A review of using thermoelasticity for structural integrity assessment

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ABSTRACT. The advances in the use of thermoelastic stress analysis (TSA) for fracture mechanics assessment are reviewed. The development of techniques to determine stress intensity factor is presented followed by the application of these techniques to fatigue crack growth, crack closure and the study of mixed mode cracks.

KEYWORDS. Thermoelastic Stress Analysis; Crack; Fatigue; Stress intensity factor range.

INTRODUCTION

Thermoelastic Stress Analysis (TSA) has been developed over last two decades to be a useful method for structural integrity assessment, Harwood and Cummins [1]. Using a sensitive infra-red detector, minute temperature changes due to the thermoelastic effect are detected from around the tip of a crack under cyclic load. The signal from the detector, S, is related to the first stress invariant by the following equation:

$$\Delta(\sigma_1 + \sigma_2) = AS \qquad (1)$$

where A is a calibration constant. An expression for this first stress invariant in the region of the crack tip can be derived from stress field equations and used to determine the stress intensity factors.
A typical un-calibrated thermoelastic map from a crack of 15 mm length loaded in mixed mode with $\Delta K_{II}/\Delta K_I = 1$ is shown in Fig. 1.

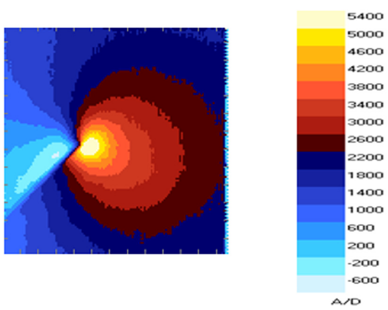


Figure 1: Thermoelastic data around a 15 mm crack loaded in mixed mode with applied mixed mode ratio $\Delta K_{II}/\Delta K_I = 1$.

The stress intensity factor value obtained from thermoelastic analysis is equal to the range of the stress intensity factor, ΔK, which occurs at the crack tip due to the applied cyclic load. This allows the actual crack driving force to be experimentally determined rather than being inferred from maximum and minimum stress intensity factors, which is the case with other experimental techniques. The direct determination of ΔK makes the technique ideal for use in structural integrity assessment. Further advantages over other experimental methods are that thermoelasticity is a non-contacting method, with minimal surface preparation required, which may be used to study cracks in both real components and models.

This paper will review the advances made in thermoelastic fracture mechanics in recent years. An outline of the experimental issues which need to be considered for determining stress intensity factors using thermoelasticity was presented by Tomlinson and Olden [2] and this paper adds to that. Methods to determine the stress intensity factor at crack tips using thermoelastic stress analysis are explored, starting from selected line techniques using the Westergaard equations to the most recent methods which utilise the full array of data available together with complex stress field equations. Crack path analysis and interaction of cracks will be explored in addition to fracture mechanics investigation of a range of materials. In order to obtain accurate results a number of areas of experimental procedure need to be considered and these are discussed in detail. The paper will present the progress made on determination of stress intensity factors for fatigue cracks, tracking the location of the crack tip during propagation under cyclic loading, and also in investigating the crack closure effect.

Recently, Risitano et al. [3], utilizing the thermographic method, made an estimations very close of the real values of stress concentration factor and fatigue-stress concentration factor.

THE DETERMINATION OF STRESS INTENSITY FACTORS FOR SHARP NOTCHES

Evaluation of mode I Stress Intensity Factors for sharp notches

There are primarily five methods available to determine the stress intensity factor of a crack using thermoelastic equipment. The techniques proposed by Stanley and Chan [4], Stanley and Dulieu-Smith [5], Lesniak [6], Tomlinson et al. [7] and Lin et al. [8] are detailed in the following sections.

In 1986 the thermoelastic technique was first used to determine ΔK_I, the opening mode stress intensity factor, by Stanley and Chan [4] who used a SPATE (Stress Pattern Analysis by Thermal Emission) system to record the thermoelastic signal, *S*. Using the first two terms of the Westergaard equations for the elastic stresses in the vicinity of a crack under mode I and mode II loading it was shown that the stress intensity factor of a crack could be related to the SPATE signal recorded using the following relationship:

$$AS = \frac{2\Delta K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} - \frac{2\Delta K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2} \tag{2}$$

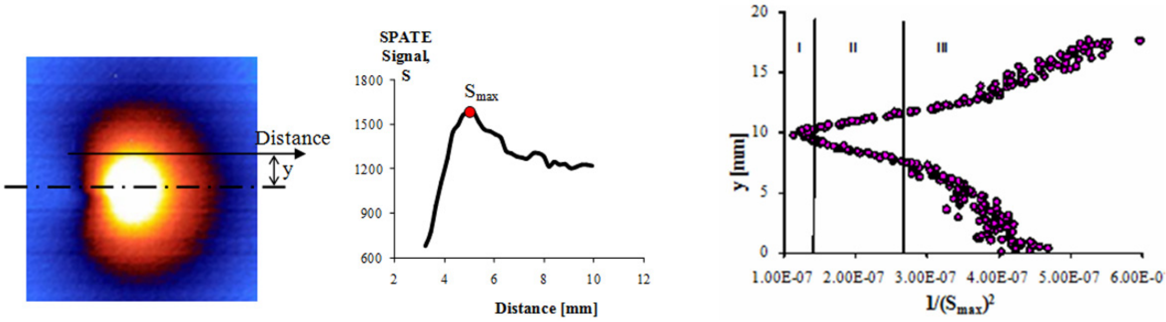
Thus the SPATE signal around the crack tip, *S*, when multiplied by a calibration factor, *A*, could be used to calculate stress intensity factors from points located at distances (r,θ) from the crack tip. This equation provides the basis of determining both single-mode stress intensity factors, and also mixed-mode. For the latter case it was explained that ΔK_I would first be generated from data along the line $\theta = 0$, before ΔK_{II} could be calculated from a general line. Emphasis was placed by the authors on the likelihood of large errors if a single point value were used, as it would not be clear whether data had been recorded from the region of validity for the Westergaard equations, and also it would be necessary to know the exact location of the crack tip. The region of validity of the Westergaard equations is discussed in a later section.

Stanley and Chan [4] also describe a graphical method to determine the opening mode stress intensity factor of a fatigue crack. This used a series of line scans either perpendicular or parallel to the crack line. In the parallel case the stress intensity factor is determined using:

$$\Delta K_I = \sqrt{\frac{4\pi A^2 Gr}{3\sqrt{3}}} \tag{3}$$

where *Gr.* is the gradient of the linear region from a graph of $1/S_{max}^2$ against distance *y*. Typical data points from which the gradient was obtained are shown in Fig. 2. The non-linearity observed close to the crack tip and also at the extremities was attributed to crack tip plasticity and finite plate size respectively. Remote stresses in a finite width specimen are higher than in an infinite specimen, for which the Westergaard equations were derived. For the fatigue cracks studied all experimental results were found to be within 10 % of the theoretical solutions.

This technique was further developed on mode II fatigue cracks, Stanley and Chan, [9].



(a) (b) (c)

Figure 2: A typical set of data points from a thermoelastic scan of a crack (Stanley and Chan [2]).

A study of simulated inclined edge and centre cracks by Stanley and Dulieu-Smith [10] showed that the isopachic contour in the crack tip region generally took the form of a cardioid curve, centred on the crack tip. By determining the area and orientation of a typical cardioid, ΔK_I and ΔK_{II} were calculated. Using Eq. (2) the following equation was derived:

$$r = \frac{\Delta K_I^2 + \Delta K_{II}^2}{\pi A^2 S^2}(1+\cos(\theta + 2\phi)) \qquad (4)$$

where $\phi = \tan^{-1}(\Delta K_I/\Delta K_{II})$.

For a constant value of S, Eq. (4) represents a closed curve in (r,θ) co-ordinates which represent a cardioid or "apple" shape. In the experimental investigation a series of line scans parallel to the crack line were used to construct a curve by recording the location(s) on each line scan where a particular value of thermoelastic signal was measured. Hand fitting a cardioid shape to experimental data did not result in a perfectly symmetrical curve and scatter in the results was noted. The agreement between experimental and theoretical $\Delta K_I/\Delta K_{II}$ values was concluded to be no better than moderate. In a later paper the omission of the non-singular stress term in the calculation of stress intensity factors in the original work of Stanley and Dulieu-Smith [10] was discussed. A method was suggested for including this factor and determining its value from a map of thermoelastic data. However, no results were presented to indicate whether experimental values were closer to theoretical predictions for ΔK_I and ΔK_{II} when account was taken of the non-singular stresses. The cardioid curve method was extended to allow inclusion of all the data from around the crack tip (Fulton et al. [11], Dulieu-Barton et al. [12]). A computer program is used to calculate the area and orientation of nine cardioid curves of constant thermoelastic signal. A potential source of error in the method was highlighted as being its sensitivity to an exact knowledge of pixel size in the map of thermoelastic data. This technique was further developed to greater accuracy by Dulieu-Barton and Worden, [13] using a curve fitting routine based on a genetic algorithm to generate the cardioid curves from thermoelastic data, which were subsequently used to determine the stress intensity factor.

An alternative approach for determining stress intensity factors for cracks subject to mixed-mode loading which had previously been used in photoelastic analysis, was developed by Tomlinson et al. [7]. A Newton-Raphson iteration combined with a least squares approach was used to fit the equations describing the stress field around the crack tip, based on Mushkelishvili's approach, to the experimental data. This approach allows different applied stress fields to be described which may include non-uniform stress fields. The computerised analysis method required the map of data obtained from each SPATE scan to be interrogated at a number of points on lines radiating from the crack tip between an inner and an outer radius in the "singularity dominated zone". The co-ordinates of these points and the SPATE signal value were input into a new computer program which calculates ΔK_I and ΔK_{II}. The mean and variance of the least-squares fit of the solution to the data points are also calculated in order to give an indication of the accuracy of the results.

A further data array method was developed by Lesniak [6] where the Airy stress function was fit to the array of thermoelastic data using a least squares method. The method uses the complete stress function, rather than just the singularity term, and therefore can account for the boundaries of a component. It is also very fast, since multiple terms of the stress field can be least-squares fit in seconds, which gives the potential to perform live crack growth analyses. The procedure gave errors of 3% for mode I cases when compared to analytical solutions. A mixed-mode example has also been published but with less accurate results, Lesniak et al. [14].

The above methods require thermoelastic data to be taken from close to the crack tip in order to determine the stress intensity factor, since these approaches use the principles of linear elastic fracture mechanics. In such methods there are restrictions on the areas of data collection due to localised plasticity and tri-axial stress effects near the crack tip. In order to overcome these problems, a hybrid method has been developed which is based on equilibrium, compatibility and the J integral and uses far-field thermoelastic data to determine the stress intensity factor, Lin et al. [8]. The value of the procedure is that it is not restricted to isotropic materials and was demonstrated by determining the stress intensity factor for cracks in both orthotropic composites and isotropic plates.
The proposed methods by Tomlinson et. al. [7], Lesniak et al. [14], Dulieu - Barton et al. [12], Diaz et al. [15] which have been developed to determine stress intensity factors at crack tips have taken advantage of computing power and data processing available on a standard personal computer. These methods have been shown to give more accurate results than previous manual methods when compared to theoretical solutions, and it is believed this work will advance in a number of areas in the near future.

Evaluation of mixed mode solutions for sharp notches

Shiratori et al. [16] used a method of stress intensity factor calculation based on the Westergaard equations and applied the SPATE technique to three different specimen types. The first specimen type was a compact tension specimen which was notched but had no fatigue crack. Results for three notch lengths showed that SPATE results were in fairly close agreement with the findings of a finite element analysis, except within 5 mm of the crack tip where they were much lower. The authors suggested that this was due to the smoothing of the data by the SPATE equipment, leading to an underestimation of the stress value in regions where the stress gradient is high. In the smoothing the value of stress at a pixel is determined from the average stress value at a pixel and also the surrounding pixels. Therefore it was recommended that the minimum load range that could be applied to obtain a clear stress plot should be used to minimise the stress gradient at the crack tip. The other specimens analysed by the authors were a surface notched specimen and also a fatigue crack at the end of a slanted through-wall crack. On the basis of the results from the three specimens it was concluded that stress intensity factors for various crack types could be calculated to within 10-20% using SPATE equipment. This was considered to be suitably accurate for use in the analysis of cracks in the complicated members of real structures where computational methods cannot be reliably used due to unknowns such as loads, crack length and other boundary conditions.
The method of Tomlinson et al. [8] was experimentally assessed using angled slots of various lengths in centre cracked specimens. An array of up to 100 discrete points was used to provide SPATE signal values to be input into the analysis. Mode I and II stress intensity factors for slots at both 90° and 45° to the loading were on average within 6.6% of theoretical solutions. Large differences in solutions for the shortest crack length at the 45° angle were explained as possibly being due to interaction of the crack tips. Although for 30° cracks mode I results were close to a theoretical solution, this was not the case for ΔK_{II}, where it was suggested that in fact the theory may not be reliable for the geometry. Spark eroded slots loaded at 45° were found to give excellent results by Dulieu-Barton et al. [12] when compared to theoretical solutions, although those for 60° slots were less accurate. The paper also included an interesting comparison with the results obtained from previous investigations.
Marsavina and Tomlinson [17] presented a study of mixed mode stress intensity factors investigating a biaxial specimen with a 45° inclined notch produced by spark erosion. By changing the loads on the two axes a large range of mode mixities where created from pure mode I to predominantly mode II. Using the staring array DeltaTherm system and the algorithm of Tomlinson el al. [8] they obtained experimental values for stress intensity factors range in good agreement with the theoretical solution 4.5 % error for ΔK_I and 6.5 % for ΔK_{II}.

DETERMINATION OF STRESS INTENSITY FACTORS FROM FATIGUE CRACKS

Mode I Stress Intensity Factors

The previous section outlines the main improvements and variations on methods to determine stress intensity factors using thermoelasticity. Other researchers have made contributions, particularly in the area of fatigue crack studies.
Both real cracks and spark eroded slots were examined in the experimental investigation of Dulieu-Barton et al. [12]. There was evidence of crack closure occurring when a fatigue crack was studied under a number of different stress ranges. The ratio of ΔK_I recorded using SPATE equipment to the value of the theoretical solution was found to decrease as the

stress range increased. Results for the real fatigue cracks with no crack closure agreed with theoretical values to within 11%. As the error was not consistently this high, it was suggested that for the worst case this error may have been due to geometric irregularities of real cracks.

An improvement in the methodology for monitoring fatigue crack growth and inferring the stress intensity factor from thermoelastic data is presented by Diaz et al. [15]. The approach is based on a multipoint over-deterministic method but uses a new fitting algorithm based of the Downhill Simplex Method to fit the equations describing the stress field around the crack tip to thermoelastic data, where the crack tip was considered as a variable to be optimised. Initially the accuracy was checked with images generated based on Westergaard's stress field equations. For the stress fields dominated by the mode I component, the new algorithm yielded mode I results within 1% of the value used to generate the image. For the mixed mode cases, the difference between the inferred SIF and the one employed in generating the images is 6% for the mode I SIF and 10 % for mode II. Processing experimental data obtained for real Mode I cracks shows that the methodology is sensitive to the crack closure effect.

In an experimental study of real cracks in compact tension specimens by Olden [18] it was found that the technique of Tomlinson et al. [8] was very sensitive to errors in the crack tip location,. This problem was easily overcome by locating the correct crack tip position by moving the relative position of the crack tip to the data points to give the smallest mean of the least-squares fit. A similar method was proposed by Lesniak et al. [14], where the crack tip location was estimated by the user and was then optimised automatically by a search routine.

Diaz et al. [19] showed that the presence of high stress gradients or local plasticity at the crack tip leads to a loss of adiabatic conditions, and consequently conduction through the specimen takes place. Investigating the phase map of the thermoelastic signal they concluded that any deviation from zero of the phase is due to the presence of non-adiabatic conditions. On all phase thermoelastic maps they observed a region surrounding the crack tip and ahead of the crack where the phase becomes negative, that is the thermal response lags behind the loading cycle. This fact can be seen in the phase profile along the crack line. This behaviour is due to the lack of adiabatic conditions and could be due to heat generation and conduction as a consequence of plastic work at the crack tip. The initial estimate of the crack tip is taken to be at the point where the phase changes from positive to negative.

In contrast, the simpler method of Stanley and Chan [4] did not require any knowledge of the crack tip location and therefore appears to be more applicable for the study of fatigue cracks.

Mixed mode cracks

Stress intensity measurements of a mixed-mode fatigue crack have been published by Lesniak [6], but very little detail of the experiment was supplied. Only one example was given and the crack was predominantly mode I with a K_{II}/K_I ratio of 0.6. Although K_I was within 5% of the analytical solution, K_{II} differed by 20.8%, therefore these results were no better than those determined from machined slots.

Dulieu-Barton et al. [12] have performed tests by growing mode I cracks in large specimens, then cutting sections out of the plates to produce a specimen in which axial loading gave mixed-mode conditions at the crack tip. The experimental results for central 30° and 45° inclined cracks show good agreement with the theoretical solution within 6%. However when edge cracks were investigated the experimental values are with 20 – 30 % higher than the theoretical ones. They also pointed out the importance of the frictional effects between the two crack surfaces.

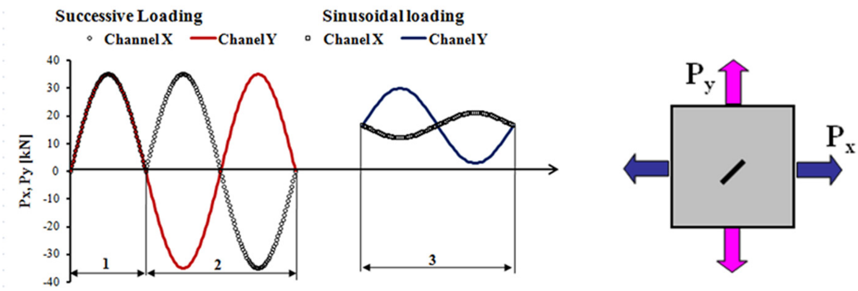


Figure 3: The loads applied to each axis of the load machine for the successive load cycle for crack growth under biaxial load, where $\Delta K_{II}/\Delta K_I = 2$ (1 – mode I loading; 2 – mode II loading), and the sinusoidal load cycle used for data collection, where $\Delta K_{II}/\Delta K_I = 2$ (3 – sinusoidal loading).

The most complete research program in investigating mixed mode cracks was published by Tomlinson and Marsavina [20]. They grow real mixed mode cracks under successive loading using method developed by Bold et al. [21] and then record thermoelastic data under a reduced sinusoidal load range, e.g. in Fig. 3, ensuring that the crack did not grow during the data collection.

They investigate the effects of the applied mixed mode and the $R=\sigma_{min}/\sigma_{max}$. The reported data at higher R (>0.5) ratios and predominantly mode I loading $\Delta K_{II}/\Delta K_I$ = 0.45, when the effect of crack closure on the values of the stress intensity factors is minimised since the crack is fully open, were obtained with an average difference between experiment and analytical solutions of 4.3% and 5% for ΔK_I and ΔK_{II} respectively which is comparable to the accuracy obtained for the specimens containing slots and also comparable to the results obtained by Dulieu-Barton et al. [11] for central crack.

Crack closure and propagating crack studies

Thermoelasticity is an ideal method for the study of crack closure, since ΔK is determined directly from the thermoelastic data. Yet very little work has been done in this area using this technique, probably due to the inaccuracy of manual data processing techniques in the past. Fulton et al. [12] have used their new computer technique to perform a study where experimentally determined stress intensity factors were found to decrease with respect to theoretical values with increasing stress range. This was stated to be possibly due to the effect of crack closure. Other tests have been performed using thermoelasticity by Batchelor [22] using the method developed by Tomlinson et al. [8], where crack closure was studied by measuring the effects on the stress intensity factor of cutting out the wake of a fatigue crack. This was found to be a useful technique, but only one crack was studied and further tests need to be carried out to give greater confidence in the results. Tomlinson et al. 1997 also concluded that if a spatial field of thermoelastic data is used, crack closure effects can be detected.

Diaz et al. [19] studying the crack propagation in single edge notched specimens found that the ΔK_I values obtained from thermoelasticity are lower than theoretical ones. The decrease in this difference at high R – ratios pointed to crack closure. Using the compliance change method for determining the opening load for a fatigue crack they found that the stress intensity factor range from thermoelastic data matched perfectly with the effective SIF calculated by compliance change.

Marsavina et al. [23] evaluated the effective stress intensity factors for a mixed mode crack, propagated using a successive loading cycle, using thermoelasticity and surface replicas. A sinusoidal loading cycle with applied $\Delta K_{II}/\Delta K_I$=0.45 was used in order to record the thermoelastic data around a crack for determining the stress intensity factors ranges. Fig. 4 shows the results normalized with a theoretical solution for the mode I SIF range. It can be observed that the thermoelastic and surface replica results are lower than the theoretical ones for lower values of R-ratio, and for R > 0.5 the experimental TSA results and the replica ones are in very good agreement with the theoretical values. They concluded that the results for the stress intensity factor range obtained by TSA represent the crack driving force.

Recent application of thermoelastic fracture mechanics has been done by Yates et al. [24] in the investigation of techniques for tracking the crack path as it grows and evaluating the strength of the mixed mode crack tip stress field. They demonstrate the suitability of TSA techniques to explore the hypothesis that the direction of fatigue cracks may be governed more strongly by directionality of crack tip plasticity rather than by the magnitude of the elastic stress field.

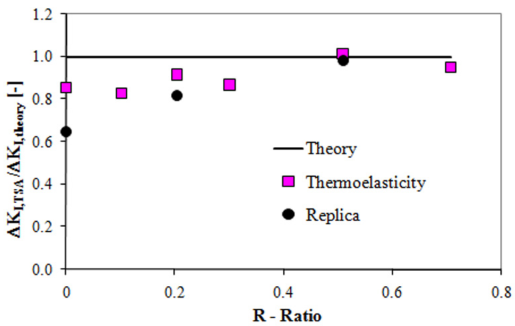


Figure 4: Theoretical, experimental TSA and replica ΔK_I values against R – ratio.

Conclusions

This review has shown the development of methods to determine stress intensity factors from thermoelastic data. With the availability of new, faster thermoelastic systems it is likely that thermoelasticity will continue to be widely used for crack tip studies. The technique has many advantages over other full-field stress analysis methods including minimal specimen preparation, equipment relatively unaffected by test environment, and perhaps most importantly it is able to provide a direct measurement of the crack driving force *ΔK*. This will undoubtedly mean that the technique will be beneficial in studying crack closure effects, mixed-mode and propagating cracks.

NOMENCLATURE

A	Calibration constant [MPa/signal]
K_I, K_{II}	Mode I, II stress intensity factors [MPa√ m]
r	Polar coordinate [mm]
R	Stress ratio [-]
S	Thermoelastic signal [signal]
Δ	Variation (maximum - minimum)
ϕ	Mode mixity [rad]
θ	Polar angle [rad]
σ_1, σ_2	Principal stresses [MPa]

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