

# Distributed event-triggered $H_\infty$ filtering over sensor networks with communication delays

Xiaohua Ge, Qing-Long Han\*

*Centre for Intelligent and Networked Systems, Central Queensland University, Rockhampton, QLD 4702, Australia*

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## Abstract

This paper is concerned with distributed event-triggered  $H_\infty$  filtering over sensor networks with communication delays. Firstly, a new distributed event-triggered communication scheme is proposed to determine whether or not each sensor's current sampled-data should be broadcast and transmitted for filter design. Each sensor node is capable to make its own decision to broadcast and to transmit current sampled-data only when its local measurement output error exceeds a specified threshold. Secondly, under the distributed event-triggered communication scheme, the resultant filtering error system, which includes information of communication delays, is transformed into a system with multiple delays. Sufficient conditions on the existence of desired distributed event-triggered  $H_\infty$  filters are derived such that the resultant filtering error system is asymptotically stable with prescribed weighting average  $H_\infty$  performance. The trade-off analysis between communication resource utilization and weighting average  $H_\infty$  performance is carried out. Thirdly, a co-design algorithm for simultaneously determining the filter parameters and the threshold parameters is developed. Finally, a benchmark example is given to show the effectiveness of the obtained theoretical results.

*Keywords:* Distributed  $H_\infty$  filtering, event-triggered communication scheme, sensor network, communication delay, co-design algorithm.

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## 1. Introduction

Recent advancements in hardware and wireless communication technologies have enabled the development of low-cost, low-battery, and multi-functional sensor nodes. A sensor network is usually composed of a large number of sensor nodes, which are dispersedly deployed in the sensor field and collaborate among themselves. Each sensor node is equipped with a radio transceiver, allowing it to communicate with certain amount of neighboring nodes and to send/receive data to/from not only itself but also its neighboring nodes in accordance with a given sensing topology. With its data processing and broadcasting ability, each node also locally carries out simple computation of aggregated data and transmits the processed data to a remote task manager node, e.g., a filter node, only when required via communication networks. Recently, distributed filtering over sensor networks has been received increasing attention from signal processing and networking communities [2, 12, 17, 21, 24, 28].

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\*Corresponding author: Qing-Long Han: Tel.: +61 7 4930 9270; Fax: +61 7 4930 6886; E-mail: q.han@cqu.edu.au

In practice, sensor nodes in sensor networks are often battery operated and usually have limited energy resources for broadcasting their sampled-data to remote task manager nodes through a communication network. The network bandwidth is also a limited resource for data transmission from nodes in the sensor field. On the one hand, it is clear that the implicit assumption made by traditional filtering schemes [12, 24] that the continuous-time measurement output signal of the plant is equal to the input signal of the filter is no longer applicable in practical network environments due to digital implementation of network components. On the other hand, when there is a little fluctuation between two consecutive sampled-data or in the case that the system's state is approaching to its equilibrium and no external disturbance is acting on the system, it is obviously a waste of scarce communication resources to broadcast and to transmit current sampled-data for filter design. In this sense, it is unnecessary to update the signals by using traditional *time-triggered communication schemes* [2, 9–11, 17, 21, 22, 28], in which data transmission is executed periodically after the elapse of a fixed time interval. In fact, time-triggered communication schemes increase the frequency with which sensors broadcast and transmit the sampled-data, thereby leading to increased consumption of limited energy and/or network bandwidth resources. Therefore, one important issue in the implementation of sensor networks is to identify an efficient communication scheme to determine when or how frequently each sensor should broadcast and transmit current sampled-data to a remote task manager node to reduce the occupancy of scarce communication resources.

To overcome the drawback of traditional time-triggered communication schemes, *event-triggered control schemes* have been proposed in the published literature to mitigate the need for unnecessary data transmission while preserving system performance and have been implemented on several different system models, see e.g., linear time-invariant systems [4, 5, 14, 18, 29], nonlinear systems [1, 13, 20, 25], wireless sensor actuator networks [15], multi-agent systems [7], and distributed networked control systems [25]. Under the paradigm of event-triggered control, the objective control task is only executed after the occurrence of a specified event which is usually generated by an event triggering condition. In this sense, network resources are occupied only when “needed” and the number of transmitted control task executions is capable of significantly reducing. However, all the works above-mentioned on event-triggered communication schemes have focused on control issues of various system models. In contrast, few results in the context of *distributed event-triggered filtering* have been reported in the existing literature. Note that most of the existing event-triggered communication schemes on control issues may be not suitable to be straightforwardly extended to address the distributed event-triggered filtering problem over sensor networks due to the facts that: (i) nodes in sensor networks collaborate with each other and each of them is capable of collecting measurement from its neighboring nodes. These sensor nodes may not have access to all the measurement. Furthermore, the measurement on different sensor nodes may have different physical characteristics, e.g., motion, temperature, and electricity. Therefore, it is preferable to design a new distributed event-triggered communication scheme which reflects measurement exchanged among neighboring nodes and triggers data transmission from different sensor nodes in a distributed manner; (ii) some existing distributed event-triggered control schemes [15, 25] need to be performed continuously since the event triggering conditions depend on continuous-time states of the system, which implies that an extra dedicated hardware is required to monitor the instantaneous states of the system so as to trigger the next event. Hence, it is of significance and necessity to propose a new distributed event-triggered communication scheme in which the event triggering conditions are only dependent on the discrete measurement outputs; and (iii) the majority of the current event-triggered control

schemes are based on a prior assumption that the event triggering functions are determined in advance and then suitable control strategies are designed in order to ensure the stability of the system. In other words, the event triggering parameters need to be preset before implementing the control laws. It is thus desirable to develop a novel distributed event-triggered filtering scheme to co-design both the filter parameters and the event triggering parameters. Recently, the problem of distributed event-triggered estimation over wireless sensor networks was studied in [26]. A global event-triggered communication policy for state estimation was established to minimize the weighted function of the network energy consumption and the number of transmissions. Nevertheless, the algorithm design was limited to a central estimator, i.e., all the sensed measurement was sent to a fusion center in which a dedicated estimation approach was applied. It is noteworthy that this centralized estimation scheme may cause high cost for the fusion center collecting measurement from every sensor node and lead to increase of computational burden for the fusion center computing the estimation in a centralized manner [6]. Consequently, a fully distributed event-triggered filtering algorithm, which uses local measurement information available from each communication link, is preferred in the setting of sensor networks. In [16], although an event-triggered communication scheme was developed to determine when each sensor should transmit its sampled measurement to a remote filter through a wireless sensor network, information exchange, which plays an important role in the process of distributed filtering, among sensors was not considered. To the best of the authors' knowledge, *there is no systematic result that addresses the distributed event-triggered  $H_\infty$  filtering problem over sensor networks in a fully distributed fashion while simultaneously preserving desired system performance and reducing communication resource utilization. This is the first motivation of the current study.*

For network-based control and filtering, network-induced communication delays inevitably occur during data transmission due to the limited network bandwidth and/or congested network traffic. There are some recent results available about network-induced delays [3, 19, 23, 31, 32]. However, the effects of communication delays are not considered in the majority of results on distributed filtering over sensor networks in the existing literature [2, 12, 21, 24, 26, 28]. It is well acknowledged that network-induced delays are regarded as one of the main sources of system performance degradation and divergence of distributed filtering algorithms implemented. Moreover, when the design issue of the event-triggering communication scheme is pursued, the effects of communication delays should be taken into account since the design procedure is performed by using the sampled-data after it is transmitted through unreliable communication networks. In this sense, communication delays may affect not only system performance but also network resource utilization. *Therefore, the second motivation of this study is to further investigate the effects of communication delays on distributed event-triggered  $H_\infty$  filtering over sensor networks in a unified framework.*

Based on the observations above, this paper aims to address the distributed event-triggered  $H_\infty$  filtering problem over sensor networks in the presence of communication delays. The main contributions are summarized as follows:

- (i) *A new distributed event-triggered communication scheme is proposed.* The sensor nodes are dispersedly deployed in the sensor field and the filter nodes are physically distributed via a communication network. Each of these sensor nodes has capability to collect measurement outputs from its all underlying neighboring nodes, to process aggregated data in accordance with a prescribed sensing topology, to sample these processed data at a constant sampling period, and to further broadcast the sampled-data to a remote filter node through the network. Whether or not current sampled-data on each sensor node should be broadcast and transmitted

is determined by an event monitor. Each node can locally make its own decision to broadcast and to transmit its sampled-data only when the local output error value between the current sampled output and the latest transmitted output exceeds a specified threshold;

- (ii) *Criteria for designing distributed event-triggered  $H_\infty$  filters are established.* Under the distributed event-triggered communication scheme, the resultant filtering error system, which includes information of communication delays, is transformed into a system with multiple delays. New sufficient conditions on the existence of desired distributed event-triggered  $H_\infty$  filters are derived such that the resultant filtering error system is asymptotically stable with prescribed weighting average  $H_\infty$  disturbance attenuation performance. Applying the obtained theoretical results, the trade-off analysis between communication resource occupancy and weighting average  $H_\infty$  performance is carried out; and
- (iii) *A new co-design algorithm is developed* to determine desired filter parameters and threshold parameters while simultaneously preserving an expected level of weighting average  $H_\infty$  performance and maintaining an expected level of communication resources occupancy for data transmission.

The effectiveness of the proposed filter design method is illustrated by a benchmark example.

*Notation:*  $diag_N\{U\}$  (or  $diag_N\{U_i\}$ ) denotes the  $N$ -block-diagonal matrix  $diag\{U, \dots, U\}$  (or  $diag\{U_1, \dots, U_N\}$ ).  $diag_N^i\{U\}$  (or  $diag_N^i\{U_i\}$ ) denotes the  $N$ -block-diagonal matrix with its  $i$ -th block  $U$  (or  $U_i$ ) and the others zero. Similarly,  $vec_N\{U\}$  (or  $vec_N\{U_i\}$ ) and  $vec_N^i\{U\}$  (or  $vec_N^i\{U_i\}$ ) denote row vectors with suitable blocks;  $col_N\{U\}$  (or  $col_N\{U_i\}$ ) and  $col_N^i\{U\}$  (or  $col_N^i\{U_i\}$ ) denote column vectors with suitable blocks.  $\otimes$  stands for the Kronecker product for matrices.  $sym(UV)$  refers to the symmetrized expression  $UV + V^T U^T$ . Other notations used in this paper are quite standard.

## 2. Problem statement and a distributed event-triggered communication scheme

### 2.1. The system description

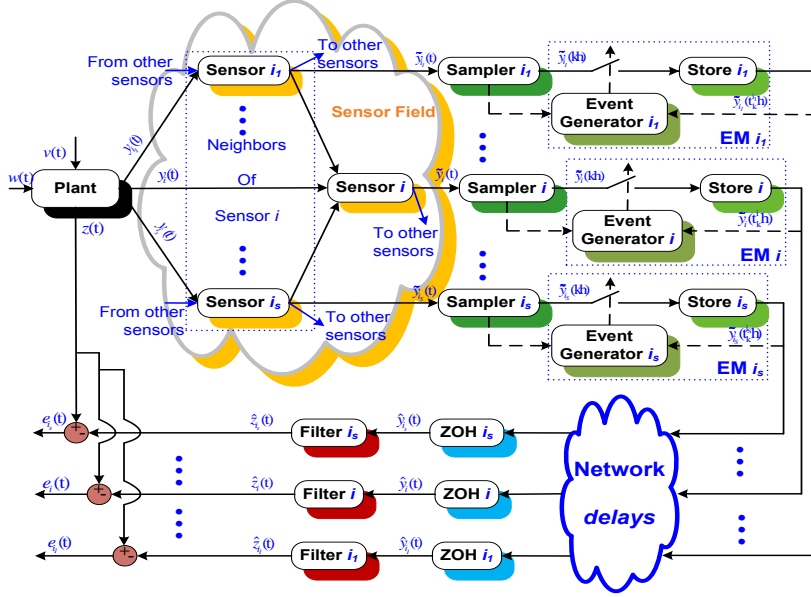
Consider the following continuous-time linear time-invariant system described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t), & x(0) = x_0 \\ z(t) = Ex(t) \end{cases} \quad (1)$$

over a sensor network with the measurement outputs on  $N$  distributed sensor nodes given by

$$y_i(t) = C_i x(t) + D_i v(t), \quad i \in \mathcal{V} = \{1, 2, \dots, N\} \quad (2)$$

where  $x(t) \in \mathbb{R}^{n_x}$  is the state;  $w(t) \in \mathbb{R}^{n_w}$  and  $v(t) \in \mathbb{R}^{n_v}$ , which belong to  $\mathcal{L}_2[0, \infty)$ , are the system external disturbance noise signal and the output measurement noise signal, respectively;  $z(t) \in \mathbb{R}^{n_z}$  is the objective output signal to be estimated;  $x_0$  is the initial state;  $y_i(t) \in \mathbb{R}^{n_y}$  is the measurement output received by the sensor  $i$  from the plant;  $A, B, E, C_i$ , and  $D_i$ ,  $i \in \mathcal{V}$ , are known matrices with appropriate dimensions.



**Figure 1:** A schematic diagram of distributed event-triggered  $H_\infty$  filtering over a sensor network

## 2.2. The sensing topology

Let  $\mathcal{V}$  be an index set of  $N$  sensor nodes,  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  be the edge set of paired sensor nodes, and  $\mathcal{W} = [w_{ij}] \in \mathbb{R}^{N \times N}$  be the weighted adjacency matrix. The directed graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$  represents the sensing topology of the sensor network. An edge of  $\mathcal{G}$  is denoted by  $(i, j)$ . The adjacency elements  $w_{ij}$  associated with the edges of the graph are positive, i.e.,  $w_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$ . Moreover, we assume that self-loops are allowed in the graph, i.e.,  $w_{ii} = 1$ ,  $i \in \mathcal{V}$ , and therefore,  $(i, i)$  can be regarded as an additional edge. The set of neighbors of node  $i$  plus the node itself is denoted by  $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$ .

## 2.3. Distributed $H_\infty$ filters

The structure of data collection surrounding the sensor node  $i$  is shown in Figure 1, where the data available on the sensor node  $i$  comes from both itself and its all underlying neighboring nodes  $i_1, \dots, i_s \in \mathcal{N}_i$ , dispersedly deployed in the sensor field. Then, the aggregated data is processed and given by

$$\tilde{y}_i(t) = \sum_{j \in \mathcal{N}_i} w_{ij} y_j(t) \quad (3)$$

in accordance with the prescribed sensing topology  $\mathcal{G}$ . After simple computation, the processed data is sampled and broadcast to a remote filter node through a communication network. Whether or not the current sampled-data on each sensor node should be broadcast through the network medium is determined by a distributed event-triggered communication scheme, i.e., the data transmission through the communication network only takes place when a specified event occurs. Based on the facts above, the following distributed  $H_\infty$  filters are proposed

$$\begin{cases} \dot{\hat{x}}_i(t) = G_i \hat{x}_i(t) + K_i \hat{y}_i(t) \\ \hat{z}_i(t) = H_i \hat{x}_i(t) + L_i \hat{y}_i(t) \end{cases} \quad (4)$$

where  $\hat{x}_i(t) \in \mathbb{R}^{n_x}$  is the state estimation of the filter  $i$ ;  $\hat{y}_i(t)$  is the input of the filter  $i$  collected from the sensor node  $i$  itself and its all possible neighboring nodes  $\mathcal{N}_i$  through the network;  $\hat{z}_i(t) \in \mathbb{R}^{n_z}$  is the output of the filter  $i$  representing an estimation of  $z(t)$ ;  $G_i, K_i, H_i$ , and  $L_i, i \in \mathcal{V}$ , are the filter parameter matrices to be determined.

#### 2.4. A distributed event-triggered communication scheme

For convenience of development, we introduce the following notations:

- (i)  $\{kh \mid k \in \mathbb{N} = \{0, 1, 2, \dots\}\}$  : *the sampling time sequence*. The samplers are assumed to be clock-driven and synchronized. The processed data  $\tilde{y}_i(t)$  on the sensor node  $i$  is sampled at discrete instants  $kh$ , where  $k \in \mathbb{N}$  and  $h > 0$  represents a constant sampling period. At each sampling instant  $kh$ , the sampled-data  $\tilde{y}_i(kh)$  and its time-stamp  $k$  are encapsulated into a data packet  $(k, \tilde{y}_i(kh))$ ;
- (ii)  $\{t_k^i h \mid t_k^i \in \mathbb{N}\}$  : *the broadcasting and transmitting time sequence*. The  $t_k^i h$  denotes the time instant when the sensor node  $i$  broadcasts its sampled-data to the network and this broadcast data is assumed to be successfully received by a logic zero-order-holder (ZOH). All transmitted packets are time-stamped;
- (iii)  $\{s_k^i h \mid s_k^i = t_k^i + m_i, m_i = 0, 1, \dots, M_i, M_i = t_{k+1}^i - t_k^i - 1\}$  : *the real-time sampling time sequence between two consecutive broadcasting instants*;
- (iv)  $\tau_{t_k^i}^i$  : *the communication delay*. Denote by  $\tau_{t_k^i}^i$  the communication delay among the data transmission from the sensor node  $i$  to the filter node  $i$  at time instant  $t_k^i h$  introduced by the network. For simplicity but without loss of generality,  $\tau_{t_k^i}^i$  is assumed to be upper-bounded, i.e.,  $0 < \tau_{t_k^i}^i \leq \bar{\tau}_i$ . Let  $\bar{\tau} = \max_{i \in \mathcal{V}} \{\bar{\tau}_i\}$ .

An *event monitor* (EM) is introduced to determine when current sampled-data packet  $(s_k^i, \tilde{y}_i(s_k^i h))$  should be broadcast to the logic ZOH  $i$  for filter design, as shown in Figure 1. Each EM consists of an *event generator* and a *storer*. The event generator is adopted to produce a series of events. The sequence of broadcasting time instants is determined recursively according to the following communication scheme

$$t_{k+1}^i h = t_k^i h + \min_{m_i \geq 0} \left\{ (m_i + 1)h \mid \|\Psi_i^{\frac{1}{2}} e_y^i(t)\|_2 \geq \delta_i \|\Psi_i^{\frac{1}{2}} \tilde{y}_i(t_k^i h)\|_2 \right\} \quad (5)$$

where  $e_y^i(t)$  is a local output error between the latest broadcasting time instant and the real-time sampling time instant on node  $i$ , i.e.,  $e_y^i(t) = \tilde{y}_i(t_k^i h) - \tilde{y}_i(s_k^i h)$ ;  $\Psi_i$  is a symmetric positive definite weighting matrix; and  $\delta_i \in (0, 1)$  is a scalar *threshold parameter* on node  $i$ . The storer is used to reserve the latest broadcast sampled-data.

**Remark 1.** Similar to [30], the ZOH  $i$  in Figure 1 is assumed to be event-driven and has the logical capability of checking the time stamps of the newly arrived data packets and choosing the newest one to actuate the filter. Even though transmitted data packets may arrive at the ZOH  $i$  with a different temporal order due to the communication delay, the ZOH  $i$  is configured to keep the arrived packet only when its time stamp is greater than that of currently stored packet. In other words, the

logic ZOH  $i$  is to store the latest filter input packet and to keep the filter input unchanged until the output of the logic ZOH  $i$  being updated by a new data packet. Therefore, the out-of-order packet in the transmission is discarded actively by the logic ZOH  $i$  and the phenomenon of packet disorder is avoidable. Furthermore, the relationship of  $t_{k+1}^i h + \tau_{t_{k+1}^i}^i > t_k^i h + \tau_{t_k^i}^i$  is ensured by the logic ZOH  $i$ . Without loss of generality, we assume that data transmission over the network is performed in a single packet manner and data packet dropouts do not happen during data transmission.

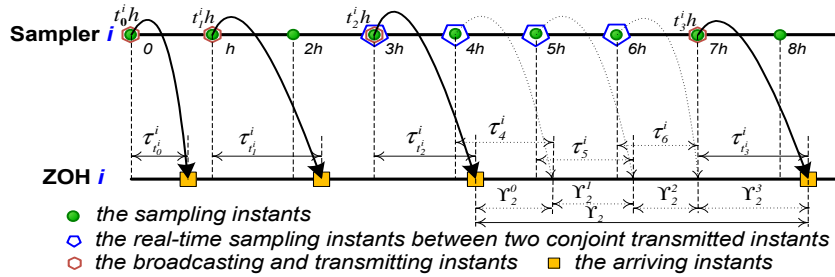
Under the proposed communication scheme (5), once the real-time sampled-data  $\tilde{y}_i(s_k^i h)$  satisfies  $\|\Psi_i^{\frac{1}{2}} e_{\tilde{y}}^i(t)\|_2 \geq \delta_i \|\Psi_i^{\frac{1}{2}} \tilde{y}_i(t_k^i h)\|_2$ , the EM  $i$  immediately broadcasts the real-time sampled-data through the communication network. Otherwise, this sampled-data is discarded right away. It is worth stressing that the logic ZOH  $i$  keeps the latest data packet and filter  $i$  will hold its input unchanged until the output of the logic ZOH  $i$  being updated by a new data packet. When the latest transmitted data packet  $(t_k^i, \tilde{y}_i(t_k^i h))$  is released from the logic ZOH  $i$ , it immediately actuates the filter  $i$  with  $\tilde{y}_i(t_k^i h)$ . In this sense,  $\forall t \in [t_k^i h + \tau_{t_k^i}^i, t_{k+1}^i h + \tau_{t_{k+1}^i}^i)$ , the filter input on node  $i$  is given by

$$\hat{y}_i(t) = \tilde{y}_i(t_k^i h) = \sum_{j \in \mathcal{N}_i} w_{ij} y_j(t_k^i h). \quad (6)$$

Figure 2 depicts the timing diagram of signal sampling, transmitting and arriving. It is clearly seen that the set of transmitting instants  $\{t_k^i h | t_k^i \in \mathbb{N}\}$  is a subset of the sampling time sequence  $\{kh | k \in \mathbb{N}\}$ . Furthermore, the holding interval of the logic ZOH  $i$  can be reconstructed as

$$\Upsilon_k \triangleq [t_k^i h + \tau_{t_k^i}^i, t_{k+1}^i h + \tau_{t_{k+1}^i}^i) = \bigcup_{m_i=0}^{M_i} \Upsilon_k^{m_i} \quad (7)$$

where  $\Upsilon_k^{m_i} = [s_k^i h + \tau_{s_k^i}^i, s_k^i h + h + \tau_{s_{k+1}^i}^i)$ . Note that the communication delays  $\tau_{s_k^i}^i$ , ( $m_i \neq 0, M_i$ ) can be properly chosen while keeping the right order of the start point and the end point of the subsets  $\Upsilon_k^{m_i}$ , which is illustrated in Figure 2 on the transmitting interval  $[3h + \tau_3^i, 7h + \tau_7^i)$ .



**Figure 2:** The timing diagram of signal sampling and transmission

### 2.5. The filtering error system

To proceed with, we define an artificial time-varying delay as  $d_k^i(t) = t - s_k^i h$ ,  $t \in \Upsilon_k^{m_i}; i \in \mathcal{V}$  for each communication link. Then from the definition of  $d_k^i(t)$ , one can see that  $d_k^i(t)$  is a function satisfying

$$0 < \tau_{s_k^i}^i \leq d_k^i(t) < h + \bar{\tau} \triangleq \tau_M, t \in \Upsilon_k^{m_i}; \quad d_k^i(t) = 1, t \in \Upsilon_k^{m_i} \text{ and } t \neq s_k^i h + \tau_{s_k^i}^i. \quad (8)$$

To facilitate subsequent discussion, we further denote an estimation error vector  $e_i(t) = z(t) - \hat{z}_i(t)$ ,  $i \in \mathcal{V}$ , and  $\bar{A} = \text{diag}_N\{A\}$ ,  $\bar{B} = \text{col}_N\{B\}$ ,  $\bar{C} = \text{vec}_N^1\{\text{col}_N\{C_i\}\}$ ,  $\bar{C} = \text{diag}_N\{C_i\}$ ,  $\bar{D} = \text{col}_N\{D_i\}$ ,  $\bar{E} = \text{diag}_N\{E\}$ ,  $\xi(t) = \text{col}_N\{\hat{x}_i(t)\}$ ,  $\bar{x}(t) = \text{col}_N\{x(t)\}$ ,  $e(t) = \text{col}_N\{e_i(t)\}$ ,  $\bar{G} = \text{diag}_N\{G_i\}$ ,  $\bar{K}_i = \text{diag}_N^i\{K_i\}$ ,  $\bar{H} = \text{diag}_N\{H_i\}$ ,  $\bar{L}_i = \text{diag}_N^i\{L_i\}$ , and  $\bar{e}_y^i(t) = \text{col}_N^i\{e_y^i(t)\}$ .

Setting  $\chi(t) = \text{col}_2\{\bar{x}(t), \xi(t)\}$  and combining the system (1)-(2), and (4), we obtain the following augmented filtering error system

$$\begin{cases} \dot{\chi}(t) = \tilde{A}\chi(t) + \sum_{i=1}^N \tilde{A}_i\chi(t - d_k^i(t)) + \tilde{B}w(t) + \sum_{i=1}^N \tilde{C}_i\bar{e}_y^i(t) + \sum_{i=1}^N \tilde{D}_i v(t - d_k^i(t)) \\ e(t) = \tilde{E}\chi(t) + \sum_{i=1}^N \tilde{E}_i\chi(t - d_k^i(t)) - \sum_{i=1}^N \tilde{L}_i\bar{e}_y^i(t) + \sum_{i=1}^N \tilde{F}_i v(t - d_k^i(t)) \end{cases} \quad (9)$$

for all  $t \in \Upsilon_k^{m_i}$ , where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{G} \end{bmatrix}, \tilde{A}_i = \begin{bmatrix} 0 & 0 \\ \bar{K}_i(\mathcal{W} \otimes I)\hat{C} & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} \bar{B} \\ 0 \end{bmatrix}, \tilde{C}_i = \begin{bmatrix} 0 \\ \bar{K}_i \end{bmatrix}, \tilde{D}_i = \begin{bmatrix} 0 \\ \bar{K}_i(\mathcal{W} \otimes I)\bar{D} \end{bmatrix} \\ \tilde{E} &= [\bar{E} \quad -\bar{H}], \tilde{E}_i = [-\bar{L}_i(\mathcal{W} \otimes I)\hat{C} \quad 0], \tilde{F}_i = -\bar{L}_i(\mathcal{W} \otimes I)\bar{D}. \end{aligned}$$

From the distributed event-triggered communication scheme (5), for all  $t \in \Upsilon_k^{m_i}$ , it is clear that

$$\|\Psi_i^{\frac{1}{2}}\bar{e}_y^i(t)\|_2 < \delta_i \|\Psi_i^{\frac{1}{2}}(\bar{e}_y^i(t) + \mathcal{C}_i\bar{x}(t - d_k^i(t)) + \mathcal{D}_i v(t - d_k^i(t)))\|_2 \quad (10)$$

where  $\mathcal{C}_i = \mathcal{I}_i((\mathcal{W} \otimes I)\hat{C})$ ,  $\mathcal{D}_i = \mathcal{I}_i((\mathcal{W} \otimes I)\bar{D})$  with  $\mathcal{I}_i = \text{diag}_N^i\{I\}$ ,  $i \in \mathcal{V}$ . The initial condition of the augmented filtering error system is supplemented as  $\chi(\theta) = \eta(\theta)$ ,  $\forall \theta \in [-\tau_M, 0]$ , where  $\eta(\theta)$  is a continuous function with  $\eta(0) = \text{col}_2\{\text{col}_N\{x_0\}, \text{col}_N\{0\}\}$ .

The **distributed event-triggered  $H_\infty$  filtering problem** to be addressed is then formulated as follows: For given scalars  $h > 0$ ,  $\bar{\tau} > 0$ , and  $\beta \in (0, 1)$ , co-design the distributed  $H_\infty$  filters in the form of (4) and the threshold parameters  $\delta_i > 0$  in (5) such that

- (i) The resultant filtering error system (9) subject to (10) with  $w(t) = 0$  and  $v(t - d_k^i(t)) = 0$ ,  $i \in \mathcal{V}$  is asymptotically stable;
- (ii) Under zero initial conditions, the estimation error signal  $e_i(t)$  satisfies the following weighting average  $H_\infty$  performance

$$\frac{1}{N} \sum_{i=1}^N \|e_i(t)\|_2^2 < \beta\gamma^2 \|w(t)\|_2^2 + \sum_{i=1}^N (1 - \beta)\gamma^2 \|v(t - d_k^i(t))\|_2^2 \quad (11)$$

for all nonzero  $w(t) \in \mathcal{L}_2[0, \infty)$ ,  $v(t - d_k^i(t)) \in l_2[0, \infty)$ ,  $i \in \mathcal{V}$ , where  $\gamma > 0$  is a desired level of  $H_\infty$  performance.

**Remark 2.** Inspired by [21], average  $H_\infty$  performance is adopted to guarantee that each filter well estimates the system's output  $z(t)$ . Note that the positive scalar  $\beta$  in (ii) refers to a prescribed weighting factor. It explicitly explains how external disturbance and measurement noise individually affect system performance at a different weighting rate. How to choose the weighting factor  $\beta$  is a trade-off between the effects of the external disturbance and the measurement noise, and depends on practical situations. In an illustrative example in Section 6, we plot the relationship between the minimal value  $\gamma_{min}$  of the weighting average  $H_\infty$  performance and the weighting factor  $\beta$ . From the figure, one can properly select the weighting factor  $\beta$  to obtain desirable weighting average  $H_\infty$  performance.



**Remark 3.** When the threshold parameter  $\delta_i \rightarrow 0^+$ , the next broadcasting time instant  $t_{k+1}^i h$  can be approximately determined by  $t_k^i h + h$  from (5), which means that each node broadcasts its sampled-data in a time-triggered or periodical fashion. As a consequence, the dynamics of the system (9) with an event-triggered communication scheme approaches to the one with a time-triggered communication scheme. Note from the formulation (9) that the resultant filtering error system is transformed into a system with multiple artificial delays  $d_k^i(t)$ ,  $t \in \Upsilon_k^{m_i}$ ;  $i \in \mathcal{V}$ , and “perturbations”  $\bar{e}_y^i(t)$  which depend on the errors between the latest transmitted data and the current sampled-data described by (10).

**Remark 4.** The proposed distributed event-triggered communication scheme has the following features: (i) there is no need to establish some extra hardware to detect and monitor the successive outputs since the event triggering condition is only dependent on the latest transmitted sampled-data and the local output error at the discrete sampling instants; (ii) the lower bound of the inter-event time (the minimum time interval between two consecutive events, i.e.,  $t_{k+1}^i h - t_k^i h$ ) is an integer multiple of the sampling period  $h > 0$ . Therefore, the “Zeno” behavior [4] is avoided; (iii) the next broadcasting time instants  $t_{k+1}^i h$  on the sensor node  $i$  are closely related to both its own and its neighbors’ sampled-data, which means that the proposed communication scheme enables collected data from the sensor node  $i$  itself and its all underlying neighboring nodes  $\mathcal{N}_i$  to be broadcast and transmitted through the network; and (iv) the threshold parameters  $\delta_i$ ,  $i \in \mathcal{V}$ , can be co-designed with the filter parameters, which will make clear later in Section 5.

### 3. Distributed event-triggered $H_\infty$ filtering performance analysis

In this section, by employing the Lyapunov-Krasovskii functional approach, an  $H_\infty$  filtering performance analysis criterion for the augmented filtering error system (9) subject to (10) is established.

**Theorem 1.** For given filter parameters  $G_i, K_i, H_i$ , and  $L_i$ , threshold parameters  $\delta_i \in (0, 1)$ , scalars  $\beta \in (0, 1), \gamma > 0, h > 0$ , and  $\bar{\tau} > 0$ , the resultant filtering error system (9) subject to (10), where  $\delta_i = \tilde{\delta}_i^{1/2}$ , is asymptotically stable with the prescribed weighting average  $H_\infty$  performance defined in (11) if there exist real matrices  $P > 0, Q > 0, \Psi_i > 0, R_i > 0, X$  and  $S_i$ ,  $i \in \mathcal{V}$  of appropriate dimensions such that

$$\Gamma = \begin{bmatrix} \Phi & \Xi^T \\ * & -NI \end{bmatrix} < 0, \quad \begin{bmatrix} R_i & S_i \\ * & R_i \end{bmatrix} \geq 0 \quad (12)$$

where

$$\Xi = \text{vec}_7\{\tilde{E}, 0, 0, \text{vec}_N\{\tilde{E}_i\}, \text{vec}_N\{-\bar{L}_i\}, 0, \text{vec}_N\{\tilde{F}_i\}\} \quad \text{and} \quad \Phi = [\Phi_{mn}]_{7 \times 7}$$

with

$$\begin{aligned} \Phi_{11} &= Q - \sum_{i=1}^N R_i + \text{sym}(X\tilde{A}), \Phi_{12} = P - X + \tilde{A}^T X^T, \Phi_{13} = -\sum_{i=1}^N S_i \\ \Phi_{14} &= \text{vec}_N\{R_i + S_i + X\tilde{A}_i\}, \Phi_{15} = \Phi_{25} = \text{vec}_N\{X\tilde{C}_i\} \\ \Phi_{16} &= \Phi_{26} = X\tilde{B}, \Phi_{17} = \Phi_{27} = \text{vec}_N\{X\tilde{D}_i\} \end{aligned}$$

$$\begin{aligned}
\Phi_{23} &= \Phi_{35} = \Phi_{36} = \Phi_{37} = \Phi_{46} = \Phi_{56} = \Phi_{67} = 0 \\
\Phi_{22} &= \tau_M^2 \sum_{i=1}^N R_i - \text{sym}(X), \Phi_{24} = \text{vec}_N\{X\check{A}_i\}, \Phi_{34} = \text{vec}_N\{R_i + S_i^T\} \\
\Phi_{33} &= -Q - \sum_{i=1}^N R_i, \Phi_{44} = \text{diag}_N\{-2R_i - \text{sym}(S_i) + \Theta_1^i\} \\
\Phi_{45} &= \text{diag}_N\{\Theta_2^i\}, \Phi_{47} = \text{diag}_N\{\Theta_4^i\}, \Phi_{55} = \text{diag}_N\{\Theta_3^i\}, \Phi_{57} = \text{diag}_N\{\Theta_5^i\} \\
\Phi_{66} &= -\beta\gamma^2 I, \Phi_{77} = \text{diag}_N\{-(1-\beta)\gamma^2 I + \Theta_6^i\} \\
\Theta_1^i &= [\check{\Xi}_i \ 0], \check{\Xi}_i = [\check{\delta}_i \mathcal{C}_i^T \Psi_i \mathcal{C}_i \ 0]^T, \Theta_2^i = [\check{\delta}_i \Psi_i \mathcal{C}_i \ 0]^T, \Theta_3^i = (\check{\delta}_i - 1)\Psi_i \\
\Theta_4^i &= [\check{\delta}_i \mathcal{D}_i^T \Psi_i \mathcal{C}_i \ 0]^T, \Theta_5^i = \check{\delta}_i \Psi_i \mathcal{D}_i, \Theta_6^i = \check{\delta}_i \mathcal{D}_i^T \Psi_i \mathcal{D}_i.
\end{aligned}$$

**Proof.** See Appendix A. □

#### 4. Distributed event-triggered $H_\infty$ filters design

In this section, for given threshold parameters  $\delta_i$ ,  $i \in \mathcal{V}$ , we use the weighting average  $H_\infty$  disturbance attenuation performance index  $\gamma$  as a metric. The problem we proposed to address is to design the filter parameters  $G_i, K_i, H_i$ , and  $L_i$  to minimize  $\gamma$ . The following theorem provides a sufficient condition on the existence of desired distributed  $H_\infty$  filters.

**Theorem 2.** *For given threshold parameters  $\delta_i \in (0, 1)$  and given scalars  $\beta \in (0, 1), \gamma > 0, h > 0$ , and  $\bar{\tau} > 0$ , if there exist real matrices  $P > 0, Q > 0, \Psi_i > 0, R_i > 0, X_1, X_3, S_i$ , and diagonal matrices  $X_2, \check{G}, \check{K}_i, \check{H}$ , and  $\check{L}_i$ ,  $i \in \mathcal{V}$  of appropriate dimensions such that*

$$\check{\Gamma} = \begin{bmatrix} \check{\Phi} & \check{\Xi}^T \\ * & -NI \end{bmatrix} < 0, \quad \begin{bmatrix} R_i & S_i \\ * & R_i \end{bmatrix} \geq 0 \quad (13)$$

where  $\check{\Phi}$  is derived from  $\Phi$  by replacing  $\Phi_{11}, \Phi_{12}, \Phi_{14}, \Phi_{15}, \Phi_{16}, \Phi_{17}, \Phi_{24}, \Phi_{25}, \Phi_{26}, \Phi_{27}$  by  $\check{\Phi}_{11}, \check{\Phi}_{12}, \check{\Phi}_{14}, \check{\Phi}_{15}, \check{\Phi}_{16}, \check{\Phi}_{17}, \check{\Phi}_{24}, \check{\Phi}_{25}, \check{\Phi}_{26}, \check{\Phi}_{27}$  with

$$\begin{aligned}
\check{\Phi}_{11} &= Q - \sum_{i=1}^N R_i + \text{sym}(\Omega), \check{\Phi}_{12} = P - X + \Omega^T, \check{\Phi}_{14} = \text{vec}_N\{R_i + S_i + \Upsilon_i\} \\
\check{\Phi}_{15} &= \check{\Phi}_{25} = \text{vec}_N\{\check{\Upsilon}_i\}, \check{\Phi}_{24} = \text{vec}_N\{\Upsilon_i\}, \check{\Phi}_{16} = \check{\Phi}_{26} = \check{\Omega}, \check{\Phi}_{17} = \check{\Phi}_{27} = \text{vec}_N\{\hat{\Upsilon}_i\} \\
\Omega &= \begin{bmatrix} X_1 \bar{A} & \check{G} \\ X_3 \bar{A} & \check{G} \end{bmatrix}, X = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_2 \end{bmatrix}, \Upsilon_i = \begin{bmatrix} \check{K}_i(\mathcal{W} \otimes I) \hat{C} & 0 \\ \check{K}_i(\mathcal{W} \otimes I) \hat{C} & 0 \end{bmatrix} \\
\check{\Upsilon}_i &= \begin{bmatrix} \check{K}_i & 0 \\ \check{K}_i & 0 \end{bmatrix}, \check{\Omega} = \begin{bmatrix} X_1 \bar{B} \\ X_3 \bar{B} \end{bmatrix}, \hat{\Upsilon}_i = \begin{bmatrix} \check{K}_i(\mathcal{W} \otimes I) \bar{D} \\ \check{K}_i(\mathcal{W} \otimes I) \bar{D} \end{bmatrix},
\end{aligned}$$

then there exist desired distributed  $H_\infty$  filters in the form of (4) such that the resultant filtering error system (9) subject to (10), where  $\delta_i = \check{\delta}_i^{1/2}$ , is asymptotically stable with the prescribed weighting average  $H_\infty$  performance defined in (11). Furthermore, the filter parameters in (4) are given by

$$\bar{G} = X_2^{-1} \check{G}, \quad \bar{K}_i = X_2^{-1} \check{K}_i, \quad \bar{H}, \quad \text{and} \quad \bar{L}_i, \quad \forall i \in \mathcal{V}. \quad (14)$$

**Proof.** Obviously,  $\Gamma < 0$  in (12) implies that  $\Phi_{22} = \tau_M^2 \sum_{i=1}^N R_i - \text{sym}(X) < 0$ . Therefore, we have  $X > 0$  since  $R_i > 0$ , which means that  $X_2$  is a nonsingular matrix. By introducing two new variables  $\check{G} = X_2 \bar{G}$  and  $\check{K}_i = X_2 \bar{K}_i$ , inequality (12) straightforwardly implies (13). This completes the proof.  $\square$

With Theorem 2, the problem to be solved in this section can be transformed into the following minimization problem (refers to as *MinProb 1*):

$$\begin{aligned} \min \quad & \varpi \\ \text{s.t.} \quad & P > 0, Q > 0, \Psi_i > 0, R_i > 0, \text{ and (13)} \end{aligned}$$

where  $\varpi = \gamma^2$ . Accordingly, the minimal weighting average  $H_\infty$  performance index  $\gamma = \sqrt{\varpi}$  and the filter parameters can be obtained by solving *MinProb 1*.

**Remark 5.** In many real-world applications, e.g., target tracking and attack, system guidance and navigation, the system performance rather than communication resources may be a major concern for system analysis and design. In this section, we use the weighting average  $H_\infty$  disturbance attenuation level  $\gamma$  as a performance index. By solving *MinProb 1*, desired filter parameters and weighting matrix parameters can be determined such that the weighting average  $H_\infty$  performance index  $\gamma$  can be minimized at the cost of consuming a certain amount of communication resources since the threshold parameters  $\delta_i$ ,  $i \in \mathcal{V}$  are given.

**Remark 6.** From Theorem 2, one can see that, for given threshold parameters and a fixed sampling period, the communication delays affect only the system performance. On the other hand, for a given weighting average  $H_\infty$  performance index, one may apply Theorem 2 to determine the maximum value of  $\tau_M$ , where  $\tau_M = h + \bar{\tau}$ . In particular, if one sets  $\bar{\tau} = 0$ , i.e., no communication delay exists or the effects of communication delays can be ignored, the obtained  $\tau_M$  denotes the maximum sampling period.

## 5. The co-design issue

According to the proposed distributed event-triggered communication scheme (5), the number of transmitted data packets through each communication link is closely related to each threshold parameter  $\delta_i$ ,  $i \in \mathcal{V}$ . In the sequel, we define the following data packet transmission ratio (DPTR) as a network performance index through the communication link  $i$ ,  $i \in \mathcal{V}$ :

$$J_i = \frac{\mathcal{N}_{tdp}}{\mathcal{N}_{tsdp}} \quad (15)$$

where  $\mathcal{N}_{tdp}$  denotes the number of transmitted data packets and  $\mathcal{N}_{tsdp}$  denotes the number of total sampled-data packets.

Generally, as  $\delta_i$  increases, less communication resources are occupied for data transmission since a large value of  $\delta_i$  has the effects of increasing the average inter-event time through the communication link  $i$ . As a result, a large value of  $\delta_i$  usually leads to a small value of DPTR  $J_i$ . On the other hand, when the utilization of communication resources is reduced, less data packets are transmitted through the network to implement filters. Therefore, deteriorative system performance may be achieved. In other words, a large value of  $\delta_i$  normally results in a large value of the weighting average

$H_\infty$  disturbance attenuation performance index  $\gamma$ . In practical situations, however, the performance index  $\gamma$  should be made as small as possible so as to minimize the effects from exogenous disturbance and noise on system output, while the performance index  $J_i \in (0, 1)$ ,  $i \in \mathcal{V}$ , should also be made as small as possible so as to reduce the communication resource occupancy. Intuitively, it is a difficult task to directly obtain both a minimal  $\gamma$  and a minimal  $J_i$  through the communication link  $i$  by solving Theorem 2. In this sense, applying Theorem 2, one may perform the *trade-off* analysis between the communication resource occupancy and the weighting average  $H_\infty$  disturbance attenuation performance.

In order to guarantee a desired weighting average  $H_\infty$  performance index, we set an expected performance level  $\gamma^*$ . To ensure satisfactory communication resource utilization, we set an expected performance level  $J_i^*$  through the communication link  $i$ . Hence, the ***distributed event-triggered  $H_\infty$  filtering problem*** to be addressed in this paper turns to *co-designing the filter parameters  $G_i, K_i, H_i, L_i$ , and the threshold parameters  $\delta_i$  while simultaneously minimizing  $|\gamma - \gamma^*|$ , the error absolute value between the actual value and the expected value of the  $H_\infty$  performance index, and  $|J_i - J_i^*|$ ,  $\forall i \in \mathcal{V}$ , the error absolute value between the actual value and the expected value of the DPTR through the communication link  $i$ . To achieve this goal, we define the following objective function*

$$f(J_i, \gamma) = \rho_0 |\gamma - \gamma^*| + \sum_{i=1}^N \rho_i |J_i - J_i^*| \quad (16)$$

where  $\rho_j > 0$ ,  $j = 0, 1, \dots, N$ , is the prescribed weighting coefficient satisfying  $\sum_{j=0}^N \rho_j = 1$ . The following co-design algorithm is proposed to tackle the problem formulated above.

---

***Co-Design Algorithm 1:*** For given  $\beta, h, \bar{\tau}$ , and preset  $J_i^*$  and  $\gamma^*$ , determine  $G_i, K_i, H_i, L_i, \Psi_i, \delta_i$ , desired  $J_i$  and  $\gamma$  such that  $f(J_i, \gamma)$  is minimized.

---

- Step 1.** Choose expected  $J_i^*$  and  $\gamma^*$ , and suitable weighting coefficients  $\rho_j > 0$ ,  $j = 0, 1, \dots, N$ .
- Step 2.** Set  $m := 0$ . Choose a sufficiently large initial  $f^{(0)}$ , a sufficiently small initial  $\delta_i^{(0)} > 0$  and choose a suitable  $\varepsilon_i$  as the step increment of  $\delta_i^{(0)}$ ,  $i \in \mathcal{V}$ .
- Step 3.** For given positive scalars  $\beta, h$ , and  $\bar{\tau}$ . Solve the LMIs defined in Theorem 2 with  $\gamma = \gamma^*$ , **if** it is feasible, determine  $\Psi_i, \check{G}, \check{K}_i, X_2, \bar{H}$ , and  $\bar{L}_i$ , and go to Step 4; **else** reset  $\gamma^*, \beta, h$ , and  $\bar{\tau}$ , repeat Step 3 again. **end**
- Step 4.** In a given simulation time, apply (5) to determine  $\delta_i^*$  with the obtained  $\Psi_i$  and the preset  $J_i^*$ . For  $\delta_i = \delta_i^*$ , **if** there exists a feasible solution satisfying LMIs defined in Theorem 2 with the obtained  $\delta_i, \Psi_i, \check{G}, \check{K}_i, X_2, \bar{H}$ , and  $\bar{L}_i$ , go to Step 5; **else** reset  $J_i^*, \beta, h$ , and  $\bar{\tau}$  and repeat Step 3 again. **end**
- Step 5.**
- While**  $\delta_1 < 1$

...

**While**  $\delta_N < 1$

- i) Solve the LMIs defined in Theorem 2 with  $\gamma = \gamma^*$  to determine  $\Psi_i, \check{G}, \check{K}_i, X_2, \bar{H}$ , and  $\bar{L}_i$ .
- ii) In a given simulation time, apply (5) to determine  $\delta_i^*$  with the obtained  $\Psi_i$  and the preset  $J_i^*$ .
- iii) Solve MinProp 1 with the obtained  $\delta_i^*, \Psi_i, \check{G}, \check{K}_i, X_2, \bar{H}$ , and  $\bar{L}_i$  to determine  $\gamma$ .
- iv) In a given simulation time, apply (5) to calculate the performance index  $J_i$  in (15).
- v) Compute  $\bar{G}, \bar{K}_i, \bar{H}$ , and  $\bar{L}_i$  according to (14), and calculate  $f = \rho_0|\gamma - \gamma^*| + \sum_{i=1}^N \rho_i |J_i - J_i^*|$ . **If**  $f^{(m)} > f$ , set  $m = m + 1$ , update  $\delta_i^{(m)} := \delta_i, f^{(m)} := f, \gamma^{(m)} := \gamma, \bar{G}^{(m)} := \bar{G}, \bar{K}_i^{(m)} := \bar{K}_i, \bar{H}^{(m)} := \bar{H}, \bar{L}_i^{(m)} := \bar{L}_i, \Psi_i^{(m)} := \Psi_i$ , and  $J_i^{(m)} := J_i$ ; **Else** keep  $\delta_i^{(m)}, f^{(m)}, \gamma^{(m)}, \bar{G}^{(m)}, \bar{K}_i^{(m)}, \bar{H}^{(m)}, \bar{L}_i^{(m)}, \Psi_i^{(m)}$ , and  $J_i^{(m)}$ .  
**End**
- vii) Update  $\delta_N = \delta_N + \varepsilon_N$ .

**End**

...

Update  $\delta_1 = \delta_1 + \varepsilon_1$ .

**End**

**Step 6.** Output  $\delta_i^{(m)}, f^{(m)}, \gamma^{(m)}, J_i^{(m)}, \bar{G}^{(m)}, \bar{K}_i^{(m)}, \bar{H}^{(m)}, \bar{L}_i^{(m)}$ , and  $\Psi_i^{(m)}$ ,  $i \in \mathcal{V}$ , and exit.

**Remark 7.** The co-design Algorithm 1 is developed to design desired distributed  $H_\infty$  filters and distributed event-triggered communication scheme such that the filtering error system is asymptotically stable simultaneously with a desired  $H_\infty$  performance level and an admissible communication resources occupancy level. In this case, the communication delays may affect both the system performance index and the network performance index since the filter parameters and the threshold parameters are designed after considering the effects of communication delays.

**Remark 8.** Note that the computational complexity of the proposed co-design Algorithm 1 relies on solving Theorem 2 and MinProp 1, which both can be formulated as linear matrix inequality (LMI) feasibility problems, for each iteration. Existing efficient interior-point algorithms are available to deal with such problems. Denote the total number of scalar decision variables of Theorem 2 and MinProp 1 as  $\mathcal{D}_1$  and  $\mathcal{D}_2$ , respectively, and the total row size of the LMIs defined in Theorem 2 and MinProp 1 as  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , respectively. As a consequence, the computational complexity of the proposed co-design Algorithm 1 is proportional to  $\mathcal{O}(\mathcal{S}_1 \mathcal{D}_1^3 + \mathcal{S}_2 \mathcal{D}_2^3)$ . It should be mentioned that the total number of scalar decision variables and the total row size of the LMIs are dependent on the system dimensions ( $n_x, n_y, n_z$ , where  $x(t) \in \mathbb{R}^{n_x}, y_i(t) \in \mathbb{R}^{n_y}, z(t) \in \mathbb{R}^{n_z}$ ) and the scale of the sensor network (the number of sensor nodes, i.e.,  $N$ ). In this sense, as the growth of the scale of the sensor network and/or the system dimensions, the overall computational complexity of the proposed co-design Algorithm 1 polynomially increases.

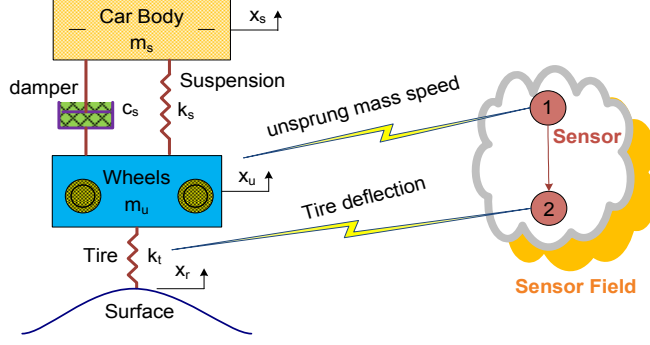


Figure 3: The quarter-car suspension system over a sensor network

## 6. An illustrative example

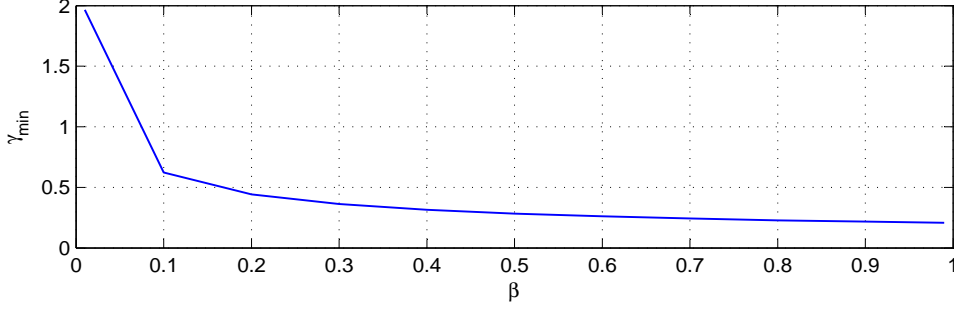
A quarter-car suspension model, depicted in Figure 3, is provided to show the effectiveness of the proposed filter design method. The sprung mass is  $m_s$ , which denotes the car chassis; the unsprung mass is  $m_u$ , which represents the wheel assembly; the spring  $k_s$  and the damper  $c_s$  represent the stiffness and damping of the uncontrolled suspension that are placed between the car body and the wheel assembly, while the spring  $k_t$  serves to the model of the compressibility of the pneumatic tire; the variables  $x_s$ ,  $x_u$ , and  $x_r$  are the displacements of the car body, the wheel, and the road disturbance input, respectively. The ideal dynamic equations for the sprung and unsprung masses of the quarter-car model are given by

$$\begin{cases} m_s \ddot{x}_s(t) + c_s(\dot{x}_s - \dot{x}_u(t)) + k_s(x_s(t) - x_u(t)) = 0 \\ m_u \ddot{x}_u(t) + c_s(\dot{x}_u - \dot{x}_s(t)) + k_s(x_u(t) - x_s(t)) + k_t(x_u(t) - x_r(t)) = 0. \end{cases}$$

Let  $x_1(t) = x_s(t) - x_u(t)$ ,  $x_2(t) = x_u(t) - x_r(t)$ ,  $x_3(t) = \dot{x}_s(t)$ , and  $x_4(t) = \dot{x}_u(t)$ , where  $x_1(t)$  denotes the suspension deflection,  $x_2(t)$  denotes the tire deflection,  $x_3(t)$  denotes the sprung mass speed, and  $x_4(t)$  denotes the unsprung mass speed. Then the quarter-car suspension model can be represented as  $\dot{x}(t) = Ax(t) + Bw(t)$  with

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_u}{m_u} & \frac{c_s}{m_u} & -\frac{c_s}{m_u} \end{bmatrix}, B = \begin{bmatrix} 0 \\ -2\pi q_0 \sqrt{G_0 v_0} \\ 0 \\ 0 \end{bmatrix}.$$

The parameters in the quarter-car model matrices are chosen as [27]:  $m_s = 973kg$ ,  $k_s = 42720N/m$ ,  $c_s = 3000Ns/m$ ,  $k_u = 101115N/m$ ,  $m_u = 114kg$ ,  $G_0 = 512 \times 10^{-6}m^3$ ,  $q_0 = 0.1m^{-1}$ , and  $v_0 = 12.5m/s$ . We consider a sensor network consisting of two nodes deployed in the sensor field. The sensing topology of the considered sensor network is characterized by a directed graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$  with the nodes  $\mathcal{V} = \{1, 2\}$ , the set of edges  $\mathcal{E} = \{(1, 1), (2, 1), (2, 2)\}$ , and the adjacency elements  $w_{ij} = 1$ ,  $(i, j) \in \mathcal{E}$ . Assume that the unsprung mass speed is measured by the sensor node 1 with noise  $v(t)$  and the tire deflection is measured by the sensor node 2 with noise  $v(t)$ . The measurement output matrices are given by  $C_1 = [0 \ 0 \ 0 \ 1]$ ,  $C_2 = [0 \ 1 \ 0 \ 0]$ ,  $D_1 = 0.1$ , and  $D_2 = 0.3$ . Choose  $E = [0 \ 0 \ 1 \ 0]$ . The objective is then to design desired distributed  $H_\infty$  filters in the form of (4)



**Figure 4:** The relationship between  $\beta$  and  $\gamma_{min}$

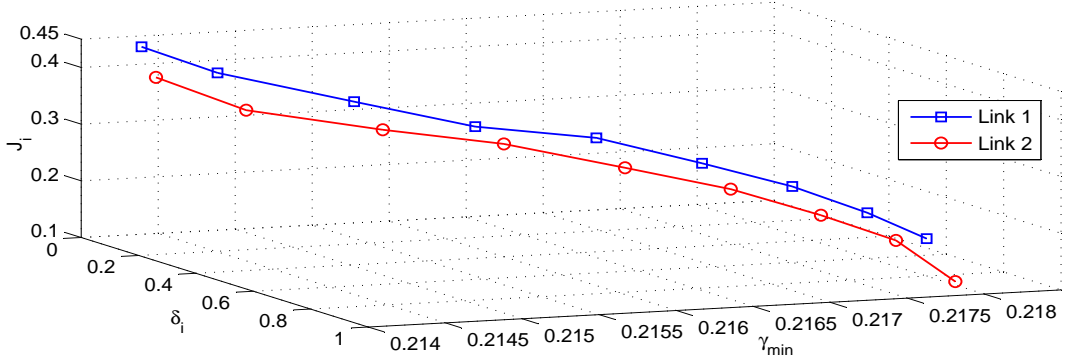
over the sensor network to estimate the sprung mass speed  $x_3(t)$  by using the measurement of the unsprung mass speed  $x_4(t)$  or the tire deflection  $x_2(t)$  on different sensor nodes.

Firstly, choosing  $h = 0.01s$  and  $\bar{\tau} = 0.06s$ , and applying Theorem 2, we obtain the relationship between the minimal value  $\gamma_{min}$  of the weighting average  $H_\infty$  performance and the weighting factor  $\beta$ , as illustrated in Figure 4, which enables us to properly select the weighting factor  $\beta$  to obtain the desirable weighting average  $H_\infty$  performance. Hence, the introduction of  $\beta$  increases the flexibility in the solution space for the formulated  $H_\infty$  optimization problem [33]. In the following simulation, we choose the weighting factor as  $\beta = 0.9$ .

Secondly, for different threshold parameters  $\delta_1$  and  $\delta_2$ , we solve MinProb 1 to obtain the trade-off curves in Figure 5, which illustrates the relationship between the DPTR  $J_i$  and the weighting average  $H_\infty$  noise attenuation performance index  $\gamma$  for each communication link. It can be seen clearly from Figure 5 that the DPTR  $J_i$  is generally inversely proportional to the  $H_\infty$  performance index  $\gamma$ , which means that, in order to preserve desired  $H_\infty$  performance, one has to occupy more communication resources to transmit sampled-data for implementing filters; otherwise, the communication resources may be saved at the cost of a tolerable  $H_\infty$  performance index. In this sense, the trade-off curves in Figure 5 provide quantitative information that when the network designer selects appropriate communication parameters to allow a certain communication resource occupancy while guaranteeing the stability of the system with a desirable level of system performance. More specifically, for given threshold parameters  $\delta_1 = 0.4$  and  $\delta_2 = 0.5$ , solving MinProb 1, we find that there exist desired distributed  $H_\infty$  filters in the form of (4) such that the resultant filtering error system (9) subject to (10) is asymptotically stable with the minimal weighting average  $H_\infty$  noise attenuation performance level  $\gamma_{min} = 0.2166$ . Moreover, the filter parameters and the weighting matrix parameters in (5) are given by

$$\begin{aligned}
 G_1 &= \begin{bmatrix} -4.1655 & 11.6945 & 0.5166 & 0.2187 \\ -2.0748 & 10.2264 & -0.3625 & 1.7327 \\ -32.6690 & -28.2439 & -2.5106 & -1.3183 \\ 241.2645 & -671.8086 & 23.5413 & -44.1784 \end{bmatrix}, K_1 = \begin{bmatrix} 0.0388 \\ 0.0287 \\ -0.2804 \\ -0.8566 \end{bmatrix} \\
 G_2 &= \begin{bmatrix} -7.3505 & 18.1277 & 0.2516 & 0.2078 \\ -3.3732 & 13.2055 & -0.5185 & 1.7298 \\ -4.7009 & -97.6669 & -0.4157 & -2.4063 \\ 295.4573 & -804.2688 & 28.8141 & -43.6425 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0018 \\ 0.0015 \\ -0.0157 \\ -0.0494 \end{bmatrix} \\
 H_1 &= [0.0437 \quad -1.8803 \quad -0.5890 \quad -0.2722], L_1 = 0.0045, \Psi_1 = 0.2822
 \end{aligned}$$

$$H_2 = [-0.4554 \quad -0.7446 \quad -0.5363 \quad -0.2717], L_2 = 0.0002, \Psi_2 = 0.0108.$$



**Figure 5:** Trade-off between the DPTR  $J_i$  and the  $H_\infty$  performance index  $\gamma$  with different threshold parameters  $\delta_i$  for communication link  $i$ ,  $i \in \mathcal{V}$

Thirdly, we consider the co-design issue. Choose an expected weighting average  $H_\infty$  noise attenuation performance index as  $\gamma^* = 0.2500$ , expected DPTRs as  $J_1^* = 0.150$  and  $J_2^* = 0.180$ . Select the initial  $\delta_i$  as 0.0025, the step increment  $\varepsilon_i$  of  $\delta_i$  as 0.0025,  $\forall i \in \mathcal{V}$ , the weighting coefficients as  $\rho_0 = 0.5$ ,  $\rho_1 = 0.25$ , and  $\rho_2 = 0.25$ , respectively. Applying Co-Design Algorithm 1, it is found that the proposed co-design problem is solvable with the value of the objective function  $f = 0.0029$ , the threshold parameters  $\delta_1 = 0.78$ ,  $\delta_2 = 0.75$ , and the actual value of the weighting average  $H_\infty$  noise attenuation performance index  $\gamma = 0.2523$ , the actual values of the DPTR  $J_1 = 0.150$  and  $J_2 = 0.173$ , respectively. Moreover, the filter parameters and the weighting matrix parameters in (5) are given by

$$G_1 = \begin{bmatrix} -3.2554 & 4.2514 & 0.6223 & -0.0637 \\ 3.9573 & -7.7284 & 0.4212 & 0.0892 \\ -27.2709 & -35.9003 & -2.0594 & 0.1229 \\ 16.9029 & -55.6897 & 3.2698 & -8.7709 \end{bmatrix}, K_1 = \begin{bmatrix} 0.0249 \\ -0.0194 \\ -0.1297 \\ -0.2127 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} -3.5070 & 4.7214 & 0.5930 & -0.0986 \\ 4.1875 & -8.3987 & 0.4499 & 0.1241 \\ -25.8198 & -39.4273 & -1.9061 & 0.2423 \\ 19.5489 & -60.5177 & 3.6879 & -8.9257 \end{bmatrix}, K_2 = \begin{bmatrix} -0.0004 \\ 0.0019 \\ -0.0118 \\ -0.0136 \end{bmatrix}$$

$$H_1 = [-0.7757 \quad 0.5148 \quad -0.7072 \quad 0.0160], L_1 = 0.0039, \Psi_1 = 0.0605$$

$$H_2 = [-0.8168 \quad 0.6115 \quad -0.7031 \quad 0.0158], L_2 = 0.0018, \Psi_2 = 0.0143.$$

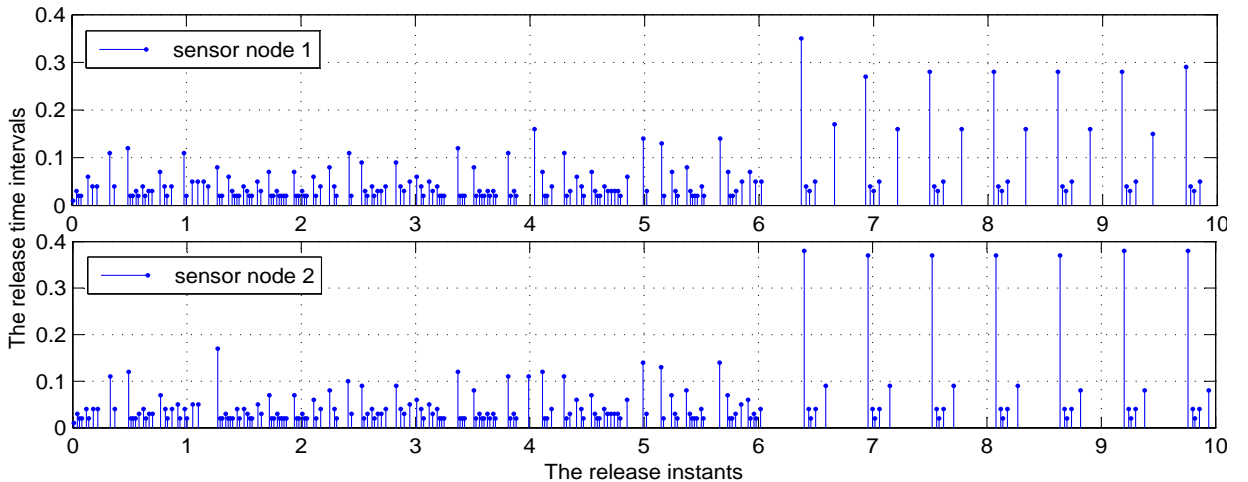
Furthermore, take an initial condition as  $x_0 = \text{col}_4\{0, 0, 0, 0\}$ . Consider the case of an isolated bump in an otherwise smooth road surface, the corresponding disturbance input signal is taken as:  $w(t) = k_0 \pi v_0 / l_0 \sin(2\pi v_0 t / l_0)$ , if  $1 \leq t < 1 + l_0 / v_0$ ;  $w(t) = 0$ , otherwise, where  $k_0 = 0.2m$  is the height of the bump and  $l_0 = 5m$  is the length of the bump. The measurement noise  $v(t)$  is assumed to be uniformly distributed within  $[0, 0.1]$  on the time interval  $[0, 6]s$ . Perform the simulation for 10s. The comparison of the number of transmitted data packets through each communication link between the conventional *time-triggered communication scheme* (**TTCS**) and the proposed *distributed event-triggered communication scheme* (**DETCS**) is illustrated in Table 1. Note that on the time interval



**Table 1:** Number of transmitted data packets through each communication link under different schemes

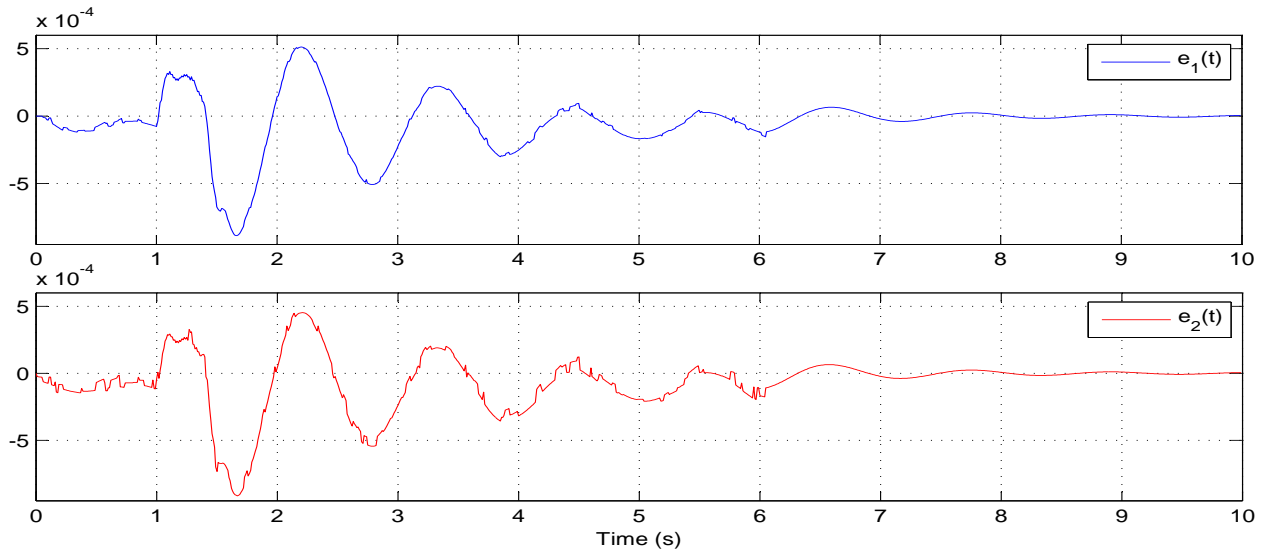
	Communication Link 1	Communication Link 2
Total Sampled-Data Packets	1000	1000
Under <b>TTCS</b>	1000	1000
Under <b>DETCS</b>	181	186

$[0, 10]s$ , the number of totally sampled-data packets is 1000. From Table 1, one can see that, by applying the **DETCS**, the number of transmitted data packets through each communication link is significantly reduced for each communication link compared with the conventional **TTCS**. The release instants and the time intervals between two adjoint transmitted instants on the sensor node  $i$ ,  $i \in \mathcal{V}$ , are shown in Figure 6. One can see that the maximum release time interval between any two consecutive release instants is  $0.35s$  on the sensor node 1 and  $0.38s$  on the sensor node 2, respectively. Connecting the obtained filters with the system under consideration and applying the determined distributed event-triggered communication scheme, Figure 7 depicts the curves of the estimation error signals  $e_i(t)$ ,  $i \in \mathcal{V}$ , on each sensor node. It can be seen from Figure 7 that the designed filters perform well.

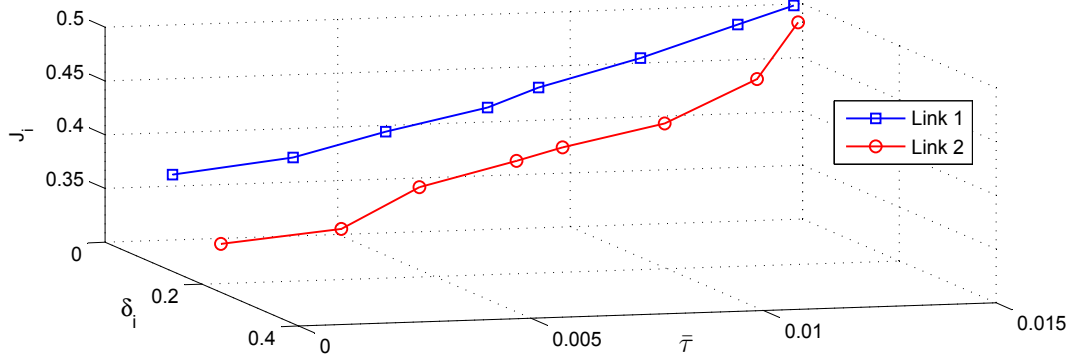


**Figure 6:** The transmitted instants and time intervals on node  $i$ ,  $i \in \mathcal{V}$

Finally, for a fixed sampling period  $h = 0.01s$  and a given weighting average  $H_\infty$  performance index  $\gamma = 0.2100$ , we apply Theorem 2 to obtain the relationship between the DPTR  $J_i$  and the delay bound  $\bar{\tau}$  with different threshold parameters  $\delta_i$  for each communication link, as illustrated in Figure 8. It can be seen that as  $\delta_i$  increases, less communication resources are occupied for data transmission through each communication link. On the other hand, communication delays may also affect the communication resource utilization and a decreased delay bound normally leads to less communication resource utilization.



**Figure 7:** The estimation error signals  $e_i(t)$  on node  $i$ ,  $i \in \mathcal{V}$



**Figure 8:** The relationship between the DPTR  $J_i$  and the delay bound  $\bar{\tau}$  under different threshold parameters  $\delta_i$  for communication link  $i$ ,  $i \in \mathcal{V}$

## 7. Conclusion

The problem of distributed event-triggered  $H_\infty$  filtering over sensor networks in the presence of communication delays has been addressed. A new distributed event-triggered communication scheme has been developed to decide when the current sampled-data of each sensor node should be broadcast and transmitted to a remote filter node through a communication network. Each sensor node is capable of making its own decision to broadcast and to transmit its current sampled-data only when its local measurement output error exceeds a specified threshold. Sufficient existence conditions for designing desired distributed  $H_\infty$  filters and threshold parameters have been derived. Applying the obtained results, the trade-off analysis between communication resource utilization and  $H_\infty$  performance has been performed. Furthermore, a new algorithm has been developed to co-design the filter parameters and the threshold parameters while simultaneously preserving a desired  $H_\infty$  performance index and maintaining satisfactory communication resource utilization. The application of the proposed results to a benchmark example has shown the effectiveness of the proposed filter design method.

## Appendix A

To begin with, the following integral inequality, which is helpful for deriving our main results, is presented.

**Lemma 1.** *For two given nonnegative scalars  $\underline{\tau}, \bar{\tau}$  satisfying  $\bar{\tau} \geq \underline{\tau}$ , a scalar time-varying function  $d(t) \in [\underline{\tau}, \bar{\tau}]$ , a vector-valued function  $\dot{\chi} : [-\bar{\tau}, -\underline{\tau}] \rightarrow \mathbb{R}^n$ , constant matrices  $R = R^T \in \mathbb{R}^{n \times n}$ ,  $S \in \mathbb{R}^{n \times n}$ , and  $\begin{bmatrix} R & S \\ * & R \end{bmatrix} \geq 0$  such that the following integration is well defined, then*

$$-(\bar{\tau} - \underline{\tau}) \int_{t-\bar{\tau}}^{t-\underline{\tau}} \dot{\chi}^T(s) R \dot{\chi}(s) ds \leq -\phi_1^T R \phi_1 - \phi_2^T R \phi_2 + \phi_1^T S \phi_2 + \phi_2^T S^T \phi_1 \quad (17)$$

where  $\phi_1 = \chi(t - d(t)) - \chi(t - \underline{\tau})$  and  $\phi_2 = \chi(t - \bar{\tau}) - \chi(t - d(t))$ .

**Proof.** When  $d(t) = \underline{\tau}$  (respectively,  $d(t) = \bar{\tau}$ ), we have  $\phi_1 = 0$  (respectively,  $\phi_2 = 0$ ). Then, the inequality (17) reduces to the Jensen integral inequality [8]. Therefore, (17) still holds. In the sequel, we discuss about the case  $\underline{\tau} < d(t) < \bar{\tau}$ . Define  $\phi_3 = \frac{d(t)-\underline{\tau}}{\bar{\tau}-\underline{\tau}}$ , and  $\phi_4 = \frac{\bar{\tau}-d(t)}{\bar{\tau}-\underline{\tau}}$ . Applying Jensen integral inequality, we obtain

$$\begin{aligned} -(\bar{\tau} - \underline{\tau}) \int_{t-\bar{\tau}}^{t-\underline{\tau}} \dot{\chi}^T(s) R \dot{\chi}(s) ds &= -(\bar{\tau} - \underline{\tau}) \int_{t-d(t)}^{t-\underline{\tau}} \dot{\chi}^T(s) R \dot{\chi}(s) ds - (\bar{\tau} - \underline{\tau}) \int_{t-\bar{\tau}}^{t-d(t)} \dot{\chi}^T(s) R \dot{\chi}(s) ds \\ &\leq -\left(1 + \frac{\phi_4}{\phi_3}\right) \phi_1^T R \phi_1 - \left(1 + \frac{\phi_3}{\phi_4}\right) \phi_2^T R \phi_2. \end{aligned} \quad (18)$$

Recalling that  $\begin{bmatrix} R & S \\ * & R \end{bmatrix} \geq 0$ , then we have

$$\begin{bmatrix} \sqrt{\frac{\phi_4}{\phi_3}} \phi_1 \\ \sqrt{\frac{\phi_3}{\phi_4}} \phi_2 \end{bmatrix}^T \begin{bmatrix} R & S \\ * & R \end{bmatrix} \begin{bmatrix} \sqrt{\frac{\phi_4}{\phi_3}} \phi_1 \\ \sqrt{\frac{\phi_3}{\phi_4}} \phi_2 \end{bmatrix} \geq 0 \quad (19)$$

which can be equivalently rewritten as

$$-\frac{\phi_4}{\phi_3}\phi_1^T R\phi_1 - \frac{\phi_3}{\phi_4}\phi_2^T R\phi_2 \leq \phi_1^T S\phi_2 + \phi_2^T S^T\phi_1. \quad (20)$$

Consequently, combining (18) and (20) yields (17). This completes the proof.  $\square$

Now we are ready to prove Theorem 1.

**Proof of Theorem 1.** Choose the Lyapunov-Krasovskii functional candidate as

$$V(t) = \chi^T(t)P\chi(t) + \int_{t-\tau_M}^t \chi^T(s)Q\chi(s)ds + \int_{-\tau_M}^0 \int_{t+s}^t \tau_M \dot{\chi}^T(\theta)\mathfrak{R}\dot{\chi}(\theta)d\theta ds \quad (21)$$

where  $\mathfrak{R} = \sum_{i=1}^N R_i$ . Then the proof is twofold. Firstly, we will prove the asymptotic stability of the resultant filtering error system (9) subject to (10). Then the weighting average  $H_\infty$  performance index will be considered.

Firstly, we assume that  $w(t) = 0$  and  $v(t - d_k^i(t)) = 0$ ,  $i \in \mathcal{V}$ . Taking the time derivative of  $V(t)$  with regard to  $t$  along the trajectory of the filtering error system (9) yields

$$\begin{aligned} \dot{V}(t) &= 2\chi^T(t)P\dot{\chi}(t) + \chi^T(t)Q\chi(t) - \chi^T(t - \tau_M)Q\chi(t - \tau_M) \\ &\quad + \tau_M^2 \dot{\chi}^T(t)\mathfrak{R}\dot{\chi}(t) - \tau_M \int_{t-\tau_M}^t \dot{\chi}^T(s)\mathfrak{R}\dot{\chi}(s)ds \end{aligned} \quad (22)$$

for all  $t \in \Upsilon_k^{m_i}$ .

Notice that for all  $d_k^i(t) \in [0, \tau_M)$ ,  $t \in \Upsilon_k^{m_i}$ , applying Lemma 1, we have the following bounding inequality

$$-\tau_M \int_{t-\tau_M}^t \dot{\chi}^T(s)R_i\dot{\chi}(s)ds \leq -\varphi_i^T R_i\varphi_i - \psi_i^T R_i\psi_i + \varphi_i^T S_i\psi_i + \psi_i^T S_i^T\varphi_i \quad (23)$$

where  $\varphi_i = \chi(t - d_k^i(t)) - \chi(t)$ ,  $\psi_i = \chi(t - \tau_M) - \chi(t - d_k^i(t))$ , and  $\begin{bmatrix} R_i & S_i \\ * & R_i \end{bmatrix} \geq 0$ .

On the other hand, it can be seen from (9) that

$$\Lambda \triangleq 2(\chi^T(t)X + \dot{\chi}^T(t)X) \left( \tilde{A}\chi(t) + \sum_{i=1}^N \tilde{A}_i\chi(t - d_k^i(t)) + \sum_{i=1}^N \tilde{C}_i\tilde{e}_y^i(t) - \dot{\chi}(t) \right) = 0 \quad (24)$$

holds for any constant matrix  $X$ .

From (10) with  $v(t - d_k^i(t)) = 0$ ,  $i \in \mathcal{V}$ , we have  $\|\tilde{e}_y^i(t)\|_2 < \delta_i \|\tilde{e}_y^i(t) + C_i\bar{x}(t - d_k^i(t))\|_2$ , which is equivalent to

$$\begin{bmatrix} \chi(t - d_k^i(t)) \\ \tilde{e}_y^i(t) \end{bmatrix}^T \begin{bmatrix} \Theta_1^i & \Theta_2^i \\ * & \Theta_3^i \end{bmatrix} \begin{bmatrix} \chi(t - d_k^i(t)) \\ \tilde{e}_y^i(t) \end{bmatrix} > 0. \quad (25)$$

Adding the zero term  $\Lambda$  to (22), and combining (22), (23), and (25) yield  $\dot{V}(t) \leq \tilde{\zeta}^T(t)\tilde{\Phi}\tilde{\zeta}(t)$ , where  $\tilde{\zeta}(t) = \text{col}_{2N+3}\{\chi(t), \dot{\chi}(t), \chi(t - \tau_M), \text{col}_N\{\chi(t - d_k^i(t))\}, \text{col}_N\{\tilde{e}_y^i(t)\}\}$ , and  $\tilde{\Phi}$  is derived from  $\Phi$  defined in (12) by removing its last two rows and columns. Obviously, (12) implies  $\tilde{\Phi} < 0$ . Then one has

that  $\dot{V}(t) \leq -\kappa\chi^T(t)\chi(t) < 0$  for  $\chi(t) \neq 0$ , where  $\kappa = \lambda_{\min}(-\tilde{\Phi})$ . Therefore, it can be concluded that the resultant filtering error system (9) subject to (10) is asymptotically stable.

Next, for all nonzero  $w(t) \in \mathcal{L}_2[0, \infty)$  and  $v(t-d_k^i(t)) \in l_2[0, \infty)$ , denote  $\zeta(t) = \text{col}_{3N+4}\{\tilde{\zeta}(t), w(t), \text{col}_N\{v(t-d_k^i(t))\}\}$ . Notice that (10) is now equivalent to

$$\begin{bmatrix} \chi(t-d_k^i(t)) \\ \bar{e}_y^i(t) \\ v(t-d_k^i(t)) \end{bmatrix}^T \begin{bmatrix} \Theta_1^i & \Theta_2^i & \Theta_4^i \\ * & \Theta_3^i & \Theta_5^i \\ * & * & \Theta_6^i \end{bmatrix} \begin{bmatrix} \chi(t-d_k^i(t)) \\ \bar{e}_y^i(t) \\ v(t-d_k^i(t)) \end{bmatrix} > 0. \quad (26)$$

Applying (22) to system (9) with nonzero  $w(t), v(t-d_k^i(t)), i \in \mathcal{V}$ , and using Schur complement, we have

$$\dot{V}(t) \leq \zeta^T(t)\hat{\Phi}\zeta(t) \quad (27)$$

for all  $t \in \Upsilon_k^{m_i}$ , where  $\hat{\Phi}$  is derived from  $\Phi$  defined in (12) by replacing its entries  $\Phi_{66}$  and  $\Phi_{77}$  with  $\hat{\Phi}_{66} = 0$  and  $\hat{\Phi}_{77} = \text{diag}_N\{\Theta_6\}$ , respectively.

Consider the following  $H_\infty$  performance  $\mathcal{J} \triangleq \int_0^\infty \mathcal{J}_w(t)dt$  with  $\mathcal{J}_w(t) = \frac{1}{N}e^T(t)e(t) - \beta\gamma^2w^T(t)w(t) - \sum_{i=1}^N(1-\beta)\gamma^2v^T(t-d_k^i(t))v(t-d_k^i(t))$ . From (27), we have that  $\dot{V}(t) + \mathcal{J}_w(t) \leq \zeta^T(t)\Gamma\zeta(t)$  for all  $t \in \Upsilon_k^{m_i}$ . Clearly, if (12) holds, one obtains

$$\dot{V}(t) + \mathcal{J}_w(t) < 0 \quad (28)$$

for all  $t \in \Upsilon_k^{m_i}$ . Under zero initial conditions, letting  $t_0^i = 0$ , integrating (28) on  $t$  from  $t_k^i h + \tau_{t_k^i}^i$  to  $t_{k+1}^i h + \tau_{t_{k+1}^i}^i$ , and summing the integration on  $k$  from 0 to  $\infty$  yield

$$\sum_{k=0}^{\infty} \int_{t_k^i h + \tau_{t_k^i}^i}^{t_{k+1}^i h + \tau_{t_{k+1}^i}^i} \mathcal{J}_w(t)dt < -V(t)|_{t=\infty}. \quad (29)$$

This implies that  $\mathcal{J} \triangleq \int_0^\infty \mathcal{J}_w(t)dt < 0$ , i.e.,  $\frac{1}{N}e^T(t)e(t) < \beta\gamma^2w^T(t)w(t) + \sum_{i=1}^N(1-\beta)\gamma^2v^T(t-d_k^i(t))v(t-d_k^i(t))$ , which can be rewritten as  $\frac{1}{N}\sum_{i=1}^N \|e_i(t)\|_2^2 < \beta\gamma^2\|w(t)\|_2^2 + \sum_{i=1}^N(1-\beta)\gamma^2\|v(t-d_k^i(t))\|_2^2$ . Therefore, the prescribed  $H_\infty$  performance (11) is satisfied provided that (12) holds. This completes the proof.  $\square$

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