Time Diversification and Contributions

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ABSTRACT

The time diversification debate examines the relationship between risk and investment horizon. The debate has generally considered wealth to be a function only of returns. This paper considers the practicalities of this assumption (particularly in light of the broader pension finance debate) by examining two further accumulation models. Our findings reveal that one particular variable – contributions – significantly impacts terminal wealth. Furthermore, we find that measures proposed as ways of studying time diversification generally ignore the influence of contributions. If the time diversification debate is truly about the risks of long term investing, considering realistic accumulation models should be the first step to understanding the relationship between risk and investment horizon.

JEL classification: G23; G11
***Key words**:* Time diversification; Risk; Investment horizon; Contributions; Defined contribution; Pensions

INTRODUCTION

Samuelson (1969) initiated the time diversification debate by observing how time horizon affected the optimal allocation to risky assets. Using an expected utility framework, he concluded that the allocation to risky assets was independent of time, and only a function of risk tolerance. Samuelson's (1969) conclusions were based on three assumptions: (1) the investor exhibits constant relative risk aversion, (2) returns follow a random walk, and (3) wealth is a function only of returns. Much of the subsequent research within the expected utility framework has considered variations to these assumptions (Kritzman, 1994; Milevsky, 1999; Gollier, 2002), and many of the competing streams of research use these assumptions as a critique of the framework itself (Fisher and Statman, 1999; Booth, 2004). In this paper, we use a critique of the last of Samuelson's (1969) assumptions – that wealth is a function only of returns – and of the time diversification literature in general, to motivate an examination of the impact of contributions on the question at the heart of the time diversification debate: Are risky assets more or less risky over longer horizons?

When considered in the light of the institutional setting observed in the pension fund industry – that is, defined contribution (DC) investing involves regular cash contributions over the working life – it is surprising that almost the entire time diversification literature takes place within an 'initial endowment' framework.[i] Of the entire time diversification literature, only a small number of relatively recent studies actually incorporate periodic cash inflows, or contributions, in any way.[ii] Jagannathan and Kocherlakota (1996), using an expected utility approach similar to that of Samuelson (1969), determine the optimal risky asset weights over various time horizons for varying degrees of constant relative risk aversion for "*a household that has $20,000 per year available for investment from its salary income* (p. 16)." Jagannathan and Kocherlakota (1996) show that the median allocation to stocks for a median household with a risk aversion coefficient of five is a decreasing function of time horizon: allocations fall non-linearly from approximately 80 per cent at age 35 to around 40 per cent at age 65.[iii]

But perhaps the most relevant study in this literature is the work of Hickman et al. (2001). Hickman et al. (2001) use six different asset classes and compare performance over various investment horizons in order to determine which assets offer the dominant investment strategy under a variety of holding-period and risk aversion assumptions. Hickman et al. (2001) find evidence supporting greater risk allocations for those investors with longer horizons, and a shift toward lower risk holdings as retirement approaches.

This study extends the typical initial endowment framework by examining an accumulation model where terminal wealth is the function of contributions, salary growth, returns and initial endowment. If one accepts that terminal wealth expectations are somehow anchored to terminal income (as argued in this paper), then we need a way of adjusting terminal wealth to account for

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[i] For example, Strangeland and Turtle (1999) state: "As in much of the time diversification literature, we do not consider the more general case of intertemporal consumption and income flows. This [Strangeland and Turtle's (1999)] framework is consistent with the notion that time diversification is typically posited as advice that is dependent solely on an investor's age and time until retirement, with little concern for future cash flows (p. 12)."

[ii] A number of studies analyse cash outflows (or withdrawals from wealth) as a way of studying the interplay between consumption and retirement investing (e.g. Samuelson, 1969; Merton, 1969).

[iii] Also see the work of Mukherji (2008), Panyagometh (2011) and Ayres and Nalebuff (2013).

differing levels of terminal income. We employ the retirement wealth ratio (*RWR*) (discussed in greater detail in the methodology section) as a way of considering this issue.

APPROACH

Data

The data used in this study are the well-known, and commonly used, monthly stock and T-bills returns maintained by French (2012). The excess return on the market (R_m-R_f) maintained by French (2012) is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks, from the Center for Research into Security Prices (CRSP), minus the one-month Treasury bill rate, obtained from Ibbotson Associates. To calculate nominal total stock returns, we add back the one-month Treasury bill rate.

Methodology

We take stock and T-bill data and, using four separate simulation methods that generate 10,000 synthetic returns paths for each of nine different investment horizons. The four simulation methods have been chosen for several reasons. Firstly, in order to contrast our findings with those of the time diversification literature we must replicate methods used in the literature, namely a parametric Monte Carlo method and the non-parametric bootstrap simulation method of Efron (1979). Secondly, we also consider non-parametric block bootstrap techniques. These block bootstrap techniques also fulfil another purpose: they are recognised as being better able to capture the time series characteristics of financial returns (Pascual and Ruiz, 2002; Mukherji, 2008). The nine investment horizons to be considered are 1, 5, 10, 15, 20, 25, 30, 35 and 40 years.

Accumulation models

In this paper, we critique the initial endowment model, where terminal wealth is a function of only returns and the magnitude of the initial endowment. The first of the two additional aspects to the methodology employed in this study, is the consideration of two further accumulation models: the constant contribution model; and, the constant percentage contribution model (Table 1).

Table 1. Accumulation models

This table presents the differences between the accumulation models studied in this paper. While the percentage rates are quoted in per annum terms, these annual rates are applied on a monthly basis in the modeling.

Accumulation model	Contribution rate (k)	Salary growth rate (g)
Initial endowment model	Zero	Zero
Constant contribution model	9% per annum	Zero
Constant percentage contribution model	9% per annum	3% per annum

Firstly, we will examine a constant contribution model where contributions are fixed at nine per cent of salary (credited monthly), and salary remains constant in nominal terms over the investment horizon. Secondly, we consider a constant percentage contribution model where contributions are again fixed at nine per cent of salary (credited monthly), but salary increases at a constant rate of three per cent per annum (applied on a monthly basis). By examining the marginal impact of contributions and salary growth, we hope to provide positive insights into the importance of contributions as a novel extension to the time diversification literature. By incorporating multiple cash flows as we observe in DC investing, we also introduce the real world to the time diversification debate.

Earnings and account balance data

In this study, we use median weekly earnings data from the US Bureau of Labor Statistics (BLS) to provide benchmark income levels for US workers of ages that correspond to the nine investment horizons. This income data is used as the basis for identifying median account balances from the Employment Benefit Research Institute (EBRI) for use as the initial wealth in the simulation of terminal wealth paths. This data is shown in Table 2, where row three reports the starting income for each investment horizon (row one) and the corresponding age (row two).

Table 2. Earnings and account balance data

This table presents earnings and related account balance data in order to approximate initial wealth (W_0) for various horizons. Row one shows the investment horizon. Row two shows the assumed investor age that corresponds to the investment horizon. Row three shows Bureau of Labour Statistics (BLS) (2009) median earnings data for the fourth quarter of 2008 (annualised, rounded). Row four shows raw Employment Benefit Research Institute (EBRI) (2009) median account balance data that corresponds to the annualised BLS earnings data in row three (Only includes 401(k) accounts. Previous employer accounts, and IRAs are excluded). Row five shows the EBRI data rounded to the nearest thousand dollars. The rounded data is used as initial wealth (W_0) in the analysis in this paper. Row six shows data that was sourced to validate the account balance data shown in rows four (in raw form) and five (in rounded form). The data was obtained from the US Census Bureau (2012) and represents the median value of retirement accounts by age (including IRAs, Keogh accounts, 401(k), 403(b)). Note that there are two major differences between the data in rows five and six: (1) row six data is more recent by around two years allowing the sampled population to accumulate more assets; and, (2) row six data includes a more complete variety of account types. These two differences would lead us to expect the row six data to be greater, an expectation that is born out in the numbers. Given these reconcilable differences, we suggest that the US

Census bureau data provides a reasonable cross check for the EBRI data. Investment horizon and assumed age are expressed in years. All other data are expressed in dollars.

Investment horizon (years)	**40**	**35**	**30**	**25**	**20**	**15**	**10**	**5**	**1**
Assumed age	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**64**
Median earnings data	**25,000**	**35,000**	**39,000**	**42,000**	**42,000**	**43,000**	**43,000**	**43,000**	**33,000**
Raw median account bal.	**4,757**	**10,108**	**15,458**	**34,176**	**52,893**	**62,242**	**71,591**	**72,713**	**73,834**
Median account balance	**5,000**	**10,000**	**15,000**	**34,000**	**53,000**	**62,000**	**72,000**	**73,000**	**74,000**
Validating account bal.	**N/A**	**10,000**	**23,000**	**36,000**	**51,500**	**67,000**	**82,500**	**98,000**	**77,000**

In this paper we focus on median earnings data, and the account balances corresponding to median wage earners, as the most relevant for a study of pension finance because it is these individuals that are most likely to have to rely on their retirement accounts to fund their consumption in retirement. In using these income levels, we seek to employ reasonable, representative income levels as the basis for calculating contributions, and as reasonable starting points to which we apply salary growth.

Throughout we assume that the income levels that correspond to the nine investment horizons (row three, Table 2) grow at a constant rate of three per cent per annum for the length of the investment horizon, applied on the same monthly basis as contributions are calculated and added to wealth at time t. As for the problem of determining an appropriate income level for a given age, arriving at a single representative "average" level of salary growth is problematic for a number of reasons. Firstly, income growth over a working life is rarely uniform or constant in nature. Income profiles are affected by both macroeconomic trends, and by factors idiosyncratic to the individual like gender, occupation, education level, employer and industry. Scholars have shown that a typical income profile is "humped" in nature, rising to its zenith in the early-to-mid fifties after which it falls as the individual transitions from full time work to semi-retirement (Byrne et al., 2006). Whilst we concede that our assumption could be subject to criticism, we take consolation from the fact that the purpose of this research does not hinge on the accuracy of the salary growth assumption (if such an assumption exists). Rather, we investigate an accumulation model incorporating salary growth (the constant percentage contribution model) to understand the marginal impact of salary growth when compared to the constant contribution and initial endowment models.

In summary, the three accumulation models evolve as follows: the initial endowment model begins with initial wealth and is affected only by returns; the constant contribution model sees terminal wealth as a function of initial wealth, returns, and constant nominal dollar contributions; and, the constant percentage contribution model generates terminal wealth from the interplay of initial wealth, returns and contributions that rise with income growth (cf. Table 1).

▌ ***Evaluating outcomes using the retirement wealth ratio***

The second additional methodological aspect in this study is the introduction of a further basis upon which to judge terminal wealth outcomes. The challenge with return- or dollar-based terminal wealth measures is that neither is particularly informative for the investor in terms of what performance means to their spending power in retirement. Baker, Logue, and Rader (2005), for example, argue that defined contribution plans should be measured in terms of their ability to generate sufficient retirement income. What we need therefore is a measure that sizes terminal wealth against some relevant benchmark (Booth, 1997; Clarkson, 1989; Booth and Yakoubov, 2000).

One such measure is the retirement wealth ratio (RWR_T) of Basu and Drew (2009), based on the work of Booth and Yakoubov (2000), which is calculated by dividing terminal wealth (W_T) by gross income at time T.[iv] Basu and Drew (2010) provide a rationale for the use of the retirement wealth ratio into the pension finance literature was because "*it is very likely that the participant's post-retirement income expectations are closely linked to their immediate income before retirement* (p. 292)." For example, a terminal wealth of one million dollars will appear much more attractive to an individual whose final salary is \$50,000 per annum when compared to another individual whose final salary in \$200,000 per annum. If we were to judge each scenario based on terminal wealth alone, performance would be equivalent with each individual retiring with one million dollars. Expressed in retirement wealth ratio terms, the worker on the lower income would retire with a RWR_T of 20 times (\$1,000,000 divided by \$50,000) versus an RWR_T of five times (\$1,000,000 divided by \$200,000) for the individual on the higher income.

The RWR_T therefore allows us to compare accumulation models where incomes at time T are not equivalent. In this study, we are able to compare terminal wealth for the initial endowment and constant contribution models because in each case salary is constant in nominal dollar terms over the investment horizon. By introducing the constant percentage contributions model, where contributions

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[iv] A similar measure – the "expected accumulation" – is used by Booth and Yakoubov (2000).

rise due to the effect of salary growth, we have a model with a different and higher final income. We therefore evaluate performance in RWR_T terms so as to avoid over-estimating the performance of the constant percentage contribution model because we have ignored the higher final salary, and hence higher post-retirement income expectations.

EMPIRICAL EVIDENCES

For all three models – initial endowment, constant contributions and constant percentage contribution – we perform Monte Carlo, Efron (1979) bootstrap, stationary bootstrap, and empirical block bootstrap simulations for each of the nine horizons (1, 5, 10, 15, 20, 25, 30, 35, 40 years), for a total of 108 models. Consequently, in the below tables we report 108 versions of the same measure in order to compare the results on three dimensions: (1) the accumulation model, which looks at different sets of determinants of terminal wealth; (2) the modeling technique, which allows for different conceptions of the asset return process (e.g. random walk); and, (3) investment horizon, which is the main subject of the time diversification literature.

▌The Retirement Wealth Ratio

The RWR_T allows us to compare accumulation models where both terminal wealth and income at time T are not equivalent. Had we restricted our analysis to the initial endowment and constant contribution accumulation models, we would be able to compare performance on a wealth basis because final salaries for the two models are equal. But by introducing the constant percentage contributions model, where contributions rise due to the effect of salary growth, we have a model with a different final income thereby adding a variable that must be controlled for. We now review the performance of these simulations in RWR_T terms.

▌Distribution of Retirement Wealth Ratios

We now summarise the simulation results of RWR_T in Tables 3 and 4.

Table 3. Distribution of retirement wealth ratios

This table presents the distribution of terminal wealth for the four simulation techniques over nine investment horizons for the three accumulation models. Initial wealth (W_0) is as per Table 2. The simulation techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. Performance measures are grouped by measure type with measures of central tendency reported in Panel A (mean retirement wealth ratio) and Panel B (median, or 50th percentile retirement wealth ratio). Measures of dispersion are reported in Panels C (standard deviation in retirement wealth ratio terms) and D (retirement wealth ratio range). Panels E through G summarise the lower half of the distribution of terminal wealth (minimum, 5th percentile and 25th percentile retirement wealth ratio respectively). Panels H through J summarise the upper half of the distribution of terminal wealth (75th percentile, 95th percentile and maximum retirement wealth ratio, respectively). Results are presented as multiples (i.e. in unit of "times" as in "*x* times final salary").

PANEL A – 95th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	15.42	15.32	13.26	15.26	77.64	80.13	71.36	79.72	30.53	30.11	27.72	30.05
35	12.58	12.55	11.18	12.74	49.76	49.53	44.97	49.21	21.41	21.21	19.36	21.60
30	9.92	10.04	9.01	9.97	31.22	30.52	28.66	31.03	15.15	15.29	14.18	15.15
25	12.23	12.24	11.09	12.13	23.75	24.35	22.35	24.37	13.06	12.99	11.85	12.74
20	10.96	10.97	10.28	10.91	17.57	17.57	16.57	17.47	10.49	10.35	9.99	10.35
15	7.28	7.35	6.96	7.40	10.74	10.64	10.34	10.68	7.25	7.34	7.01	7.21
10	4.89	4.89	4.80	4.92	6.53	6.67	6.38	6.58	5.04	5.09	4.93	5.06
5	2.91	2.91	2.89	2.90	3.51	3.51	3.48	3.51	3.04	3.06	3.05	3.05
1	2.48	2.48	2.49	2.50	2.58	2.58	2.58	2.59	2.51	2.50	2.51	2.52

PANEL B – 95th percentile retirement wealth ratio

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**	**mc**	**b**	**bs**	**bb**
40	7.35	7.66	8.10	8.44	46.80	47.90	50.05	51.40	19.22	18.98	20.41	20.54
35	6.81	6.98	7.16	7.53	31.97	32.02	33.18	34.37	14.20	14.22	14.88	15.68
30	5.76	6.03	6.26	6.54	21.26	21.09	22.58	23.10	10.74	10.73	11.31	11.71
25	7.97	7.83	8.34	8.61	16.98	17.63	18.10	18.72	9.47	9.48	9.77	10.20
20	7.77	7.89	8.23	8.39	13.11	13.15	13.96	14.33	8.16	7.95	8.45	8.54
15	5.63	5.62	5.91	6.22	8.69	8.68	9.19	9.40	5.87	5.93	6.24	6.35
10	4.12	4.13	4.38	4.51	5.63	5.74	5.93	6.03	4.37	4.39	4.59	4.71
5	2.67	2.68	2.85	2.86	3.25	3.25	3.41	3.47	2.84	2.86	2.99	3.00
1	2.44	2.46	2.52	2.53	2.55	2.55	2.63	2.63	2.48	2.48	2.55	2.55

PANEL C– 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	27.14	26.28	16.07	21.54	99.68	116.84	69.19	90.15	37.55	38.68	25.38	31.01
35	19.14	18.62	12.62	16.30	60.65	57.52	40.40	50.13	24.93	23.74	15.88	20.81
30	13.47	13.01	8.92	10.98	32.53	31.77	22.62	28.26	15.77	16.36	10.64	12.93
25	14.37	14.26	9.80	11.84	23.27	23.52	16.37	20.74	12.39	12.00	8.32	9.75
20	10.86	10.60	8.03	9.39	15.42	15.48	11.17	12.73	8.41	8.52	6.43	7.30
15	5.99	6.37	4.59	5.31	7.73	7.61	5.78	6.54	5.10	5.48	3.86	4.34
10	3.11	3.10	2.51	2.80	3.78	3.83	2.96	3.39	2.85	2.97	2.28	2.53
5	1.26	1.26	1.07	1.10	1.42	1.43	1.20	1.23	1.20	1.20	1.04	1.06
1	0.45	0.44	0.45	0.46	0.46	0.46	0.47	0.47	0.44	0.44	0.45	0.45

PANEL D– 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	660.34	704.17	202.97	484.60	2,340.57	4,523.30	719.40	1,495.86	732.95	1,174.90	377.37	453.27
35	590.16	460.37	186.54	288.94	1,342.02	998.98	714.42	739.59	542.19	531.12	229.14	303.27
30	236.24	256.51	100.28	132.10	422.93	743.69	251.76	390.98	467.80	527.20	110.34	206.62
25	337.49	184.43	121.95	127.61	533.66	505.18	180.08	275.64	249.30	178.80	97.86	108.42
20	141.11	204.29	91.06	118.23	208.74	353.01	108.92	108.90	96.13	132.82	69.84	96.67
15	125.83	144.99	60.88	50.56	115.09	117.12	60.96	55.52	69.67	172.46	36.24	41.44
10	41.87	47.81	22.68	28.11	46.52	38.78	24.56	37.66	36.16	38.90	17.92	21.73
5	11.62	12.84	8.62	8.33	12.59	20.13	11.91	8.56	11.74	10.79	9.31	8.59
1	3.78	4.36	4.30	4.30	3.42	4.88	4.89	4.40	3.68	4.53	4.27	4.27

PANEL E – 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	0.12	0.08	0.01	0.00	2.03	1.80	0.73	0.84	1.27	1.30	0.71	0.64
35	0.10	0.08	0.04	0.02	1.58	1.82	1.06	0.70	1.03	1.30	0.68	0.44
30	0.16	0.07	0.06	0.02	1.13	1.46	0.93	0.49	0.99	0.97	0.52	0.50
25	0.26	0.23	0.16	0.06	1.23	1.06	0.91	0.36	1.00	1.06	0.54	0.45
20	0.35	0.31	0.20	0.05	0.91	1.09	0.76	0.44	0.98	0.37	0.57	0.30
15	0.28	0.26	0.18	0.03	0.91	0.97	0.48	0.24	0.68	0.64	0.44	0.30
10	0.57	0.39	0.13	0.16	1.04	0.81	0.43	0.30	0.80	0.75	0.58	0.22
5	0.46	0.58	0.15	0.10	0.77	0.81	0.37	0.23	0.59	0.47	0.29	0.20
1	1.10	1.08	0.77	0.77	1.23	1.11	0.82	0.82	1.19	1.18	0.80	0.80

PANEL F– 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	1.08	1.08	1.06	0.77	10.71	10.43	10.90	8.60	4.73	4.81	4.97	4.14
35	1.10	1.12	1.06	0.77	7.61	7.76	7.94	6.54	3.91	3.87	4.08	3.36
30	1.11	1.06	1.07	0.85	5.96	5.70	5.98	4.86	3.24	3.21	3.21	2.89
25	1.66	1.67	1.61	1.28	5.04	5.01	5.17	4.36	3.13	3.01	2.96	2.63
20	1.95	1.94	1.97	1.54	4.22	4.18	4.29	3.64	2.73	2.72	2.77	2.45
15	1.64	1.69	1.60	1.33	3.17	3.13	3.22	2.81	2.30	2.27	2.27	1.94
10	1.56	1.56	1.48	1.24	2.46	2.44	2.33	2.09	1.89	1.90	1.84	1.65
5	1.35	1.34	1.20	1.15	1.74	1.71	1.60	1.52	1.52	1.52	1.46	1.38
1	1.80	1.80	1.74	1.74	1.88	1.88	1.81	1.82	1.85	1.84	1.76	1.76

PANEL G– 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	3.40	3.46	3.67	3.40	24.64	25.17	27.37	25.63	10.65	10.36	11.44	10.91
35	3.30	3.29	3.49	3.30	17.51	17.52	18.83	18.00	8.09	8.27	8.88	8.60
30	2.88	3.00	3.15	3.02	12.19	12.18	13.25	12.83	6.38	6.40	6.86	6.72
25	4.22	4.10	4.43	4.27	10.07	10.42	10.92	10.81	5.81	5.83	6.10	6.02
20	4.38	4.44	4.74	4.52	8.14	8.27	8.77	8.64	5.13	5.09	5.41	5.29
15	3.41	3.42	3.65	3.64	5.69	5.60	6.14	6.05	3.93	3.97	4.20	4.11
10	2.78	2.80	2.97	2.93	3.98	4.02	4.19	4.16	3.12	3.10	3.26	3.23
5	2.02	2.04	2.13	2.15	2.51	2.49	2.63	2.67	2.17	2.21	2.32	2.33
1	2.16	2.18	2.21	2.22	2.26	2.27	2.29	2.30	2.20	2.22	2.22	2.25

PANEL H – 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	16.56	16.96	16.62	18.89	91.24	93.26	90.25	100.64	36.00	35.65	35.22	38.03
35	14.61	14.60	14.11	15.76	58.86	58.87	58.22	62.19	25.51	25.52	24.80	27.28
30	11.60	11.91	11.72	13.05	38.00	36.89	37.27	39.80	18.54	18.25	18.27	19.33
25	15.01	14.78	14.71	15.80	29.02	29.82	28.92	31.33	15.62	16.00	15.21	16.56
20	13.59	13.69	13.49	14.47	21.59	21.77	21.22	22.84	13.04	12.78	12.99	13.38
15	9.08	9.19	9.20	9.73	13.38	13.26	13.27	13.74	9.00	9.08	8.92	9.33
10	6.12	6.11	6.21	6.43	8.03	8.35	8.16	8.44	6.21	6.25	6.24	6.45
5	3.52	3.54	3.55	3.61	4.22	4.22	4.23	4.29	3.64	3.68	3.68	3.69
1	2.76	2.75	2.76	2.77	2.86	2.85	2.86	2.88	2.79	2.77	2.79	2.79

PANEL I – 95th percentile retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	56.07	53.21	42.68	51.18	246.98	249.62	208.07	243.80	91.72	91.80	75.54	86.08
35	42.62	41.23	34.75	42.08	148.99	148.79	119.39	143.33	60.62	60.01	49.55	60.45
30	32.24	31.86	26.45	30.29	90.90	86.74	71.65	83.53	40.93	42.17	34.80	38.94
25	36.43	37.29	30.04	35.61	65.41	66.23	53.71	63.52	35.16	34.26	27.72	31.76
20	30.86	30.60	25.79	29.06	46.02	45.12	38.33	42.17	26.03	25.62	22.24	24.33
15	18.54	18.61	15.72	17.58	25.16	24.65	21.31	22.95	16.64	16.76	14.42	15.34
10	10.80	10.58	9.44	10.12	13.62	14.01	11.75	12.81	10.35	10.51	9.02	9.74
5	5.31	5.22	4.66	4.80	6.18	6.18	5.49	5.63	5.30	5.31	4.81	4.91
1	3.28	3.25	3.15	3.16	3.39	3.37	3.26	3.27	3.29	3.26	3.17	3.18

PANEL J – Maximum retirement wealth ratio

Investment	Initial endowment				Constant contributions				Constant percentage contributions			
horizon	mc	b	bs	bb	mc	b	bs	bb	mc	b	bs	bb
40	660.46	704.25	202.98	484.60	2,342.60	4,525.10	720.13	1,496.70	734.22	1,176.20	378.08	453.91
35	590.26	460.45	186.58	288.96	1,343.60	1,000.80	715.48	740.29	543.22	532.42	229.82	303.71
30	236.39	256.58	100.34	132.12	424.06	745.15	252.69	391.47	468.79	528.17	110.86	207.13
25	337.75	184.66	122.11	127.67	534.89	506.25	180.99	276.00	250.30	179.86	98.40	108.88
20	141.46	204.59	91.26	118.28	209.65	354.10	109.69	109.34	97.11	133.19	70.40	96.97
15	126.11	145.25	61.05	50.59	116.00	118.09	61.44	55.77	70.35	173.10	36.68	41.73
10	42.44	48.20	22.81	28.27	47.56	39.59	24.99	37.96	36.96	39.65	18.50	21.95
5	12.08	13.42	8.77	8.43	13.35	20.95	12.28	8.79	12.33	11.26	9.60	8.79
1	4.88	5.44	5.07	5.07	4.65	5.99	5.71	5.22	4.87	5.72	5.07	5.07

The first point to note from Table 3 is that the large absolute differences in the results for the constant contribution and constant percentage contribution models when compared to the initial endowment model persist when measured in RWR_T terms. This result emphasises that contributions are an extremely important variable in pension finance problems. All measures reported in Table 3 for the constant contribution and constant percentage contribution models increase in magnitude with investment horizon. For some measures, like the standard deviation of RWR_T reported in Panel C, we see evidence that supports the time diversification literature. Hickman et al. (2001), for example, also found that the standard deviation of terminal wealth increased with time horizon. The range of RWR_T estimates increases in investment horizon confirming the results of McEnally (1985) and others (e.g. Mukherji, 2008).

Other measures appear to yield opposite results when we move beyond the initial endowment model. Compare, for example, the minimum RWR_T estimates (Panel E, Table 3) for the initial endowment model with those of the other two accumulation models. For the initial endowment model, estimates tend to fall (i.e. worsen) as investment horizon lengthens. For the two accumulation models that incorporate contributions, minimum RWR_T estimates tend to rise (i.e. improve) with investment horizon. Whilst this evidence suggests that the near universal absence of contributions from the time diversification literature may be a significant deficiency, caution is required for two reasons. Firstly, because of the low absolute value of wealth for the minimum paths, the differences in initial wealth for the various horizons obscure the results somewhat. And, secondly, while different trends appear to exist for different accumulation models, in economic terms, all outcomes are equally poor.

The noticeable (and expected) difference in the RWR_T estimates for the constant percentage contribution model reported in Table 3 is that they are now lower than the estimates for the constant contribution model. This is because the denominator in the calculation of RWR_T (i.e. final salary) is also growing with time hence the ratio of terminal wealth to final salary will be lower. Whilst this might at first suggest an inferior terminal wealth outcome for the plan member, this is not the case as the hypothetical plan member has also earned higher incomes throughout their working life because of salary growth. Thus, whilst terminal wealth isn't lower in absolute terms, the terminal wealth relative to the plan member's expectations is lower, where expectations are assumed to be related to final salary.

▌Downside Risk Measures

We now consider the estimates for value-at-risk and expected tail loss (see Table 4). For the initial endowment model, downside risk increases (i.e. RWR_{T}s fall) with time. For the two models with contributions, downside risk decreases (i.e. RWR_{T}s rise) with time due to the positive effect of ongoing contributions and, for the constant percentage contributions model, of salary growth. The effect of contributions and salary growth can be observed by comparing expected tail loss estimates (in RWR_{T} terms) through time for the initial endowment and constant percentage contribution models.

Table 4. Downside risk measures

This table presents selected downside risk measures for the four simulation techniques over nine investment horizons for the three accumulation models. The simulation techniques (with their codes in brackets) are: Monte Carlo simulation (mc), Efron (1979) bootstrap (b), stationary bootstrap (bs) and the empirical block bootstrap (bb). For ease of comparison the results are divided into panels. The measures are: 95% value-at-risk (VaR) is reported in Panel A, and 95% expected tail loss is reported in Panel B. All results are expressed in thousands of dollars.

PANEL A – 95% value-at-risk

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	1.08	1.08	1.06	0.78	10.71	10.43	10.90	8.60	4.74	4.81	4.97	4.14
35	1.10	1.12	1.06	0.77	7.62	7.76	7.94	6.55	3.91	3.87	4.08	3.36
30	1.11	1.06	1.07	0.86	5.96	5.70	5.98	4.86	3.24	3.21	3.21	2.89
25	1.66	1.67	1.61	1.28	5.04	5.01	5.17	4.36	3.13	3.01	2.96	2.63
20	1.95	1.94	1.97	1.54	4.22	4.18	4.30	3.64	2.73	2.72	2.79	2.45
15	1.64	1.69	1.60	1.33	3.17	3.13	3.23	2.81	2.30	2.27	2.27	1.94
10	1.56	1.56	1.48	1.24	2.46	2.44	2.33	2.09	1.89	1.90	1.84	1.65
5	1.35	1.34	1.20	1.15	1.74	1.71	1.60	1.52	1.52	1.52	1.46	1.39
1	1.80	1.80	1.74	1.74	1.89	1.88	1.81	1.82	1.86	1.84	1.76	1.76

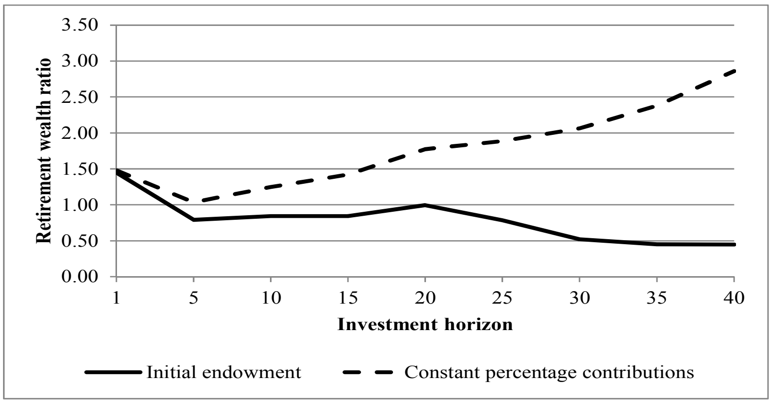
PANEL B – 95% expected tail loss

Investment	**Initial endowment**				**Constant contributions**				**Constant percentage contributions**			
horizon	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**	**mc ($)**	**b ($)**	**bs ($)**	**bb ($)**
40	0.71	0.70	0.66	0.45	7.63	7.63	7.64	5.75	3.55	3.65	3.55	2.86
35	0.74	0.74	0.68	0.45	5.72	5.81	5.67	4.40	3.03	3.03	2.96	2.38
30	0.76	0.72	0.70	0.52	4.56	4.35	4.36	3.36	2.58	2.52	2.43	2.06
25	1.19	1.15	1.05	0.78	3.91	3.86	3.73	3.04	2.47	2.37	2.27	1.89
20	1.43	1.41	1.35	0.99	3.31	3.22	3.16	2.58	2.18	2.15	2.10	1.77
15	1.25	1.28	1.14	0.84	2.56	2.50	3.45	2.04	1.90	1.83	1.76	1.42
10	1.26	1.25	1.08	0.84	2.06	2.03	1.79	1.56	1.58	1.57	1.42	1.25
5	1.14	1.13	0.88	0.79	1.49	1.46	1.22	1.13	1.32	1.30	1.10	1.03
1	1.68	1.65	1.47	1.45	1.75	1.72	1.53	1.53	1.72	1.68	1.48	1.48

Figure 1 shows that over short horizons of one to five years, contributions make little difference with ETL estimates for the two accumulation models differing by only a small amount. As horizon lengthens, however, the combined effects of contributions, salary growth, and returns (and associated compounding) leads to significantly different outcomes with the constant percentage contribution model yielding a 40-year RWR_{T} over six times larger than the initial endowment model equivalent. Thus, we see the difference that contributions and salary growth make even for the average of the five per cent worst portfolio outcomes.

Figure 1. Expected tail loss with investment horizon

Using the empirical block bootstrap (bb) simulation method, this figure presents estimates of the 95% expected tail loss in RWR_{T} terms for the initial endowment (heavy line) and constant percentage contribution (dashed line) models against investment horizon.



CONCLUDING REMARKS

This study demonstrates that the time diversification literature suffers from two related challenges. Firstly, the literature largely ignores at least one variable – contributions – that significantly impacts terminal wealth. Contributions are a well-documented feature of most developed systems of retirement savings. We therefore have both empirical and practical reasons to reject our null hypothesis: That alternative accumulation models have no bearing on the relationship between risk and investment horizon. Secondly, the measures proposed by the literature as ways of measuring time diversification, with few exceptions, ignore the influence of contributions. These exceptions are where performance is expressed in terms of terminal wealth. This distinction between wealth-based measures and return-based measures provides further support for the findings of related research which highlighted that resolution of the time diversification debate may be more related to the risk measure's measurement basis (return- or wealth-based), than the particular risk measure of choice.

Because of these issues, the time diversification literature has, to paraphrase Kritzman (2000), become a "*referendum on the meaning of risk*" for a mis-specified problem. From the perspective of retirement savings, whether risk rises or falls with investment horizon in the absence of contributions is not a particularly interesting question because it is devoid of important context. We can see this by comparing, say, the 95% value-at-risk estimates for the initial endowment and constant percentage contribution models for the 40-year horizon in Table 4. In retirement wealth ratio terms, the risk for the constant percentage contribution model – arguably the most realistic under consideration – is more than four times larger than that of the initial endowment model suggesting better downside performance. But this model also generally brings with it a wider range of outcomes depending on one's view of the asset return process.

If the time diversification debate really is about long term investing, then it should be reframed to allow realistic accumulation models to be examined, after which conclusions about the relationship between risk and investment horizon can be reached. Recent research by Basu and Drew (2009) has shown that, as a plan member approaches retirement, portfolio size rises rapidly - the portfolio size effect - magnifying the potential impact of a large negative return. Macqueen and Milevsky (2009) describe this as sequencing risk, or the risk of experiencing a poor sequence of returns. Risk is thus highly path-dependent and virtually unique to the individual. The key implication of the work of Basu and Drew (2009) and Macqueen and Milevsky (2009) is that, because of the effect of the compounding of returns, contributions and salary growth, a negative 25 per cent return five years into one's working life is not the same as an equivalent return five years prior to retirement. Another closely related way of perceiving risk was proposed by Fabozzi et al. (2006) who suggested risk be thought of as episodic, rather than as a durable trend with time. So rather than either rising or falling with time as is claimed in the time diversification literature, risk according to this view manifests itself suddenly, persists for a period, then disappears only to return later. We posit that by ignoring contributions, we may be debating a phenomenon in a context devoid of reality. Without this reality, and in the face of emerging research, the generality of such findings, particularly to the broader pension finance debate, may be limited.

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