Geometric Study of Surface Finishing of Selective Laser Melting Moulds

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Abstract

Selective laser melting, which is based on the principle of material incremental manufacturing, has been recognised as a promising additive manufacturing technology. The principle of additive manufacturing lies in fabricating a part or an assembly of parts, layer by layer through a bottom to top approach. The technology is suited for creating geometrically complex components that can not possibly or feasibly be made by any other means. This technique has a weak point related to the surface finishing. Therefore, during the construction of layer by layer, there is a need to use techniques such as milling to remove material. This hybrid approach allows the fabrication of parts with internal complex structures and very good surface finishing. To plan and optimize the successive additive and subtractive phases, we need a quick tool to determine when the geometry of a piece is suitable for surface finishing by a 3 axes milling machine. This problem can be reduced to a layer by layer subproblem of approximately covering a slice of the object by circles of the diameter of the smallest drill available that can reach its depth. This reduction to the plane allows us to use a medial axis approach. The medial axis of a planar domain, defined as the set of centers of maximal circles contained in the domain, relates very closely to the notion of generalized Voronoi diagram, and has been proposed in several milling applications that involve motion planning. We propose to use it, and certain extensions of it, as a practical way of determining the best possible finishing quality at a slice. To that end we have to find which of the available construction strategies best suits our needs to determine exactly or approximately the medial axis of a polygon and its extensions.

Keywords: Selective Laser Melting; medial axis; truncated medial axis; milling.

1. Introduction

The appearance of new technologies in different scientific areas has been boosting and deeply changing the industrial world. The globalization has been exposing the industry to a very competitive market. The clients are increasingly selective, which compel the industry to seriously invest in high-end technologies in order to be more efficient, economically sustainable and to be able to satisfy the full needs of its clients. In recent years the mold industry has been investing in new technologies, such as Fused Deposition Modeling (FDM), Selective Laser Sintering (SLS), Electron Beam Melting (EBM) and Selective Laser Melting (SLM). Their differences are essentially in mechanical details (for more details about these technologies see [1]).

In this work we are concerned with the SLM technology which is nowadays considered one of the most important technologies for metal processing. It is an additive manufacturing technique that uses digital information to produce a 3D metallic object with a *laser* from a metal powder. This technique is characterized by a successive addition of material to a specific area, layer by layer, in opposition to what happens in the traditional manufacturing. In the last years there has been an increasing interest in the use of this technique, mainly, because it offers a relative freedom in the geometry of the objects to be manufactured, giving the possibility of producing objects that would be difficult or even impossible to produce using other techniques (see Fig. 1). Furthermore, the fact that the SLM technology produces objects with high geometric complexity makes it easier to produce personalized objects, pieces with very small dimensions or other products that would be economically inviable with convectional manufacturing.

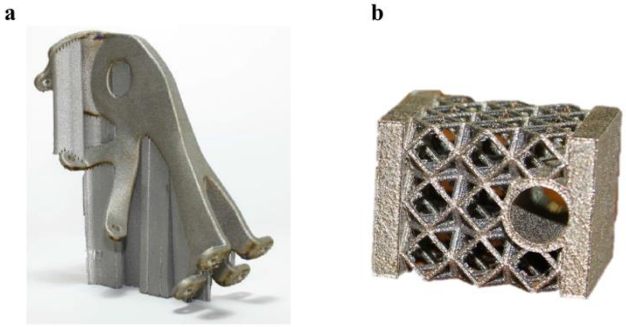

Fig. 1. Objects produced with SLM technology: a) titanium part with support structure produced for aeronautics; b) steel lattice structure to optimize the heat flow inside a mould

Usually a machine of SLM technology has a computer interface that gives to the operator the possibility of taking control of the production process inside the machine. To use the SLM technology, we start with a 3D digital model of the geometry of the object to be produced. The object is then vertically produced regarding a choice of z axis, layer by layer, and therefore it is necessary to know the geometry of each layer, in particular, its contour.

Assume that a figure is given in three dimensions in the xyz axes, where z axis is the height. Let us cut the figure with the plans $z = z_n,\ z_n \geq 0,\ n =$0, 1, 2… We call *slice* to each layer between $z = z_n$ and $z = z_{n+1}$. The Fig. 2(b) displays sections of the object in Fig. 2(a). The idea is to build the object slice by slice where the thickness of each slice is a parameter of the additive process. The thickness is usually small enough so that the slices can be considered flat for practical geometric purposes.

One of the limitations of this technology is related with the efficiency in surface finishing and for this reason, some objects need to be milled to remove the roughness. Therefore, after a first phase of production it is often applied a second phase that consists of a subtractive process. It is in this context that the industry is searching to bring together both processes, additive and subtractive, to be executed in just one process, step by step or layer by layer. Here, the additive process gives the desired form to the object while it is executed and then, the subtractive process removes the surplus material. One should notice that the surface finishing of an object is an important issue, because it can influence the cost of production. The main goal of this work is to use mathematical tools to construct algorithms that will help us to control part of the production process.

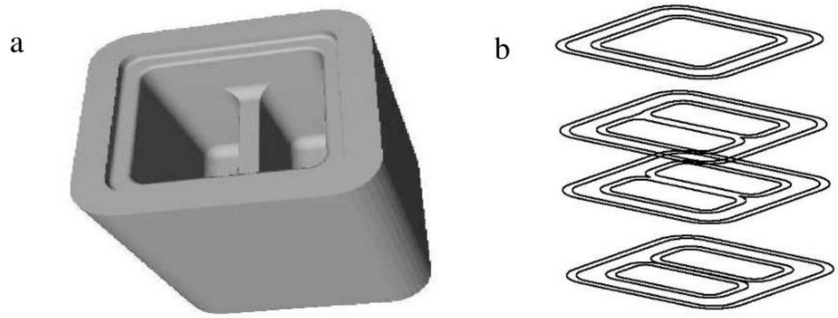

Fig. 2. An example of an object to be produced with SLM technology. In (b) we have sections of the object in (a)

The milling cutters have a fixed length and thickness, and if the object has deep cavities it may not be easy to mill the deepest regions after the fabrication is complete. To avoid this problem, one solution is to interrupt the additive process while it is still possible to reach such regions with the milling cutters. After the partial object has been milled, it returns to the additive process from the last quota where it was interrupted. These interruptions are expensive and time-consuming and therefore it is important to decide how many interruptions we need and when we should make them in order to minimize the costs.

We assume that the object is given in STL format, and this kind of files stores the geometry of the surface as a set of connected triangles. Therefore the contour of each section is polygonal. More details about these files and its slicing can be found in [2]. For a milling cutter to machine a section two things need to be taken into consideration: the length of the milling cutter must be sufficiently long to reach the section in question, and the diameter sufficiently small in order that the disk defined by the horizontal section of the milling cutter can approximately cover all the points in the polygon. Assuming that the milling cutter is cylindrical, its sections are disks, and hence the geometrization of this problem formally consists in how well can we cover a given polygon by disks of a fixed given radius. The covered region will be called reachable region or area.

The structure of this work is the following. In Section 2 we introduce the concept of medial axis of a polygon, a type of polygon skeleton. We cover some of its relevant properties and propose the new concept of truncated medial axis that will allow us to determine the reachable areas. We end the section with an algorithmic approach to construct the medial axis that can be generalized to its truncated version. Finally, in Section 3, we sketch the way of how to apply the developed geometrical theory to solve the original problem and present some final conclusions.

2. A skeleton based solution

While solving the problem of covering a polygon with circles of a fixed radius, we are led to the concept of skeletonization or thinning process of a polygon. Skeletonization is a process of transforming a polygon into a short caricature that is called medial axis. The medial axis of a polygon is the locus of points inside it that have at least two closest points on the boundary. This skeletonization process can be applied for almost any kind of shape, that is, we can find the medial axis for any sufficiently smooth planar region. In this work we will focus solely on the polygonal case, since we are assuming triangulated surfaces given in STL format, but it should be possible to apply this theory even for solids given in more complex forms. For more about the skeletonization process see [12]. Now we formally define the medial axis of a polygon.

2.1. Medial axis

In this section we introduce the definition of the medial axis of a polygon. This notion follows naturally from the idea of covering a polygon by disks. We start by defining a maximal disk in a polygon.

Definition 3.1. A disk is said to be maximal in a polygon P if it is not contained in any other disk enclosed in P.

An example of maximal disks can be seen in Fig. 3(a). Next we define the medial axis, as a way of capturing the most important information given by the maximal disks.

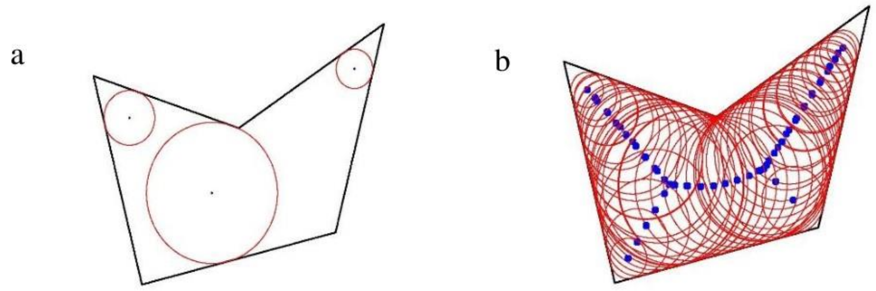

Fig. 3. The disks inside the polygon are maximal disks. In (b) the blue points are centers of the maximal disks.

Definition 3.2. For a polygon P, the *medial axis* is defined as the set of all centers of maximal disks of P.

In Fig. 3(b) the blue points are centers of some (not all) maximal disks and hence they must be part of the medial axis of P. It is already visible in that figure that they seem to form a one-dimensional structure. In Fig. 4(a) we show the complete medial axis for that polygon, while in Fig. 4(b) we see an example with a more complex geometry.

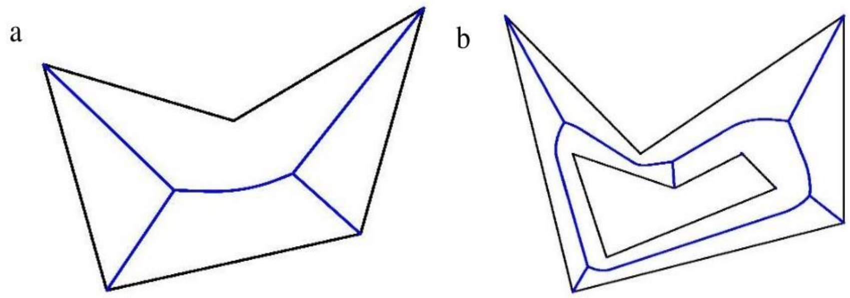

Fig. 4. The blue lines are the medial axis of the corresponding polygon. In (b) we have a polygon with a hole.

The medial axis can be shown to be a union of straight line segments and parabolic arcs and it preserves the topology (shape) of the original object. In particular, the object has the same number of holes as its medial axis and if the object is connected so it is the medial axis. More properties can be found in [3] and [4].

The medial axis is not a new tool. It was first introduced by Blum [13] in 1967 and since then many applications have been found for it. In image processing it has been useful to analyze images and match objects by reducing shapes to short caricatures given by their medial axes [5]. In motion planning for robotics, the medial axis can be used to find a free path to move a point, a circle, a segment or a polygon in the plane [6, 7]. Virtual endoscopy produces images in a non-invasive way and the medial axis has been proposed as a tool for automated path planning for this procedure [8].

As we mentioned before, the medial axis preserves the shape topology. Moreover we can actually use the medial axis to recover precisely the original object. We just need an additional piece of information: for each point of the medial axis, we need the radius of the maximal disk centered on it.

Theorem 3.1: A given polygon P is equal to the closure of the union of the maximal disks of P centered at a point of the medial axis. In other words

$$P = \overline{\bigcup_{x \in M(P)} B(x)_{r_{max}}}$$

where $B(x)_{r_{max}}$ denotes the maximal disk in P of center $x \in M(P)$ and radius r_{max}.

To illustrate this theorem we will return to our previous example. In Fig. 5 (a) we have the medial axis and some maximal disks. As the density of the disks increases, as seen in Fig. 5(b), we tend to recover the original polygon, as seen in Fig. 5 (c).

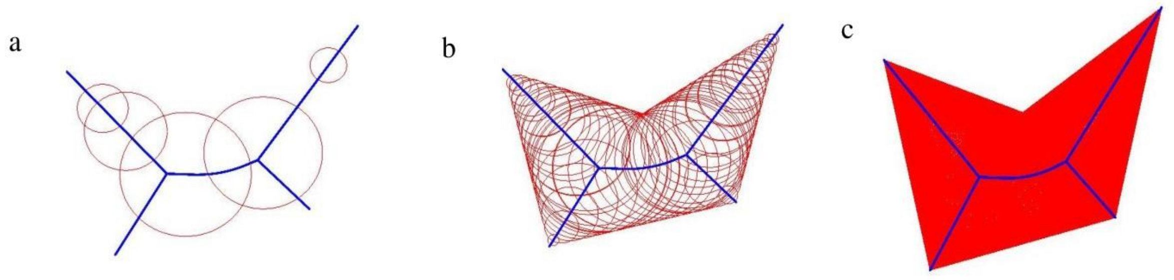

Fig. 5. In (a) we have the medial axis and some maximal disks. As the density of these disks increases we obtain (c) which approximates the original polygon.

2.2. Truncated medial axis

In this section we introduce the definition of truncated medial axis, which will help us to find the reachable region of a polygon. If we go back to the original problem, presented in the introduction, it is clear that using disks of arbitrarily small radius in not realistic. The milling cutters have a fixed radius, and while we can use them to cover any disk of larger radius, they will not be able to mill very small details. This basic consideration leads us to introduce the concept of truncated medial axis.

Definition 3.3. Given a disk of fixed radius s, we define the *truncated medial axis* $M_s(P)$ of a polygon P to be the set of points in the medial axis that are centers of maximal disks with radius greater or equal than s.

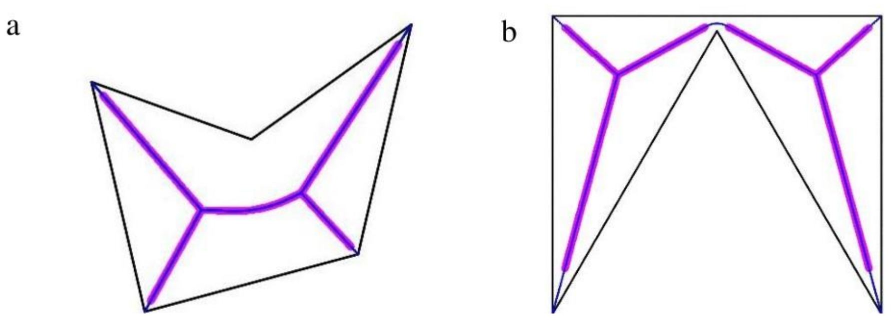

Fig. 6. The blue lines are the original medial axes and the magenta lines are truncated medial axis for the corresponding polygon.

In Fig. 6 we have depicted in magenta the truncated medial axes for each polygon whereas the original medial axes are in thinner blue lines. Note that the truncated medial axis does not need to be connected even if the original polygon is, as illustrated in Fig. 6(b).

We aim to find the region that is reachable with a milling cutter of some fixed radius. The relation between this region and the truncated medial axis is explained in the next theorem.

Theorem 3.2: The region of a polygon that can be reached by a milling cutter of radius s is

$$R_s = \overline{\bigcup_{x \in M_s(P)} B(x)_{r_{max}}}$$

where $B(x)_{r_{max}}$ denotes the maximal disk in P of center $x \in M(P)$ and radius r_{max}.

In Fig. 7, we display the reachable region for each polygon and the truncated medial axes that has been already shown in Fig. 6. The red area denotes the reachable region for the corresponding polygon. One interesting observation is that while the truncated medial axis shape is strongly related to the reachable region, the region can be connected even if the truncated medial axis is disconnected, suggesting that the relationship between the truncated medial axis and the reachable region is a little bit more subtle than that of the medial axis and the polygon.

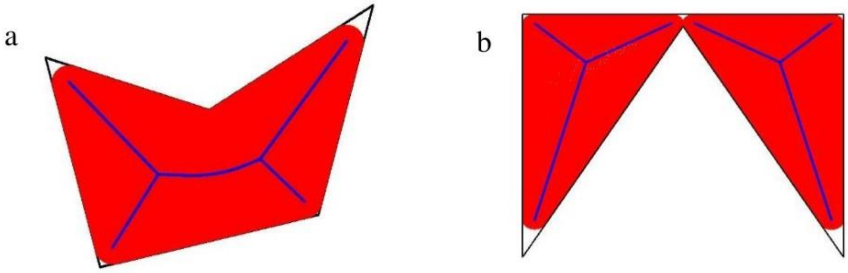

Fig. 7. The red area is the region in the respective polygon that can be reached by a milling cutter of some fixed radius.

The pieces that were truncated from the medial axis correspond precisely to the regions that will not be covered by any disk.

2.3. Algorithms

In order to use this approach in practice we need a feasible way of computing the truncated medial axis. There are several efficient algorithms to compute the medial axis that can generally be classified in two classes. Semi-continuous methods, where the boundary is approximated by a sample of points and the medial axis is extracted using discrete Voronoi Diagrams, and continuous (exact) methods, where the medial axis is extracted from the full boundary, for instance, by using generalized Voronoi diagrams. For a survey on these methods see [3, 9, 10, 11].

Continuous methods applied to a polygon of n vertices can give an exact description of the medial axis in $O(n^{3/2})$ time (see [9]), but are not extendable to very general shapes, and are very delicate to implement. Semi-continuous methods, on the other hand, are very easy to implement, and can be applied to any kind of shape, but only extract a discrete approximation of the medial axis whose quality is hard to control due to the medial axis instability with respect to small perturbations (see [3]). Our need to introduce some adaptations to the algorithms in order to construct the truncated medial axis led us to the use of the more versatile semi-continuous approach.

In order to introduce the semi-continuous approach, we need to recall the concept of Voronoi diagram. Given a finite set of points, a Voronoi cell of one of them is the set of points in the plane that are closer to it than to any of the other points in the set, and the collection of the boundaries of all such polygonal cells is what we call the Voronoi diagram. Another way of thinking of the Voronoi diagram is as the medial axis of the complement of a finite set of points. This discrete object is a classic construction and can be computed very efficiently in $O(n\log(n))$ time, where

n is the number of points. The idea is then to discretize our problem and use the fact that we can compute the discretization exactly in a fast and stable manner.

Let us suppose that we have a polygon P. To approximate the medial axis we follow the following steps, as proposed in [10].

- We sample the polygon boundary guaranteeing that the distance between two consecutive points is no bigger than some fixed ε.
- We construct the Voronoi diagram of the sample points.
- We keep the edges and vertices of the Voronoi diagram that are contained in the original polygon and discard the remaining.

What we get is an approximation of the medial axis, obtained in $O((n/\varepsilon)\log(n/\varepsilon))$ time, that tends to the medial axis as the space between sample points, ε, gets closer to zero. In Fig. 8(a) we have an example of a sample of points in the boundary of a polygon and in Fig. 8(b) we have the Voronoi diagram of the sample, with the edges that are kept for the approximation of the medial axis (those that are contained in the polygon) highlighted in magenta. We can see that in this simple case the approximation is already very good.

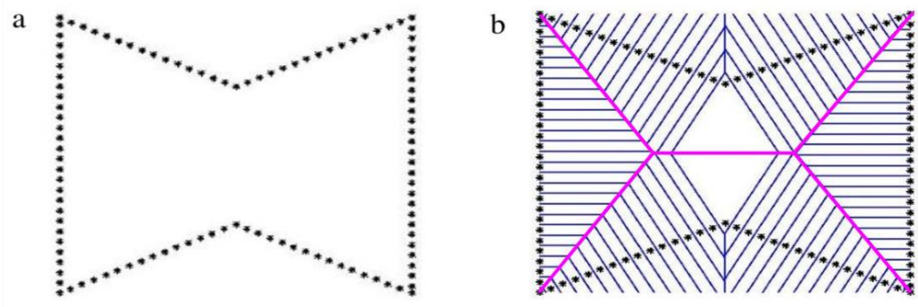

Fig. 8. In (a) we have a sampling of a polygon and the blue and magenta lines in (b) represent the Voronoi diagram of this sample.

To find the truncated medial axis we just need the radius s of the milling cutter to be used and then to remove the points in the medial axis associated to the maximal disks with the radius less than s. For every vertex of the Voronoi diagram, checking if the maximal disk associated to it has radius less than s is very quick, and if that is the case we discard the vertex and the edges adjoining it. The only other thing to check is the edges that come from reflex vertices of P, such as the longer edge in Fig. 8(b). These tend to be longer than the other edges, since they will not be broken in pieces by the discretization, and can have both vertices respecting the radius property while some middle points do not. A fast way to study these edges is to check only for edges with length above a certain tolerance threshold, verify if they are formed by reflex vertices and, if so, look at the vertices of the polygon that generate them and see the distance between them. If it is larger than $2s$, the entire edge is kept, if it is lower, a simple computation can determine where the edge should be cut, and what portion remains. It can be showed that if we treat in this manner the results attained by the medial axis discrete approximation, we get an approximation to the truncated medial axis that will converge to the true truncated medial axis as ε converges to zero as intended. For each ε the corresponding approximate reachable region can easily be computed and will be an inner approximation to the true region.

From the truncated medial axis one can immediately read the points on the boundary that will not be reached by the milling process, and hence get a measure on the attainable finishing quality for the slice. Other measures of quality, such as the area of the non-reachable surface, the topology of the reachable region (making sure that no crevice or hole is unreachable) can easily be implemented.

3. Conclusions

The application of the medial axis technique gives us the reachable region for each section. We then are left with the question of how to go from applying this technique to each section to applying it to the original problem of determining where the additive problem should be interrupted. A first possible approach is to consider all fabrication slices, study each of them to determine the maximum tool diameter permitted in order to get an acceptable quality and also to find the maximum depth that such tool can reach. At each slice we obtain some stop restrictions that must be verified and it is easy to find an optimal fabrication strategy to satisfy them all. Schematically, consider the object shown in Fig. 2 with its sections. We will study each section, as shown in Fig. 9, to scan for delicate details with our automatic procedure.

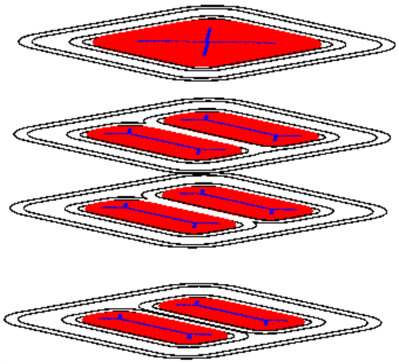

Fig. 9. The red regions are regions that can be reached by a milling cutter of fixed radius s.

There are however some simplifications that can be made, due to the framework we are considering. Since moulds need to be monotonous (meaning that the sections of the solid decrease in size when the height increases, without the creation of overhangs) one section being reached means all the above sections are reached, unless there is some change in the topology of the section (shallower details appear at certain heights). This allows us to not study every section but only those where there are interesting changes in the topology, something that can also be easily detected by the use of medial axis.

Some future work to be developed is the possibility of the desired finishing quality to be specified a priori by an operator and differ for different parts of the piece. Another point where improvements need to be introduced is on the handling of spherical drills, necessary to satisfactorily drill edges that are not vertical. However it is our belief that the properties of the medial axis make it again a perfect tool for handling these cases, possibly even by dealing with the 3-dimensional case by the direct use of the 3-dimensional structure of the medial axis of a polyhedron.

In conclusion, in this paper we propose the use of the medial axis and a variation of it to quickly deal with questions of access of a drill to a given surface. This allows us to measure finishing quality or even feasibility, and use it to plan the interruptions for a hybrid additive and subtractive fabrication method.

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