Foreign aid in a dynamic world with transboundary pollution*

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Formulating a dynamic general equilibrium model of small open economies suffering from global pollution, welfare effects of foreign aid are studied. We show that aid raises the global pollution and worsens the donor’s welfare in both the steady state and transition. On the other hand, aid deteriorates the recipient’s steady state welfare, whereas the recipient’s time path of welfare can improve under certain conditions.

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1 Introduction

This paper studies welfare effects of foreign aid in a dynamic context. While there is an extensive literature in this field, this paper is differentiated from the predecessors in two respects. First, we consider the effects of foreign aid in the presence of transboundary pollution. Second, we exclusively focus on dynamic effects of aid by building a dynamic general equilibrium model. In the first respect, Naito (2003) proves the possibility of Pareto-improving aid under transboundary pollution. Chao and Yu

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(1999) and Hatzipanayotou et al. (2002) make welfare evaluations of tied aid in a polluted world. All of these works rest on static models. In the second respect, Shimomura (2007) and Naito and Ohdoi (2007) are notable contributions. They show the possibility of Pareto-improving (untied) aid in a two-country dynamic model. While the growth rate is exogenous in Shimomura (2007), it is endogenous in Naito and Ohdoi (2007).

While environmental and dynamic aspects are important in discussing foreign aid in the modern world, there is no work that incorporates both. This paper makes a small step toward this direction. Constructing a dynamic two-country model of international trade with transboundary pollution, this paper revisits the welfare effects of aid.

We deviate from Shimomura (2007) and Naito and Ohdoi (2007) in two respects. First, we assume away the terms of trade effect by presuming that the international commodity prices are constant over time. It is well-known that the terms of trade effect plays a central role in the argument of foreign aid. However, it drastically makes the analysis quite intractable. In order to overcome this analytical difficulty, we suppose two small open countries.1) No terms of trade effect enables us to restrict attention to how foreign aid affects welfare through accumulation of capital and pollution. Second, our model is too specified as compared to Naito (2003) and Shimomura (2007). In this sense, we recognize that the scope of our analysis should not be overestimated, but our purpose in this paper is not to pursue the generality of results but to derive definite results in a simple framework.

This paper proceeds as follows. Section 2 presents a model. Section 3

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1) We never insist that the terms of trade is unimportant. The strategic use of the assumption of small countries is made in Yano and Nugent (1999), Hatzipanayotou et al. (2002), Chatterjee et al. (2003), and Chatterjee and Turnovsky (2004, 2007) as well.
makes comparative static and dynamic analyses. Based on them, Section 4 considers welfare effects of aid. Section 5 concludes the paper.

2 A model

Consider two small open countries, Home and Foreign, which consume two consumption goods, Goods 1 (non-numeraire) and 2 (numeraire). Let \( p \) denote the relative price of Good 1 and be constant over time since the two countries are too small to affect the world price. Home is a donor and Foreign a recipient. An asterisk (*) is attached to the Foreign variables.

2.1 Home’s fundamentals

Home’s preference and technology are specified. The production side is captured by a GDP function. We assume a Heckscher-Ohlin technology, which makes the GDP function take a form of

\[
\text{Eq. (1)}
\]

where \( K \) and \( L \) are the endowment of capital and labor, and \( r \) and \( w \) are the reward of capital and labor, respectively. Eq. (1) is obtained by assuming that both goods are positively produced from capital and labor.\(^2\)

An important property of the GDP function is that the partial derivative of the GDP function with respect to \( p \) gives the equilibrium supply of Good 1.

A pollutant is emitted proportionally to the output of Good 1. Given the assumptions made so far, Home’s representative consumer solves a dynamic utility maximization problem:

\[\text{Letting } K_i/L_i, \ i = 1, 2 \text{ denote the factor intensity of Good } i, \text{ incomplete specialization is justified under } \max\{K_1/L_1, K_2/L_2\} > K/L > \min\{K_1/L_1, K_2/L_2\}. \]

Under this condition, the two factor prices are determined solely by the commodity prices as \( r(p) \) and \( w(p) \).
\[
\max_{C_1, C_2} \int_0^\infty e^{-\rho t}[\alpha \ln C_1 + (1 - \alpha) \ln C_2 - sZ]dt, \quad s, \rho > 0
\]
subject to \[\dot{K} = r(p)K + w(p)L - pC_1 - C_2 - \delta K - T, \quad \delta \in (0, 1)\]
\[\dot{Z} = r'(p)K + w'(p)L - \gamma Z, \quad \gamma \in (0, 1),\]

where \(C_i, i = 1, 2\) is consumption of Good \(i\), \(Z\) the pollution stock, \(T\) the income transfer, \(\rho\) the discount rate, \(s\) the marginal damage from pollution, and \(\delta\) and \(\gamma\) are respectively the rate of depreciation of capital and natural purification of pollution. This problem is solved with the aid of a Hamiltonian:

\[
H = \alpha \ln C_1 + (1 - \alpha) \ln C_2 - sZ + \lambda [r(p)K + w(p)L - pC_1 - C_2 - \delta K - T] + \mu [r'(p)K + w'(p)L - \gamma Z],
\]

where \(\lambda\) and \(\mu\) are the costate variable associated with two constraints. Then, the first-order necessary conditions for optimality are

\[
\frac{\alpha}{C_1} = \lambda p \quad \text{(2)}
\]
\[
\frac{1 - \alpha}{C_2} = \lambda \quad \text{(3)}
\]
\[
\dot{\lambda} = \lambda(\rho + \delta - r) - \mu r' \quad \text{(4)}
\]
\[
\dot{\mu} = \mu(\rho + \gamma) + s \quad \text{(5)}
\]
\[
\dot{K} = rK + wL - \frac{1}{\lambda} - \delta K - T \quad \text{(6)}
\]
\[
\dot{Z} = r'K + w'L - \gamma Z \quad \text{(7)}
\]

where \(\dot{K}\) is obtained by substituting (2) and (3) into the flow budget constraint. The present dynamic general equilibrium system consists of (4)-(7) whose endogenous variables are \(\lambda, \mu, K\) and \(Z\). However, we can reduce them into a two-dimensional system.

Note that the time path of \(\mu\) is solely determined by (5). Eq. (5) implies that \(\mu(t) = -s/(\rho + \gamma)\) for any \(t \geq 0\) if the initial value of \(\mu\) is chosen as
\[ \mu(0) = -s/\rho + \gamma. \] Then, substituting \( \mu = -s/(\rho + \gamma) \) into (4) and making the same argument as right above, \( \lambda \) also becomes time-invariant so that \( \lambda(t) = -sr'/\rho + \gamma)(\rho + \delta - r) \) for any \( t \geq 0 \). Accordingly, all we have to do is to consider (6) and (7).

### 2.2 Foreign’s fundamentals

The Foreign economy is described. Pollution emitted in Home is transboundary and \( \beta Z \) arrives in Foreign where \( \beta \in [0, 1] \). As in Shimomura (2007) and Doi, Iwasa and Shimomura (2007), Foreign is assumed to specialize to Good 2 by using labor only. Hence, letting the labor/output coefficient of Good 2 in Foreign be denoted by \( a_2 \), the Foreign consumer’s problem is formulated as follows.

\[
\max_{C_1, C_2} \int_0^{\infty} e^{-\rho t} [u^*(C_1^*, C_2^*) - \beta Z], \quad \rho^*, \beta > 0
\]

subject to

\[
\dot{A}^* = r^* A^* + \frac{L^*}{a_2} - pC_1^* - C_2^* + T,
\]

where \( A^* \) is the net credit in Foreign, and \( r^* \) the interest rate. Note here that \( L^*/a_2 \) is the income from production of Good 2. The associated Hamiltonian is

\[
H^* = u^*(C_1^*, C_2^*) - \beta^* Z + \lambda^* \left( r^* A^* + \frac{L^*}{a_2} - pC_1^* - C_2^* + T \right),
\]

where \( \lambda^* \) is the costate variable. The maximum principle yields

\[
\frac{\partial u^*(C_1^*, C_2^*)}{\partial C_1^*} \equiv u^*_1(C_1^*, C_2^*) = \lambda^* p
\]

\[
\frac{\partial u^*(C_1^*, C_2^*)}{\partial C_2^*} \equiv u^*_2(C_1^*, C_2^*) = \lambda^* \]

\[
\dot{\lambda}^* = \lambda^* (\rho^* - r^*)
\]

\[
\dot{A}^* = r^* A^* + \frac{L^*}{a_2} - pC_1^* - C_2^* + T
\]

\[
0 = \lim_{t \to \infty} e^{-\rho t} \lambda^* A^*.
\]

3) For a more detailed argument, see Kemp, Long and Shimomura (2001).
Based on the standard assumption in trade theory such that there is no international financial market, the credit market in Foreign must be cleared *domestically*, which means that $A^*(t) = 0$ for any $t \geq 0$. In view of that, it follows that $\lambda(t) = 0$ for any $t \geq 0$. Consequently, the demand function of each good is obtained by the following ‘static’ optimality conditions:

$$
\frac{u_1(C_1^*, C_2^*)}{u_2(C_1^*, C_2^*)} = p \\
pC_1^* + C_2^* = \frac{L^*}{a_2^*} + T.
$$

The resulting demand functions are a function of $p$ and income $L^*/a_2^* + T$. Substituting them into the utility function, Foreign’s welfare is measured by

$$
W^* \equiv \int_0^\infty e^{-\rho^* t} \left[ V^* \left( p, \frac{L^*}{a_2^*} + T \right) - v^* (\beta Z) \right] dt,
$$

where $V^*(\cdot)$ is a standard indirect utility function.

3 **Comparative statics and dynamics**

While our model is quite specified, the specificity allows us to make comparative static and dynamic analyses easily. As already mentioned, the core system consists of (6) and (7). We apply the comparative dynamics technique initiated by Oniki (1973) to them. Differentiating (6) and (7) with respect to $T$, we have a system of variational equations:

$$
\dot{K}_T = (r - \delta) K_T - 1 \\
\dot{Z}_T = r' K_T - \gamma Z_T,
$$

where $K_T \equiv dK/dT$ and $Z_T \equiv dZ/dT$. If the economy is initially in the steady state, the coefficients in (13) and (14) are all time-invariant and hence they are a system of ordinary differential equations with constant coefficients. Resorting to the standard technique for solving differential equations, the general solutions to (13) and (14) become
where $A_1$ and $A_2$ are arbitrary constants which correspond to the two eigenvalues $-\gamma$ and $r - \delta$, respectively. $K_T$ and $Z_T$ capture the effect of $T$ on the steady state value of $K$ and $Z$. In other words, they are obtained by differentiating the system of equations with $\dot{K} = \dot{Z} = 0$ and explicitly represented by

\begin{align*}
K_T(t) &= \frac{r - \delta + \gamma}{r'} A_2 e^{(r - \delta)t} + \overline{K_T} \\
Z_T(t) &= A_1 e^{-\gamma t} + A_2 e^{(r - \delta)t} + \overline{Z_T},
\end{align*}

(15) (16)

As is clear from (15) and (16), we make the following assumption on stability.

**Assumption 1.** $r - \delta < 0.$

Moreover, noting that $\lambda = -sr'/(\rho + \delta - r)(\rho + \gamma)$ is a marginal utility of $K$, which must be positive from an economic viewpoint, we need:

**Assumption 2.** *Good 1 is labor-intensive,* i.e., $r' < 0.$

It follows from Assumptions 1 and 2 that $\overline{K_T} < 0$ and $\overline{Z_T} > 0$, namely, foreign aid reduces the steady state stock of capital and increases the steady state pollution. The mechanism behind this observation comes from the well-known Rybczynsky theorem. A reduction in capital decreases the supply of the capital-intensive good (*Good 2*) and increases that of the labor-intensive good (*Good 1*). This, in turn, enhances pollution since only the production of *Good 1* generates pollutants.

The two constants $A_1$ and $A_2$ are determined by the initial conditions.
Noting that $K_T(0) = Z_T(0) = 0$ because the initial value of both is historically given, and letting $t \to 0$ in (15) and (16) yield

$$K_T(0) = 0 = \frac{r - \delta + \gamma}{r'} A_2 + \frac{1}{r - \delta}$$

$$Z_T(0) = 0 = A_1 + A_2 + \frac{r'}{\gamma(r - \delta)},$$

by substituting (17) and (18) into (15) and (16). Solving for $A_1$ and $A_2$, we have

$$A_1 = -\frac{r'}{\gamma(r - \delta + \gamma)}$$

$$A_2 = -\frac{r'}{(r - \delta + \gamma)(r - \delta)}.$$

Accordingly, the pair of the solutions to (13) and (14) is derived as

$$K_T(t) = -\frac{1}{r - \delta} e^{(r - \delta) t} + K_T$$

$$= \frac{1}{r - \delta} \left[ 1 - e^{(r - \delta) t} \right] > 0$$

(19)

$$Z_T(t) = -\frac{r'}{\gamma(r - \delta + \gamma)} e^{-\gamma t} - \frac{r'}{\gamma(r - \delta + \gamma)} e^{(r - \delta) t} + Z_T.$$ (20)

Summarizing the results obtained, we establish:

**Proposition 1.** Under Assumptions 1 and 2, foreign aid reduces both the steady state stock and the time path of capital. On the other hand, the steady state stock of pollution increases, whereas the effect on time path of pollution is ambiguous.

Proposition 1 is of some interest according to which an acceptance of foreign aid necessarily reduces the capital not only in the steady state but also in the transition. In other words, the effect on the steady state stock and the time path has the same direction. In contrast, the effect on pollution can fluctuate between the steady state stock and the transition path. As (18) shows, the steady state pollution increases as a result of aid. However, such an incremental effect is not always the case in the transition.
(see (20)). This implies that the effect on the steady state welfare can be different from the effect on the lifetime utility.

4 Welfare effects of foreign aid

Having identified the impacts of foreign aid on the steady state stock and the dynamic path of capital and pollution, we readily examine the welfare effects. We will consider Home’s welfare first and then proceed to Foreign’s welfare.

4.1 Home’s welfare

In order to address the welfare effect on Home’s welfare, we need to express welfare as a function of $T$. Note that Home’s instantaneous welfare is

$$U \equiv \alpha \ln C_1(\lambda p) + (1 - \alpha) \ln C_2(\lambda p) - sZ(T),$$

where $C_1$ and $C_2$ are the solution to (2) and (3), both of which are independent from $T$. Therefore, the effect of $T$ becomes

$$\frac{dU}{dT} = -s \frac{dZ}{dT} = -\frac{sr'}{\gamma(r - \delta)} < 0,$$

from Assumptions 1 and 2 and (18). This is a quite natural result because aid increases the steady state pollution stock, which is harmful.

While the above finding is restricted to the steady state, what effect on the lifetime welfare follows? To answer this question, let us define

$$W \equiv \int_0^\infty e^{-\rho t} [\alpha \ln C_1(\lambda p) + (1 - \alpha) \ln C_2(\lambda) - sZ(T)]dt.$$

Then, if we assume that the economy is initially in the steady state, the effect of $T$ on $W$ is calculated as follows.
Thus, while the effect on the dynamic path of $Z$ is ambiguous, the effect on Home’s welfare is the same between the steady state and the transition. This is formally stated in:

**Proposition 2.** Foreign aid lowers not only the steady state level but the lifetime level of Home’s welfare.

### 4.2 Foreign’s welfare

Turn to the effect on Foreign’s welfare. In view of that (12) defines the lifetime welfare, the steady state welfare is defined by

$$U^* = V^* \left( p, \frac{L^*}{a_2^*} + T \right) - v^*(\beta Z).$$

Hence, differentiating with respect to $T$ yields

$$\frac{dU^*}{dT} = V_I^* - \frac{\beta v^* r'}{\gamma (r - \delta)} dZ,$$

where $V_I^*(\cdot)$ denotes the partial derivative of $V^*(\cdot)$ with respect to $L^*/a_2^* + T$ and the second equation follows from (18). While the first term in (21) captures the direct income effect of transfer and is positive, the second is the indirect effect through transboundary pollution and is negative. Consequently, the total effect is ambiguous and it is trivial that Foreign gains from aid if and only if the former effect dominates the latter. We will give
a simple example to ensure such a welfare improvement for Foreign.

Before presenting an example which leads to an improvement in Foreign’s welfare, let us address the effect of aid on the lifetime welfare. The discounted sum of utility is given in (12). Making use of it, the impact of aid on \( W^* \) is

\[
\frac{dW^*}{dT}
\bigg|_{T=0} = \int_0^\infty e^{-\rho^* t} \left[ V^*_I - \beta v^* Z_T \right] dt
\]

\[
= \int_0^\infty e^{-\rho^* t} \left\{ V^*_I - \beta v^* \left[ -\frac{r'}{\gamma(r-\delta+\gamma)} e^{-\gamma t} \right. \right.
\]

\[
- \frac{r'}{(r-\delta)(r-\delta+\gamma)} e^{(r-\delta t) + Z_T} \left. \right] \right\} dt
\]

\[
= \int_0^\infty e^{-\rho^* t} \left\{ V^*_I - \beta v^* \left[ -\frac{r'}{\gamma(r-\delta+\gamma)} e^{-\gamma t} \right. \right.
\]

\[
- \frac{r'}{(r-\delta)(r-\delta+\gamma)} e^{(r-\delta t) + \frac{r'}{\gamma(r-\delta)}} \left. \right] \right\} dt
\]

\[
= \frac{V^*_I}{\rho^*} - \frac{\beta v^* r'}{\rho^*(\rho^* + \gamma)(r-\delta-\rho^*)}.
\]

(22)

As in the case of the steady state welfare, the first term in (22) gives a direct income effect, whereas the second the losses from pollution magnified by aid.

Eqs. (21) and (22) convince us that Foreign gains from aid if and only if the direct income effect overweighs the losses from pollution. Then, one may naturally ask whether there is an example which ensures Foreign’s gain. The rest of this section presents a possibly simplest example. Suppose that Foreign’s felicity function is quasi-linear:

\[
u^*(C^*_1, C^*_2) - v^*(\beta Z) = \tilde{u}(C^*_1) + a^* C^*_2 - s^* \beta Z,
\]

\[\tilde{u}'(\cdot) > 0, \quad \tilde{u}''(\cdot) < 0, \quad a^*, s^* > 0.\]

Then, the conditions for welfare improvements are obtained as follows.

steady state welfare : \( \frac{dU^*}{dT} > 0 \iff a^* > \frac{s^* \beta r'}{\gamma(r-\delta)} \) (23)

lifetime welfare : \( \frac{dW^*}{dT} > 0 \iff a^* > \frac{s^* \beta r'}{(\rho^* + \gamma)(r-\delta-\rho^*)} \) (24)
Therefore, all we can state is:

**Proposition 3.** The effects on Foreign’s welfare is ambiguous. However, supposing a quasi-linear preference and the marginal utility of income is large enough to satisfy (23) and (24), foreign aid is beneficial to Foreign both in the steady state and in the transition.

While Proposition 3 concerns the sufficient condition for aid to be gainful to the recipient in the steady state and transition, we have another possibility. Under Assumptions 1 and 2, one can verify that

\[
\frac{s^* \beta r'}{\gamma (r - \delta)} > \frac{s^* \beta r'}{(\rho^* + \gamma)(r - \delta - \rho^*)} > 0,
\]

and hence the following result can be established.

**Corollary.** Suppose a quasi-linear preference of Foreign. Then, an acceptance of aid reduces Foreign’s steady state welfare, but raises its time path of welfare under

\[
\frac{s^* \beta r'}{\gamma (r - \delta)} > a^* > \frac{s^* \beta r'}{(\rho^* + \gamma)(r - \delta - \rho^*)}.
\]  

(25)

The above corollary is of some use because it makes clear that the effect on steady states can not necessarily carry over to the effect on time paths. If we look at reality, the direct effect of transfer dominates in the short-run. In this sense, aid should be accepted. However, it gradually increases world pollution, which is likely to dominate in the long-run. Therefore, the Foreign government may have an incentive to refuse the aid from a long-run point of view.
5 Concluding remarks

Making use of a simple dynamic model of international trade with environmental externalities, we have analytically examined welfare effects of foreign aid. Under plausible assumptions, we have shown that (i) aid raises both the steady state level and the time path of pollution, that (ii) the donor necessarily loses from giving aid, and that (iii) the welfare effect on the recipient is ambiguous. Result (iii) follows since the direct income effect is positive but the indirect effect through pollution expansion is negative. However, we have provided a simple example with which the recipient gains from aid.

Unlike the welfare effects on the donor, there emerges an interesting contrast on the welfare effects on the recipient. Depending on the level of marginal utility of income, it proves possible that the steady state welfare decreases but the time path of welfare increases due to aid. This observation captures an importance of dynamic considerations since the comparative static results focusing on the steady state can not necessarily apply to the change in time path.

However, despite the above virtues, we should recognize limitations in this paper. As mentioned in Introduction, we have ignored the terms of trade effect and made numerous specifications and assumptions. In discussing the welfare effects of foreign aid more properly, it is needed to relax these restrictions. They are left as our future work.
References


