Developing a New Data Envelopment Analysis Methodology for Supplier Selection in the Presence of both Undesirable Outputs and Imprecise Data

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Abstract

Supplier selection plays a key role in an organization because the cost of raw material constitutes the main cost of the final product. Selecting an appropriate supplier is now one of the most important decisions of the purchasing department. This decision generally depends on a number of different criteria. The objective of this paper is to propose a Data Envelopment Analysis (DEA) methodology that considers both undesirable outputs and imprecise data simultaneously. The proposed model is applied in supplier selection problem. A numerical example demonstrates the application of the proposed method.

Keywords: Data envelopment analysis, Undesirable outputs, Imprecise data.

1- Introduction

Supplier selection plays a key role in an organization because the cost of raw material constitutes the main cost of the final product. A typical manufacturer spends 60% of its total sales on purchased items such as raw materials, parts, subassemblies and components. In automotive industries, these costs may be more than 50% of the total revenues. That can go up to 80% of the total product costs for high technology firms. Many experts believe that the supplier selection is the most important activity of a purchasing department. Therefore, selecting the most appropriate supplier appears to have significant cost-cutting opportunities (Kokangul and Susuz, 2009).

Some mathematical programming approaches have been used for supplier selection in the past. A sample of recently published papers is presented as below.

Lee et al. (2009) proposed a model to select the factors for evaluating green suppliers, and to evaluate the performance of suppliers. The Delphi method is applied first to select the most important sub-criteria for traditional suppliers and for green suppliers. The results for green supplier are applied next to construct a hierarchy for green supplier evaluation problem. A Fuzzy Extended Analytic Hierarchy Process (FEAHP) model is constructed next based on the hierarchy to evaluate green suppliers for an anonymous manufacturer in Taiwan, and the most suitable supplier can be selected. Lin (2009) suggested applying Analytic Network Process (ANP) technique for identifying top suppliers by considering the effects of interdependence among the selection criteria. As well, to achieve optimal allocation of orders among the selected suppliers, a Multi-Objective Linear Programming (MOLP) method was proposed. Hsu and Hu (2009) presented an ANP approach to incorporate the issue of Hazardous Substance Management (HSM) into supplier selection. In this study, identification of criteria of HSM competence is categorized into four dimensions, a multi-criteria decision model is proposed. ANP is then applied to supplier selection and is characterized by interdependencies among decision structure components. Kokangul and Susuz (2009) suggested an integrated Analytic Hierarchy Process (AHP) and non-linear integer programming and multi-objective integer programming models for maximizing total value of purchase, minimizing total cost of purchase and maximizing total value of purchase and minimizing total cost of purchase simultaneously. Their model applies the AHP which uses pair-wise comparison to make trade-offs between tangible and intangible factors and calculate a weight of suppliers. Applying these weights as coefficients of an objective function in the proposed models determines the best suppliers and assigns optimal order quantities to the determined suppliers under constraints such as quantity discounts and the total capacity of the selected suppliers.

Guneri et al. (2009) presented an integrated fuzzy and linear programming approach to the supplier selection problem. Firstly, linguistic values expressed in trapezoidal fuzzy numbers are applied to assess weights and ratings of supplier selection criteria. Then a hierarchy multiple model based on fuzzy set theory is expressed and fuzzy positive and negative ideal solutions are used to find each supplier's closeness coefficient. Finally, a linear programming model based on the coefficients of suppliers, buyer's budgeting, suppliers' quality and capacity constraints is developed and order quantities assigned to each supplier according to the linear programming model. Wu (2009b) proposed a new method to solve group decision making problems with fuzzy numbers based on grey related analysis and Dempster–Shafer theory. The proposed method involves two steps: (1) the individual aggregation using grey related analysis and (2) the group aggregation using Dempster–Shafer rule of combination. Wu et al. (2010) discussed a possibility approach to solve a fuzzy multi-objective programming model. They applied their model to supplier selection when various risks were considered. They modeled a supply chain consisting of three levels and used simulated data, which can be collected from the company's trading history with distributions empirically derived. They proposed an algorithm to solve the proposed fuzzy multi-objective programming model.

Wu (2009a) developed a hybrid supplier evaluation model using Data Envelopment Analysis (DEA), Neural Networks (NN), and Decision Trees (DT). The model deals with multiple criteria including intangible criteria embedded in the supplier selection problem. The model can function as both a classification model and a regression model. The model consists of two modules: Module 1 applies DEA and classifies suppliers into efficient and inefficient clusters based on the resulting efficiency scores. Module 2 utilizes firm performance-related data to train DT, NN model and apply the trained DT model to new suppliers. To select the best suppliers in the presence of both cardinal and ordinal data, Farzipoor Saen (2007) proposed a method, which is based on Imprecise Data Envelopment Analysis (IDEA). Farzipoor Saen (2009a) proposed a new pair of nondiscretionary factors-imprecise data envelopment analysis (NF-IDEA) models for selecting the best suppliers in the presence of nondiscretionary factors and imprecise data. Again, Farzipoor Saen (2009b) proposed a model for ranking suppliers in the presence of weight restrictions, nondiscretionary factors, and cardinal and ordinal data.

1.1 Undesirable outputs

DEA measures the relative efficiency of Decision Making Units (DMUs) with multiple performance factors that are grouped into outputs and inputs. Once the efficient frontier is determined, inefficient DMUs can improve their performance to reach the efficient frontier by either increasing their current output levels or decreasing their current input levels. In conducting efficiency analysis, it is often assumed that all outputs are "good". However, such an assumption is not always justified, because outputs may be "bad". For example, if inefficiency exists in production processes where final products are manufactured along with the production of waste and pollutants, then the respective outputs of waste and pollutants are undesirable (bad) and should be reduced in order to improve performance.

Lu and Lo (2007) and Farzipoor Saen (in press) classified the alternatives for dealing with undesirable outputs in the DEA framework as below. The first is to simply ignore the undesirable outputs. The second is either to treat the undesirable outputs in terms of a non-linear DEA model or to treat the undesirable outputs as outputs and adjust the distance measurement in order to restrict the expansion of the undesirable outputs (Färe et al., 1989). The third is either to treat the undesirable outputs as inputs or to apply a monotone decreasing transformation (e.g., $1/y^b$, where y^b represents the undesirable output). Seiford and Zhu (2002) have proposed an approach which deals with undesirable outputs in the DEA framework. The approach is invariant to the data transformation within the DEA model.

Undesirable factors have been grown substantially since Färe et al. (1989) firstly introduced a non-linear programming problem for efficiency evaluation in the existence of undesirable factors. Scheel (2001) presented some radial measures which assume that any change of output level will involve both desirable and undesirable outputs. Seiford and Zhu (2002) developed a radial model to improve the efficiency via increasing desirable outputs and decreasing undesirable outputs. Hadi Vencheh et al. (2005) developed a model for efficiency evaluation incorporating undesirable inputs and undesirable outputs, simultaneously. For more related researches, please see Jahanshahloo et al. (2004), Korhonen and Luptacik (2004), Jahanshahloo et al. (2005), Zhang et al. (2008) and Liang et al. (2009). Farzipoor Saen (in press) presented an Additive model for considering imprecise data and undesirable factors. However, as Farzipoor Saen (2005) addressed, the Additive model has no scalar measure (ratio efficiency). Although Additive model can discriminate between efficient and inefficient DMUs by the existence of slacks, it has no means of gauging the depth of inefficiency, similar to radial models of DEA.

1.2 Imprecise data

DEA, proposed by Charnes et al. (1978) (Charnes, Cooper, Rhodes (CCR) model) and developed by Banker et al. (1984) (Banker, Charnes, Cooper (BCC) model), is an approach for evaluating the efficiencies of DMUs. This evaluation is generally assumed to be based on a set of cardinal (quantitative) output and input factors. As addressed by Farzipoor Saen (2009c), in many real-world applications (especially supplier selection problems), however, it is essential to take into account the existence of bounded data when rendering a decision on the performance of a DMU. Very often, it is the case that for a factor such as number of bills received from the supplier without errors, one can, provide an interval data of the suppliers.

The capability to provide a more precise, crisp measure reflecting such a factor might be beyond the realm of reality. Therefore, the data may be imprecise. Note that a factor such as number of bills received from the supplier without errors is a desirable output and imprecise factor, simultaneously. To deal with imprecise data in DEA, IDEA models and methods have been developed. Imprecise data implies that some data are known only to the extent that the true values lie within prescribed bounds while other data are known only in terms of ordinal relations.

In summary, the contributions of this paper are as below.

- The proposed model does not demand weights from the decision maker.
- The proposed model considers multiple undesirable outputs for supplier selection.
- The proposed model considers imprecise data for supplier selection.
- A model in the presence of both undesirable outputs and imprecise data is introduced.

The objective of this paper is to develop a model for selecting suppliers in the presence of both undesirable outputs and imprecise data.

This paper proceeds as follows. In Section 2 the model is proposed. Numerical example and concluding remarks are discussed in Sections 3 and 4, respectively.

2- Proposed model

Here, to consider both undesirable outputs and imprecise data, a model is developed. It is assumed that there are n DMUs to be evaluated. Each DMU consumes varying amounts of m different inputs to produce p different outputs. The outputs corresponding to indices $1, 2, \ldots, k$ are desirable and the outputs corresponding to indices $k+1, k+2, \ldots, p$ are undesirable outputs. It is preferred to produce desirable outputs as much as possible and not to produce undesirable outputs. Let $X \in R_{+}^{m\times n}$ and $Y \in R_{+}^{p\times n}$ be the matrices, consisting of non-negative elements, containing the observed input and output measures for the DMUs. The vector of inputs consumed by DMU$_{j,}$ is denoted by $\boldsymbol{x}_j$ (the jth column of $\boldsymbol{X}$). The quantity of input i consumed by DMU$_j$ is denoted by x_{ij}. A similar notation is used for outputs. DMU$_o$ is the DMU under consideration.

To consider undesirable factors, Korhonen and Luptacik (2004) introduced a model. Their model is based on the idea of presenting all outputs as a weighted sum, but using negative weights for undesirable outputs. The model is as below.

$$
\begin{aligned}
&max \quad \frac{\sum_{r=1}^{k}\mu_r y_{ro} - \sum_{s=k+1}^{p}\mu_s y_{so}}{\sum_{i=1}^{m}v_i x_{io}} \\
&s.t. \\
&\qquad \frac{\sum_{r=1}^{k}\mu_r y_{rj} - \sum_{s=k+1}^{p}\mu_s y_{sj}}{\sum_{i=1}^{m}v_i x_{ij}} \le 1, \qquad j = 1,2,...,n \qquad\qquad (1) \\
&\qquad \mu_r, v_i \ge \varepsilon, \qquad r = 1,2,...,p; \qquad i = 1,2,...,m.
\end{aligned}
$$

where ε is the non-Archimedean infinitesimal. μ_r and μ_s are the weights given to desirable outputs and undesirable outputs, respectively. v_i is the weight given to input *i*.

At this juncture, to select the efficient suppliers, a new model that considers both undesirable outputs and imprecise data is proposed. The final efficiency score for each DMU (supplier) will be characterized by an interval bounded by the best lower bound efficiency and the best upper bound efficiency of each DMU. The model is based on the interval arithmetic.

Let

$$\theta_j = \frac{\sum_{r=1}^{k}\mu_r y_{rj} - \sum_{s=k+1}^{p}\mu_s y_{sj}}{\sum_{i=1}^{m}v_i x_{ij}} \qquad j = 1,2,...,n \qquad (2)$$

be the efficiency of DMU$_j$. According to the operation rules on interval data, there is

$$\theta_j = \frac{\sum_{r=1}^{k}\mu_r\left[y_{rj}^L, y_{rj}^U\right] - \sum_{s=k+1}^{p}\mu_s\left[y_{sj}^L, y_{sj}^U\right]}{\sum_{i=1}^{m}v_i\left[x_{ij}^L, x_{ij}^U\right]}$$

$$= \frac{\left[\sum_{r=1}^{k}\mu_r y_{rj}^L, \sum_{r=1}^{k}\mu_r y_{rj}^U\right] - \left[\sum_{s=k+1}^{p}\mu_s y_{sj}^L, \sum_{s=k+1}^{p}\mu_s y_{sj}^U\right]}{\left[\sum_{i=1}^{m}v_i x_{ij}^L, \sum_{i=1}^{m}v_i x_{ij}^U\right]}$$

$$= \frac{\left[\sum_{r=1}^{k}\mu_r y_{rj}^L, \sum_{r=1}^{k}\mu_r y_{rj}^U\right]}{\left[\sum_{i=1}^{m}v_i x_{ij}^L, \sum_{i=1}^{m}v_i x_{ij}^U\right]} - \frac{\left[\sum_{s=k+1}^{p}\mu_s y_{sj}^L, \sum_{s=k+1}^{p}\mu_s y_{sj}^U\right]}{\left[\sum_{i=1}^{m}v_i x_{ij}^L, \sum_{i=1}^{m}v_i x_{ij}^U\right]}$$

$$= \left[\frac{\sum_{r=1}^{k}\mu_r y_{rj}^L}{\sum_{i=1}^{m}v_i x_{ij}^U}, \frac{\sum_{r=1}^{k}\mu_r y_{rj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L}\right] - \left[\frac{\sum_{s=k+1}^{p}\mu_s y_{sj}^L}{\sum_{i=1}^{m}v_i x_{ij}^U}, \frac{\sum_{s=k+1}^{p}\mu_s y_{sj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L}\right] \qquad j = 1,2,\ldots,n \tag{3}$$

It is obvious that θ_j should be an interval number, which is denoted by $\left[\theta_j^L, \theta_j^U\right]$ $\left(j = 1,\cdots,n\right)$. Let

$$\theta_j = \left[\theta_j^L, \theta_j^U\right] = \left[\frac{\sum_{r=1}^{k}\mu_r y_{rj}^L}{\sum_{i=1}^{m}v_i x_{ij}^U}, \frac{\sum_{r=1}^{k}\mu_r y_{rj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L}\right] - \left[\frac{\sum_{s=k+1}^{p}\mu_s y_{sj}^L}{\sum_{i=1}^{m}v_i x_{ij}^U}, \frac{\sum_{s=k+1}^{p}\mu_s y_{sj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L}\right] \subseteq (0,1], \quad j = 1,\cdots,n \tag{4}$$

Then

$$\theta_j^U = \frac{\sum_{r=1}^{k}\mu_r y_{rj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L} - \frac{\sum_{s=k+1}^{p}\mu_s y_{sj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L} \le 1, \qquad j = 1,\cdots,n$$

$$\theta_j^L = \frac{\sum_{r=1}^{k}\mu_r y_{rj}^L}{\sum_{i=1}^{m}v_i x_{ij}^U} - \frac{\sum_{s=k+1}^{p}\mu_s y_{sj}^L}{\sum_{i=1}^{m}v_i x_{ij}^U} > \varepsilon, \quad j = 1,\cdots,n \tag{5}$$

In order to measure the upper and lower bounds of the efficiency of DMU$_o$, the following pair of fractional programming models for DMU$_o$ is constructed:

$$
\begin{aligned}
& Max\,\theta_j^U = \frac{\sum_{r=1}^{k} \mu_r y_{ro}^U}{\sum_{i=1}^{m} v_i x_{io}^L} - \frac{\sum_{s=k+1}^{p} \mu_s y_{so}^U}{\sum_{i=1}^{m} v_i x_{io}^L} \\
& s.t. \\
& \qquad \frac{\sum_{r=1}^{k} \mu_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} - \frac{\sum_{s=k+1}^{p} \mu_s y_{sj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \le 1, \quad j = 1, \cdots, n \\
& \qquad \mu_r, \mu_s, v_i \ge \varepsilon \qquad \forall r, s, i
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
& Max\,\theta_j^L = \frac{\sum_{r=1}^{k} \mu_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U} - \frac{\sum_{s=k+1}^{p} \mu_s y_{sj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U} \\
& s.t. \\
& \qquad \frac{\sum_{r=1}^{k} \mu_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} - \frac{\sum_{s=k+1}^{p} \mu_s y_{sj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \le 1, \quad j = 1, \cdots, n \\
& \qquad \mu_r, \mu_s, v_i \ge \varepsilon \qquad \forall r, s, i
\end{aligned}
\tag{7}
$$

Using Charnes-Cooper transformation, the above pair of fractional programming models can be simplified as the following equivalent linear programming models:

$$
\begin{aligned}
& Max\theta_j^U = \sum_{r=1}^{k} \mu_r y_{ro}^U - \sum_{s=k+1}^{p} \mu_s y_{so}^U \\
& s.t. \\
& \qquad \sum_{i=1}^{m} v_i x_{io}^L = 1 \\
& \qquad \sum_{r=1}^{k} \mu_r y_{rj}^U - \sum_{s=k+1}^{p} \mu_s y_{sj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0, \quad j = 1, \cdots, n \\
& \qquad \mu_r, \mu_s, v_i \ge \varepsilon \qquad \forall r, s, i
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
& Max\theta_j^L = \sum_{r=1}^{k} \mu_r y_{rj}^L - \sum_{s=k+1}^{p} \mu_s y_{sj}^L \\
& s.t. \\
& \qquad \sum_{i=1}^{m} v_i x_{ij}^U = 1, \\
& \qquad \sum_{r=1}^{k} \mu_r y_{rj}^U - \sum_{s=k+1}^{p} \mu_s y_{sj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0, \quad j = 1, \cdots, n \\
& \qquad \mu_r, \mu_s, v_i \ge \varepsilon \qquad \forall r, s, i
\end{aligned} \tag{9}
$$

where θ_o^U stands for the best possible relative efficiency achieved by DMU$_o$ when all the DMUs are in the state of best production activity, while θ_o^L stands for the lower bound of the best possible relative efficiency of DMU$_o$. They constitute a possible best relative efficiency interval $\left[\theta_o^L, \theta_o^U\right]$. Note that model (8) determines the production frontier for all the DMUs and model (9) uses the production frontier as a benchmark to measure the lower bound efficiency of each DMU.

In order to judge whether a DMU is DEA efficient or not, the following definition is given.

Definition 1. A DMU, DMU$_o$, is said to be DEA efficient if its best possible upper bound efficiency $\theta_o^{U*} = 1$; otherwise, it is said to be DEA inefficient if $\theta_o^{U*} < 1$.

Therefore, one unified approach that deals with imprecise data and undesirable outputs in a direct manner have been introduced.

Now, the method of transforming ordinal preference information into interval data is discussed, so that the interval DEA models presented in this paper can still work properly even in these situations (Wang et al., 2005; Farzipoor Saen, 2006, 2008, 2009a, 2009b, 2009c).

Suppose some input and/or output data for DMUs are given in the form of ordinal preference information. There may exist strong ordinal preference information such as $y_{rj}>y_{rf}$ or $x_{ij}>x_{if}$, which can be further expressed as $y_{rj} \geq \chi_r y_{rf}$ and $x_{ij} \geq \eta_i x_{if}$, where $\chi_r > 1$ and $\eta_i > 1$ are the parameters on the degree of preference intensity provided by decision maker. At this point, consider the transformation of ordinal preference information about the output y_{rj} ($j=1,\ldots, n$) for example. The ordinal preference information about input and undesirable output data can be converted in the same way. For strong ordinal preference data, the resultant permissible interval for each $\hat{y}_{rj}$ can be derived as follows:

$$\hat{y}_{rj} \in \left[\sigma_r \chi_r^{n-j}, \chi_r^{1-j}\right], \qquad j=1,\cdots,n \quad \text{with} \quad \sigma_r \leq \chi_r^{1-n}. \tag{10}$$

where σ_r is a small positive number reflecting the ratio of the possible minimum of $\{y_{rj} | j=1,\ldots, n\}$ to its possible maximum. It can be approximately estimated by the decision maker. It is referred as the ratio parameter for convenience.

Through the scale transformation above and the estimation of permissible intervals, all the ordinal preference information is converted into interval data and can thus be incorporated into Models (8) and (9).

In interval efficiency assessment, since the final efficiency score for each DMU is characterized by an interval, a simple yet practical ranking approach is thus needed for ranking the efficiencies of different DMUs. Here the Minimax Regret-based Approach (MRA) applied by Wang et al. (2005), Farzipoor Saen (2006), Farzipoor Saen (2008), and Farzipoor Saen (2009d) is introduced. The approach is summarized as follows:

Let $A_i = \left[a_i^L, a_i^U\right] = \langle m(A_i), w(A_i) \rangle$ $(i=1,\cdots,n)$ be the efficiency intervals of n DMUs, where $m(A_i) = \frac{1}{2}(a_i^U + a_i^L)$ and $w(A_i) = \frac{1}{2}(a_i^U - a_i^L)$ are their midpoints (centers) and widths. Without loss of generality, suppose $A_i = [a_i^L, a_i^U]$ is chosen as the best efficiency interval. Let $b = \max_{j \neq i}\{a_j^U\}$. Obviously, if $a_i^L < b$, the decision maker might suffer the loss of efficiency (also called the loss of opportunity or regret) and feel regret. The maximum loss of efficiency he/she might suffer is given by

$$\max(r_i) = b - a_i^L = \max_{j \neq i}\{a_j^U\} - a_i^L.$$

If $a_i^L \geq b$, the decision maker will definitely suffer no loss of efficiency and feel no regret. In this situation, his/her regret is defined to be zero, i.e. $r_i = 0$. Combining the above two situations, there is

$$\max(r_i) = \max\left[\max_{j \neq i}(a_j^U) - a_i^L, 0\right].$$

Thus, the minimax regret criterion will choose the efficiency interval satisfying the following condition as the best (most desirable) efficiency interval:

$$\min_i\{\max(r_i)\} = \min_i\left\{\max\left[\max_{j \neq i}(a_j^U) - a_i^L, 0\right]\right\}.$$

Based on the above analysis, the following definition for ranking efficiency intervals is given.

Definition 2. Let $A_i = [a_i^L, a_i^U] = \langle m(A_i), w(A_i)\rangle$ $(i = 1, \cdots, n)$ be a set of efficiency intervals. The maximum loss of efficiency (also called maximum regret) of each efficiency interval A_i is defined as

$$R(A_i) = \max\left[\max_{j \neq i}(a_j^U) - a_i^L, 0\right] = \max\left[\max_{j \neq i}\left\{m(A_j) + w(A_j)\right\} - (m(A_i) - w(A_i)), 0\right], \quad i = 1, \cdots, n.$$

It is evident that the efficiency interval with the smallest maximum loss of efficiency is the most desirable efficiency interval.

To be able to generate a ranking for a set of efficiency intervals using the maximum losses of efficiency, the following eliminating steps are suggested:

Step 1: Calculate the maximum loss of efficiency of each efficiency interval and choose a most desirable efficiency interval that has the smallest maximum loss of efficiency (regret). Suppose A_{i_1} is selected, where $1 \le i_1 \le n$.

Step 2: Eliminate A_{i_1} from the consideration, recalculate the maximum loss of efficiency of every efficiency interval and determine a most desirable efficiency interval from the remaining (*n-1*) efficiency intervals. Suppose A_{i_2} is chosen, where $1 \le i_2 \le n$ but $i_2 \ne i_1$.

Step 3: Eliminate A_{i_2} from the further consideration, re-compute the maximum loss of efficiency of every efficiency interval and determine a most desirable efficiency interval A_{i_3} from the remaining (*n-2*) efficiency intervals.

Step 4: Repeat the above eliminating process until only one efficiency interval A_{i_n} is left. The final ranking is $A_{i_1} \succ A_{i_2} \succ \cdots \succ A_{i_n}$, where the symbol "$\succ$" means “is superior to”.

The above ranking approach is referred to as the MRA. In the next section, a numerical example is presented.

3- Numerical Example

The data set for this example is taken from Farzipoor Saen (in press) and contains specifications on 18 suppliers (DMUs). The cardinal input considered is Total Cost of shipments (TC). The desirable output utilized in the study is Number of Bills received from the supplier without errors (NB). NB will serve as the bounded output. The undesirable output is Parts Per Million (PPM) of defective parts. Supplier Reputation (SR) is included as a qualitative input. SR is an intangible factor that is not usually explicitly included in evaluation model for supplier. This qualitative variable is measured on an ordinal scale so that, for instance, reputation of supplier 18 is given the highest rank, and supplier 17, the lowest. Note that, the measures selected in this paper are not exhaustive by any means, but are some general measures that can be utilized to evaluate suppliers. In an application of this methodology, decision makers must carefully identify appropriate inputs and outputs to be used in the decision making process. Table 1 depicts the supplier's attributes.

Table 1. Related attributes for 18 suppliers

Supplier No. (DMU)	Inputs		Desirable output	Undesirable output
	TC (1000 $) x_{1j}	SR* x_{2j}	NB y_{1j}	PPM y_{2j}
1	253	5	[50, 65]	1
2	268	10	[60, 70]	5.3
3	259	3	[40, 50]	4.6
4	180	6	[100, 160]	30
5	257	4	[45, 55]	30
6	248	2	[85, 115]	30
7	272	8	[70, 95]	30
8	330	11	[100, 180]	13.8
9	327	9	[90, 120]	4
10	330	7	[50, 80]	30
11	321	16	[250, 300]	26.4
12	329	14	[100, 150]	25.8
13	281	15	[80, 120]	25.8
14	309	13	[200, 350]	21.9
15	291	12	[40, 55]	9
16	334	17	[75, 85]	7
17	249	1	[90, 180]	6.3
18	216	18	[90, 150]	28.8

* Ranking such that 18 ≡ highest rank,…, 1 ≡ lowest rank ($x_{2,18} > x_{2,16} \ldots > x_{2,17}$)

Suppose the preference intensity parameter and the ratio parameter about the strong ordinal preference information are given (or estimated) as $\eta_2 = 1.12$ and $\sigma_2 = 0.01$, respectively. Using the transformation technique described in previous section, an interval estimate for SR of each supplier can be derived, which is shown in the Table 2.

Table 2. Interval estimate for the 18 suppliers after the transformation of ordinal preference information

Supplier No. (DMU)	SR
1	[.01574, .22917]
2	[.02773, .40388]
3	[.01254, .1827]
4	[.01762, .25668]
5	[.01405, .20462]
6	[.0112, .16312]
7	[.02211, .32197]
8	[.03106, .45235]
9	[.02476, .36061]
10	[.01974, .28748]
11	[.05474, .79719]
12	[.04363, .63552]
13	[.04887, .71178]
14	[.03896, .56743]
15	[.03479, .50663]
16	[.0613, .89286]
17	[.01, .14564]
18	[.06866, 1]

Therefore, all the input and output data are now transformed into interval numbers and can be evaluated using proposed models. Table 3 reports the results of efficiency assessments for the 18 suppliers obtained by using proposed models (8) and (9). The positive non-Archimedean infinitesimal, ε has been set to 0.0001.

Based on the definition 1, suppliers 1, 14, and 17 have the possibility to be DEA efficient. If they are able to use the minimum inputs to produce the maximum outputs, they are DEA efficient (efficient in scale); otherwise, they are not DEA efficient. Although suppliers 1, 14, and 17 have the possibility to be DEA efficient, due to the differences in the lower bound efficiencies, their performances are in fact different. The remaining 15 suppliers with relative efficiency scores of less than 1 are considered to be inefficient.

In order to rank the efficiencies of the 18 suppliers (DMUs), the MRA is employed to compute the maximum loss of efficiency for each supplier (see appendix). As computations show, supplier_{11} is selected as the best supplier.

As discussed earlier, Farzipoor Saen (in press) proposed an Additive model for considering imprecise data and undesirable factors. To test the validity of the efficiency results obtained in this paper, results are compared with Farzipoor Saen (in press). Table 4 shows the efficiency scores obtained by the Additive model. In Table 4, suppliers 1, 14, and

17 are efficient[1]. To compare the results, Spearman test using SPSS software is employed. Notice that to make two column vectors comparable, the upper bounds of efficiency interval in Table 3 have been considered. The test result is depicted in Table 5. It can be seen that the results show a high correlation. In addition, the testing value under the hypothesis that all efficiency scores are totally independent is 0.000, that is to say, the independent hypothesis should be rejected. So we can conclude that the efficiency scores are distinctly correlated.

Table 3. The efficiency interval for the 18 suppliers

Supplier No. (DMU)	Efficiency Interval
1	[0.653, 1]
2	[0.197, 0.255]
3	[0.136, 0.253]
4	[0.488, 0.888]
5	[0.152, 0.267]
6	[0.3, 0.618]
7	[0.225, 0.378]
8	[0.267, 0.556]
9	[0.334, 0.586]
10	[0.131, 0.294]
11	[0.686, 0.824]
12	[0.266, 0.401]
13	[0.249, 0.375]
14	[0.57, 1]
15	[0.121, 0.171]
16	[0.198, 0.225]
17	[0.319, 1]
18	[0.366, 0.611]

[1] In Additive model, a DMU is said to be efficient if its objective value be zero, otherwise, it is said to be inefficient.

Table 4. The efficiency scores obtained by Farzipoor Saen (in press)

Supplier No. (DMU)	Efficiency scores
1	0
2	-207
3	-217
4	-57
5	-238
6	-155
7	-217
8	-175
9	-135
10	-289
11	-67
12	-225
13	-204
14	0
15	-252
16	-262
17	0
18	-108

Table 5. Spearman non-parametric correlation result between models of Farzipoor Saen (in press) and current paper

	model of current paper
model of Farzipoor Saen (in press)	0.91** (.000)

** Correlation is significant at the 0.01 level (2-tailed)

4- Concluding Remarks

As Akarte et al. (2001) discussed, purchasing is one of the most crucial and vital activities of business, as it has a significant impact on finance, operations and competitiveness of the organization. Selecting an appropriate supplier is now one of the most important decisions of the purchasing department. This decision generally depends on a number of different criteria. Traditionally, cost has been the main criterion used in selecting a supplier, but slowly non-price criteria such as quality, delivery and overall capability are becoming equally important.

To select the best suppliers, this paper has proposed a methodology for dealing with undesirable outputs in the presence of imprecise data.

The problem considered in this research is at initial stage of investigation and further studies can be done based on the results of this paper. Some of them are as below.

- Similar research can be repeated in the presence of stochastic data.
- Similar study can be replicated in the context of cross-efficiency evaluation.
- This study used the proposed model for supplier selection. It seems that more fields (e.g. technology selection, personnel selection, market selection, etc) can be applied.

Appendix

R(supplier$_1$)= .347, R(supplier$_2$)= .803, R(supplier$_3$)= .864, …, R(supplier$_{18}$)= .634

Obviously, supplier$_{11}$ has the smallest maximum loss of efficiency. So, supplier$_{11}$ is rated as the best supplier and eliminated from the further consideration. Therefore for the remaining suppliers, maximum losses of efficiency are recalculated.

Repeating the above process, the ranking order of 18 suppliers is obtained as follows:

Supplier$_{11} \succ$ supplier$_1 \succ$ supplier$_{14} \succ$ supplier$_4 \succ$ supplier$_{18} \succ$ supplier$_{17} \succ$ supplier$_9 \succ$ supplier$_6 \succ$ supplier$_8 \succ$ supplier$_{12} \succ$ supplier$_{13} \succ$ supplier$_7 \succ$ supplier$_{16} \succ$ supplier$_2 \succ$ supplier$_{10} \succ$ supplier$_5 \succ$ supplier$_3 \succ$ supplier$_{17}$.

Therefore, supplier$_{11}$ is selected as the best supplier.

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