

**NEW DOCTORAL DEGREES  
IN THE DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF OSIJEK**

**Dr. Ljerka Jukić** received her PhD in Mathematics from the Department of Mathematics of the University of Zagreb on 7th July 2011 with the dissertation entitled “TEACHING AND LEARNING OUTCOMES IN UNDERGRADUATE CALCULUS COURSES FOR STUDENTS OF TECHNICAL AND SCIENCE STUDIES IN CROATIA AND DENMARK” (Mentors: Prof. Bettina Dahl Søndergaard, Prof. Franka Miriam Brückler).

**Abstract**

The issue of retention of knowledge is an important area of research that has the potential to improve teaching practices and achieve curricular goals. This issue is a matter of great concern since many educators and mathematics lecturers noticed a low level of students retention of definitions, concepts and theorems after mathematics course instruction and examination. Only small number of studies investigated retention of university students' knowledge, especially in calculus, therefore this study provides valuable insights into the retained knowledge in calculus, focusing on the non-mathematics students. We investigated retained calculus concepts for students coming from two universities that differ in various aspects, one in Croatia and one in Denmark.

In the first chapter we give motivation for this study. Also we provide historical overview of the most important mathematicians who gave their contribution to the first notions and findings in integral and differential calculus. The chapter ends with the core topics of differential and integral calculus introduced to the first year university students.

We compared the calculus teaching at a Croatian and Danish university in terms of students' retention of core calculus concepts in integrals and derivatives two months after instructions. We focused on the procedural and conceptual knowledge. The results showed that for both universities, a large portion of taught subject matter was forgotten. Investigating students grades and scores on the questionnaire, we found that the passing grades of Calculus 1 course did not predict the results in the test two months later. Students with the lowest passing grades had better results two month later, or there was no difference.

Next we investigated the retention of core procedural and conceptual concepts in integral calculus among Croatian students in physics, electrical engineering, and civil engineering study programs. The students were surveyed three times in several time periods: two, six and ten months after instructions in integral calculus, combining that with the beginning and the end of Calculus 3. The result showed that the students improved their answers to the conceptual questions. In the second investigation that took place six months after examination and at the beginning of Calculus 3, students answers showed deterioration in most of the procedural questions. In the third investigation that took place ten months after instructions in integrals and at the end of Calculus 3, procedural knowledge improved.

This strongly indicated that students' procedural knowledge was very fragile, and that students might forget procedural knowledge quickly when this knowledge is not used any more. We discovered that students generalize every integral of the form

$$\int \frac{dx}{f(x)} \text{ to } \ln(f(x)) + C.$$

We investigated students' knowledge 14 months after instructions, we found effect of mental structures called met-befores and met-afters that affected students' knowledge in differential calculus. When we investigated students' concept image and concept definition of definite integral two months after instructions and examination, we discovered that students retained highly incoherent knowledge of definite integral as an area. Here we also added to Tall-Vinner discussion claiming that it differs from student to student if concept definition is part of the concept image. Also, we found that improper use of technology leads to retention of superficial conceptual knowledge and deterioration of procedural knowledge.

### Published papers

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- [3] F. M. Brückler, **Lj. Jukić**, *How science students understand, remember and use mathematics*, Monograph of 4th International Conference Research in Didactics of the Sciences, Poland, Krakow, (2010).
- [4] **Lj. Jukić**, B. Dahl Søndergaard, *What affects retention of core calculus concepts among university students? A study of different teaching approaches in Croatia and Denmark*, Proceedings of 7th Congress of the European Society for Research in Mathematics Education (CERME), Rzeszow, Poland, (2011).
- [5] **Lj. Jukić**, B. Dahl Søndergaard, *University students' concept image and retention of the definite integral*, Proc. 35th Conf. of the Int. Group for the Psychology of Mathematics Education, Turkey, Ankara: PME, (2011).