Wirtschaftswissenschaftliche Fakultät der Eberhard-Karls-Universität Tübingen

An Overreaction Implementation of the Coherent Market Hypothesis and Option Pricing

Rainer Schöbel Jochen Veith

Tübinger Diskussionsbeitrag Nr. 306 April 2006

Wirtschaftswissenschaftliches Seminar Mohlstraße 36, D-72074 Tübingen

An Overreaction Implementation of the Coherent Market Hypothesis and Option Pricing

Rainer Schöbel and Jochen Veith*

April 2006

Abstract

Inspired by the theory of social imitation (Weidlich 1970) and its adaptation to financial markets by the Coherent Market Hypothesis (Vaga 1990), we present a behavioral model of stock prices that supports the overreaction hypothesis. Using our dynamic stock price model, we develop a two factor general equilibrium model for pricing derivative securities. The two factors of our model are the stock price and a market polarization variable which determines the level of overreaction. We consider three kinds of market scenarios: Risk-neutral investors, representative Bernoulli investors and myopic Bernoulli investors. In case of the latter two, risk premia provide that herding as well as contrarian investor behaviour may be rationally explained and justified in equilibrium. Applying Monte Carlo methods, we examine the pricing of European call options. We show that option prices depend significantly on the level of overreaction, regardless of prevailing risk preferences: Downward overreaction leads to high option prices and upward overreaction results in low option prices.

JEL Classification: G12, G13

Keywords: behavioral finance, coherent market hypothesis, market polarization, option pricing, overreaction, chaotic market, repelling market

^{*}Department of Corporate Finance, Faculty of Economics and Business Administration, Eberhard Karls University Tübingen, Mohlstrasse 36, 72074 Tübingen, Germany. Phone: +49-7071-2977088 or -2978206. E-Mail: rainer.schoebel@uni-tuebingen.de or jochen.veith@uni-tuebingen.de

1 Introduction

Since more than thirty years, the widely accepted hypothesis about the behavior of asset prices in perfect markets is the Efficient Market Hypothesis (EMH) or, in its continuous-time form, the "geometric Brownian motion" hypothesis. This implies that asset prices are log-normally distributed. However, in the last twenty years, new studies of security prices have reversed some of the earlier evidence favoring the EMH. Behavioral finance has emerged as an alternative view of the financial markets suggesting, that investing in speculative assets is a social activity. It thus seems plausible that investors' behavior and hence prices of speculative assets would be influenced by social movements. A historically important study in this context is Shiller's (1981) work on stock market volatility. Shiller found, that even though the price of the aggregate stock market is importantly linked to its dividends, stock prices seem to show far too much volatility to be in accordance with a simple dividend discount model. He offered the following explanation: "This behavior of stock prices may be consistent with some psycological models. Psychologists have shown in experiments that individuals may continually overreact to superficially plausible evidence even when there is no statistical basis for their reaction". Other studies by DeBondt and Thaler (1985, 1987), Campbell and Shiller (1988) or Poterba and Summers (1988) provided some evidence that stock returns are mean reverting. De Bondt and Thaler also propose an overreaction hypothesis, whereby financial markets weigh recent information more heavily than prior information. Markets eventually realize that they formed biased expectations and prices revert back to their fundamental value.

In this paper, we follow the intensions of Shiller and DeBondt et al. and suggest a two-factor stock price model that supports the overreaction hypothesis whereby overreaction is due to herd behavior. The first factor of our model may be regarded as the present value of future dividends as a proxy of the fundamental value of the stock market. The second one is of psycological nature describing the behavior of investors and how they esteem the fundamental value in their current psycological environment. To model the behavior of investors, we use a theory of social imitation originally developed by Weidlich (1970). The theory was adopted to the financial markets by Vaga (1990) called the coherent market hypothesis (CMH). The CMH is a nonlinear statistical model using terms such as order parameters to describe the behavior of investors at a macroscopic level. Introducing two key parameters, the level of crowd behavior and the prevailing economic fundamentals, the model allows for a variety of associated market states ranging from efficient over coherent to chaotic. However, Vaga assumed a direct proportional relationship between market polarization and stock returns. The distributions of the polarization are therefore simply viewed as steady state return distributions, which is in contradiction with empirical evidence.

In this paper, we take a step further and formulate a continuous-time version of the CMH. This opens the door to apply dynamical financial theory, especially option pricing. Starting from the Fokker Planck equation describing investor sentiment in terms of polarization levels, we are able to extract and analyze the underlying stochastic process. We also find that the process exhibits a special kind of mean reversion, which can be used to model overreaction in stock prices when superimposed on a geometric Brownian motion. The resulting return distributions offer the same risk-return characteristics associated to the various states of investor sentiment as proposed by Vaga, but overcome the deficiency of a steady state perspective. Based on our stock price process, we develop a general equilibrium model for valuing derivative securities using the CIR (1985a) framework and apply it to the valuation of European call options.

The remaining part of the paper is organized as follows: In the second Section, we present the main ideas and properties of Weidlichs's theory of social imitation and Vaga's adoption to financial markets, the coherent market hypothesis. In section 3, we introduce a continuous-time version of the CMH and formulate a stock price process in accordance with the overreaction hypothesis. Furthermore, we take this stock price process and develop three different equilibrium models for valuing derivative securities. Section 4 illustrates the characteristics of these models and reports some results of a comparative static analysis. In section 5 we summarize and provide some concluding remarks.

2 The Coherent Market Hypothesis

2.1 A Theory of Social Imitation

The starting point of our work is a theory of social imitation, originally provided by W. Weidlich (1970). The general idea of Weidlich was, that there is a structural similarity between a physical ensemble of interacting individual systems in thermal equilibrium and the interactions in a society of human individuals. In particular, Weidlich extends the well known Ising model (1925) of ferromagnetism to the phenomenon of polarization of opinion in social groups.

Two important characteristics determine the behavior of an Ising system: The interactions (e.g. attracting forces) between fluctuating individual subsystems and a collective parameter determining the degree of the fluctuations. As a consequence, the ensemble as a whole behaves macroscopically in a complete different way above and below a certain critical phase transition parameter.

To transfer this basic approach to human behavior, Weidlich applies the following framework: There is a group of n individuals who may influence each other with respect to their decisions and each individual can decide between two attitudes: Up (+) or down (-). The probability of finding, at time t, n_+ individuals with attitude +, and n_- with attitude - is $f[n_+, n_-; t]$ and the transition probabilities per unit time for an individual of changing from attitude + to - or vice versa are $p_{+-}(n_+, n_-)$ and $p_{-+}(n_+, n_-)$, respectively. Then, the rate of change of the probability density function $f[n_+, n_-; t]$ can be expressed by a master equation of the form¹

$$\frac{\Delta f(n_{+},n_{-};t)}{\Delta t} = (n_{-}+1) p_{-+}(n_{+}-1,n_{-}+1) f[n_{+}-1,n_{-}+1;t] + (n_{+}+1) p_{+-}(n_{+}+1,n_{-}-1) f[n_{+}+1,n_{-}-1;t] (1) - n_{+} p_{+-}(n_{+},n_{-}) f[n_{+},n_{-};t] - n_{-} p_{-+}(n_{+},n_{-}) f[n_{+},n_{-};t].$$

which simply sums up all probability currents to and from the considered state (n_+, n_-) within the short time interval Δt . To describe the market's excess opinion (positive or negative), we introduce a new state variable which we call the market polarization q defined as

$$q = (n_{+} - n_{-})/2n; \ q \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$
 (2)

and normalised to 2n.

After some algebraic manipulation, equation (1) can be expressed in terms of q and a Taylor expansion up to the second order finally yields the Fokker Planck equation²

$$f_t(q,t) = -\frac{\partial}{\partial q} \left[K(q) f(q,t) \right] + \frac{1}{2} \frac{\partial^2}{\partial q^2} \left[Q(q) f(q,t) \right].$$
(3)

Note, that the drift K(q) and the diffusion coefficient Q(q) still depend on the yet unspecified transition probabilities p_{+-} and p_{-+} . The stationary solution of equation (3) can be found by standard methods with the result

$$f_{st}(q) = \frac{c}{Q(q)} \exp\left[2\int_{-\frac{1}{2}}^{q} \frac{K(q)}{Q(q)}dq\right]$$
(4)

where c is a normalization constant.

The general time-dependent solution of (3) describes the evolution of the probability density from its initial condition (e.g. Dirac's delta function at t = 0) to the stationary distribution. Since there exists no closed expression, a series expansion of the form

$$f(q;t) = f_{st}(q) + \sum_{i=1}^{\infty} c_i \exp\left[-\lambda_i(t-t_0)\right] \phi_i(x)$$
(5)

may be applied. Here, the λ_i form a denumerable sequence of eigenvalues and $\phi_i(x)$ are an orthonormal system of eigenfunctions to the Fokker Planck operator $-\frac{1}{2}\frac{\partial^2}{\partial q^2}Q(q) + \frac{\partial}{\partial q}K(q)$.

In order to further evaluate $f_{st}(q)$ we need explicit assumptions about the transition probabilities $p_{+-}(q)$ and $p_{-+}(q)$. At this point, Weidlich employes the intuitively

¹For an illustration see figure 8 in appendix A.1.

²For a detailed derivation see appendix A.1.

appealing analogy between the behavior of individuals in a social group and the elementary magnets within a ferromagnet. As in the Ising model, he assumes that the individuals are subject to two forces: An internal field created by the individuals themselves and proportional to the excess opinion of the group and an external field independent of the behavior of the individuals. The transition probabilities then depend exponentially on these force fields and adapted to our case, they are³

$$p_{+-}(q) = \alpha \exp[-(kq+h)]$$

$$p_{-+}(q) = \alpha \exp[+(kq+h)].$$
(6)

The adaption parameter k describes the willingness of the individuals to align to the group and preference parameter h measures the influence of the external force. Applying these transition probabilities, explicit expressions for the drift K(q) and diffusion coefficient Q(q) can be derived as

$$K(q) = \alpha \left[\sinh(kq+h) - 2q\cosh(kq+h)\right]$$

$$Q(q) = \frac{\alpha}{n} \left[\cosh(kq+h) - 2q\sinh(kq+h)\right].$$
(7)

Substituting (7) into (4), the solution for the stationary case can be evaluated easily using numerical methods. The resulting distributions are shown in figure 1. There are two regimes of behavior, ordered and disordered. For low adaption of the individuals to the group $k \ll k_{crit}$, we find almost balanced states (q around zero) with a centered distribution. If the adaption parameter k approaches its critical value, a phase transition to more ordered states occures and the distribution undergoes a bifurcation. For an increasing preference parameter h the distributions are distorted additionally whereby the response to a change in h is largest near the transition.

[Insert figure 1]

2.2 Five Market States

The coherent market hypothesis was suggested by Vaga (1990) in contrast to the efficient market hypothesis. Vaga carried over Weidlich's concepts to the financial markets. He recognized, that the theory was able to drop the premise of rational investors and therefore relaxes the assumption of approximately normally distributed stock returns. Vaga re-interpreted the control parameters k and h and proposed to use k as a measure of the level of crowd behavior or market sentiment and h as a measure of the prevailing economic fundamentals. Each set of parameters (k, h) corresponds to a different market state. He assumed a direct proportional relationship between market polarization and stock returns. The distributions of the polarization derived by Weidlich are therefore simply viewed as return distributions.

According to Vaga, there are four types of market states which can be described by the model under reasonable assumptions about the control parameters:

³For a detailed explanation see appendix A.2.

- 1. Efficient market $(0 \le k \ll k_{crit}, h = 0)$: This market state can be obtained in choosing neutral fundamentals and market sentiment well below the critical threshold. In the limiting case k = 0, the return distribution is similar to the normal distribution (fig. 2.2 a).
- 2. Coherent market $(k \approx k_{crit}, h \ll 0 \text{ or } h \gg 0)$: If we presume crowd behavior in conjunction with strong bullish or bearish fundamentals, the model allows to create coherent market states. As we have seen in the last section, the return distribution will be skewed to the left or to the right. The expected return is different from zero, accompanied with an unusual small standard deviation. In this state, the traditional risk-return tradeoff is inverted and investors can earn above-average returns with below-average risk (fig. 2.4 a).
- 3. Chaotic market $(k \approx k_{crit}, h \leq 0 \text{ or } h \geq 0)$: This market state can be modeled in supposing crowd behavior with only slightly bearish or bullish fundamentals which leads to a bimodal return distribution. This is the worst of all worlds with low return for above-average risk (fig. 2.5 a and fig. 2.6 a).
- 4. Unstable transitions: These are all intermediate market states that cannot be assigned to one of the former states (fig. 2.3 a).

We would like to extend this list of market states by the following case:

5. Repelling market (k < 0, h = 0): In this market state, we would observe just the opposite of crowd behavior, namely a repelling behavior among investors. Any investor would try to avoid to have the same opinion than the majority. As a result, the return distribution becomes concentrated around q = 0 and eventually a Dirac's delta function as k approaches minus infinity.

(fig. 2.1 a).

All things considered, the CMH offers a framework to evaluate the risk-reward potential for a given market. However, this is only achieved in a qualitative sense. Our main point of criticism is the rigorous re-interpretation of Weidlichs polarization distributions as return distributions. It is obvious, that Vaga's distributions f(q, t)are unable to fulfill the most basic demands on financial return distributions since they end up in a steady state.

3 The Continuous-Time CMH

3.1 Dynamics of Market Polarization

The Fokker Planck equation (3) describes a diffusion process for the polarization q(t) with diffusion coefficients K(q) and Q(q). There are two basically different approaches to the class of diffusion processes. On the one hand, one can define them in terms of the conditions on the transition probabilities f(q, t), which is what we did in the previous sections. On the other hand, one can study the variable q(t) itself and its variation with respect to time. This leads to a stochastic differential equation for q. In fact, under the usual regularity conditions, it can be shown that there exists a Wiener process w(t) such that the polarization q(t) follows the stochastic differential equation

$$dq = K(q) dt + \sqrt{Q(q)} dw(t).$$
(8)

To gain further insight into the behavior of the stochastic process dq, we have to analyze the drift K(q), the diffusion coefficient Q(q) and the associated sample paths in more detail. In figure 2 we plot the graphs of the functions K(q) and Q(q)for different market states together with the associated distributions f(q,t). The drift exhibits kind of mean reversion towards the stable nodes which in turn leads to the maxima of the distributions f(q,t). The diffusion coefficient has its maximum at q = 0 and diminishes for growing |q|. This assures that the distribution is bounded on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Figure 3 shows some sample paths, which are stationary. As is apparent from figure 3 b), the sample path for the "efficient" market state (k = 0, h = 0) has nothing in common with a random walk (e.g. no growing variance over time), except that it has the same type of distribution at a given point in time.

[Insert figure 2]
[Insert figure 3]

3.2 Stock Price Dynamics

In this section, we formulate a stock price process in accordance with the overreaction hypothesis using the properties of the coherent market hypothesis. Two empirical discoveries are the guidelines of our stock price model: The observed variation in expected discounted dividends is too low to justify the volatility in stock price movements and stock prices tend to mean revert. To address these characteristics, we assume that stock price changes are driven by two factors: The first one describes the fundamental value D of the company or market index and can be regarded as the value of all future dividends discounted to the present. The second factor is the market polarization q, which we use to describe how investors esteem the fundamental value in their current psycological environment. Since the fundamental value is not a stationary target, we model its dynamics by a geometric Brownian motion

$$dD = \mu_D D \, dt + \sigma_D D \, dw_2 \tag{9}$$

a commonly used assumption in asset pricing. The return of the asset is then modeled by superimposing the process dq in the following way⁴:

$$d(\ln S) = d(\ln D) + \kappa dq \tag{10}$$

Here, S denotes the market price of the asset and κ describes the impact of the subjective perception of the investors. From Itô's lemma, we derive the joint Markov process for polarization q and stock price S as follows:

$$dq = K(q) dt + \sqrt{Q(q)} dw_1$$
$$dS = \left[\mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q)\right] S dt + \kappa \sqrt{Q(q)} S dw_1 + \sigma_D S dw_2. \tag{11}$$

As can be seen in figure 4 (solid lines), the return distributions assigned to the process dS retain the characteristics of the density distributions of the process dq. Furthermore, dS shows the desired growing variance over time and therefore overcomes the deficiencies of Vaga's return distributions without loosing its main attractions. We can see this from the contour plots in figure 5 which illustrate the evolution of the return distributions over a time period of two years.

]	Insert figure 4]
[Insert figure 5]

3.3 Pricing Derivative Securities

In the following, we develop several equilibrium models for valuing derivative securities based on our stock price process. We consider an economy with a fixed number of identical investors which receive only capital income and can choose between immediate consumption or investment. The investors trade in perfectly competitive, continuous markets with no transaction costs. Trading takes place only at

$$S = D \cdot \exp(\kappa q) \tag{10a}$$

⁴In terms of absolute values, equation (10) can be written as

so we can see that a high level of positive coherence leads to high stock valuations and vice versa. A loose interpretation may be that the stock price is most overvalued when everybody is too optimistic about it (q at high levels). Eventually, no more buyers can be found and it is more probable for the stock price to decrease than to increase. One major advantage of this assumption about the stock price is that D drops out of the sde for S.

equilibrium prices and the representative investor is assumed to have time-additive preferences of the form

$$E_t \left[\int_t^\infty U(C(s), s) ds \right].$$
(12)

 E_t is an expectation operator conditional on current wealth W at state (q, S) of the economy and C(s) is the consumption flow at time s. There are three investment assets: A stock which underlies the speculative forces described above, a risk free asset and a derivative security written on the stock price S.

The representative investor's decision problem is equivalent to maximizing (12) subject to the budget constraint

$$dW = \left[\left(1 - \omega_P - \omega_F\right) rWdt + \omega_P W \frac{dS}{S} + \omega_F W \frac{dF}{F} - Cdt$$
(13)

by selecting an optimal level of consumption and fractions ω_P and ω_F of wealth invested in stock and the derivative instrument respectively.

Assuming, that the value of the derivative security will not depend on wealth W, it can be shown, that the price of the derivative security F must satisfy the fundamental valuation equation

$$\frac{1}{2}tr(F_{\underline{x}\underline{x}'}\underline{\sigma}\underline{\sigma}') + F_{\underline{x}}\left[\underline{\mu} - \underline{\sigma}\underline{\lambda}\right] + F_t - rF = 0$$
(14)

with $\underline{x} = \begin{bmatrix} q \\ S \end{bmatrix}$, drift vector $\underline{\mu}(\underline{x}, t) = \begin{bmatrix} K(q) \\ [\mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q)]S \end{bmatrix}$ and correlation matrix $\underline{\sigma}(\underline{x}, t) = \begin{bmatrix} \sqrt{Q(q)} & 0 \\ \kappa \sqrt{Q(q)}S & \sigma_DS \end{bmatrix}$. $\underline{\lambda}$ denotes the market price of risk and

r is the riskless rate of return.

Since the market polarization q is not a traded asset, we cannot use arbitrage arguments to fully eliminate investors' risk preferences from the derivative pricing problem. To close the model, we make additional assumptions regarding risk preferences by considering the following three types of investors:

Case 1 General equilibrium and risk neutrality.

Setting $\underline{\lambda} = 0$, equation (14) does no longer depend on risk preferences explicitly. Thus we can calculate the price of the derivative security by assuming that risk neutrality prevails. The riskless rate of return then simply equals the drift of the stock price process

$$r(q) = \mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q) \tag{15}$$

and the fundamental partial differential equation (PDE) for valuing derivative securities reduces to

$$0 = \frac{1}{2}Q(q)F_{qq} + \frac{1}{2}(\kappa^2 Q(q) + \sigma_D^2)S^2 F_{SS} + K(q)F_q + r(q)SF_S + F_t - r(q)F.$$
(16)

This case corresponds to the general equilibrium case of CIR (1985b) with r derived endogenously. We assume, that investors are endowed with a logarithmic utility function over consumption

$$U(C(t), t) = \exp(-\rho t) \ln C(t) \tag{17}$$

where ρ is the parameter of time preference. Since in macroeconomic equilibrium all investments must be held with a positive fraction in the representative investors portfolio, the market clearing condition is $\omega_P = 1$. That is, the aggregated wealth is always fully invested in stock. The risk free asset as well as the derivative security are both in zero net supply and serve for risk allocation purposes only. Solving the corresponding Bellman equation for this problem leads to the risk free interest rate

$$r(q) = \mu_D + \kappa K(q) - \frac{1}{2}\kappa^2 Q(q) - \sigma_D^2$$
(18)

and the vector of factor risk prices

$$\underline{\lambda}(q) = \begin{bmatrix} \kappa \sqrt{Q(q)} \\ \sigma_D \end{bmatrix}.$$
(19)

The required risk premium $\underline{\lambda\sigma}_{S}(q)$ regarding stock price risk becomes the lower, the larger |q| and the higher the prevailing crowd behavior k (see figure 6a). Thus, optimal guidance forces investors to reduce their risk aversion in times of high coherence and strong crowd behavior. The fundamental PDE is now given by

$$0 = \frac{1}{2}Q(q) F_{qq} + \frac{1}{2}(\kappa^2 Q(q) + \sigma_D^2)S^2 F_{SS} + [K(q) - \kappa Q(q)] F_q + r(q)SF_S + F_t - r(q)F.$$
(20)

Note that in both cases so far, r is endogenous and represents the instantaneously riskless real interest rate in a pure barter economy, which can become negative in a recession or a depression.

Case 3 Partial equilibrium and a myopic Bernoulli investor with log utility.

In this case, there is a small individual investor who seeks to optimize the composition of his entire portfolio. His transactions exert no influence on the formation of prices and he acts as a price taker. As above, this investor has logarithmic utility but the risk free interest rate is exogenously given and elastically supplied:

$$r = const$$
 (21)

Accordingly, the market clearing condition is now given by $\omega_P + \omega_r = 1$. On the macroeconomic level, only the derivative security is in zero net supply ($\omega_F = 0$). Solving the Bellman equation under this clearing condition, we get

$$\underline{\lambda}(q) = \frac{\left(\mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q) - r\right)}{\kappa^2 Q(q) + \sigma_D^2} \begin{bmatrix} \kappa \sqrt{Q(q)} \\ \sigma_D \end{bmatrix}$$
(22)

for the vector of factor risk prices and the fundamental PDE becomes

$$0 = \frac{1}{2}Q(q) F_{qq} + \frac{1}{2}(\kappa^2 Q(q) + \sigma_D^2)S^2 F_{SS} + \left[K(q) - \frac{\kappa Q(q)}{\kappa^2 Q(q) + \sigma_D^2} \left(\mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q) - r\right)\right] F_q \qquad (23) + rSF_S + F_t - rF.$$

We can see from (22) that the vector of factor risk prices (as well as optimal demand ω_P^*) is independent of the instantaneous covariance between realized returns and state variables. Therefore, investors decisions do not take into account the effect of economic state variables on future realized returns and their behavior is described as myopic⁵.

The risk premium $\underline{\lambda}\sigma_S(q)$ is shown in figure 6b. We can see that below the critical threshold k_{crit} , investors require high risk premia at negative polarization levels q, indicating high risk aversion. By contrast, investors would even pay to take risk (negative risk premia), if q is at positive levels. Above the critical threshold k_{crit} , investors get more courageous in pessimistic polarization states by lowering their required risk premia and they become cautious in optimistic ones by raising the risk premia.

[Insert figure 6]

4 Model Analysis

In this section, we solve the various fundamental valuation equations for the case of European call options using Monte Carlo methods.

4.1 Computational considerations

According to Feynman and Kac, the solution of (14) is equivalent to the discounted expectation of the boundary condition under the equivalent martingale measure:

$$F(S,q,t) = e^{-r(T-t)} \widehat{E}_t \left[g(S(T)) \right]$$
(24)

⁵See Feldman (1992). Albeit, we should keep in mind that investors still use this covariance to calculate the risk-adjusted drift through the risk premium $\sigma \lambda$.

Since we want to price European call options, the boundary condition is given by $g = \max(S(T) - X, 0)$. Using equation (24), we are able to derive a solution of (14) doing Monte Carlo simulations. For this purpose, we employ the Euler approximation of the stochastic processes dq and dS:

$$q_{t} = q_{t-1} + \left[K(q_{t-1}) - \lambda_{q}(q_{t-1}) \sqrt{Q(q_{t-1})} \right] \Delta t + \sqrt{Q(q_{t-1})} \Delta w_{1,t}$$
(25)
$$S_{t} = S_{t-1} + r(q_{t-1}) S_{t-1} \Delta t + \kappa \sqrt{Q(q_{t-1})} S_{t-1} \Delta w_{1,t} + \sigma_{D} S_{t-1} \Delta w_{2,t}$$

To improve the efficiency of the simulation procedure, we use the antithetic variates technique. For every pair of random variables $(\Delta w_{1,t}, \Delta w_{2,t})$ the four stock prices $S_{t1}(\Delta w_{1,t}, \Delta w_{2,t})$, $S_{t2}(\Delta w_{1,t}, -\Delta w_{2,t})$, $S_{t3}(-\Delta w_{1,t}, \Delta w_{2,t})$ and $S_{t4}(-\Delta w_{1,t}, -\Delta w_{2,t})$ are calculated to get the antithetic variates estimate

$$F(S_t) = \frac{1}{4} \left\{ F(S_{t1}) + F(S_{t2}) + F(S_{t3}) + F(S_{t4}) \right\}.$$

For our nonlinear problem, it prooved to be efficient to increase the number of sample paths at the expense of time step Δt .⁶ Therefore we use a time step of $\Delta t = 1/252$ corresponding to one trading day and to ran 100'000 sample paths for each option series.

4.2 Sample option prices

For each of the three considered types of investors, sample option prices based on the stock price distributions S(q, t) are presented in tables 1-3. The maturity of the options is six month, but general results appear to be true for all maturities. The tables are divided into panels, labeled A-E, covering a broad range of parameter values including most of the scenarios introduced in chapter 2. Panel A investigates the influence of the degree of crowd behavior k on option prices. Panels B and C contrast option prices in different economic climates h. Panels D and E examine the effect of varying initial overreaction. For each option, three numbers are calculated: A "new" price corresponding to our simulated "true" distribution, a Black-Scholes price equivalent and the Black-Scholes implied volatility associated with the new price. The latter serves as a proxy for the skewness of the implied return distribution. To calculate the Black-Scholes price, we use for all three models the volatility of the simulated terminal distribution. In case of the risk neutral investor and representative Bernoulli investor, we use the risk free rate that is consistent with put-call parity. In case of the myopic Bernoulli investor, we fix r at ln(1.1) = 0.0953. Independent of risk preferences, we set S = 100, $\mu_D = 0.1$, $\sigma_D = 0.1$, $\rho = 0$, $\alpha = 50$.

⁶For example, the standard error of F for an at the money option (myopic Bernoulli investor, efficient market state) using 100'000 sample paths at time step $\Delta t = \frac{1}{252}$ is 0.291%. The standard error of F using only 10'000 sample paths at the finer time step $\Delta t_2 = \frac{1}{10}\Delta t$ is 0.710%. Since the number of calculated increments is the same in both settings, they take the same amount of computing time.

Case 1 General equilibrium and risk neutrality.

Here, our aim is to find out, how options would be priced in the original sense of the coherent market hypothesis. By assuming that risk neutrality prevails, no risk-adjustment is needed and we can simply use the original return distributions to price options. On panel A of table 1, we present option prices based on repelling market, efficient market and chaotic market environments. Our first observation is, that rising crowd behavior leads to higher implied volatilities but also higher interest rates. As a result, option prices are the higher, the higher crowd behavior k. Second, option prices in repelling and efficient markets don't differ much from their Black-Scholes equivalents, while in chaotic markets, deviations are significant. Third, implied volatilities exhibit a strong volatility from for positive k and a slight volatility smile for negative k, indicating the presence of short and fat tails respectively. In figure 7a, we have plotted the differences between the new option price and the Black-Scholes equivalent at different levels of crowd behavior. The more positive k, the more overprices Black-Scholes far-in- and far-out-of-the-money options while at-the-money options are underpriced. For negative k, effects of mean reversion and stochastic volatility are mostly offsetting each other.

[Insert Figure 7]

Panels B and C encompass the values that appear to characterize options in coherent market states, allowing h to range from -0.1 to 0.1 while crowd behavior is either below (k = 1.8) or above (k = 2.1) the critical threshold. As we can see, coherent bear markets are associated with low interest rates and low option prices while coherent bull markets come along with high interest rates and high option prices⁷. In weak coherent markets, deviations from the Black-Scholes equivalent are rather small. In strong coherent bear markets, out of-the-money options are underpriced by Black-Scholes, whereas in strong coherent bull markets, out of-the-money options are overpriced.

The most interesting results are obtained by varying initial overreaction by the choice of polarization q_0 . As indicated in panels D and E, we find, that a downside overreaction (q < 0) exerts an upward influence on option prices while an upside overreaction (q > 0) has the opposite effect. Figure 7b shows the difference between the new option price and the Black-Scholes price in a downward versus an upward overreaction. Upward overreaction leads to positive differences in-the-money and negative differences out-of-the-money, indicating the presence of a left skewed implied return distribution. In a downward overreaction, the reverse is true.

⁷This is not an unusual quality. Our model suggests, that positive fundamentals should come along with high interest rates and vice versa. Thus, our model is in accordance with the commonly observable behaviour of a central bank, which regulates interest rates by switching to expansive fiscal policy in poor economic climates and to restrictive fiscal policy in boom times.

Case 2 General equilibrium and a representative Bernoulli investor with log utility.

The difference between the representative Bernoulli investor and the risk neutral investor lies in the risk adjustment of expected returns. Since the representative Bernoulli's risk premium is always positive, risk adjusted expected returns are lower (see dotted lines in figure 4). This in turn leads to a reduction of all option prices (compare option prices of table 2 with those of table 1). Surprisingly, risk adjustment has no influence on the qualitative pricing features already discussed above (figures 7c and 7d). Implied volatility patterns as well as general pricing behavior regarding variations in k, h and q remain the same as for the risk neutral investor.

Case 3 Partial equilibrium and a myopic Bernoulli investor with log utility

The myopic Bernoulli investor accounts for overreaction by expanding the riskadjusted return distribution beyond the borders of the original return distribution, both, to the left and to the right (see dashed lines in figure 4). Risk preferences provide, that drift uncertainty is fully transferred into volatility uncertainty. The drift of the adjusted stock price process becomes a constant and we have to deal with a purely stochastic volatility model.

As can be seen from panel A of table 3, volatility and hence option prices are decreasing with rising crowd behavior k. Thus, options are priced lower in chaotic markets than in efficient markets. Second, observations regarding implied volatility remain the same as discussed above, except that the volatility smile for negative k is more pronounced while the volatility frown for positive k is reduced. If we have a look at figure 7e, we can see how this translates into differences between the new option price and equivalent Black-Scholes price. Since there is no mean reversion, deviations between the model price and Black-Scholes price extend over a much larger range of strike prices.

Looking at Panels B and C, we find that the influence of economic bias h on option prices is rather small, independent of prevailing crowd behavior. Thus, coherent markets don't matter much in the risk adjusted world of the myopic Bernoulli investor.

Regarding overreaction, we have similar pricing patterns as for the other two types of investors: Downward (upward) overreaction is associated with right (left) skewed return distributions and high (low) option prices (see panels D and E). Here, the intuition behind can be explained isolated from effects due to mean reversion: Consider the case in which q_0 is below zero. Higher q (still negative) is associated with higher volatilities $\sqrt{Q(q)}$ but also with higher stock prices. As q rises, Srises and the probability of large positive changes in S increases. This means, that very high stock prices become more probable. On the other hand, lower qis associated with lower stock price volatilities and stock prices. If q and hence Sfalls, it becomes less likely that large changes take place and terminal stock price will be low. The net effect is, that the terminal return distribution is positively skewed. When q_0 is positive, the reverse is true. Thus, the underlying mechanism is very similar to traditional stochastic volatility models where positive or negative correlation between volatility and stock price leads to right or left skewed return distributions respectively⁸.

5 Conclusion

Motivated by the theory of social imitation originally developed by Weidlich, we present a behavioral model of stock prices that supports the overreaction hypothesis. According to Weidlichs theory, group polarization can be described in a probabilistic manner. Vaga was the first who recognized the theory's usefulness to describe return distributions of financial markets. He re-interpreted Weidlich's control parameters as level of crowd behavior and prevailing economic fundamentals respectively and formulated the so called coherent market hypothesis. This hypothesis offers a rich variety of market states where the relationship between risk and return is solely explained by investor sentiment.

In this paper, we take a step further and formulate the continuous-time version of the CMH. This opens the door to established formalism of financial theory, especially option pricing. Starting from the Fokker Planck equation describing investor sentiment in terms of polarization levels, we are able to extract and analyze the underlying stochastic process. We find that the associated distributions of this diffusion end in a steady state and therefore are unable to represent return distributions. We also find that the process exhibits a special kind of mean reversion, which can be used to model overreaction in stock prices when superimposed on a geometric Brownian motion. The resulting return distributions offer the same risk-return characteristics associated to the various states of investor sentiment as proposed by Vaga, but overcome the deficiency of a steady state perspective.

Using our overreaction stock price model, we develop a two factor general equilibrium model for pricing derivative securities. The two factors of our model are the stock price and market polarization which determines the level of overreaction. We examine three competing hypothesis about investor preferences under which either the market price of risk is zero or derived endogenously and option prices are consistent with the absence of arbitrage: Risk-neutral investor, representative Bernoulli investor and myopic Bernoulli investor. In case of the latter two types of investors, we are able to draw conclusions about their pattern of behavior from their required risk premia. The representative Bernoulli investor's risk premium diminishes as market polarization and hence overreaction increases (in either direction). This implicates that he acts contrarian in pessimistic but contagious in

⁸In traditional stochastic volatility models, skewness is introduced by assuming an instantaneous correlation between the driving Wiener processes: $dw_S dw_{\sigma} = \rho dt$. For an analytical approach, where volatility follows an Ornstein-Uhlenbeck process, see Schöbel and Zhu (1999).

optimistic market environments. Optimal behavior in case of the myopic Bernoulli investor is quite different. Below the critical level of crowd behavior, this investor exhibits strong herding behavior: When market polarization is negative, he wants to be compensated for risk, whereas when it is positive, he even pays for taking risk. By contrast, above the critical threshold he mostly takes contrarian positions by reducing the risk premium at positive polarization levels and increasing it at negative levels. Overall, we could show that herding as well as contrarian investor behavior may be the optimal course of action in response to an apparently irrational stock price process driven by investor sentiment.

Applying Monte Carlo simulations, we examine European call options. Although our three model specifications behave quite multi-faceted regarding the various pricing parameters, they exhibit one common feature: Option prices depend significantly on the level of overreaction. Upward overreaction leads to low option prices and downward overreaction leads to high option prices. The main reason behind this lies in the model's intrinsic assumption about stochastic volatility: Upward overreaction is accompanied by a negative correlation between stock price and volatility, whereas in a downward overreaction, this correlation is positive. As a result, the skewness of the terminal return distribution can be negative or positive, providing the observed pricing biases.

6 Appendices

6.1 Derivation of the Fokker-Planck equation and its stationary solution

Let $p_{+-}(n_+, n_-)$ and $p_{-+}(n_+, n_-)$ be the transition probabilities per unit time for an individual of changing from attitude + to - and vice versa. The temporal change of the probability $f(n_{+}, n_{-}; t)$ to find the group in the state (n_{+}, n_{-}) is the difference between gains and losses of probability per unit time. As we see from figure 8, this is kind of master equation and can be written as

$$\frac{\Delta f(n_{+},n_{-};t)}{\Delta t} = \frac{gains - losses}{\Delta t}
= (n_{-}+1) p_{-+}(n_{+}-1,n_{-}+1) f[n_{+}-1,n_{-}+1;t]
+ (n_{+}+1) p_{+-}(n_{+}+1,n_{-}-1) f[n_{+}+1,n_{-}-1;t] (26)
-n_{+} p_{+-}(n_{+},n_{-}) f[n_{+},n_{-};t] - n_{-} p_{-+}(n_{+},n_{-}) f[n_{+},n_{-};t].$$

[Insert Figure 8]

If we introduce

$$q = (n_+ - n_-)/2; \ q \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

as a measure of the polarization of the group, we can rewrite the transition probabilities p in terms of q:

$$w_{+-}(q) := n_{+}p_{+-}(n_{+}, n_{-}) = n(\frac{1}{2} + q)p_{+-}(q)$$
$$w_{-+}(q) := n_{-}p_{-+}(n_{+}, n_{-}) = n(\frac{1}{2} - q)p_{-+}(q)$$

With these definitions and $\Delta q = \frac{1}{n}$, equation (26) simplifies to

$$\frac{\Delta f(q;t)}{\Delta t} = w_{+-}(q + \Delta q)f(q + \Delta q, t) + w_{-+}(q - \Delta q)f(q - \Delta q, t) - w_{+-}(q)f(q, t) - w_{-+}(q)f(q, t) .$$

A series expansion of the first two terms on the right hand side up to the second order in Δq leads to

$$\begin{aligned} \frac{d}{dt}f(q,t) &= w_{+-}(q)f(q,t) + \frac{\partial}{\partial q}w_{+-}(q)f(q,t)\Delta q + \frac{1}{2}\frac{\partial^2}{\partial q^2}w_{+-}(q)f(q,t)\Delta q^2 \\ &+ w_{-+}(q)f(q,t) - \frac{\partial}{\partial q}w_{-+}(q)f(q,t)\Delta q + \frac{1}{2}\frac{\partial^2}{\partial q^2}w_{-+}(q)f(q,t)\Delta q^2 \\ &- w_{+-}(q)f(q,t) - w_{-+}(q)f(q,t) \\ &= \frac{\partial}{\partial q}[w_{+-}(q) - w_{-+}(q)]f(q,t)\Delta q + \frac{1}{2}\frac{\partial^2}{\partial q^2}[w_{+-}(q) + w_{-+}(q)]f(q,t)\Delta q^2 \end{aligned}$$

If we define the drift K(q) and the diffusion coefficient Q(q) as follows

$$K(q) = [w_{-+}(q) - w_{+-}(q)]\Delta q$$

$$Q(q) = [w_{+-}(q) + w_{-+}(q)]\Delta q^{2}$$

we get in the limit $\Delta q \rightarrow 0$ a Fokker Planck equation for the probability density:

$$\frac{d}{dt}f(q,t) = -\frac{\partial}{\partial q}\left[K(q)f(q,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial q^2}\left[Q(q)f(q,t)\right]$$
(27)

To solve this partial differential equation, it is convenient to introduce the probability 'current'

$$j(q;t) := K(q)f(q,t) + \frac{1}{2}\frac{\partial}{\partial q}\left[Q(q)f(q,t)\right].$$

Using j(q; t), equation (27) becomes a continuity equation for the probability density function f(q, t)

$$\frac{d}{dt}f(q,t) + \frac{\partial}{\partial q}j(q;t) = 0.$$

Since there is no probability current across the boundaries at $q = \pm \frac{1}{2}$, the boundary conditions are

$$j(q = \pm \frac{1}{2}; t) = 0.$$
 (28)

The condition for the steady state solution is

$$\frac{d}{dt}f_{st}(q,t) = 0.$$

But this means that $\frac{\partial}{\partial q} j_{st}(q;t) = 0$ and therefore $j_{st}(q;t) = const.$. Together with the boundary conditions (28) it follows that

$$j_{st}(q;t) = 0 = K(q)f_{st}(q,t) + \frac{1}{2}\frac{\partial}{\partial q}\left[Q(q)f_{st}(q,t)\right].$$

This is an ordinary differential equation and the solution can be obtained easily by integration:

$$f_{st}(q) = \frac{c}{Q(q)} \exp\left[2\int_{-\frac{1}{2}}^{q} \frac{K(q)}{Q(q)}dq\right]$$

with the normalization constant c.

6.2 Derivation of the transition probabilities p_{+-} and p_{-+}

Ising introduced an idealized model of the ferromagnet that consists of a lattice of elementary magnets (more precise magnetic moments, so-called spins σ_i), which can adopt only two orientations: Parallel ($\sigma_i = 1$) to an external field H or antiparallel ($\sigma_i = -1$). The total energie E_{λ} of a magnet configuration λ is assumed to depend on the external field and on an internal field created by the magnet-magnet interaction in the following way:

$$E_{\lambda} = \sum_{i} E_{\lambda i} = \sum_{i} \left(-\mu H \sigma_{i} - \sum_{j \neq i} I_{ij} \sigma_{i} \sigma_{j} \right)$$

 I_{ij} are the spin-spin interaction parameters and μ is the magnitude of a magnetic moment. As in every thermodynamic system, the total energie E_{λ} is the central item that determines various other thermodynamic variables. For example, the probability of finding the system in the spin-configuration λ is given by the Boltzmann distribution

$$w(E_{\lambda}) = c \exp(-E_{\lambda}/k_B T)$$

with the Boltzmann constant k_B and temperature T. Even though the system is in thermal equilibrium it is not in a static state. Spins may flip from one orientation to the other, changing the total energy. The probability per unit time of such a flipping process is governed by the equation

$$\Pr(\sigma_i \to -\sigma_i) = \frac{A}{\tau} \exp\left[\frac{-W + E_{\lambda i}}{k_B T}\right] = \frac{A}{\tau} \exp\left[\frac{-(W + \mu H \sigma_i + \sum_{j \neq i} I_{ij} \sigma_i \sigma_j)}{k_B T}\right]$$
(29)

where A, τ and W are appropriately chosen constants.

The characteristics of the Ising model are well documented⁹: There exists a phase transition temperature T_c defined by the formula

$$\sinh(2I/k_BT_c) = 1.$$

Therefore, three phases can be distinguished: For high temperatures $(T > T_c)$, the spins fluctuate almost independently of their neighbours entailing a mean polarization of zero. For temperatures around the phase transition temperature $(T \approx T_c)$ there are big clusters of aligned spins, while the mean polarization is still zero. In the case of very low temperatures $(T < T_c)$, one of the clusters grows at the cost of the oppositely orientated spins until all spins point in the same direction. The mean polarisation is then close to one.

The basic idea of Weidlich was to formulate the transition probabilities p_{+-} and p_{-+} in complete analogy to the Ising model. Setting $\alpha = \frac{A}{\tau} \exp(-\frac{W}{k_B T}), \pm \mu H = \mu H \sigma_i$ and $\pm I(n_+ - n_-) = \sum_{j \neq i} I_{ij} \sigma_i \sigma_j$ we get from (29)

$$p_{+-}(q) = \alpha \exp\left(\frac{-I(n_{+} - n_{-}) - \mu H}{k_{B}T}\right)$$
$$= \alpha \exp\left(\frac{-\frac{I}{2n}q - \mu H}{k_{B}T}\right) = \alpha \exp\left[-(kq + h)\right]$$

$$p_{-+}(q) = \alpha \exp\left(\frac{+I(n_{+} - n_{-}) + \mu H}{k_{B}T}\right)$$
$$= \alpha \exp\left(\frac{+\frac{I}{2n}q + \mu H}{k_{B}T}\right) = \alpha \exp\left[+(kq + h)\right].$$

For further simplification we introduced the adaption parameter $k = \frac{I}{2nk_BT}$ and the preference parameter $h = \frac{\mu H}{k_BT}$.

6.3 Derivation of the Fundamental Valuation Equation

According to (11), the returns of the risky asset are governed by the stochastic differential equation

$$\frac{dS(\underline{x},t)}{S} = \alpha(\underline{x},t)dt + \underline{\eta}(\underline{x},t)d\underline{w}(t)$$
(30)

with drift

$$\alpha = \mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q)$$

 $^{^{9}}$ For an analytic investigation see Cohen (1968) and for a computer simulation see Ogita et al. (1969).

and diffusion vector

$$\underline{\eta} = \left[\begin{array}{cc} \kappa \sqrt{Q(q)} & \sigma_D \end{array} \right].$$

Let $\underline{x} = \begin{bmatrix} q \\ S \end{bmatrix}$ be the state vector which follows the two-dimensional Markov process

$$d\underline{x} = \underline{\mu}(\underline{x}, t)dt + \underline{\sigma}(\underline{x}, t)d\underline{w}(t)$$
(32)

where

$$\underline{\mu}(\underline{x},t) = \begin{bmatrix} K(q) \\ \left[\mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q) \right] S \end{bmatrix}$$

is the drift vector and

$$\underline{\sigma}(\underline{x},t) = \begin{bmatrix} \sqrt{Q(q)} & 0\\ \kappa \sqrt{Q(q)}S & \sigma_DS \end{bmatrix}$$

is the correlation matrix of the subordinated processes dS and dq. Let further

$$\frac{dF(\underline{x},t)}{F} = \beta(\underline{x},t)dt + \underline{\psi}(\underline{x},t)d\underline{w}(t)$$
(33)

be the stochastic process of the returns of the derivative security which may depend on all state variables, but not on wealth W.

We want to maximize

$$J(W, \underline{x}, t) = E_t \left[\int_t^\infty U(C) ds \right]$$
(34)

with the utility function

$$U(C) = \exp(-\rho s) \ln C(s)$$
(35)

subject to the budget constraint

$$dW = \left[\left(1 - \omega_P - \omega_F\right] rWdt + \omega_P W \frac{dS}{S} + \omega_F W \frac{dF}{F} - Cdt.$$
(36)

The last equation can be rewritten using equations (30) and (33), so we get

$$dW = [r + \omega_P(\alpha - r) + \omega_F(\beta - r)]Wdt + [\omega_P\underline{\eta} + \omega_F\underline{\psi}]Wd\underline{w} - Cdt.$$
(37)

The necessary optimality condition for (34) is the Bellman equation

$$\max_{C,\omega_P,\omega_F} \left[U(C) + \frac{E_t[dJ]}{dt} \right] = 0.$$
(38)

We can write $E_t[dJ]$ as follows:

$$E_t[dJ] = J_t dt + J_W E_t[dW] + J_{\underline{x}} E_t[d\underline{x}] + \frac{1}{2} tr(J_{\underline{x}\underline{x}'} E_t[d\underline{x}d\underline{x}']) + J_{\underline{x}W} E_t[d\underline{x}dW] + \frac{1}{2} J_{WW} E_t[dW^2] + o(dt)$$

with the expectations

$$E[d\underline{x}] = \underline{\mu}dt$$

$$E_t[d\underline{x}d\underline{x}'] = \underline{\sigma}\sigma'dt$$

$$E_t[d\underline{x}dW] = (\omega_P \underline{\sigma}\underline{\eta}' + \omega_F \underline{\sigma}\underline{\psi}')Wdt$$

$$E_t[dW] = [r + \omega_P(\alpha - r) + \omega_F(\beta - r)]Wdt - Cdt$$

$$E_t[dW^2] = (\omega_P^2 \eta \eta' + 2\omega_P \omega_F \psi \eta' + \omega_F^2 \psi \psi')W^2dt.$$

Ignoring terms of order o(dt), we can now substitute into the Bellman equation to get

$$0 = \max_{C,\omega_{P},\omega_{F}} [U(C) + J_{t} + J_{W} ([r + \omega_{P}(\alpha - r) + \omega_{F}(\beta - r)]W - C) + J_{\underline{x}\underline{\mu}} + J_{\underline{x}W} (\omega_{P}\underline{\sigma}\underline{\eta}' + \omega_{F}\underline{\sigma}\underline{\psi}')W + \frac{1}{2} J_{WW} (\omega_{P}^{2}\underline{\eta}\underline{\eta}' + 2\omega_{P}\omega_{F}\underline{\psi}\underline{\eta}' + \omega_{F}^{2}\underline{\psi}\underline{\psi}')W^{2} + \frac{1}{2} tr(J_{\underline{x}\underline{x}'}\underline{\sigma}\underline{\sigma}')].$$

The first order conditions are

$$U_C - J_W = 0 \tag{39}$$

$$J_W(\alpha - r) + \underline{\eta}\underline{\sigma}' J'_{\underline{x}W}W + J_{WW}(\omega_P \underline{\eta}\underline{\eta}' + \omega_F \underline{\eta}\underline{\psi}')W = 0$$
(40)

$$J_W(\beta - r) + \underline{\psi}\underline{\sigma}' J'_{\underline{x}W}W + J_{WW}(\omega_P \underline{\psi}\underline{\eta}' + \omega_F \underline{\psi}\underline{\psi}')W = 0.$$
(41)

As shown in Merton (1971), the investor's value function J is partially separable and has the form

$$J(W, \underline{x}, t) = \frac{\exp(-\rho t)}{\rho} \ln(W) + H(\underline{x}, t).$$
(42)

With this value function, Equation (39) becomes the usual consumption optimality condition derived by Merton

$$C = \rho W. \tag{43}$$

Case 1 Market clearing condition $\omega_P = 1$:

In this case, $\omega_r = \omega_F = 0$ and we can solve equation (40) for r:

$$r = \alpha - \underline{\eta} \eta' \tag{44}$$

Substituting this result into (41) yields

$$\beta = r + \psi \underline{\lambda}_2 \tag{45}$$

with market price of risk

$$\underline{\lambda}_2 = \eta'. \tag{46}$$

Case 2 Market clearing condition $\omega_P + \omega_r = 1$:

In this case, only the derivative security is in zero net supply ($\omega_F = 0$) and we obtain the optimal portfolio decision ω_P^* from (40):

$$\omega_P^* = \frac{1}{\underline{\eta\eta'}} (\alpha - r). \tag{47}$$

If we substitute ω_P^* into (41), the drift of the derivative security can be written as

$$\beta = r + \underline{\psi}\underline{\lambda}_3 \tag{48}$$

where

$$\underline{\lambda}_3 = \frac{\underline{\eta'}}{\underline{\eta\eta'}} (\alpha - r) \tag{49}$$

is the vektor of factor risk prices.

To derive the fundamental valuation equation we have to apply Ito's lemma to $F(\underline{x}, t)$ with the result

$$dF = \left(F_t + F_{\underline{x}\underline{\mu}} + \frac{1}{2}tr(F_{\underline{x}\underline{x}'}\underline{\sigma}\underline{\sigma}')\right)dt + F_{\underline{x}}\underline{\sigma}d\underline{w}$$
(50)
$$= \beta(\underline{x}, t)Fdt + \underline{\psi}(\underline{x}, t)Fd\underline{w}(t).$$

Setting the identities $\beta(\underline{x},t) = \frac{1}{F} \left(F_t + F_{\underline{x}\underline{\mu}} + \frac{1}{2} tr(F_{\underline{x}\underline{x}'} \underline{\sigma} \underline{\sigma}') \right)$ and $\underline{\psi}(\underline{x},t) = \frac{1}{F} \left(F_{\underline{x}\underline{\sigma}} \right)$ into equations (45) and (48) respectively, we get

$$\frac{1}{2}tr(F_{\underline{xx}'}\underline{\sigma\sigma}') + F_{\underline{x}}\left(\underline{\mu} - \underline{\sigma\lambda}_i\right) + F_t - rF = 0.$$
(51)

Resubstituting the expressions for \underline{x} , $\underline{\mu}$, $\underline{\sigma}$, α , $\underline{\eta}$ and $\underline{\lambda}_i$ finally leads to our valuation equations for derivative securities.

References

- [1] CAMPBELL, JOHN Y. AND ROBERT L. SHILLER (1988): "Stock Prices, Earnings, and Expected Dividends", *Journal of Finance 43*, 661-676
- [2] COHEN, E. G. D. (ED.) (1968): "Fundamental Problems in Statistical Mechanics, Vol. 2", Amsterdam: North-Holland.
- [3] COX JOHN C., JONATHAN E. INGERSOLL JR. AND STEPHEN A. ROSS (1985a): "An Intertemporal General Equilibrium Model of Asset Prices", *Econometrica* 53 (2), 363-384
- [4] COX JOHN C., JONATHAN E. INGERSOLL JR. AND STEPHEN A. ROSS (1985b):"A Theory of the Term Structure of Interest Rates", *Econometrica* 53 (2), 385-407

- [5] DE BONDT, WERNER F. M. AND RICHARD H. THALER (1985): "Does the Stock Market Overreact ?", Journal of Finance 40, July, 793-808
- [6] DE BONDT, WERNER F. M. AND RICHARD H. THALER (1987): "Further Evidence on Investor Overreaction and Stock Market Seasonality", *Journal of Finance* 42, July, 557-581
- [7] FELDMAN, D. (1992): "Logarithmic Preferences, Myopic Decisions and Incomplete Information", Journal of Financial and Quantitative Analysis 27, December, 619-29
- [8] MERTON, ROBERT C. (1971): "Optimum Consumption and Portfolio Rules in a Continuous-Time Model", *Journal of Economic Theory* 3, 373-413
- [9] OGITA, N., ET AL. (1969). Suppl. Jap. phys. Soc. 26, 145
- [10] POTERBA JAMES M. AND LAWRENCE H. SUMMERS (1988): "Mean Reversion in Stock Prices", Journal of Financial Economics 22, 27-59
- [11] SHILLER, ROBERT J. (1981): "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?", American Economic Review 71, 421-436
- [12] SCHÖBEL, RAINER AND JIANWEI ZHU. (1999): "Stochastic Volatility with an Ornstein-Uhlenbeck Process: An Extension", *European Finance Review* 2, 23-46
- [13] VAGA, TONIS (1990): "The Coherent Market Hypothesis", Financial Analysts Journal, November-December, 36-49.
- [14] WEIDLICH, WOLFGANG (1971): "The Statistical Description of Polarization Phenomena in Society", British Journal of Math. Statist. Psychology 24, 251-266

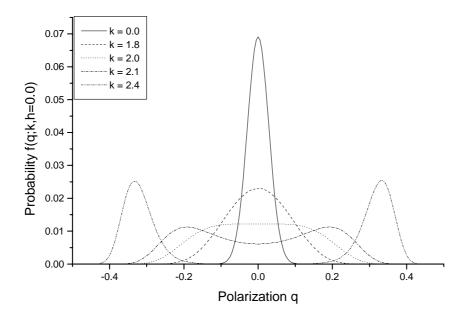


Figure 1a: Phase transition in the case of no preferences (h = 0.0).

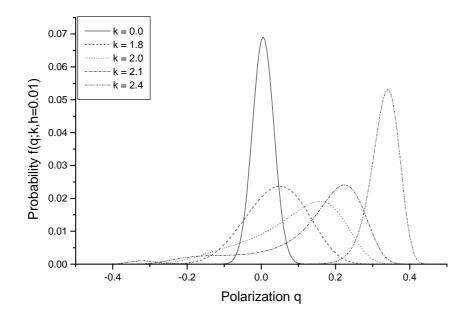
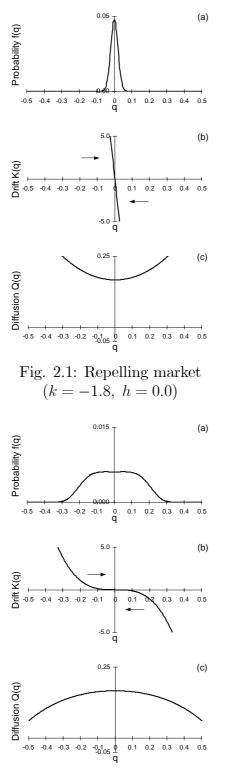
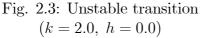
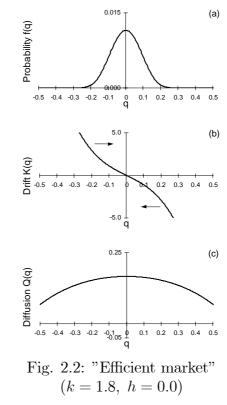


Figure 1b: Phase transition in the case of positive preferences (h = 0.01).







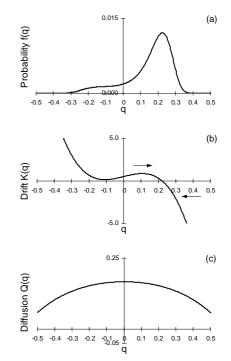


Fig. 2.4: Coherent bull market (k = 2.1, h = 0.01)

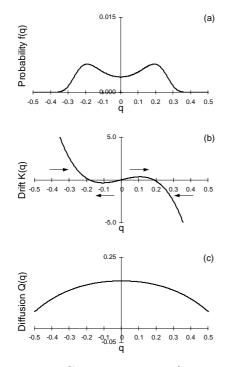


Fig. 2.5: Chaotic market (neutral) $(k=2.1,\ h=0.0)$

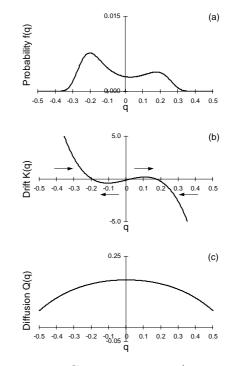
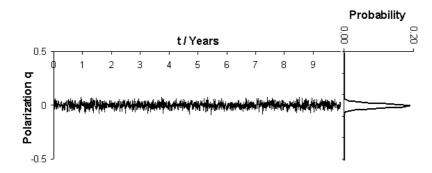
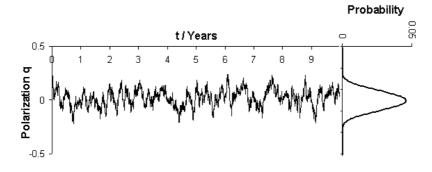


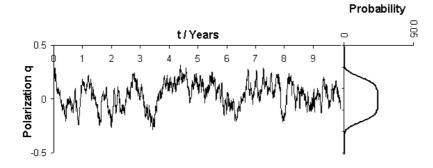
Fig. 2.6: Chaotic market (bearish) $(k=2.1,\ h=-0.003)$



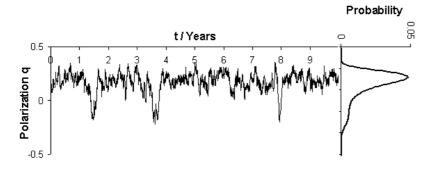
a) Repelling market



b) "Efficient market"

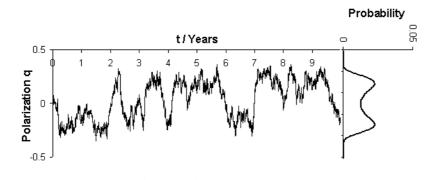


c) Unstable transition

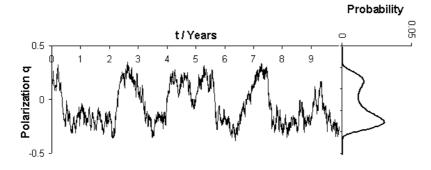


d) Coherent bull market

Figure 3: Trajectories of the stochastic process dq.



e) Chaotic market (neutral)



f) Chaotic market (bearish)

Figure 3: Trajectories of the stochastic process dq (continued).

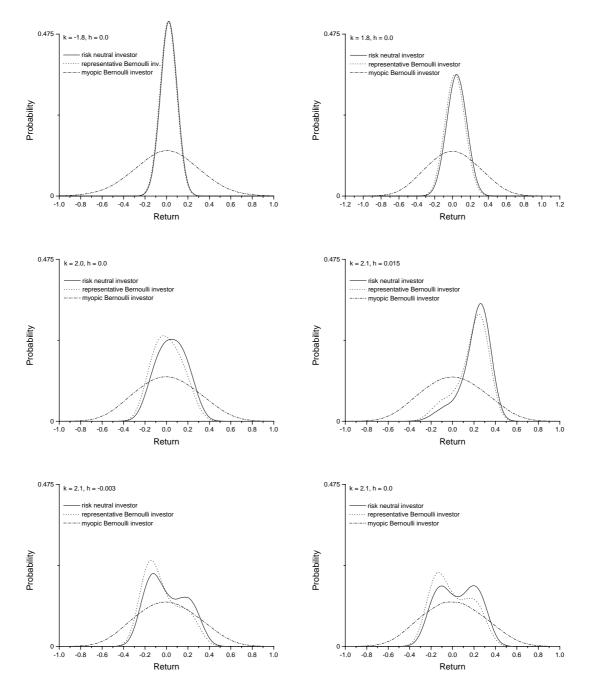


Figure 4: Implied return distributions for risk-neutral investor (original), representative Bernoulli investor and myopic Bernoulli investor in different market states.

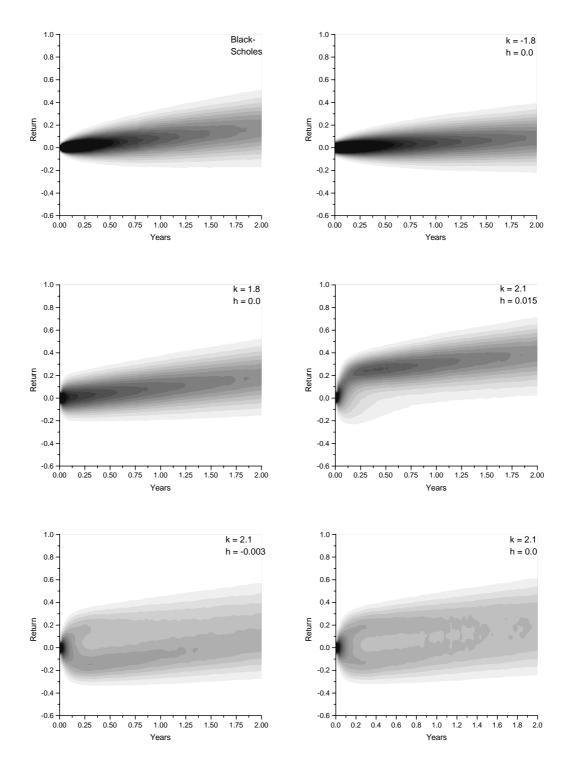


Figure 5: Contourplot of the evolution of the return distribution.

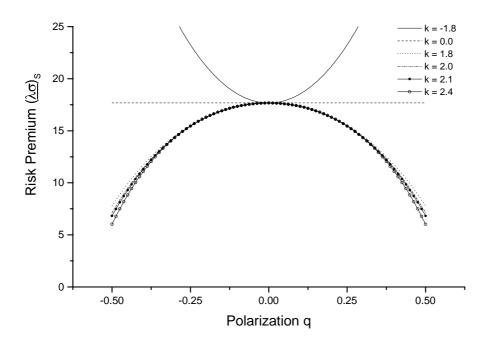


Fig. 6a: Risk premium subject to market polarization q and crowd behavior k for a representative Bernoulli investor.

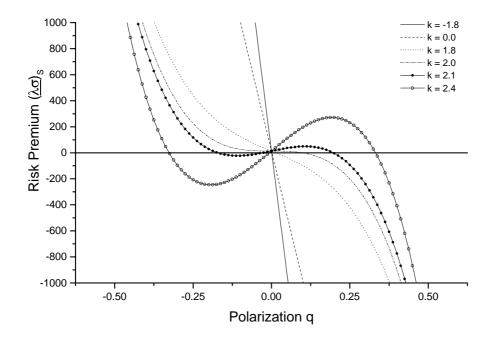


Fig. 6b: Risk premium subject to market polarization q and crowd behavior k for a myopic Bernoulli investor.

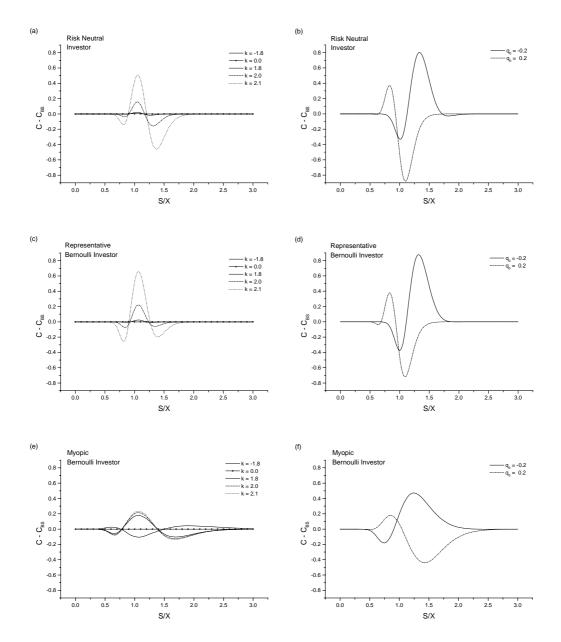


Figure 7: Differences between the model option prices and corresponding Black-Scholes prices.

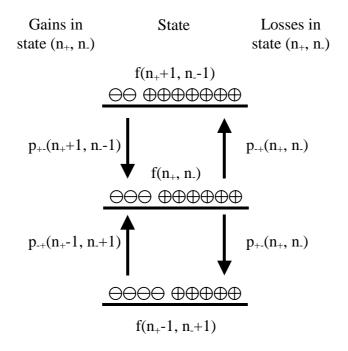


Figure 8: Probability currents to and from state (n_+, n_-) .

X	New	B/S	I. vol.	New	B/S	I. vol.	New	B/S	I. vol.		
A .			$(\mathbf{k}, \mathbf{h} = 0.0,$	$q_0 = 0.0, \sigma_d$	$= 0.1, \mu_d$	$= 0.1, \rho = 0.0$	$(0, \alpha = 50)$				
	k = -	1.8, r = 9.3	81%	k = 1	.8, r = 12.	03%	k = 2	.1, r = 15.	.73%		
80	23.84	23.84	0.1055	24.69	24.69	0.1504	26.28	26.42	0.2441		
90	14.36	14.36	0.1050	15.53	15.53	0.1538	18.35	18.29	0.2720		
00	5.89	5.89	0.1049	7.77	7.76	0.1545	12.07	11.60	0.2850		
10	1.23	1.23	0.1049	2.86	2.85	0.1542	7.18	6.74	0.2802		
20	0.11	0.11	0.1050	0.75	0.76	0.1531	3.57	3.60	0.2627		
30	0.01	0.00	0.1055	0.13	0.15	0.1509	1.37	1.78	0.2412		
	repo	elling mar	·ket	effi	cient mar	ket	cha	otic mar	ket		
3.			(k = 1.8, h ,	$q_0 = 0.0, \sigma_d$	$= 0.1, \mu_d$	$= 0.1, \rho = 0.1$	$0, \alpha = 50)$	$\alpha = 50$			
	h = -0.1, r = 3.93%			h = 0	h = 0.0, r = 12.03%			h = 0.01, r = 19.98%			
80	21.59	21.60	0.1484	24.69	24.69	0.1504	27.62	27.62	0.1523		
90	12.35	12.37	0.1515	15.53	15.53	0.1538	18.67	18.67	0.1538		
00	5.32	5.29	0.1533	7.77	7.76	0.1545	10.50	10.48	0.1533		
10	1.64	1.59	0.1544	2.86	2.85	0.1542	4.50	4.51	0.1517		
20	0.35	0.33	0.1541	0.75	0.76	0.1531	1.38	1.43	0.1498		
30	0.05	0.05	0.1523	0.13	0.15	0.1509	0.29	0.33	0.1472		
	weak co	ar mkt.	effi	efficient market			weak coherent bull mkt.				
Ζ.			(k = 2.1, h ,	$q_0 = 0.0, \sigma_d$	$= 0.1, \mu_d$	$= 0.1, \rho = 0.1$	$0, \alpha = 50)$				
	h = -(0.1, r = -9.	48%	h = 0.0, r = 15.73%			h = 0.1, r = 38.94%				
80	16.79	17.22	0.2041	26.28	26.42	0.2441	34.21	34.19	0.2539		
90	9.61	9.58	0.2372	18.35	18.29	0.2720	26.30	26.12	0.2661		
00	5.39	4.57	0.2646	12.07	11.60	0.2850	18.96	18.56	0.2606		
10	2.84	1.88	0.2767	7.18	6.74	0.2802	12.32	12.08	0.2410		
20	1.27	0.68	0.2742	3.57	3.60	0.2627	6.74	7.15	0.2156		
30	0.44	0.22	0.2622	1.37	1.78	0.2412	2.86	3.85	0.1932		
	strong	g coh. bea	r mkt.	cha	chaotic market strong coh. bull mkt						
D.			(k = 1.8, h	= 0.0, q_0 , σ_d	$= 0.1, \mu_d$	$= 0.1, \rho = 0.1$	$0, \alpha = 50)$				
	$q_0 = -$	0.1, r = 32	.00%	$q_0 = 0.0, r = 12.03\%$			$q_0 = 0.1, r = -7.93\%$				
80	31.84	31.84	0.1517	24.69	24.69	0.1504	16.95	16.95	0.1536		
90	23.33	23.34	0.1523	15.53	15.53	0.1538	8.13	8.12	0.1544		
00	15.10	15.10	0.1539	7.77	7.76	0.1545	2.70	2.69	0.1541		
10	8.06	8.05	0.1545	2.86	2.85	0.1542	0.59	0.60	0.1526		
20	3.36	3.34	0.1543	0.75	0.76	0.1531	0.08	0.09	0.1504		
30	1.06	1.06	0.1534	0.13	0.15	0.1509	0.01	0.01	0.1484		
	over	reaction d	lown	fai	ir valuatio	n	ove	rreaction	up		
	(efficient market)			(effi	cient mar	ket)	(efficient market)				
Ξ.			(k = 2.1, h)	= 0.0, q ₀ , σ _d	$= 0.1, \mu_d$	$= 0.1, \rho = 0.1$	$0, \alpha = 50)$				
	$q_0 = -1$	0.2, r = 39	.57%	$q_0 = 0$	0.0, r = 15	.73%	$q_0 = 0$	0.2, r = -9	.93%		
80	34.37	34.40	0.1719	26.28	26.42	0.2441	17.59	17.23	0.2654		
90	26.21	26.39	0.1943	18.35	18.29	0.2720	9.88	9.74	0.2467		
	18.58	18.92	0.2178	12.07	11.60	0.2850	4.19	4.79	0.2196		
00	12.44	12.52	0.2423	7.18	6.74	0.2802	1.17	2.07	0.1963		
00 10			0.2628	3.57	3.60	0.2627	0.20	0.80	0.1792		
10		/.61	0.2020	5.57							
10 20	8.09	7.61 4.25									
10	8.09 5.06	4.25 reaction d	0.2742	1.37	1.78 r valuatio	0.2412	0.02	0.28 rreaction	0.1680		

Table 1: Call option prices in case of a risk neutral investor.

Х	New	B/S	I. vol.	New	B/S	I. vol.	New	B/S	I. vol.		
A.			$(\mathbf{k}, \mathbf{h} = 0.0,$	$q_0 = 0.0, \sigma_d$	$= 0.1, \mu_d$	$= 0.1, \rho = 0.0$	$0, \alpha = 50)$				
	k = -	1.8, r = 8.0	53%	k = 1	1.8, r = 8.3	31%	k = 2	k = 2.1, r = 5.98%			
80	23.38	23.38	0.1063	23.28	23.28	0.1406	22.74	23.00	0.2148		
90	13.86	13.86	0.1055	14.07	14.08	0.1519	15.02	14.99	0.2478		
100	5.48	5.48	0.1049	6.60	6.59	0.1539	9.41	8.83	0.2750		
110	1.08	1.08	0.1049	2.24	2.22	0.1545	5.34	4.71	0.2858		
120	0.09	0.09	0.1049	0.54	0.54	0.1542	2.52	2.30	0.2796		
130	0.00 rene	0.00 elling mar	0.1055 ket	0.09 effi	0.09 cient mar	0.1528 ket	0.91 cha	1.04 Iotic marl	0.2629		
D	repe	and mar									
В.	h	0.1 = 0		-		$= 0.1, \rho = 0.0$		h = 0.01, r = 16.37%			
		-0.1, r = 0.			0.0, r = 8.3						
80	20.17	20.18	0.1445	23.28	23.28	0.1406	26.30	26.30	0.1520		
90	10.96	10.98	0.1484	14.07	14.08	0.1519	17.25	17.24	0.1533		
100	4.37	4.33	0.1514	6.60	6.59	0.1539	9.22	9.21	0.1543		
110	1.22	1.17	0.1533	2.24	2.22	0.1545	3.69	3.70	0.1534		
120	0.24	0.22	0.1542	0.54	0.54	0.1542	1.05	1.08	0.1520		
130	0.03 weak.co	0.03 herent be	0.1538 ar mkt.	0.09 effi	0.09 cient mar	0.1528 ket	0.20 weak co	0.23 herent bu	0.1500 ill mkt.		
C.						$= 0.1, \rho = 0.0$					
с.	h = -0	.1, r = -17			1.0, r = 5.9			.1, r = 31.	06%		
80	13.68	14.10	0.1741	22.74	23.00	0.2148	31.60	31.60	0.2402		
90	7.04	6.87	0.2023	15.02	14.99	0.2478	23.63	23.43	0.2402		
100	3.65	2.73	0.2350	9.41	8.83	0.2750	16.52	16.09	0.2040		
110	1.81	0.89	0.2594	5.34	4.71	0.2750	10.32	10.09	0.2676		
120	0.76	0.05	0.2687	2.52	2.30	0.2796	5.42	5.85	0.2469		
130	0.24	0.06	0.2654	0.91	1.04	0.2629	2.18	3.11	0.2222		
150		coh. bea			otic marl			g coh. bull			
D.			(k - 1.8 h)	h = 0.0, \mathbf{q}_0 , σ_d = 0.1, μ_d = 0.1, ρ = 0.0, α = 50)							
D.	$a_0 = -$	0.1 r = 28		$q_0 = 0.0, \mathbf{q}_0, 0_d = 0.1, \mu_d = 0.1, \mu = 0.0$ $q_0 = 0.0, \mathbf{r} = 8.31\%$			$q_0 = 0.1, r = -11.65\%$				
		,									
80	30.55	30.55	0.1441	23.28	23.28	0.1406	15.48	15.49	0.1514		
90 100	21.90 13.63	21.90 13.64	0.1484 0.1521	14.07 6.60	14.08 6.59	0.1519 0.1539	6.94 2.11	6.93 2.09	0.1538 0.1545		
110	6.87	6.86	0.1521	2.24	2.22	0.1539	0.41	0.42	0.1545		
120	2.66	2.64	0.1539	0.54	0.54	0.1542	0.41	0.42	0.1541		
130	0.77	0.77	0.1544	0.09	0.09	0.1528	0.00	0.00	0.1504		
	over	reaction d	own	fai	r valuatio	on	ove	rreaction	up		
	(effi	cient mar	ket)	(effi	cient mar	ket)	(effic	cient mar	ket)		
			(k = 2.1, h)	= 0.0, q ₀ , σ_{d}	$= 0.1, \mu_d$	$= 0.1, \rho = 0.0$	$0, \alpha = 50)$				
E.			(2.1.,			000/	a = 0	2 r - 18	.15%		
E.	$q_0 = -0$	0.2, r = 31	.25%	$q_0 =$	0.0, r = 5.9	98%	$q_0 = 0$.2, 1 = 10			
E. 80		0.2, r = 31 31.62		$q_0 = 22.74$	$\frac{0.0, r = 5.9}{23.00}$	0.2148	$\frac{q_0 = 0}{15.06}$	14.69	0.2714		
	$q_0 = -0$ 31.58 23.11		.25%								
80	31.58	31.62	.25% 0.1520	22.74	23.00	0.2148	15.06	14.69	0.2714		
80 90	31.58 23.11	31.62 23.33	.25% 0.1520 0.1719	22.74 15.02	23.00 14.99	0.2148 0.2478	15.06 8.06	14.69 7.90	0.2714 0.2702		
80 90 100	31.58 23.11 15.39	31.62 23.33 15.77	.25% 0.1520 0.1719 0.1914	22.74 15.02 9.41	23.00 14.99 8.83	0.2148 0.2478 0.2750	15.06 8.06 3.22	14.69 7.90 3.73	0.2714 0.2702 0.2491		
80 90 100 110	31.58 23.11 15.39 9.60	31.62 23.33 15.77 9.62	.25% 0.1520 0.1719 0.1914 0.2145	22.74 15.02 9.41 5.34	23.00 14.99 8.83 4.71	0.2148 0.2478 0.2750 0.2858	15.06 8.06 3.22 0.84	14.69 7.90 3.73 1.56	0.2714 0.2702 0.2491 0.2231		
80 90 100 110 120	31.58 23.11 15.39 9.60 5.87 3.49	31.62 23.33 15.77 9.62 5.26	.25% 0.1520 0.1719 0.1914 0.2145 0.2387 0.2579	22.74 15.02 9.41 5.34 2.52 0.91	23.00 14.99 8.83 4.71 2.30	0.2148 0.2478 0.2750 0.2858 0.2796 0.2629	15.06 8.06 3.22 0.84 0.13 0.01	14.69 7.90 3.73 1.56 0.59	0.2714 0.2702 0.2491 0.2231 0.2012 0.1846		

Table 2: Call option prices in case of a representative Bernoulli investor.

	New	B/S	I. vol.	New	B/S	I. vol.	New	B/S	I. vol.	
A.			$(\mathbf{k}, \mathbf{h} = 0.0,$	$q_0 = 0.0, \sigma_d$	$= 0.1, \mu_d$	$= 0.1, \rho = 0.0$	$(0, \alpha = 50)$			
	k = -	1.8, r = 9.5	53%	k = 1	1.8, r = 9.5	53%	k = 2	2.1, r = 9.5	53%	
80	26.42	26.42	0.4299	26.10	26.08	0.4105	26.05	26.03	0.4070	
90	19.65	19.70	0.4277	19.34	19.22	0.4140	19.29	19.15	0.4120	
100	14.19	14.27	0.4268	13.88	13.71	0.4156	13.85	13.62	0.4143	
110	10.00	10.09	0.4268	9.68	9.50	0.4156	9.64	9.40	0.4143	
120	6.92	6.99	0.4275	6.57	6.42	0.4145	6.52	6.33	0.4127	
130	4.72	4.76	0.4286	4.33	4.25	0.4124	4.27	4.18	0.4097	
	repe	elling mar	·ket	effi	cient mar	ket	cha	otic marl	ket	
B.			(k = 1.8, h,			$= 0.1, \rho = 0.0$), $\alpha = 50$)			
	h = -	0.1, r = 9.5		h = (0.0, r = 9.5	53%	h = 0.01, r = 9.53%			
80	26.09	26.07	0.4098	26.10	26.08	0.4105	26.11	26.08	0.4113	
90	19.33	19.22	0.4136	19.34	19.22	0.4140	19.34	19.23	0.4144	
100	13.88	13.70	0.4155	13.88	13.71	0.4156	13.89	13.72	0.4157	
110	9.69	9.49	0.4157	9.68	9.50	0.4156	9.68	9.50	0.4154	
120	6.58	6.41	0.4148	6.57	6.42	0.4145	6.55	6.43	0.4141	
130	4.35	4.25	0.4130	4.33	4.25	0.4124	4.32	4.26	0.4118	
	weak coherent bear mkt.			effi	cient mar	ket	weak coherent bull mkt.			
C.			(k = 2.1, h,	$q_0 = 0.0, \sigma_d$	$= 0.1, \mu_d$	$= 0.1, \rho = 0.0$	$0, \alpha = 50)$			
	h = -0).1, r = -9.	53%	h = 0.0, r = 9.53%			h = 0.1, r = 9.53%			
80	26.03	26.02	0.4061	26.05	26.03	0.4070	26.06	26.03	0.4079	
90	19.28	19.14	0.4115	19.29	19.15	0.4120	19.30	19.16	0.4125	
100	13.84	13.61	0.4141	13.85	13.62	0.4143	13.85	13.63	0.4144	
110	9.65	9.39	0.4144	9.64	9.40	0.4143	9.64	9.41	0.4141	
120	6.53	6.32	0.4131	6.52	6.33	0.4127	6.50	6.34	0.4122	
130	4.29	4.17	0.4105	4.27	4.18	0.4097	4.25	4.18	0.4090	
		coh. beau			otic marl					
	~ e	,		-						
D.		0.1				$= 0.1, \rho = 0.0$. 1	5000	
	$q_0 = \cdot$	-0.1, r = 9.1	53%	$q_0 =$	0.0, r = 9.3	53%	$q_0 =$	0.1, r = 9.1	53%	
80	25.97	26.01	0.4022	26.10	26.08	0.4105	26.16	26.08	0.4144	
90	19.20	19.13	0.4080	19.34	19.22	0.4140	19.36	19.23	0.4151	
100	13.79	13.59	0.4119	13.88	13.71	0.4156	13.84	13.71	0.4141	
110	9.64	9.38	0.4142	9.68	9.50	0.4156	9.57	9.50	0.4118	
120	6.59	6.31	0.4152	6.57	6.42	0.4145	6.41	6.42	0.4087	
130	4.40	4.15	0.4151	4.33	4.25	0.4124	4.16	4.26	0.4049	
	overreaction down			fair valuation			overreaction up			
	(offi	cient mar	ket)	(effi	cient mar	ket)	(effi	cient mar	ket)	
	(em			-00 a. a.	= 0.1, μ_d	$= 0.1, \rho = 0.0$	$0, \alpha = 50)$			
E.	(em		(k = 2.1, h)	$-0.0, \mathbf{q}_0, 0_0$			a –	0.2, r = 9.3	53%	
E.		0.2, r = 9.	(k = 2.1, h = 53%)		0.0, r = 9.3	53%	$q_0 -$,		
E. 80		0.2, r = 9. 25.78			0.0, r = 9.2 26.03	0.4070	$\frac{q_0 - 1}{26.11}$	25.94	0.4112	
	q ₀ = -		53% 0.3798	q ₀ =			-			
80 90	$q_0 = -25.63$ 18.79	25.78 18.79	53% 0.3798 0.3895	q ₀ = 26.05 19.29	26.03 19.15	0.4070 0.4120	26.11 19.20	25.94 19.03	0.4112	
80 90 100	$q_0 = -$ 25.63 18.79 13.40	25.78 18.79 13.19	53% 0.3798 0.3895 0.3975	$q_0 =$ 26.05 19.29 13.85	26.03 19.15 13.62	0.4070 0.4120 0.4143	26.11 19.20 13.55	25.94 19.03 13.47	0.4112 0.4080 0.4032	
80 90 100 110	$q_0 = -$ 25.63 18.79 13.40 9.34	25.78 18.79 13.19 8.95	53% 0.3798 0.3895 0.3975 0.4034	$\begin{array}{r} q_0 = \\ \hline 26.05 \\ 19.29 \\ 13.85 \\ 9.64 \end{array}$	26.03 19.15 13.62 9.40	$\begin{array}{c} 0.4070 \\ 0.4120 \\ 0.4143 \\ 0.4143 \end{array}$	26.11 19.20 13.55 9.16	25.94 19.03 13.47 9.25	0.4112 0.4080 0.4032 0.3972	
80 90 100	$ q_0 = - 25.63 18.79 13.40 9.34 6.38 $	25.78 18.79 13.19 8.95 5.91	53% 0.3798 0.3895 0.3975 0.4034 0.4076	$\begin{array}{r} q_0 = \\ 26.05 \\ 19.29 \\ 13.85 \\ 9.64 \\ 6.52 \end{array}$	26.03 19.15 13.62 9.40 6.33	0.4070 0.4120 0.4143 0.4143 0.4127	26.11 19.20 13.55 9.16 5.93	25.94 19.03 13.47 9.25 6.19	0.4112 0.4080 0.4032	
80 90 100 110 120	$ \begin{array}{r} q_0 = - \\ 25.63 \\ 18.79 \\ 13.40 \\ 9.34 \\ 6.38 \\ 4.28 \\ \end{array} $	25.78 18.79 13.19 8.95	53% 0.3798 0.3895 0.3975 0.4034 0.4076 0.4103	$\begin{array}{c} q_0 = \\ 26.05 \\ 19.29 \\ 13.85 \\ 9.64 \\ 6.52 \\ 4.27 \end{array}$	26.03 19.15 13.62 9.40	0.4070 0.4120 0.4143 0.4143 0.4127 0.4097	26.11 19.20 13.55 9.16 5.93 3.68	25.94 19.03 13.47 9.25	0.4112 0.4080 0.4032 0.3972 0.3907 0.3841	

Table 3: Call option prices in case of a myopic Bernoulli investor.

Die Liste der hier aufgeführten Diskussionsbeiträge beginnt mit der Nummer 252 im Jahr 2003. Die Texte können direkt aus dem Internet bezogen werden. Sollte ein Interesse an früher erschienenen Diskussionsbeiträgen bestehen, kann die vollständige Liste im Internet eingesehen werden. Die Volltexte der dort bis Nummer 144 aufgeführten Diskussionsbeiträge können nur direkt über die Autoren angefordert werden.

- 252. **McKinnon, Ronald und Gunther Schnabl:** The East Asian Dollar Standard, Fear of Floating, and Original Sin, Januar 2003.
- 253. Schulze, Niels und Dirk Baur: Coexceedances in Financial Markets A Quantile Regression Analysis of Contagion, Februar 2003.
- 254. **Bayer, Stefan:** Possibilities and Limitations of Economically Valuating Ecological Damages, Februar 2003.
- 255. **Stadler, Manfred:** Innovation and Growth: The Role of Labor-Force Qualification, März 2003.
- 256. **Licht, Georg und Manfred Stadler:** Auswirkungen öffentlicher Forschungsförderung auf die private F&E-Tätigkeit: Eine mikroökonometrische Evaluation, März 2003.
- 257. **Neubecker, Leslie und Manfred Stadler:** Endogenous Merger Formation in Asymmetric Markets: A Reformulation, März 2003.
- 258. **Neubecker, Leslie und Manfred Stadler:** In Hunt for Size: Merger Formation in the Oil Industry, März 2003.
- 259. **Niemann, Rainer:** Wie schädlich ist die Mindestbesteuerung? Steuerparadoxa in der Verlustverrechung, April 2003.
- 260.
- 261. Neubecker, Leslie: Does Cooperation in Manufacturing Foster Tacit Collusion?, Juni 2003.
- 262. **Buchmüller, Patrik und Christian Macht:** Wahlrechte von Banken und Aufsicht bei der Umsetzung von Basel II, Juni 2003.
- 263. **McKinnon, Ronald und Gunther Schnabl:** China: A Stabilizing or Deflationary Influence in East Asia? The Problem of Conflicted Virtue, Juni 2003.
- 264. **Thaut, Michael:** Die individuelle Vorteilhaftigkeit der privaten Rentenversicherung Steuervorteile, Lebenserwartung und Stornorisiken, Juli 2003.
- 265. **Köpke, Nikola und Jörg Baten:** The Biological Standard of Living in Europe During the Last Two Millennia, September 2003.
- 266. **Baur, Dirk, Saisana, Michaela und Niels Schulze:** Modelling the Effects of Meteorological Variables on Ozone Concentration A Quantile Regression Approach, September 2003.
- 267. **Buchmüller, Patrik und Andreas Marte:** Paradigmenwechsel der EU-Finanzpolitik? Der Stabilitätspakt auf dem Prüfstand, September 2003.
- 268. **Baten, Jörg und Jacek Wallusch:** Market Integration and Disintegration of Poland and Germany in the 18th Century, September 2003.
- 269. **Schnabl, Gunther:** De jure versus de facto Exchange Rate Stabilization in Central and Eastern Europe, Oktober 2003.
- 270. Bayer, Stefan: Ökosteuern: Versöhnung von Ökonomie und Ökologie?, Oktober 2003.
- 271. Köhler, Horst: Orientierungen für eine bessere Globalisierung, November 2003.
- 272. Lengsfeld, Stephan und Ulf Schiller: Transfer Pricing Based on Actual versus Standard Costs, November 2003.
- 273. Lengsfeld, Stephan und Thomas Vogt: Anreizwirkungen kostenbasierter Verrechnunspreise bei externen Effekten –Istkosten– versus standardkostenbasierte Verrechnungspreise bei Kreuzinvestitionen -, November 2003.

- 274. **Eisele, Florian und Andreas Walter**: Kurswertreaktionen auf die Ankündigung von Going Private-Transaktionen am deutschen Kapitalmarkt, Dezember 2003.
- 275. Rall, Wilhelm: Unternehmensstrategie für den globalen Wettbewerb, Februar 2004.
- 276. **Niemann, Rainer:** Entscheidungswirkungen von Verlustverrechnungsbeschränkungen bei der Steuerplanung grenzüberschreitender Investitionen, Februar 2004.
- 277. **Kirchner, Armin**: Verringerung von Arbeitslosigkeit durch Lockerung des Kündigungsschutzes – Die entscheidende Einflussgröße, März 2004.
- 278. **Kiesewetter, Dirk und Andreas Lachmund:** Wirkungen einer Abgeltungssteuer auf Investitionsentscheidungen und Kapitalstruktur von Unternehmen, April 2004
- 279. **Schanz, Sebastian:** Die Auswirkungen alternativer Gewinnverwendung von Kapitalgesellschaften im Rahmen des Halbeinkünfteverfahrens auf die Vermögenspositionen Residualanspruchsberechtigter, Mai 2004.
- 280. Stadler, Manfred: Bildung, Innovationsdynamik und Produktivitätswachstum, Mai 2004.
- 281. **Grupp, Hariolf und Manfred Stadler:** Technological Progress and Market Growth. An Empirical Assessment Based on the Quality Ladder Approach, Mai 2004.
- 282. **Güth, Werner und Manfred Stadler:** Path Dependence without Denying Deliberation. An Exercise Model Connecting Rationality and Evolution, Mai 2004.
- 283. **Duijm, Bernhard:** Offener Regionalisums als pareto-verbessernde Integrationsform, Juni 2004.
- 284. **Pitterle, Ingo und Dirk Steffen:** Welfare Effects of Fiscal Policy under Alternative Exchange Rate Regimes: The Role of the Scale Variable of Money Demand, Juni 2004.
- 285. Molzahn, Alexander: Optimale Fiskalpolitik und endogenes Wachstum, Juli 2004.
- 286. **Jung, Robert, Kukuk, Martin und Roman Liesenfeld:** Time Series of Count Data: Modelling and Estimation, August 2004.
- 287. **De Grauwe, Paul und Gunther Schnabl:** Nominal versus Real Convergence with Respect to EMU Accession. EMU Entry Scenarios for the New Member States, August 2004.
- 288. **Kleinert, Jörn und Farid Toubal:** A Structural Model of Exports versus Production Abroad, Dezember 2004.
- 289. **Godart, Olivier und Farid Toubal:** Cross the Border and Close the Gap? How do Migrants Enhance Trade, Januar 2005.
- 290. Schnabl, Gunther und Christian Danne: The Changing Role of the Yen/Dollar Exchange Rate for Japanese Monetary Policy, Februar 2005.
- 291. **Schnabl, Gunther:** Der Festkurs als merkantilistische Handelspolitik Chinas Währungsund Geldpolitik im Umfeld globaler Ungleichgewichte, Februar 2005.
- 292. **Starbatty, Joachim:** Anmerkungen zum Woher und Wohin der Europäischen Union, Februar 2005.
- 293. **Wagner, Franz W.:** Steuervereinfachung und Entscheidungsneutralität konkurrierende oder komplementäre Leitbilder für Steuerreformen?, April 2005.
- 294. **Yu, Peiyi und Werner Neus:** Market Structure, Scale Efficiency, and Risk as Determinants of German Banking Profitability, Juni 2005.
- 295. Schüle, Tobias und Manfred Stadler: Signalling Effects of a Large Player in a Global Game of Creditor Coordination, Juni 2005.
- 296. Zaby, Alexandra: Losing the Lead: Patents and the Disclosure Requirement, August 2005.
- 297. Hager, Svenja und Rainer Schöbel: A Note on the Correlation Smile, Dezember 2005.
- 298. **Starbatty, Joachim:** Zum Zusammenhang von Politik, Ethik und Ökonomik bei Aristoteles, Dezember 2005.
- 299. **Rostek, Stefan und Rainer Schöbel:** Risk Preference Based Option Pricing in a Fractional Brownian Market, Januar 2006.
- 300. **Hager, Svenja und Rainer Schöbel:** Deriving the Dependence Structure of Portfolio Credit Derivatives Using Evolutionary Algorithms, Februar 2006.

- 301. **Töpfer, Klaus:** Offene Fragen und wissenschaftliche Herausforderungen der Entwicklungsund Umweltpolitik, Februar 2006.
- 302. Stadler, Manfred: Education and Innovation as Twin-Engines of Growth, März 2006.
- 303. **Schüle, Tobias:** Forbearance Lending and Soft Budget Constraints in a Model of Multiple Heterogeneous Bank Financing, März 2006.
- 304. **Buch, Claudia und Jörn Kleinert:** Exchange Rates and FDI: Goods versus Capital Market Frictions, February 2006.
- 305. Felbermayr, Gabriel und Toubal Farid: Cultural Proximity and Trade, März 2006.
- 306. Schöbel, Rainer und Jochen Veith: An Overreaction Implementation of the Coherent Market Hypothesis and Option Pricing, April 2006.