# Experimental measurement and model validation of COD in pipe under bending with off-centered circumferential crack 

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#### Abstract

The leak area of circumferential through-thickness crack in pipe under bending depends on the position of the crack with respect to the bending plane. In leak-before-break (LBB) analysis, the assumption that the crack is symmetrically placed with respect to the bending plane is not necessarily conservative. In this work, the crack opening of circumferential cracks, off-centered with respect to the bending plane, was investigated experimentally. Here, three pipe geometries and two crack lengths were investigated. For each crack, the centred and two off-centered configuration were examined. The crack opening displacement (COD) distribution along the crack length was measured for two selected bending load levels using digital image correlation (DIC) technique. These measurements have been used for verifying the solution provided by the hodograph cone method (HCM) as proposed by Bonora [1].


Sommario. In una tubazione con difetto passante circonferenziale soggetta a carico di moment flettente, l'area di efflusso o area di leakage dipende dalla posizione del difetto rispetto al piano di momento. Nell'analisi leak-before-break (LBB), l'assunzione che il difetto sia sempre posizionato simmetricamente rispetto al piano di momento non è necessariamente un'ipotesi conservativa. In questo lavoro, l'effettiva apertura di difetti circonferenziali, in tubazioni soggette flessione e non posizionati simmetricamente rispetto al piano di momento flettente, è stata determinata attraverso opportuna indagine sperimentale. Sono stati analizzati tre geometrie di tubazione e due lunghezze di difetto. Per ogni configurazione di difetto sono state poi esaminate tre posizioni: una in asse e due fuori asse. Il COD, ovvero l'apertura del difetto lungo tutta la lunghezza del difetto, è stato misurato per mezzo di estensimentria senza contatto con tecnica speckle (digital image correlation -DIC).Le misure sono state utilizzate per verificare l'accuratezza della soluzione ottenuta con il metodo del cono odografo (HCM) proposta da Bonora [1].

Keywords. Crack; Pipe; Bending; COD; Off-axis; Hodograph Cone Method.

## INTRODUCTION

Leak Before Break criteria (LBB) provide a design route to assess if a crack can critically affect the structural integrity of a pipe or pressure vessel [2]. In this approach, the leak rate, the shape and size of the leak area (COA), or alternatively the crack opening displacement (COD) distribution along the flaw length under the acting load, is needed to estimate the associated crack size [3]. A reference configuration for the structural integrity assessment of flawed pipe is the through-thickness circumferential crack under bending and/or internal pressure load [4]. Here, the crack is
always assumed to be placed symmetrically with respect to the bending plane because the maximum crack opening and COA occur for this configuration [5].
In the literature, based on finite element analyses, several solutions for the maximum crack opening have been reported and the COA is calculated assuming an elliptical distribution of the COD along the crack length. An example of these solutions can be found in design code such as GE-EPRI [4, 6].
In real cases and for many reasons, such as fabrication imperfections, vibrations, load transient, etc., the crack can be offcentered and located in different positions around the pipe circumference. In these conditions, for leak rate and applied load identical to that of symmetrical centered crack, these solutions lead to:

1. a smaller crack opening area that drive to a larger detectable flaw size (detrimental effect).
2. a higher pipe capability to carrying load (beneficial effect).

Therefore, finite element analysis becomes mandatory to determine the effective leak area for a generic off-centered crack configuration [7].
When the crack is off-centered with respect to the bending plane, the COA is reduced with respect to that of the reference centered configuration and the shape of the COD distribution along the crack is no longer elliptical.
Bonora [1] proposed the Hodograph Cone Method (HCM) in which the COD distribution (shape and amplitude) and the leak area are predicted, as a function of the crack length and off-center angle, only by means of geometrical considerations. Later, Rahman et al. [8] performed further numerical investigations on off-centered crack configurations, and Firmature and Rahman [9] investigated the behavior of off-center cracks in elastic-plastic regime.
At the present, the HCM method is the only available solution that does not require extensive use of finite element simulation and allows quick estimation of the COD distribution and associated COA, given the crack length and the offaxis angle, and the maximum COD for the prescribed load in the centered crack configuration. For this reason, it is attractive for its potential use in design-by-analysis procedures.
This model has been validated by means of finite element simulation on selected flawed pipe configurations [10]. More recently, experimental measurements of the COD profile for off-centered circumferential crack under bending have been presented [11]. In this work, an extensive experimental work, to investigate the evolution of COD profile with the offcenter angle in different pipe configurations (diameter and thickness) under bending, was carried out. These results have been used to provide a physical evidence for the qualification of the HCM.

## MATERIAL AND METHODS

In this study, the pipe selected for the COD measurement were made of two materials: FE360 steel for the pipe 3.4 mm thick, and ST52 for 2 mm and 5 mm thick pipes. Three mean pipe radius/thickness ratios have been investigated. These ratios were selected to compare the results with the GE-EPRI solution which has the following limit $5<\mathrm{R} / \mathrm{t}<20$. A summary of the pipe geometries, off axis angle and selected load is given in Tab. 1.

| Test Configuration | Mean radius <br> $[\mathrm{mm}]$ | t <br> $[\mathrm{mm}]$ | $\mathrm{R}_{\mathrm{m}} / \mathrm{t}$ | $\mathrm{a} / \pi$ | off-axis <br> angle $\phi\left({ }^{\circ}\right)$ | LOAD <br> $[\mathrm{N}]$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2 \alpha 50 \mathrm{~T} 2$ | 29 | 2 | 14.5 | 0.139 | 0 | 30 | 60 |
| $2 \alpha 90 \mathrm{~T} 2$ | 29 | 2 | 14.5 | 0.25 | 0 | 30 | 60 |
| $2 \alpha 90 \mathrm{~T} 3.4$ | 28.3 | 3.4 | 8.32 | 0.25 | 0 | 30 | 60 |
| $2 \alpha 50 \mathrm{~T} 5$ | 27.5 | 5 | 5.5 | 0.139 | 0 | 3060 | 1500 |
| $2 \alpha 90 \mathrm{~T} 5$ | 27.5 | 5 | 5.5 | 0.25 | 0 | 3060 | 1500 |

Table 1: Test configuration: three different $\mathrm{R}_{\mathrm{m}} / \mathrm{t}$ and two different crack length.
Pipe used in this work were obtained by cold drawing ensuring constant thickness and small ovalization. On each pipe the cracks was machined using a circular saw which generate a squared crack tip with an initial opening of 1.0 mm . No fatigue pre-cracking was performed to sharp the tip radius since the interest is in the COD distribution along the crack length. However, preliminary 3D FEM analysis was performed in order to verify if an initial blunting could influence the COD
profile. Results showed that the COD profile is not affected by the shape of the initial blunting since the initial crack opening, due to the cut, scales the overall opening displacement [10].
Each pipe was loaded under four point bending. Tests were performed with Instron 300 kN electromechanical testing machine, and the COD was measured for two applied load values: 500 N and 1500 N . A pre-load of 40 N was used in order to compensate loading fixture compliance, Fig. 1.
The measure of the COD distribution along the crack length was performed by means of digital image correlation (DIC). The system used is ARAMIS by GOM, which ensures an accuracy on strain measurements of $0.01 \%$. In order to proceed with DIC, the pipe surface is initially sprayed with white coating and successively sprayed with black coating to produce the reference speckle surface. Two calibrated cameras were used in order to account for the 3D nature of the measurement. In Fig. 2 an example of the acquired and processed images from DIC is shown.
For each flawed pipe configuration, the COD profile was measured initially for the in-axis (symmetric) position with respect to the bending moment. Successively, the measure was repeated, at the same prescribed load, rotating the pipe around its longitudinal axis by an off-axis angle.


Figure 1: Four point bend test set-up.


Figure 2: Example of processed DIC images: a) in green the analysis area red dots are the virtual clip gauge ends; b) in blu the crack and green squares shows the grid for strain measurements; c) the resulting strain map.

The crack opening displacement was measured defining a series of virtual clip gauges applied on the computational grid generate by Aramis software. Since the COD profile is a 3 D curve in the coordinate space, two cameras were used to determine the coordinates of the physical points, which are mapped onto the image pixels, to obtain deformation and strain comparing digital images at different times.

## THE HODGRAPH CONE METHOD

The hodograph cone method (HCM) is a geometry approach for the estimation of the COD profile for a generic circumferential part-through crack in a pipe under bending and/or internal pressure. The method and the derivation of the solution is given in details elsewhere [1]. In the following the HCM is summarized.
According to the HCM, the geometrical representation of the COD of a circumferential crack in a pipe can be seen as the intersection of two surfaces: a cylindrical surface, representative of the pipe, and a cone, with elliptical base and the vertex insisting on the cylinder circumference. The base of the cone is an ellipse with its minor axis equal to the maximum crack opening displacement for the in axis configuration. The cone has an opening angle that is half of the physical crack opening angle. A sketch of the HCM is shown in Fig. 3.


Figure 3: Geometric representation of the hodograph cone for the reference in-axis configuration.
For the centered crack configuration, the intersection of the cone and the cylinder return in the 3 D space the COD profile as the elliptical profile projected onto the pipe surface. For the off-centered crack, the COD profile can still be obtained as the intersection of the cylinder and the shifted cone, moving its vertex along the cylinder circumference by the same off axis angle $\phi$, Fig. 4. For a circumferential crack length $2 \alpha$ R and a generic off-centered angle $\phi$, the COD curve in the 3D space can is given as:

$$
\left\{\begin{array}{l}
y=\frac{\delta^{*}}{2}\left\{[\cos (\beta)+\cos (\phi)]^{2}-\right.  \tag{1}\\
\left.[\sin (\beta)-\sin (\phi)]^{2} \tan ^{-2}(\alpha / 2)\right\}^{1 / 2} \\
x=R \sin (\beta) \\
z=R \cos (\beta)
\end{array}\right.
$$

where $\beta \in[-\alpha ;+\alpha]$ and $\delta^{*}$ is the maximum COD at the center crack length for the in axis configuration. Off course this value is function of the pipe geometry and load and can be determined either by FEM or estimated according to GEEPRI or similar solutions.
According to this, crack closure stars when the crack goes in the compression region that is predicted to occurs at an angle that depends on the crack length according to,

$$
\begin{equation*}
\phi=\frac{1}{2}(\pi-\alpha) \tag{2}
\end{equation*}
$$

Therefore, provided the pipe reference radius (inner, outer or thickness average), the crack opening angle $\alpha$, and the maximum COD at the center of the crack for the in axis configuration, Eqn. 1 provides the equation of the COD in the 3D space.


Figure 4: HCM for the generic off-center crack configuration.

## RESULTS AND DISCUSSION

In Fig. 5 the comparison of the measured and calculated COD profile for the centered crack configuration and two different pipe thicknesses is shown. Here, experimental data could be obtained only for half of the crack length since in the centered crack configuration the crack length was exceeding the DIC cameras window. However, it was possible to measure the COD beyond the point for which the maximum opening occurs to confirm the symmetry of the COD profile. The maximum COD was measured for the centered crack configuration also for $2 \alpha=90^{\circ}$ and for the three pipe thicknesses. These results have been used to validate available solutions based on FEM such as those reported in the GE-EPRI. It was found that in some cases, these solutions do not agree well with present experimental results. The HCM assumes that, given the maximum COD for the center crack, the COD distribution for the off-center crack configuration is given by

$$
\begin{equation*}
\delta(\phi)=\delta_{0}\left\{[\cos (\beta)+\cos (\phi)]^{2}-[\sin (\beta)-\sin (\phi)]^{2} \tan ^{-2}(\alpha / 2)\right\}^{1 / 2} \tag{1.1}
\end{equation*}
$$

This solution overestimates the effective maximum opening for given off-axis angle. It has been found that applying the following correction usually leads to much better agreement between the HCM and FEM results,

$$
\delta_{0}^{*}=\delta_{0} \cos \left(\phi / \phi_{0}\right)
$$

where $\phi_{0}$ is a factor that depends on the amplitude of the applied bending and crack length and it is equal to the ratio of half crack angle $\alpha$ and the off center angle $\tau$ for which there is half closure of the crack. This correction is applied only for half crack angle greater than $45^{\circ}$.
In Fig. 7-Fig. 11 the comparison of the HCM and the measured COD profile for the off-axis configuration is shown. In all the cases, the agreement is very good indicating that the HCM is accurate enough to predict the effective modification of the COD profile with the increasing off-axis angle even for very large cracks.
In Tab. 2, the comparison between the measured COD and calculated values determined with GE-EPRI solution for the centered crack configuration, [6] is given. It is interesting to note that for smaller thickness and shorter crack length, the difference is larger. A possible explanation is that GE-EPRI numerical simulation were performed using shell elements which do not correctly take into account 3D effects and rotation of the through-thickness crack faces. Much better agreement is found when using 3D finite element models as shown in by the author in [10].


Figure 5: COD distribution for specimen in the canonical configuration, $\left(\phi=0^{\circ}\right)$, crack length $2 \alpha=50^{\circ}$.


Figure 6: COD distribution for pipe with different thickness: crack length $2 \alpha=90^{\circ}$.


Figure 7: $\mathrm{t}=2 \mathrm{~mm}$ crack length $2 \alpha=50^{\circ} ;\left(\phi=30^{\circ}\right)$ and $\left(\phi=60^{\circ}\right)$.


Figure 8: t=2 mm crack length $2 \alpha=90^{\circ} ;\left(\phi=30^{\circ}\right)$ and $\left(\phi=60^{\circ}\right)$.


Figure 9: $\mathrm{t}=5 \mathrm{~mm}$; crack length $2 \alpha=50^{\circ} ;\left(\phi=30^{\circ}\right)$ and $\left(\phi=60^{\circ}\right)$.


Figure 10: $\mathrm{t}=5 \mathrm{~mm}$; crack length $2 \alpha=90^{\circ} ;\left(\phi=30^{\circ}\right)$ and $\left(\phi=60^{\circ}\right)$.


Figure 11: $\mathrm{t}=3.4 \mathrm{~mm}$; crack length $2 \alpha=50^{\circ} ;\left(\phi=30^{\circ}\right)$ and $\left(\phi=60^{\circ}\right)$.

| CONFIGURATION | GE-EPRI[6] [mm] | EXPERIMENTAL <br> MEASURE [mm] | ERROR \% |
| :---: | :---: | :---: | :---: |
| $2 \alpha 50 \mathrm{~T} 2$ | $4.9 \mathrm{E}-3$ | $7.2 \mathrm{E}-3$ | 32 |
| $2 \alpha 90 \mathrm{~T} 2$ | $1.5 \mathrm{E}-3$ | $1.8 \mathrm{E}-3$ | 17 |
| $2 \alpha 90 \mathrm{~T} 3.4$ |  |  |  |
| $2 \alpha 50 \mathrm{~T} 5$ | $5.4 \mathrm{E}-3$ | $6.5 \mathrm{E}-3$ | 14 |
| $2 \alpha 50 \mathrm{~T} 5$ | $1.4 \mathrm{E}-3$ | $1.5 \mathrm{E}-3$ | 7 |

Table 2: Comparison between maximum COD calculated by GE-EPRI and the experimental values.

## Conclusions

In this work, an experimental investigation on the shape of the COD profile for off-centered circumferential crack in bended pipe was carried out. COD measures were performed on pipes with different thickness, different crack length, and for different values of the off-crack angle. These measures have been used to validate the hodograph cone method which provided a geometrical solution for the determination of the COD modification as a function of the pipe geometry, crack length and off-axis angle. The comparison of the HCM solution and present COD measures indicates that the HCM is very accurate and rubout tools for predicting the COD for off-centered crack configuration suggesting its use as valid alterative to extensive finite element analysis in LBB analysis. The measures of the maximum COD for the centered crack configuration were compared with GE-EPRI solutions. It was found that for the selected $\mathrm{R} / \mathrm{t}$ pipe ratio and crack opening angle, the GE-EPRI provides always a maximum CTOD that is larger than that measured in the experiments. This difference, although conservative, becomes larger when considering shorter crack length and thinner pipe.

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