.63470	13.60294	−0.64860	12.95434
V	20.57280	31.12295	6.33009	3.69255	3.69255	100.22646	5.27508	105.50153	51.31579	1.43498	52.75077
Total	127.34569	78.26879	137.80249	89.28373	15.55532	448.26182		581.97303			
Consumption		29.41989		14.35097	89.94622		133.71701				

TABLE Ⅷ

	A_{1i}	A_{2i}	A_{3i}	A_{4i}	A_{5i}	C_i	V_i	A_i	Q_i	ΔQ_i	$Q_i+\Delta Q_i$
I	35.02004	25.46912	31.83640	7.64074	1.91018	101.87629	25.46912	127.34562	$20.\dot{4}\dot{5}$	2.69920	23.15375
II	0.51587	0.23449	0.21886	0.01563	0.10943	1.09428	0.46898	1.56326	24.28571	-23.97306	0.31265
III	26.67038	1.77803	35.56050	40.89458	1.77803	106.68150	71.12100	177.80250	$13.\dot{3}$	4.44692	17.78025
Ⅳ	26.83672	7.15646	98.40132	17.89115	1.78911	152.07477	26.83672	178.91149	13.60294	8.76100	22.36394
V	38.30264	57.94501	11.78543	71.69468	6.87483	186.60258	9.82119	196.42377	51.31579	46.89610	98.21189
Total	127.34565	92.58311	177.80251	138.13677	12.46158	548.32942		682.04664			
Consumption		-91.01985		40.77472	183.96219		133.71701				

TABLE Ⅸ

	A_{1i}	A_{2i}	A_{3i}	A_{4i}	A_{5i}	C_i	V_i	A_i	Q'_i	ΔQ_i	$Q'_i+\Delta Q_i$
I	35.02004	25.46912	31.83640	7.64074	1.91018	101.87629	25.46912	127.34562	$20.\dot{4}\dot{5}$	2.69920	23.15375
II	40.30664	18.32120	17.09979	1.22141	8.54989	85.49893	36.64240	122.14133	24.28571	0.14255	24.42827
III	20.67038	1.37803	27.56050	31.69458	1.37803	82.68150	55.12100	137.80250	$13.\dot{3}$	0.44692	13.78025
Ⅳ	14.80720	3.94859	54.29307	9.87147	0.98715	83.90747	14.80720	98.71467	13.60294	-1.26361	12.33933
V	6.54143	9.89602	2.01275	12.24422	1.17410	31.86852	1.67729	33.54581	51.31579	-34.54288	16.77291
Total	117.34569	56.01295	132.80251	62.67241	13.99935	385.83271		519.54993			
Consumption	9.99992	63.12838	5.	36.04225	19.54646		133.71701				

TABLE IX can never exist. Thus to this example, other sectors have to change their production to satisfy these constraints. Returning to the TABLE VI, if we give some limits such as $R_{c1}^* \geqq 5$ and $R_{c3}^* \geqq 2.5$ to products produced in sector 1 and 2, we will obtain results shown in TABLE IX.

Conclusion :

In this chapter I detailed non-negative constraints as in case of changes in the parameters of synchronization economics without taking the realm of finance in consideration. I proved that, even when the level of technology does not change or the organic composition of capital (α_i) does not change either during a certain period, the so-called technological coefficients (a_{ij}) frequently used in the analysis of the input-output type are superficially changed into (a_{ij}') in *ex post* with changes in the rate of surplus value (β_i) and the capitalists' propensity to consume (γ_i). As for the organic composition of capital to which I have related, there exist some different interpretations of it. Especially, L. R. Klein defined the organic composition of capital as $\frac{C}{C+V}$ in his paper "Effective Demand and Employment."[13] But S. Tsuru defined it as "the ratio of $\frac{C}{V}$" in his paper "Keynes versus Marx."[14] Tsuru's interpretation about the concept is correct, because Karl Marx said that "The composition of capital is to be understood in a two-fold sense. On the side of value, it is determined by the proportion in which it is divided into constant capital or value of the means of production, and variable capital or value of labour-power, the sum total of wages. On the side of material, as it functions in the process of production, all capital is divided into means of production and living labour-power. This latter composition is determined by

13) Lawrence R. Klein, *Marx and Modern Economics* edited by David Horowitz, (London: MacGibbon & Kee Ltd., 1968), p. 155.
14) Shigeto Tsuru, *Marx and Modern Economics, op. cit.*, p. 184.

the relation between the mass of the means of production employed, on the one hand, and the mass of labour necessary for their employment on the other. I call the former the *value-composition,* the latter the *technical composition* of capital. Between the two there is a strict correlation. To express this, I call the value-composition of capital, in so far as it is determined by its technical composition and mirrors the changes of the latter, the *organic composition* of capital."[15]

According to this definition of the organic composition of capital, it means only that the ratio of $\frac{C_i}{V_i}$ is equal to α_i, $\left(\frac{C_i}{V_i}=\alpha_i\right)$, as Tsuru defined. Thus, there may be a case that some progress in technology is thought to work for saving the volume of invariable capital (C_i) compared with variable capital (V_i). Therefore α_i may be thought to decrease. But as a general tendency, the technological progress involves an increase in capital. Can we not solve this contradiction? Generally speaking, the technological progress mainly works for machineries, equipments and administrative systems of production, not for raw materials or intermediate products and machineries that require more raw materials than before. If these assumptions are admitted to be used, by the technological progress some a_{ij} s belonging to machineries or equipments will increase. Therefore a_i depends on the difference between the volume of the former and that of the latter. Considering the problem of technological progress as I had discussed in my paper "Changes in Variables and Parameters in the Capital Formation,"[16] in the illustrations of this paper I assumed that α_i and β_i were constant in order to clarify the effects of changes in γ_i on a_{ij}.

Through the six papers I have written on the generalized reproduction scheme based on Keynes' most important premise of saving being equal to investment, if we get rid of the concept "advance" and in-

15) Marx, K. *op. cit.,* Vol. I, p. 574.

16) Yuichi Shinzawa, "Changes in Variables and Parameters in the Capital Formation," *Waseda Business Review* (IRBA Waseda University) No. 4, 1978, pp. 125–154.

stead introduce the concept of iteration processes into an economic system, it can be proved to be entirely correct in connection with various kinds of reproduction schemes of advance economics: as those theories of Marx's, Tsuru's and Lange's. In connection with concepts of Leontief's input-output analysis, of Kalecki's model and of Sraffa's *post factum* model, the generalized reproduction scheme was proved to have no contradictions in its structure. By having done so, I was assured that many different types of economics were synthesized in my generalized reproduction scheme. However, the reader of my paper may have the following questions concerning assumptions of my scheme, which I have placed in order to avoid unnecessary confusion and misunderstanding:

1. Is the relative value (μ_i) equal to the relative price (ν_i)?
2. Are relations among relative prices constant?
3. How can we explain financial phenomena after throwing away the concept of "advance"?

and so on. Particularly in connection with problem 3, I will discuss these in accordance with financial phenomena in "ADVANCE vs. FINANCE."[17]

17) Yuich Shinzawa, "Advance vs. Finance," *WBES*, No. 15, 1979.