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# OZI violating eight-quark interactions as a thermometer for chiral transitions

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**Abstract**

This work is a follow-up of our recent observation that in the $SU(3)$ flavor limit with vanishing current quark masses the temperature for the chiral transition is substantially reduced by adding eight-quark interactions to the Nambu–Jona-Lasinio Lagrangian with $U_A(1)$ breaking. Here we generalize the case to realistic light and strange quark masses and confirm our prior result. Additionally, we demonstrate that depending on the strength of OZI violating eight-quark interactions, the system undergoes either a rapid crossover or a first order phase transition. The meson mass spectra of the low lying pseudoscalars and scalars at $T = 0$ are not sensitive to the difference in the parameter settings that correspond to these two alternatives, except for the singlet-octet mixing scalar channels, mainly the $\sigma$ meson.


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The proposal to include eight-quark interactions to stabilize the ground state of the combined three flavor Nambu–Jona-Lasinio (NJL) [1] and 't Hooft [2] Lagrangians [3–9] has been done in [10], where the most general spin zero combinations have been worked out. The bosonized Lagrangian and the corresponding scalar effective potential and gap equations have been derived at leading order of the $1/N_c$ expansion, embracing effects of the different current quark masses $m_u \neq m_d \neq m_s$.

Since then a number of physical applications of the enlarged model has been discussed at length. Among them the spectra and properties of the low lying pseudoscalar and scalar meson nonets [11], the behavior of the hadronic vacuum in a constant magnetic field [12], or at finite temperature [13]. This latter study has been done in the $SU(3)$ flavor limit for massless quarks. In particular, it has been realized that the critical temperature $T_c$ at which transitions occur from the dynamically broken chiral phase to the symmetric phase decreases in presence of the eight-quark interactions. The part of the multi-quark Lagrangian that violates the Okubo–Zweig–Iizuka (OZI) rule is mainly responsible for this effect.

Although eight-quark interactions are not required to stabilize the vacuum of the $SU(2)$ NJL model, their effect on the critical temperature of the chiral transition has been studied as well for this case [14]. The conclusion remained unchanged.

The OZI violating interactions have a strong impact on the type of chiral transition with massless quarks. If their strength is relatively weak, the phase transition restoring chiral symmetry at finite temperatures is of second order, as in the standard NJL model. However, with increase of the strength at $T = 0$ the effective potential develops a minimum at the origin [11]. In this case there is the possibility of coexistence with another minimum obtained by spontaneously breaking the symmetry from a certain critical value of the strength of the 't Hooft interaction. These starting configurations with double vacua at $T = 0$ [15] lead to a lowering of the critical temperature as well, but a new interesting feature is that one deals now with a chiral transition of first order.

We stress that the usual picture associated with spontaneous breakdown of chiral symmetry in NJL models is the one related with a maximum of the potential at the origin, induced by the large value of the four-quark coupling strength. The alternative picture that we promote is only possible due to the $U_A(1)$

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breaking terms and eight-quark interactions. The latter guarantee global stability of the vacuum and do not alter the quality of mesonic spectra obtained in the conventional case.

The reported decrease in temperature is welcome in view of recent lattice calculations [16], obtained for finite values of the quark masses. In this case there is evidence that a rapid crossover occurs at $T \sim 150$ MeV (note that this value is not reached in the framework of the standard NJL approach). In the massless case one expects a first order transition for the three flavor case [17–20].

In this Letter we study the combined impact of realistic values for the current quark masses and eight-quark interactions on the occurrence of extrema in the thermodynamic potential of the model and their behavior as functions of the temperature. Beside a rapid crossover the model also allows for first order transitions to occur. The latter is a new attribute of NJL models, induced by the eight-quark forces. We elucidate how the strength of eight-quark interactions versus the four-quark coupling constant gauges the occurrence of the different allowed solutions. The temperature dependence of meson spectra for the low lying pseudoscalars and scalars is calculated in the rapid crossover case to illustrate the result.

We do not consider here the effect of the Polyakov loop [21–23], which is known to increase the transition temperature by $\sim 25$ MeV [16]. In relation with the NJL model the role of the Polyakov loop has been investigated in several papers, see, e.g., [24–26]. The present study can be extended likewise.

The central object of the present analysis are the gap equations at finite temperature. The corresponding expressions at $T = 0$ were derived in [11] within a generalized heat kernel scheme which takes into account quark mass differences in a symmetry preserving way at each order of the long wave-length expansion [27], and we will consider from now on the isospin limit, $m_u = m_d \neq m_s$

$$\begin{cases} h_u + \frac{N_c}{6\pi^2} M_u (3I_0 - \Delta_{us} I_1) = 0, \\ h_s + \frac{N_c}{6\pi^2} M_s (3I_0 + 2\Delta_{us} I_1) = 0. \end{cases} \tag{1}$$

This system must be solved selfconsistently with the stationary phase equations

$$\begin{cases} G h_u + \Delta_u + \frac{\kappa}{16} h_u h_s + \frac{g_1}{4} h_u (2h_u^2 + h_s^2) + \frac{g_2}{2} h_u^3 = 0, \\ G h_s + \Delta_s + \frac{\kappa}{16} h_u^2 + \frac{g_1}{4} h_s (2h_u^2 + h_s^2) + \frac{g_2}{2} h_s^3 = 0. \end{cases} \tag{2}$$

Here $\Delta_{us} = M_u^2 - M_s^2$, $\Delta_l = M_l - m_l$, $l = u, d, s$, and $M_l$ denote the constituent quark masses. The factors $I_i$, are given by the average $I_i = [2J_i(M_u^2) + J_i(M_s^2)]/3$, $i = 0, 1, \ldots$, and represent one-quark-loop integrals

$$J_i(M^2) = \int_0^\infty \frac{dt}{t^{2-i}} \rho(t\Lambda^2) e^{-tM^2}, \tag{3}$$

with the Pauli–Villars regularization kernel [28]

$$\rho(t\Lambda^2) = 1 - (1 + t\Lambda^2) \exp(-t\Lambda^2), \tag{4}$$

where $\Lambda$ is an ultra-violet cutoff (the model is not renormalizable). In the approximation considered we need only to know

$$J_0(M^2) = \Lambda^2 - M^2 \ln\left(1 + \frac{\Lambda^2}{M^2}\right), \tag{5}$$

and

$$J_1(M^2) = \ln\left(1 + \frac{\Lambda^2}{M^2}\right) - \frac{\Lambda^2}{\Lambda^2 + M^2}. \tag{6}$$

The model parameters are the four-quark coupling $G$, the 't Hooft interaction coupling $\kappa$, the eight-quark couplings $g_1$, $g_2$ ($g_1$ multiplies the OZI violating combination), the current quark masses $m_l$, and the cutoff $\Lambda$. The stability of the effective potential is guaranteed if the couplings fulfill the following inequality [10]

$$g_1 > 0, \qquad g_1 + 3g_2 > 0, \qquad G > \frac{1}{g_1}\left(\frac{\kappa}{16}\right)^2. \tag{7}$$

We need additional input to fix parameters. The mass formulae of the pseudoscalar and scalar mesons, and the weak decay couplings $f_\pi$, $f_K$ obtained in [11] are used for that. The fit shows the interesting correlation between $G$ and the OZI violating coupling $g_1$: $G$ goes down, when $g_1$ increases. The mass of $f_0(600)$ is the main observable responsive to these changes, diminishing with increasing strength of the $g_1$ coupling. The point is that although spectra are essentially not sensitive to this correlation, the effective potential is. We will see that finally the pattern of the finite temperature restoration of chiral symmetry depends on how strong the OZI violating forces are.

In Tables 1–3 the low lying pseudoscalar and scalar meson characterisitics are displayed at $T = 0$ (stars denote input). One sees from Table 1 how a smaller value of the parameter $G$ is accompanied with an increase of the eight-quark coupling $g_1$.

The generalization to finite temperature of these expressions occurs in the quark loop integrals $J_0$, $J_1$. Introducing the Mat-

Table 1
Parameters of the model: $m_u = m_d$, $m_s$, and $\Lambda$ are given in MeV. The couplings have the following units: $G$ (GeV$^{-2}$), $\kappa$ (GeV$^{-5}$), $g_1$, $g_2$ (GeV$^{-8}$). We also show the values of constituent quark masses $M_u = M_d$ and $M_s$ in MeV (only the case of global minima)

| Sets | $m_u$ | $m_s$ | $M_u$ | $M_s$ | $\Lambda$ | $G$ | $-\kappa$ | $g_1$ | $g_2$ |
|---|---|---|---|---|---|---|---|---|---|
| a | 5.8 | 183 | 348 | 544 | 864 | 10.8 | 921 | 0* | 0* |
| b | 5.8 | 181 | 345 | 539 | 867 | 9.19 | 902 | 3000* | −902 |
| c | 5.9 | 186 | 359 | 544 | 851 | 7.03 | 1001 | 8000* | −47 |
| d | 5.8 | 181 | 345 | 539 | 867 | 5.00 | 902 | 10000* | −902 |

Table 2
The masses, weak decay constants of light pseudoscalars (in MeV), the singlet-octet mixing angle $\theta_p$ (in degrees), and the quark condensates $\langle\bar{u}u\rangle$, $\langle\bar{s}s\rangle$ expressed as usual by positive combinations in MeV

| | $m_\pi$ | $m_K$ | $m_\eta$ | $m_{\eta'}$ | $f_\pi$ | $f_K$ | $\theta_p$ | $-\langle\bar{u}u\rangle^{\frac{1}{3}}$ | $-\langle\bar{s}s\rangle^{\frac{1}{3}}$ |
|---|---|---|---|---|---|---|---|---|---|
| a | 138* | 494* | 480 | 958* | 92* | 118* | −13.6 | 237 | 191 |
| b | 138* | 494* | 480 | 958* | 92* | 118* | −13.6 | 237 | 192 |
| c | 138* | 494* | 477 | 958* | 92* | 117* | −14.0 | 235 | 187 |
| d | 138* | 494* | 480 | 958* | 92* | 118* | −13.6 | 237 | 192 |

Table 3
The masses of the scalar nonet (in MeV), and the corresponding singlet-octet mixing angle $\theta_s$ (in degrees)

| Sets | $m_{a_0(980)}$ | $m_{K_0^*(800)}$ | $m_{f_0(600)}$ | $m_{f_0(980)}$ | $\theta_s$ |
|---|---|---|---|---|---|
| a | 963.5 | 1181 | 707 | 1353 | 24 |
| b | 1024* | 1232 | 605 | 1378 | 20 |
| c | 980* | 1201 | 463 | 1350 | 24 |
| d | 1024* | 1232 | 353 | 1363 | 16 |

subara frequencies

$$J_0(M^2) \to J_0(M^2, T) = 16\pi^2 T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} \times \int_0^\infty ds\, \rho(s\Lambda^2) e^{-s[(2n+1)^2\pi^2T^2+\vec{p}^2+M^2]}, \tag{8}$$

and using the Poisson formula

$$\sum_{n=-\infty}^{\infty} F(n) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{+\infty} dx\, F(x) e^{i2\pi mx}, \tag{9}$$

where $F(n) = \exp[-s(2n+1)^2\pi^2T^2]$, one integrates over the 3-momentum $\vec{p}$, getting the result

$$J_0(M^2, T) = \int_0^\infty \frac{ds}{s^2} \rho(s\Lambda^2) e^{-sM^2} \times \left[1 + 2\sum_{n=1}^{\infty} (-1)^n \exp\left(\frac{-n^2}{4sT^2}\right)\right]. \tag{10}$$

Similarly one gets

$$J_1(M^2, T) = -\frac{\partial}{\partial M^2} J_0(M^2, T). \tag{11}$$

It is then easy to verify that $\lim_{T\to\infty} J_{0,1}(M^2, T) = 0$, and at $T = 0$ one recovers the starting expressions (3).

Using these expressions in Eq. (1), we solve the system (1)–(2) numerically. We henceforth assume that the model parameters $G, \kappa, g_1, g_2, m_l, \Lambda$ do not depend on the temperature. As a result we obtain the temperature dependent solutions $M_l(T)$, representing the extrema of the thermodynamic potential.

The result is show in Fig. 1 (set c) and Fig. 2 (set d). There are either one or three $(M_u^{(i)} = M_d^{(i)}, M_s^{(i)})$, $i = 1, 2, 3$ couples of solutions at fixed values of $T$. For set (c) (as well as (b)) only one branch of solutions is physical, i.e. positive valued. The other two have negative values for the light quark masses. This is in contrast with the $SU(3)$ limit case with zero current quark masses, where one branch collapses to the origin $M_u = M_d = M_s = 0$ for all values of the remaining model parameters and $T$.

For finite values of the current quark masses the corresponding branch moves in the $M_u, M_s$ plane: only below a certain critical value of $G$ does it remain positive valued at all $T$. This is the case shown in Fig. 2 for set (d), where all three branches are positive valued.

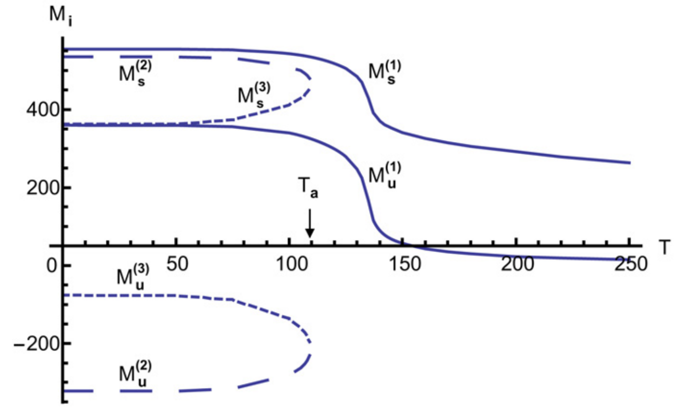


Fig. 1. Branches of $M_u^{(i)}(T)$, $M_s^{(i)}(T)$ pairs, denoting extremal points of the thermodynamic potential. Solid lines (start at $T = 0$ as deepest minima), dashed lines (start as relative minima at $T = 0$) and dotted lines (saddle at $T = 0$) for the parameter set (c). Only one branch is in the physical (positive masses) region (solid curves). All units are in MeV.

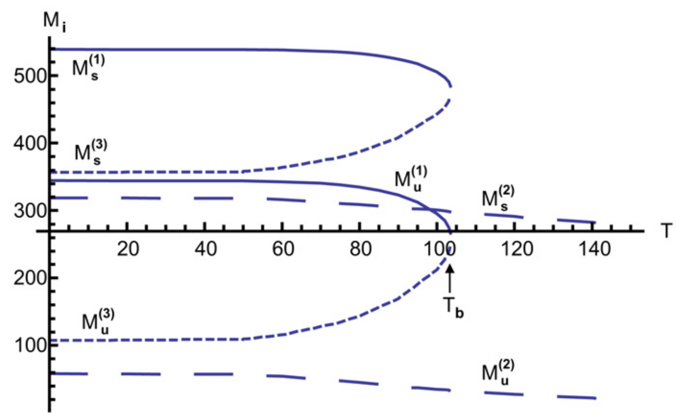


Fig. 2. The same as Fig. 1 but for set (d). All branches lie in the physical region and coexist up to $T = T_b$, from this value on only one branch survives, with much lower values of $M_l$.

In case (c) the physical changes in the values of the quark masses as functions of $T$ are traced back to a single branch. One sees however that the onset of the transition occurs at a value of $T = T_a$ for which the other unphysical two branches meet and cease to exist. The rapid crossover occurs in the short temperature interval 125 MeV $< T <$ 140 MeV.

As opposed to this scenario all three branches of set (d) are positive valued. Two of the branches (starting from the stable minimum and the saddle solution at $T = 0$) merge in the physical region at a certain $T_b$ and the surviving branch has a significantly lower mass value. Therefore in case (d) the changes involve a jump from one branch to the other, and lead to discontinuities in the observables. This is a first order phase transition.

At present lattice QCD data have not unambiguously settled the question about the order of the chiral transition. For physical values of the quark masses, calculations with staggered fermions favor a smooth crossover transition [18], while calculations with Wilson fermions predict the transition to be first order [29].

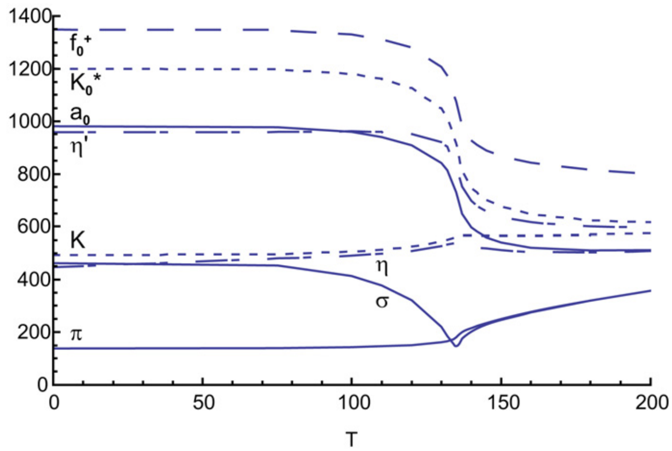


Fig. 3. The masses of pion, $\sigma = f_0(600)$, $\eta$, kaon, $\eta'$, $a_0$, $K_0^*$ and $f_0^+ = f_0(980)$ from bottom to top, for set (c) as functions of the temperature (all in MeV).

Our calculations show that OZI violating interactions can be important for the issue. If they are small (as in set (c)) the chiral symmetry restoring transition is crossover. However, above some critical value of the eight-quark coupling $g_1$ (as in set (d)) the pattern of the transition is switched to first order. This correlation can be used to set an upper (or lower) bound for the strength of the OZI violating coupling $g_1$. Without eight-quark interactions (set (a)) the physical branch evolves as function of $T$ qualitatively as in Fig. 1, but the crossover takes place at much larger temperatures, $T \simeq 210$ MeV, and the transition is much smoother.

The masses of scalar and pseudoscalar mesons at finite temperature obtained for the set (c) are shown in Fig. 3. As can be seen there is a rapid crossover for all meson masses in the same temperature interval as in Fig. 1. Strictly speaking, neither this rapid crossover nor the first order transition case (d) do imply restoration of chiral or $U_A(1)$ symmetry, but only the recovery of a distorted Wigner–Weyl phase, with the minimum of the thermodynamic potential shifted to finite quark mass values due solely to flavor breaking effects.

To get an insight in the role played by the different multi-quark interactions we analyze two limits, with the parameter set (c) as starting condition.

Case 1: We set $g_1 = g_2 = \kappa = 0$ and remaining parameters as in (c). In this limit the gap equation has only one solution for the considered parameter set, thus the system is in a distorted Wigner–Weyl phase.

Case 2: We set $\kappa = 0$ and all other parameters fixed as in (c). In this case there is no $U_A(1)$ breaking, but OZI violating effects are present. We verify that in this limit the gap equation has also only one solution, being again in a distorted Wigner–Weyl phase.

Thus the spontaneous symmetry breakdown seen in the full set (c) at $T = 0$ (and also in sets b and d) is driven exclusively by the 't Hooft interaction strength $\kappa$. We wish not to include case (a) in the present discussion, as it violates the stability conditions of (7).

In conclusion, the present study reveals that eight-quark interactions, whose effects are in an almost "dormant" state as far as the low lying scalar and pseudoscalar mesonic spectra are concerned, are of great importance in the study of temperature effects in chiral multi-quark interaction Lagrangians. It turns out that the mesonic spectra built on the spontaneously broken vacuum induced by the 't Hooft interaction strength, as opposed to the commonly considered case driven by the four-quark coupling, undergo a rapid crossover to the unbroken phase, with a slope and at a temperature which is regulated by the strength of the OZI violating eight-quark interactions. This strength can be adjusted in consonance with the four-quark coupling (keeping the remaining model parameters fixed), and leaves the spectra unchanged, except for the sigma meson mass, which decreases. This effect also explains why in the crossover region the sigma meson mass drops slightly below the pion mass. A first order transition behavior is also a possible solution within the present approach. Additional information from lattice calculations and phenomenology is necessary to fix finally the strength of interactions. We expect that the role of eight-quark interactions are of equal importance in studies involving a dense medium and extensions of the model with the Polyakov loop.

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