Threshold Models for Trended Time Series

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Abstract

This paper presents the theoretical development of new threshold autoregressive models based on trended time series. The theoretical arguments underlying the models are outlined and a nonlinear economic model is used to derive the specification of the empirical econometric models. Estimation and testing issues are considered and analysed. Additionally we apply the models to the empirical investigation of U.S. GDP. The results are encouraging and warranty further research.

Keywords: Nonlinearity, Threshold Models, EDTAR models, Forecasting.
JEL Classification: C22, C53.

*This paper is the result of work carried out for the author's Ph.D. thesis. I would like to thank Hashem Pesaran for his help, encouragement and insights during the preparation of this paper. I also thank Gary Koop and Sean Holly for helpful comments. Financial assistance from the Economic and Social Research Council is gratefully acknowledged.

1 Introduction

Investigation of nonlinear reduced form models of macroeconomic time series has lately received considerable attention. Work by Neftçi (1984) , Sichel (1989, 1993), Rothman (1991), Potter (1995), Holly and Stannett (1995) and Arden, Holly, and Turner (1997) among others[1] indicates that many features of macroeconomic times series cannot be adequately described and analysed using linear techniques. These features include asymmetric behaviour of the series over the business cycle and regime switching. As a result of this evidence, a large number of nonlinear models have been proposed in the literature. Focus has concentrated on regime-switching models. Theoretical plausibility as well as relative computational tractability have helped their wider application in macroeconomics. As a result, there is a sizeable and growing literature on reduced form regime-switching models of macroeconomic time series and especially output and unemployment.

Some of the most important contributions in this area are by Hamilton (1989), Teräsvirta and Anderson (1992), Beaudry and Koop (1993), Potter (1995) and Pesaran and Potter (1997). Hamilton (1989) investigates U.S. output using a Markov switching model. In his model, the stochastic process underlying the evolution of output switches between regimes according to a first order Markov process. Teräsvirta and Anderson (1992) use two regime logistic and exponential smooth transition autoregressive (STAR) models to analyse industrial production in a number of OECD countries and find evidence of nonlinearity for many of them. Potter (1995) applies a two regime self exciting threshold autoregressive (SETAR) model to U.S. GNP. The regimes provide alternative linear specifications for the series depending on whether lagged GNP is expanding or contracting.

The tests he carries out reject linearity in the series in favour of the SETAR model. Analysis carried out using generalised impulse response functions provide ample evidence of asymmetry in the evolution of output during recessions and expansions. Beaudry and Koop (1993) provide a model for the evolution of output where any deviation of the series below its historical maximum, used to indicate a slowdown in the economy, introduces a nonlinear dampening force which reduces the impact of negative shocks on output changes. These empirical models distinguish two phases in the evolution of business cycles which can be, roughly, characterised as recessions and expansions.

However, evidence has accumulated to indicate that a multi-regime characterisation of the business cycle might be more appropriate. Sichel (1994) distinguishes three regimes: contractions, high-growth recoveries and moderate growth periods that usually follow recoveries. An alternative multi-regime characterisation is provided by Pesaran and Potter (1997), who extend the Beaudry and Koop specification. They propose a new class of threshold models, referred to as Endogenous Delay Threshold Autoregressive (EDTAR) models, which provides a flexible framework for modelling multi-regime systems. Their specification allows for three regimes corresponding to low, normal and high output growth regimes. They apply their model to U.S. GDP with encouraging results. Rejection of linearity and evidence of asymmetry between expansions and recessions is found. Work on multi-regime characterisation of the business cycle has also been carried out by Tiao and Tsay (1994) and Boldin (1996). Tiao and Tsay specify a four regime SETAR model for U.S. GNP where the regimes distinguish between worsening/improving contraction and expansion in the series allowing for a variety

[1]For a review, see Granger and Teräsvirta (1993).

of dynamics in the evolution of the business cycle. Boldin investigates the three regime characterisation of the business cycle suggested by Sichel, using a Markov-switching model.

One major shortcoming of most previous research is that the models are not explicitly linked to economic theory but simply aim to model stylised facts and investigate possible nonlinearities in economic times series. On the other hand, economic theory provides many alternatives for the nonlinear modelling of macroeconomic time series. Our approach attempts to bridge this gap by applying the core ideas of the theoretical work of Hicks (1949) on the evolution of output over the business cycle. The key idea of Hicks is to model the economy as a linear explosive system which is subject to dampening forces when it deviates considerably from its steady state. These dampening forces take the form of a ceiling which restricts output during an upswing and a piecewise linear investment function specifying that investment is not responsive to changes in output during a downswing. Such a system would be oscillating and thus, mimic the business cycle without needing exogenous stochastic shocks to activate business cycle fluctuations. Hicks' approach shares common elements with the work of Goodwin (1951, 1955) who also considers nonlinear investment functions to provide nonlinear deterministic models of the business cycle. The model by Hicks is used here because it demonstrates forcefully the underlying ideas for our approach. Basing the empirical econometric analysis on a nonlinear economic model provides both a theoretical justification for the analysis and a valid interpretation for the nonlinear features of the data.

The layout of the paper is as follows: Section 2 presents the theoretical basis of the model following the work by Hicks. Sections 3 and 4 provide the setup of the model. Section 5 discusses estimation and linearity testing. Section 6 presents the estimation results. Section 7 analyses the predictive ability of the models. Section 8 presents the generalised impulse response function analysis. Section 9 investigates the robustness of the results to an alternative trend specification. Section 11 concludes.

2 Theoretical background

Hicks' model of the business cycle comes out of comments he made, in an influential paper (1949), on a book on economic dynamic theory by Harrod (1948). The model was analysed further in a book on trade cycle theory published by Hicks (1950). In this model, output is specified to switch between different linear laws of motion depending on its past evolution.

We denote the observed level of output by Y_t. Then, we introduce the process Y_t^τ, which represents the long-run growth process of the economy and depends on factors determining the growth of labour and technology and investment in physical and human capital. Hicks specifies this process to be given by $(1+g)^t$ where $g > 0$ is the long run growth rate of output. During the presentation of our empirical model, we will consider generalisations of the specification of the growth process. The model by Hicks is simply a multiplier-accelerator model with a 'ceiling' and a 'floor'. The model for the detrended series $X_t = Y_t / Y_t^\tau$ may be written in the following

$$X_t = \begin{cases} X_t^c & \text{if } X_t^{\text{cor}} > X_t^c \\ X_t^{\text{cor}} & \text{if } X_t^{\text{cor}} \leq X_t^c \text{ and } X_{t-1} \geq \frac{1}{1+g} X_{t-2} \\ X_t^f & \text{if } X_{t-1} < \frac{1}{1+g} X_{t-2} \end{cases} \quad (1)$$

where

$$X_t^c = c, \quad X_t^{\text{cor}} = h + \frac{1-s+v}{1+g} X_{t-1} - \frac{v}{(1+g)^2} X_{t-2}, \quad X_t^f = h + \frac{1-s}{1+g} X_{t-1}$$

$v > 0$ is the ratio of induced investment to change in output in equilibrium, $0 < s < 1$ is the ratio of savings to output in equilibrium, h is autonomous investment at time 0, and $c > 1$.

Recently, there has been some theoretical work carried out on the asymptotic dynamical properties of the Hicks model. We will report the results which are relevant for our purposes. Hommes (1993) states that, for sufficiently large values[2] of v, the map in (1) possesses an unstable fixed point and a globally attracting set which is a piecewise linear closed curve. Depending on the rotation number[3], the system converges to either a periodic or a quasi-periodic[4] orbit. A periodic orbit is obtained if the rotation number is rational and a quasi-periodic orbit otherwise.

Of course, the simplicity of the economic structure behind the model makes it unrealistic. Many extensions are possible. A straightforward one is to consider a flexible accelerator model in the place of the fixed one used to derive the above model. This extension is analysed in Chapter 1 of Kapetanios (1998a).

Despite its simplicity, the importance of the model outlined in this Section lies in providing the insight that systems with simple nonlinearities can provide the basis for meaningful economic models. In the next Section, the ideas outlined above will be used to develop the new empirical model.

3 Empirical model

In order to make the transition from the theoretical model presented in the previous Section to an empirical econometric model, it is important to articulate the main insights of Hicks' model. A key element is the specification of a growth process, Y_t^τ, which determines the evolution of output in the long run. Hicks chooses to specify this as $(1+g)^t$. However, the essence of the analysis is not affected if an alternative specification for the growth process is adopted. The fact that the equilibrium level of output changes when output switches from one linear law of motion to another does not mean that there are two long-run growth processes. Only the intercepts change between the two equilibria. Therefore, we can consider the process Y_t^τ, to be a trend, around which output evolves.

[2]For $g = 0$, $v > 1$ is the necessary condition.

[3]The rotation number measures the average number of turns taken by an orbit of the curve per unit of time.

[4]An orbit is said to be quasi-periodic if it is the result of two distinct oscillations with frequencies whose ratio is an irrational number.

Hicks seeks to specify a model which produces fluctuations in output without the need for exogenous shocks. He suggests that output follows an explosive process around its trend implying that deviations from this trend grow with time. However, these deviations cannot grow indefinitely. They must be bounded both from above and from below. Therefore, dampening forces come into effect to prevent deviations from becoming too large. These take the form of a hard ceiling when upward deviations occur, or of a nonlinear investment function which does not allow disinvestment, when output falls. The ceiling does not affect the evolution of output before it is reached but is completely binding when it is hit. This is too restrictive[5]. It is more sensible to assume that dampening forces come into play when the deviation reaches a certain magnitude but permit a further rise in the deviation. Similarly, we choose to generalise the dampening effect during a downswing by specifying that output switches to an alternative linear law of motion when negative deviations from the trend become too large. This allows for effects unrelated to the investment function to initiate a recovery of the economy. Finally, to obtain an empirical model, we need to allow for random unobserved influences of shocks to the evolution of output. It is clear that this model need not be confined to the investigation of output. Other macroeconomic series which are driven by the business cycle, such as imports or industrial production, may be modelled similarly.

The above discussion suggests the use of the threshold autoregressive (TAR) class of models to implement the theoretical structure developed above. In the next subsection a self-exciting threshold autoregressive (SETAR) model will be presented. The limitations of the model will be discussed and, in the final subsection, an alternative model belonging to the class of endogenous delay threshold autoregressive (EDTAR) models will be presented.

3.1 The self-exciting threshold autoregressive model

The canonical form of a self-exciting threshold autoregressive (SETAR) model with m regimes belonging to the class of TAR models , introduced and analysed extensively[6] by Tong (1978, 1983, 1995), for a stochastic process $\{x_t\}$ is

$$x_t = \phi_{J_t,0} + \Phi_{J_t}(L)x_{t-1} + \sigma_{J_t}\epsilon_t, \quad t = p, \ldots, T \tag{2}$$

where $\Phi_{J_t}(L) = \sum_{k=1}^{p} \phi_{J_t,k} L^{k-1}$, $J_t = a'\mathbf{I}_t$, $a = (1, 2, \ldots m)'$, $\mathbf{I}_t = (I(x_{t-d} \in A_1), I(x_{t-d} \in A_2), \ldots, I(x_{t-d} \in A_m))'$, $A_i \in \mathcal{B}$, $i = 1, \ldots, m$ where $\mathcal{B}$ is the Borel σ-field[7] of $\mathbb{R}$. The sets, A_i, $i = 1, \ldots, m$, define a partition of the real line. d is referred to as the delay parameter. The basic idea is that the state of the system, at a specific point in the past, influences the current regime of the system.

Our aim is to model the stochastic process $\{Y_t\}$ with trend $\{Y_t^{\tau}\}$. We will assume that a trend process has been obtained already. We will discuss possible empirical specifications for

[5]The restrictive nature of the specification has been acknowledged by Ichimura (1954) who states that the ceiling may be more realistically viewed as a zone where dampening forces exert downward pressure on output. Additionally, a recent paper by Saura, Vasquez, and Vegas (1998) has analysed a smoothened version of the simple Hicks model where the ceiling and floor restrictions arise through the use of smooth functions and concluded that the qualitative dynamic behaviour of the system does not change under smoothing.

[6]See also Tong and Lim (1980) and Tsay (1989).

[7]Given a set Ω, a σ-field, $\mathcal{F}$, is a class of subsets of Ω satisfying (a) $\Omega \in \mathcal{F}$, (b) if $A \in \mathcal{F}$ then the complement of A, $A^c \in \mathcal{F}$, (c) If $\{A_n, n = 1, \ldots\}$ is a sequence of sets belonging to $\mathcal{F}$ then $\cup_{n=1}^{\infty} A_n \in \mathcal{F}$. The Borel σ-field of $\mathbb{R}$ is the set of all intervals of $\mathbb{R}$.

this process in Section 4. The analysis is motivated by the Hicks output model, but, as we mentioned above, need not be confined to output series only. We choose to take logarithms of the original series. We define $y_t = 100\log(Y_t)$, $y_t^{\tau} = 100\log(Y_t^{\tau})$ and $x_t = 100\log(X_t) = 100\log(Y_t/Y_t^{\tau}) = y_t - y_t^{\tau}$. x_t may then be interpreted as the percentage deviation of y_t from its trend. The theoretical analysis suggests a SETAR model with three regimes. One regime is activated when the process evolves near its trend level. In this regime no dampening forces appear. When x_t is positive and exceeds a given level, the process enters the second regime where a nonlinear dampening effect appears. If, on the other hand, x_t is negative and large in absolute value the process enters the third regime where a similar force pushes the process up towards its trend. The three regimes will be referred to as the 'corridor', 'ceiling' and 'floor' regime respectively. In mathematical notation, the activation of the regimes will be regulated by the following indicator functions:

$$\begin{array}{rll} \text{Floor Regime} & I_{f,t} = \mathbf{1}(y_t < y_t^{\tau} - r_f), & r_f > 0 \\ \text{Corridor Regime} & I_{\text{cor},t} = \mathbf{1}(I_{f,t} + I_{c,t} = 0), & \\ \text{Ceiling Regime} & I_{c,t} = \mathbf{1}(y_t > y_t^{\tau} + r_c), & r_c > 0 \end{array} \tag{3}$$

We note that the parameters r_f and r_c as defined in (3) represent the percentage deviation of y_t from y_t^{τ} needed for the 'floor' and 'ceiling' regime to be activated respectively.

The SETAR model is then given by

$$x_t = \phi_{\text{cor},0} + \Phi_{\text{cor}}(L)x_t + I_{f,t-1}(\phi_{f,0} + \Phi_f(L)x_t) + I_{c,t-1}(\phi_{c,0} + \Phi_c(L)x_t) + h_t\epsilon_t \tag{4}$$

where $\{\epsilon_t\}$ is assumed to be an independent and identically distributed (i.i.d.) sequence of disturbances with zero mean and unit variance, $\Phi_{\text{cor}}(L)$, $\Phi_f(L)$ and $\Phi_c(L)$ are polynomials in the lag operator L with orders p_{cor}, p_f and p_c respectively, $h_t = \sigma_{\text{cor}} I_{\text{cor},t-1} + \sigma_f I_{f,t-1} + \sigma_c I_{c,t-1}$ and σ_{cor}, σ_f, σ_c are parameters to be estimated. The above specification allows for regime specific heteroscedasticity to account for possible changes in the variance of the series in different regimes. This pattern of conditional heteroscedasticity is referred to as Qualitative Threshold Autoregressive Conditional Heteroscedasticity (QTARCH)[8]. In later Sections, the following notation will be used. $\boldsymbol{\phi} = (\phi_{\text{cor},0}, \phi_{\text{cor},1}, \ldots, \phi_{\text{cor},p_{\text{cor}}}, \phi_{f,0}, \phi_{f,1}, \ldots, \phi_{f,p_f}, \phi_{c,0}, \phi_{c,1} \ldots \phi_{c,p_c})'$, where $\phi_{\text{cor},i}$, $\phi_{f,i}$ and $\phi_{c,i}$ denote the coefficients of the lag polynomials $\Phi_{\text{cor}}(L)$, $\Phi_f(L)$ and $\Phi_c(L)$. Also, $\boldsymbol{\sigma} = (\sigma_{\text{cor}}, \sigma_f, \sigma_c)'$, $\boldsymbol{\gamma} = (r_f, r_c)'$, $\mathbf{I}_t = (I_{\text{cor},t-1}, I_{f,t-1}, I_{c,t-1})'$ and $\mathbf{z}_t = (1, x_{t-1}, \ldots, x_{t-p_{\text{cor}}}, I_{f,t-1}, I$

It is instructive to note the connections between this model and the SETAR model developed by Potter (1995) for U.S. output. Potter's model has two regimes, regulating output during expansions and contractions respectively. The model is cast in differences. To obtain Potter's setup from our own we must set $y_t^{\tau} = y_{t-1}$ and $r_f = r_c \equiv r$. Therefore, our setup provides a significant generalisation in terms of the detrending transformation applied to the original series[9].

The SETAR model presented incorporates a number of features of the theoretical structure analysed in the previous Section. However, it is not clear how the different linear autoregressive mechanisms underlying the 'floor' and 'ceiling' regimes act as dampening forces on the

[8] See also, Altissimo and Violante (1996).
[9] See also the conclusions of Holly and Stannett (1995).

evolution of output. For example, when $x_t = y_t - y_t^\tau$, one would expect that the coefficients, $\phi_{f,i}$, would be negative so that, combined with negative lagged x_t's, they would push x_t upwards. But, if some $\phi_{f,i}$'s are negative and others are positive, it is not clear if dampening forces are present or not. Additionally, the model is too restrictive. Nonlinear effects are constrained to come through lagged values of the x_t. If, for example, we use $x_t = \Delta y_t$, we cannot specify nonlinear effects which depend on the magnitude of the deviation from the trend. Further, the floor and the ceiling regime must be subject to the same nonlinear effects. Consequently, a more flexible model, which is designed to investigate dampening effects in the 'floor' and 'ceiling' regimes, is needed. This will be provided in the next subsection.

3.2 The endogenous delay threshold autoregressive model

Before presenting the endogenous delay threshold autoregressive (EDTAR) model we propose, we give a brief account of the class of EDTAR models proposed by Pesaran and Potter (1997) and extended further by Altissimo and Violante (1996). The main strength of EDTAR models is their ability to deal with feedbacks from the past realisations of the system to its current dynamics in a flexible and intuitive manner. A general specification for EDTAR models which includes the specific case we will consider later may be given by the following setup where $\{x_t\}$ is the focus of interest. Let there be m index variables of the following form

$$H_{j,t} = \mathbf{1}(b_j(x_t, \ldots, x_{t-s_j}, \mathbf{v}_t, \ldots, \mathbf{v}_{t-s_j}) \in A_j)), \quad j = 1, \ldots, m$$

where $b_j : \mathbb{R}^{(s_j+1)\times(q+1)} \to \mathbb{R}$, $A_j \in \mathcal{B}$ and $\{\mathbf{v}_t\}$ is a sequence of q-dimensional vectors of random variables. Then, the following feedback variables are constructed

$$U_{j,t} = G_j(x_t, \ldots, x_{t-l_j}, \mathbf{v}_t, \ldots, \mathbf{v}_{t-l_j}, H_{j,t}, \ldots, H_{j,t-l_j})$$

where $G_j : \mathbb{R}^{(l_j+1)\times(q+2)} \to \mathbb{R}$, $j = 1, \ldots, m$. Then, the dynamics of $\{x_t\}$ are defined by

$$\Phi(L)x_t = \alpha + \sum_{j=1}^{m} \Theta_j(L)U_{j,t} + \epsilon_t$$

where $\Phi(L)$ and $\Theta_j(L)$, $j = 1 \ldots, m$, are polynomials, of order $p, p_1, \ldots, p_m$ respectively, in the lag operator and ϵ_t is a random disturbance.

Once again the aim is to model the series $\{y_t\}$ with trend $\{y_t^\tau\}$. As before, the basic equation of the model will be expressed in terms of x_t. The indicator functions defined in (3) will be the index variables of the EDTAR model defining the 'corridor', 'ceiling' and 'floor' regimes as before. The next step involves constructing the feedback variables which will enter the difference equation that determines x_t. The definition of these variables together with the main equation of the model are given by

$$F_t = \sum_{i=0}^{p_r} \left[(y_{t-i}^\tau - r_f - y_{t-i}) \prod_{j=0}^{i} I_{f,t-j} \right], \quad C_t = \sum_{i=0}^{p_e} \left[(y_{t-i} - y_{t-i}^\tau - r_c) \prod_{j=0}^{i} I_{c,t-j} \right] \qquad (5)$$

$$x_t = \phi_0 + \Phi(L)x_t + \theta_f F_{t-1} + \theta_c C_{t-1} + h_t \epsilon_t \qquad (6)$$

where $h_t = \sigma_{\text{cor}} I_{\text{cor},t-1} + \sigma_f I_{f,t-1} + \sigma_c I_{c,t-1}$, $\{\epsilon_t\}$ is an i.i.d. sequence of disturbances following the standard normal distribution[10], p_r, p_e are the lag orders for the effects of past deviations

[10] The assumption of normality is not essential for the specification and estimation of the model. However, as maximum likelihood will be used for the estimation, a distributional assumption has to be made.

from the trend on the current x_t and $\Phi(L)$ is a lag polynomial of order p. For later Sections[11] we define $\boldsymbol{\phi}$ to be a vector comprising of ϕ_0, the coefficients of the lag polynomial $\Phi(L)$ and the coefficients θ_f, θ_c, $\boldsymbol{\sigma} = (\sigma_f, \sigma_c, \sigma_{\text{cor}})'$, $\boldsymbol{\gamma} = (r_f, r_c)'$, $\mathbf{I}_t = (I_{f,t-1}, I_{c,t-1} I_{\text{cor},t-1})'$ and $\mathbf{z}_t = (1, x_{t-1}, \ldots, x_{t-p}, F_{t-1}, C_{t-1})'$.

As in the SETAR model, the error is assumed to follow a QTARCH process. Only the first lag of F_t and C_t are included in the model. To compensate for this, the variables have been constructed so as to include the cumulative past deviations that have occurred while the economy has been in the current regime up to a certain specified lag. Both the feedback variables are constructed to be either positive or zero. Each extra time period spent in the 'floor' or 'ceiling' regime leads to a rise in the value of F_t and C_t respectively. Therefore, the role of the feedback variables is to measure the dampening effects on the economy during contractions and expansions. In the EDTAR framework, it is possible to examine alternative definitions for the feedback variables and to introduce different nonlinear effects for the floor and ceiling regimes. This is undertaken in the empirical part of the thesis[12]

This EDTAR model provides a significant generalisation over previously specified EDTAR models. The model by Beaudry and Koop (1993) may be used to illustrate this. Their model is specified as follows[13]:

$$\Delta y_t = \alpha + A(L)\Delta y_{t-1} + \theta_f \text{CDR}_{t-1} + \epsilon_t$$

$$I_{f,t} = \mathbf{1}(\text{CDR}_{t-1} - \Delta y_t > 0), \quad \text{CDR}_t = (\text{CDR}_{t-1} - \Delta y_t) I_{f,t}$$

$$\text{CDR}_t = \max(y_t, y_{t-1}, \ldots) - y_t$$

where $I_{f,t}$ is the index variable and CDR_t is the feedback variable. If, in our model, we specify $y_t^\tau = \max(y_t, y_{t-1}, \ldots)$, $p_r = p_e = 0$, $\theta_c = 0$, $r_f = 0$, $h_t = \sigma$ then $\text{CDR}_t = F_t$ and the two models are equivalent. Therefore, Beaudry and Koop's model is nested within our setup. From the above comparison it can be seen that incorporating the theoretical insights of Hicks' structure in the empirical framework of the EDTAR class of models provides both a significant generalisation over previous work and a flexible setup for modelling nonlinear features of economic time series.

4 Specification of the trend

The trend or long-run growth process of y_t, y_t^τ, is unobserved. Therefore, we need to discuss its estimation. There is a number of suggestions in the literature on the estimation of trend processes. A basic task in the investigation of the empirical models will be the determination of the optimal method to estimate the trend. As models using two different trend estimates will, in general be nonnested, it is theoretically possible to consider nonnested hypothesis testing to evaluate alternative trend estimates. This will not be undertaken in this thesis but

[11]Some notation is shared between the SETAR and EDTAR models. The aim is to unify treatment of the models where possible and reduce the notational burden. No confusion should arise from this practice.

[12]Note that the model can be recast in an expanded TAR form. This form may be found in Chapter 1 of Kapetanios (1998a).

[13]In their case, $x_t = \Delta y_t$.

will be left for future research. Nevertheless, in later Sections we will investigate two different trend estimates. We now mention a number of possibilities for the estimation of the trend.

Firstly, we can use a recursive version of the Hodrick and Prescott (1980) (HP) filter which decomposes the series into a trend and a cyclical component by minimising the sum of the squares of the cyclical component plus the squares of the second differences of the trend component. We propose a recursive filter since we only want past observations to be used in the construction of the trend at each point in time. The HP filter is one of the trend estimates used in Section 9.

Alternatively, we can use the Beveridge and Nelson (1981) decomposition which assumes that the series may be represented by an ARIMA process and decomposes the series into a trend and cyclical component. A third possibility would be to postulate a deterministic function form for the trend, in terms of time and/or other variables. This form may be linear or nonlinear and may include unknown parameters which will be estimated together with the other parameters of the models. Further, the trend may simply be a moving average of the observed series. The moving average would be required to be considerably longer than a typical business cycle to provide information on the underlying trend. Finally, a structural time series approach may be used. In that context, a time series is decomposed into a number of components which, depending on the characteristics of the series, may include a trend, cyclical, seasonal and irregular component. Following specification of the components, the Kalman filter is used to provide the optimal decomposition. This is the second approach considered in Section refch4:robust. Note that the Hodrick-Prescott filter and the Beveridge Nelson decomposition may be obtained through a structural time series approach and, thus, are special cases of this approach. It should be noted that it would be interesting to consider the specification of an empirical model, incorporating the nonlinear effects considered in this paper, in the context of a threshold structural time series model, (see Harvey (1989, pp.348)). This is left to future research. All the above specifications construct the trend based on the actual time series, following the spirit of a reduced form approach. On the other hand, a structural approach, where the trend is specified to be a function of other economic factors, could be used, in the context of alternative theories of economic growth.

5 Estimation and tests for linearity

The issues addressed and the methods proposed in this Section are relevant for both the SETAR and the EDTAR model. As a result the notation which has been introduced earlier and is common to both models will be used throughout this Section. The log-likelihood function of both models has discontinuities with respect to the threshold parameters $\boldsymbol{\gamma}$ which arise because of the QTARCH structure of the error process in both the EDTAR and SETAR models. Additionally, in the SETAR model, discontinuities arise in the conditional mean as well. The log-likelihood function of the EDTAR model is not differentiable with respect to the threshold parameters even if the error process does not have a QTARCH structure.

The above affect estimation on a practical level. Standard maximum likelihood algorithms cannot be applied since the log-likelihood cannot be differentiated with respect to $\boldsymbol{\gamma}$. The usual procedure to deal with this problem is to carry out a grid search for the estimation of the threshold parameters. For each point in the grid, the threshold parameters are taken to be fixed and maximum likelihood estimation is used to obtain the other parameters. The value of $\boldsymbol{\gamma}$ which maximises the log-likelihood over the grid is the maximum likelihood estimate. The use of the data quantiles as possible estimates of the threshold parameters, as suggested by Tong and Lim (1980) and Tsay (1989), is not valid for EDTAR models since any point in the range of the parameter space may maximise the log-likelihood. We will use a grid of evenly spaced points for the estimation of the models.

The discontinuity and nondifferentiability of the log-likelihood function raise theoretical issues as well. These concern the consistency and the asymptotic distribution of the estimates. For SETAR models it has been proven by Chan (1993) that, under certain conditions for x_t, including stationarity and geometric ergodicity, the estimates of $\boldsymbol{\phi}$ and $\boldsymbol{\sigma}$ are $\sqrt{T}$-consistent and asymptotically normal. Their distribution is the same as that for the case when the threshold parameters are known. Additionally, the estimate of $\boldsymbol{\gamma}$ is T-consistent and has a non-standard distribution.

For EDTAR models consistency of the maximum likelihood estimates may be obtained directly given geometric ergodicity of the model. In order to obtain conditions under which the EDTAR model is geometrically ergodic, we need to find its Markovian representation. Then, if a drift condition derived by Tweedie (1975) is satisfied, the process is geometrically ergodic. The drift condition requires that the process moves, on average, towards the center of its state space at each point in time. The Markovian representation of the EDTAR model and a sufficient condition for Tweedie's drift condition to hold are derived in Appendix B of Kapetanios (1998a)). The statement of the sufficient condition may be found in theorem B.1 in the same Appendix. This condition allows for the possibility of an explosive linear structure in the 'corridor' regime, for the EDTAR model as implied by the theoretical structure of the Hicks model. Strong consistency of the estimates is proven in Appendix C of Kapetanios (1998a) along the lines proposed by Altissimo and Violante (1996). As far as the asymptotic distribution of the ML estimates is concerned we note two things. Firstly, the conditional mean of the EDTAR model is not discontinuous. Secondly, it can be represented by the expanded TAR model as in equation (1.18) of Kapetanios (1998a). Then, the results obtained in Chan and Tsay (1998) for TAR models with continuous autoregressive functions are relevant. Chan and Tsay (1998) prove that the least squares parameter estimates of a continuous TAR model, including the threshold parameter are $\sqrt{T}$-consistent and asymptotically normally distributed. This conclusion is radically different from that obtained for discontinuous threshold models. However, the EDTAR model we are investigating is considerably more complex that the simple TAR model analysed by Chan and Tsay. More specifically, the specification of the EDTAR model imposes a number of restrictions across regimes to the expanded TAR model. It is not clear how these restrictions affect the asymptotic distribution of the threshold parameters. Further, the existence of restrictions across regimes makes the use of maximum likelihood estimation necessary in small samples as opposed to conditional least squares considered by Chan and Tsay (1998). Consequently, we follow the treatment of Pesaran and Potter (1997) and consider the threshold parameters as given, in the estimation of the standard errors of the rest of the parameters.

Another issue concerns the choice of the lag order in the autoregressive components of the SETAR and EDTAR models and in the specification of the feedback variables in the EDTAR model. Information criteria, such as those proposed by Akaike (1973) or Schwarz (1978) may be used. However, their validity in a nonlinear context must be investigated. This topic is dealt with in Kapetanios (1999). There, both theoretical arguments and Monte Carlo evidence are given concerning their validity.

Following the estimation of the models, evidence must be provided in favour of them compared to standard linear autoregressive models. This amounts to testing the null of $\phi_{f,i} = \phi_{c,j} = 0$, $i = 1, \ldots, p_f$, $j = 1, \ldots, p_c$, in the SETAR model and the null of $\theta_f = \theta_c = 0$, in the EDTAR model[14]. As it is well known, testing for nonlinearity in a threshold model setup is not as straightforward as carrying out an F-test of the restrictions implied by the null. This happens because threshold parameters are not identified under the null. This is commonly known as the Davies problem following an article by Davies (1977) where the problem was initially tackled in a general way. In testing situations where the Davies problem arises, standard asymptotic theory does not hold. Although the problem is more fundamental, it manifests itself through identically zero score vectors and singular information matrices.

A number of solutions to the general problem of unidentifiability under the null have been proposed. Davies, in his 1977 paper, suggests viewing the set of test statistics, used to test the null, indexed by the underidentified parameter as a random process over that parameter. Then, he provides an upper bound for the probability distribution of the supremum of the test statistics when they are normally distributed. Following upon Davies' seminal work, other contributions to this problem include Davies (1987), Hansen (1992), Andrews and Ploberger (1994) and Hansen (1996). In the context of threshold autoregressive models the problem has been investigated in Chan (1990, 1991), Chan and Tong (1990) and Hansen (1997). Tong (1995) provides an overview of some available theoretical results. A survey on the general problem of unidentifiability under the null is provided by Kapetanios (1998b).

Following Pesaran and Potter (1997), we adopt the testing procedure proposed by Hansen (1996). In his paper, Hansen views the set of test statistics, $W_T(\boldsymbol{\gamma})$, obtained by carrying out a Wald test of the null hypothesis, for each pair of threshold parameter values in the grid, as an empirical process indexed by the threshold parameters. This is, asymptotically, a χ^2 process under the null. He suggests considering a scalar summary statistic, $\hat{\varpi}$, of this process, e.g. the supremum or average Wald test and proposes a simulation algorithm for obtaining critical values for the test. The simulation algorithm involves constructing K replications of a χ^2 process with the same covariance kernel as the original empirical process. Then, the summary statistics from these processes, $\hat{\varpi}^k$, $k = 1, \ldots, K$, are obtained. If the proportion of $\hat{\varpi}^k$, $k = 1, \ldots, K$, lying above $\hat{\varpi}$ is lower than a given significance level the null is rejected. Otherwise it is accepted. The summary statistics of the sets of Wald statistics considered, are the average (AVE), exponential average[15] (EXP) and supremum (SUP) of the Wald statistics. The AVE statistic puts little weight on outlying isolated large Wald statistics. On the other hand, the EXP statistics places heavier weights on large values of the Wald statistics than on

[14]The above restrictions imply linearity in the conditional mean. We also impose linearity in the conditional variance by assuming that, under the null, $\sigma_f = \sigma_c = \sigma_{\text{cor}} = \sigma$.

[15]This statistic is motivated by the contribution of Andrews and Ploberger (1994) of asymptotically optimal tests in the case of underidentified nuisance parameters. In that paper it is shown that the exponential average is the asymptotically locally most powerful test for a model where underidentified nuisance parameters exist. The test is defined as $\ln(\frac{1}{\#\boldsymbol{\Gamma}} \sum_{\boldsymbol{\Gamma}} \exp(\frac{W_T(\boldsymbol{\gamma})}{2}))$ where $\#\boldsymbol{\Gamma}$ is the number of elements in the set $\boldsymbol{\Gamma}$ whose elements are the threshold parameter grid points.

low ones.

6 Estimation results

The series[16] used for the investigation is seasonally adjusted quarterly real U.S. GDP from 1960 to 1994.

6.1 SETAR model

The theoretical discussion in the previous sections suggested a SETAR model with three regimes for the modelling of output.

It is important to note that the detrending transformation renders the series stationary, at least according to the ADF tests carried out. These are given in Table 1. Following initial experimentation with the data, we restrict the SETAR model to have the same intercept term for different regimes. The trend for most of the analysis will be constructed using a recursive Hodrick-Prescott filter[17] as suggested in Section 4. In Section 9 we will consider a structural time series trend. Obtaining the lag orders involves searching over ten different specifications[18] and using the five information criteria presented in Kapetanios (1999). The specifications are as follows[19]:

i. $p_{\text{cor}} = 1$, $p_f = p_c = 1$

ii. $p_{\text{cor}} = 2$, $p_f = p_c = 1$

iii. $p_{\text{cor}} = 2$, $p_f = p_c = 2$, $\phi_{c,1} = \phi_{f,1} = 0$

iv. $p_{\text{cor}} = 2$, $p_f = p_c = 2$

v. $p_{\text{cor}} = 3$, $p_f = p_c = 1$

vi. $p_{\text{cor}} = 3$, $p_f = p_c = 2$, $\phi_{c,1} = \phi_{f,1} = 0$

vii. $p_{\text{cor}} = 3$, $p_f = p_c = 3$, $\phi_{c,1} = \phi_{f,1} = \phi_{c2} = \phi_{f2} = 0$

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[16]The logarithm of the original series multiplied by 100 is used.

[17]The algorithm we use, for the recursive Hodrick Prescott filter works as follows: It is assumed that the vector of past observations, $\mathbf{y}_t = (y_1, \dots, y_t)'$, is used for the construction of $\hat{y}_t^\tau$. Then $\hat{\mathbf{y}}_t^\tau = A^{-1}\mathbf{y}_t$ where $A = I + \zeta K'K$, $K = \begin{pmatrix} 1 & -2 & 1 & \dots & 0 & 0 & 0 \\ 0 & 1 & -2 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -2 & 1 \end{pmatrix}$ and ζ is the Hodrick-Prescott parameter which takes the value 1600. The last element of $\hat{\mathbf{y}}_t^\tau$ is used as an estimate for y_t^τ. The filter must be initialised. So for the first 20 observations of the sample, the simple HP filter is used to obtain the trend. As up to 6 initial observations are lost during the estimation of the models due to the presence of lags, this should not significantly affect the estimation. For more details on the derivation of this algorithm see Danthine and Girardin (1989).

[18]Estimation of the models is by maximum likelihood since restrictions across regimes are imposed on some specifications. It is assumed that the disturbances $\{\epsilon_t\}$ follow the standard normal distribution.

[19]Results for the specification $p_{\text{cor}} = p_f = p_c = 3$ are not reported since the iterative ML estimation did not converge for some threshold parameter grid points.

viii. $p_{\text{cor}} = 3$, $p_f = p_c = 2$

ix. $p_{\text{cor}} = 3$, $p_f = p_c = 3$, $\phi_{c,2} = \phi_{f,2} = 0$

x. $p_{\text{cor}} = 3$, $p_f = p_c = 3$, $\phi_{c,1} = \phi_{f,1} = 0$

The threshold parameter grid was constructed so as to include regimes which occurred for 5% of the observations but not lower. Table 2 presents the chosen specifications[20], parameter estimates, standard errors and supremum, average and exponential Wald tests for nonlinearity[21], presented in page 10 for the five information criteria.

As we can see from the nonlinearity tests, the null of linearity is rejected overwhelmingly. The t-ratios of the 'ceiling' and 'floor' coefficients, although, strictly speaking, not directly applicable as tests of nonlinearity, point to the same direction[22]. The signs of the 'ceiling' and 'floor' coefficients are as expected. During a slowdown of the economy indicated by $y_t < \hat{y}_t^{\tau}$, the deviations from the trend are negative. Negative 'floor' coefficients, provide a nonlinear effect which dampens the recession in the 'floor' regime. In an upturn of the economy, indicated by $y_t > \hat{y}_t^{\tau}$, the deviations are positive. In this case, negative 'ceiling' coefficients provide a dampening effect for the expansion.

All information criteria pick specifications where the deviations at $t-2$ determine the magnitude of the nonlinear effect. It is interesting to compare this result to that obtained by Potter (1995). He, too, finds that the two period ago differences affect nonlinearly the present value of output. In his setup, this is demonstrated by the estimate of the delay parameter. It is also noted that the estimate of the 'floor' threshold is high. As it was mentioned earlier, the threshold estimate may be interpreted as a percentage change in output. So, given that the average growth rate of U.S. GDP is 0.72 %, it turns out that a significant absolute fall in output is needed to trigger a nonlinear dampening effect.

6.2 EDTAR model

The index variables used in the SETAR model will be used here as well.

Two versions of the EDTAR model are considered. Following initial investigations, an alternative specification for the 'ceiling' feedback variable is used. The alternative specification is given by:

$$C_t^* = \sum_{i=0}^{p_e} \left[\Delta y_{t-i} \prod_{j=0}^{i} I_{c,t-j} \right]$$

The second version of the EDTAR model uses this specification. Once again a search is undertaken to determine the orders, p, p_r, p_e. This search is more extensive than the one undertaken for SETAR models and involves 48 different specifications for all possible combinations of $p = 1, 2, 3$ $p_r, p_e = 1, 2, 3, 4$. To facilitate the legibility of the Tables presenting

[20]Note that the criterion values are the result of maximising the opposite of the objective functions of the information criteria.

[21]The number of replications for the construction of the tests is set to 1000 (See page 10)

[22]Note that even though the t-tests are not valid as tests of nonlinearity they are valid in providing evidence of nonlinearity in the conditional mean under the null hypothesis of a model with no nonlinearity in the mean but with heteroscedsticity of the QTARCH form since then the threshold parameters are identified under the null hypothesis.

the results, we note that the specifications are numbered 1 to 48. The first block of 16 are for $p = 1$ and so forth. Within these blocks of specifications for each p, there are 4 blocks. Each block refers to one value of p_r in ascending order. Finally, the smaller blocks contain 4 specifications, one for each value of p_e in ascending order. The threshold parameter grid has a similar structure to that used for the SETAR model. The results for the EDTAR models are presented in Tables 3 and 4.

Rejection of linearity is found for both EDTAR models. The only case where linearity is not rejected at the 5 % level is for model version 1 when the SUP test is used. However, the AVE and EXP tests unambiguously reject linearity. The signs of the feedback variables are as expected. The nonlinear effects dampen deviations from the trend. t-ratios of the 'ceiling' feedback variables indicate that C_t^* is more significant than C_t. The 'floor' threshold parameter estimates are similar for the models considered. The 'ceiling' threshold parameter, obviously, depends on the specification of the 'ceiling' feedback variable. In general, models have lag orders equal to 3. All information criteria agree in that selection. The lag orders p_r and p_e seem to be low overall, indicating that at higher order lags, deviations or differences are of little relevance for the specification of the dampening effects. It is interesting to note that when ICOMP and GIC pick a higher lag order for the 'floor' feedback variable for the first version of the EDTAR model, the estimated coefficients change very slightly, indicating the fact that the 'floor' regime occurs infrequently. On the other hand, when a higher lag order is chosen for the 'ceiling' feedback variable the estimated coefficients change moderately.

Before ending this Section we present, in Table 5, the results of some specification tests carried out on the SETAR and EDTAR models. The tests presented are: the Jarque and Bera (1987) test for residual normality, the LM test for serial correlation in the residuals (Godfrey (1978)) and the LM test for ARCH effects in the residuals (Engle (1982)). All three tests are designed for linear regression models. We claim that their use is justified, asymptotically, for the SETAR models, by the fact that the threshold parameters converge to their true values at a higher rate that the other parameters of the models, and thus may be considered known in large samples, effectively reducing the model to a linear one. For the EDTAR models, we refer to the discussion in page 9. Pending further research on the asymptotic distribution of the maximum likelihood estimates of the parameters of the EDTAR model we provide no theoretical justification for the use of the above specification tests on EDTAR models. When all the residuals are considered we see that normality is overwhelmingly rejected. However, once two outlier residuals[23] are dropped, normality cannot be rejected at all standard significance levels for all but one models[24] Additionally, for all models apart from the first version of the SETAR model the tests find no evidence of serial correlation or ARCH effects.

7 Forecasting performance

This Section investigates the issue of predictive ability of the models considered. Two criteria will be used. Firstly, in-sample predictive ability as measured by the root mean square error

[23] The outliers are the residuals for 1978Q2 and 1980Q2.

[24] One could consider reestimating the model with dummy variables for the two observations. However, note that these outliers do not appear in the actual data but only in the residuals once the models have been estimated.

of one and two step ahead predictions and secondly out-of-sample predictive ability measured similarly[25].

7.1 In-sample predictive ability

Unlike linear models, obtaining predictions from nonlinear models is not straightforward. One step ahead predictions are simple to obtain. At time t $x_{t-1}, \ldots, x_{t-p}$ are used together with the feedback and index variables dated at $t-1$ for the EDTAR and SETAR models respectively to generate the predictions. The construction of two step ahead predictions poses a major problem since the nonlinear feedback and index variables at time t are needed, but are not available and have to be constructed. They depend in a nonlinear way on the shock at time t , ϵ_t. Using a random number generator, K standard normal variates are obtained. These are then used together with $x_{t-1}, \ldots, x_{t-p}$ and the nonlinear variables at time $t-1$ to provide K values for x_t and the nonlinear variables at time t. These are in turn used to obtain K realisations for x_{t+1}. Averaging over these K values gives the final two-step ahead prediction. Asymptotically, this average should converge, in probability and almost surely, to the conditional expectation of x_{t+1} at time $t-1$, as $K \rightarrow \infty$. The in-sample predictive ability of the two models is compared to that of an AR(1), an AR(2) and an AR(3) model. Parameter estimates obtained from the whole of the dataset are used. The measures of performance are the root mean square error for the mean and the variance of the predictions and the correlation between the true and the predicted values of x_t. The formulae used for the RMSE for the mean and the variance are given below

$$\sqrt{\frac{1}{N} \sum_{t=\tau}^{N+\tau-1} \left(x_{t+n} - \hat{E}[x_{t+n}|\omega_{t-1}]\right)^2}$$

$$\left\{ \frac{1}{N} \sum_{t=\tau}^{N+\tau-1} \left\{ \left(x_{t+n} - \hat{E}[x_{t+n}|\omega_{t-1}]\right)^2 - \frac{1}{N} \sum_{t=\tau}^{N+\tau-1} \left[\left(x_{t+n} - \hat{E}[x_{t+n}|\omega_{t-1}]\right)^2 \right] \right\}^2 \right\}^{\frac{1}{2}}$$

where $n = 0, 1$. $\hat{E}[x_{t+n}|\omega_{\tau-1}]$ denotes the simulation estimate of the conditional expectation of x_{t+n} at time $t-1$, described above[26]. N is the number of observations which are being predicted and is set to 100 in our case. τ is the index of the first observation which is predicted. In our case, this observation is 1970Q1. K is set to 10000. The results are presented in Table 6. Note that the last four rows of the Table in this and the next subsection give the ratios of the RMSEs of each model compared to the RMSE of the AR(3) model.

The threshold models are performing better than the linear models on all measures. In terms of mean RMSE the second version of the EDTAR model reaches a 16 % improvement in performance compared to the AR(3) model. In terms of variance RMSE, the improvement reaches 20% for the second version of the EDTAR model and the SETAR model. Overall,

[25] It is a well known problem of linear models that they fail to produce predictions of negative growth. Pesaran and Potter (1997) have provided an EDTAR model capable of producing more, and more accurate, predictions of negative growth than the linear models considered for comparison. This was achieved using a nonlinear model in second differences of output whereas their nonlinear model in first differences could not produce better predictions of negative growth.

[26] ω_{t-1} denotes the realised history of the system at time $t-1$ and is a realisation of the random element Ω_{t-1}.

SETAR and EDTAR models perform similarly.

The above results are encouraging but it would be helpful if a formal statistical procedure were used to verify the claim that predictions from threshold models are significantly better than predictions from linear ones. This may be provided by a test of predictive performance proposed by Diebold and Mariano (1995). The procedure is designed to test the null of equal predictive ability between two models by considering the mean of the differences of squared prediction errors of the two competing models. This mean, suitably normalised, has a standard normal distribution under the null. The test statistic is given by

$$S_{DM} = \frac{\bar{d}}{\sqrt{V(\bar{d})}} \xrightarrow{d} N(0,1), \quad V(\bar{d}) = N^{-1}\left(\hat{\gamma}_0 + 2\sum_{i=1}^{n-1}\hat{\gamma}_i\right)$$

where $\bar{d} = \frac{1}{N}\sum_{i=1}^{N}\hat{d}_i$, $\hat{d}_i = \hat{\eta}^2_{\mathrm{TAR},i} - \hat{\eta}^2_i$, $i = 1, \dots, N$, $\hat{\eta}_{\mathrm{TAR},i}$ are the prediction errors from the nonlinear model, $\hat{\eta}_i$ are the prediction errors from the linear model, N is the number of prediction errors used, $\hat{\gamma}_i$, $i = 0, 1, \dots, n-1$ are the estimated autocovariances of the series of prediction error differences and n is the prediction horizon[27]. Harvey, Leybourne, and Newbold (1997) have proposed a small sample correction for the above test statistic[28]. The revised statistic is given by

$$S^*_{DM} = \left(\frac{N + 1 - 2n + N^{-1}n(n-1)}{N}\right) S_{DM}$$

and the critical values are taken from the t distribution with $N-1$ degrees of freedom. Both tests are used here. The correction makes little difference for this setup but when out-of-sample prediction will be considered in the next subsection it will be important as N will be small. Results are presented in Table 7.

As we can see the tests provide evidence for the superiority of predictions obtained by threshold models[29]

The above results do not address a significant issue on the predictive ability of the threshold models. This is whether threshold models can perform better when the system is in the extreme regimes (i.e. either in the 'floor' or in the 'ceiling' regime). To investigate this issue we present similar results to the above for the three subsets of observations in the period under consideration belonging to the 'floor', 'corridor' and 'ceiling' regime respectively, given the estimated threshold parameters for each model. The results are presented in Table 8. The linear model is an AR(3) model. Note that a different column of results from the AR model correspond to each threshold model. This is because the subsets of observations belonging to specific regimes change as the threshold parameters change.

[27] Note that for $n = 1$ only $\hat{\gamma}_0$ is used in the variance of $\bar{d}$.

[28] For an application of the testing procedure by Harvey, Leybourne, and Newbold (1997), see also Mills and Pepper (1997)

[29] In Chapter 4 of Kapetanios (1998a) where versions of the SETAR and EDTAR models with $x_t = \Delta y_t$ are considered, the Pesaran and Timmermann (1992) nonparametric test is used as a qualitative measure of forecasting ability. This test is a direction of change test where the null is that the direction of the changes in the forecasts are independent from the direction of the changes in the original series. For the in-sample forecasts, the null cannot be rejected for any linear model. On the other hand, the null is rejected in favour of all the nonlinear models.

The most obvious conclusion is that the performance of threshold models in the 'ceiling' and particularly the 'floor' regimes is significantly better than the linear model. However, even in the 'corridor' regime the nonlinear models have a slightly superior performance. This result is expected given the predominance of the 'corridor' regime in the data. The linear model tries to fit best the dominant regime and as a result cannot accommodate the different dynamics of recessions and expansions. Finally, threshold models outperform the linear one by a larger extent in two-step ahead predictions.

7.2 Out-of-sample predictive ability

Out-of-sample predictive ability is perhaps the most important aspect of a successful model. Previous evidence suggests that nonlinear reduced form models provide few advantages, as far as predictive ability is concerned, over simple linear autoregressive models. In this subsection we will compare the predictive performance of the EDTAR and SETAR models against AR(1), AR(2) and AR(3) linear models as before. Recessionary periods are especially important in this investigation because prediction during such periods tends to be less accurate. The rare occurrence of recessionary episodes is partly to blame for that. Thus we will concentrate on the period 1990Q1-1993Q3, which includes the recession of the early 90's. The procedure followed to construct the predictions is as follows:

i. The observations 1990Q1-1994Q4 are removed and the model is estimated from the reduced dataset to obtain the parameter estimates[30]

ii. The parameter estimates are used to provide one-step ahead and two-step ahead predictions in the same way as in the previous subsection.

iii. Successive observations are added to the dataset and steps 1 and 2 are repeated until observation 1993Q2 has been added to the dataset.

The formulae for the RMSEs are as before. As previously, the correlation coefficient between actual and predicted values is also presented. The number of replications used to get the two-step predictions is set to 10000. The results are given in Table 9.

EDTAR models are doing better than linear and SETAR models on all measures. As expected from a nonlinear model, the predictions are more consistent in that the RMSEs of the variances of the predictions are smaller for the EDTAR models than for the linear ones. EDTAR models perform better two steps ahead compared to linear models. The SETAR model performs worse than EDTAR models, overall. In some cases it performs worse that linear models. Given the fact that EDTAR and SETAR models were performing similarly in-sample we can conclude that EDTAR models provide a more valid predictive framework. The Diebold-Mariano statistics, given in Table 10, indicate that the evidence in favour of the threshold models is not as strong as that obtained from in-sample predictions. However, the first version of the EDTAR model performs significantly better than an AR(3) model two steps ahead.

The period under investigation includes a recession. It is possible that the nonlinear models perform better because they are designed to take account of recessions. To test the robustness

[30]The threshold parameters are estimated as well.

of the results we repeat the experiment with the period 1992 Q1 to 1994 Q4 and consider the third and fourth versions of the EDTAR model only. The results, presented in Kapetanios (1998a), indicate that the superior forecasting performance of the nonlinear models is retained.

The above results may be juxtaposed with the work of Clements and Smith (1998), in which the authors provide Monte Carlo evidence suggesting that standard SETAR models may not be able to predict better that linear models, at least according to the RMSE criterion, even if the data analysed are generated according to a SETAR model. This inability is accentuated in the case where the parameters of the models are not assumed known but are estimated from the generated data. Although a proper Monte Carlo investigation similar to that undertaken by Clements and Smith is the right approach to investigate the issues involved, it is possible that the nonlinearity underlying the EDTAR models we have been investigating, is more difficult to approximate using linear structures than that underlying simple SETAR models. However, this is a conjecture and needs to be backed up by future research.

Overall, it seems likely that the feedback variables, as measures of the dampening forces acting dynamically on the system, provide an indicator of its future evolution during recessions and expansions.

8 Generalised impulse response analysis

In this Section the models will be analysed using Generalised Impulse Response functions (GIRF). GIRFs were proposed and discussed by Koop, Pesaran, and Potter (1996) as a way of extending traditional impulse response functions (IRF) in order to analyse nonlinear and multivariate models in a theoretically valid way. Traditional IRFs are only appropriate for use in univariate linear models. Nonlinearity introduces, firstly, a bias when all future shocks are set to zero and secondly, history and shock dependence. For more details on the drawbacks of IRFs see Koop, Pesaran, and Potter (1996).

GIRFs overcome all the drawbacks of IRFs. The basic characteristic of GIRFs is that future shocks are not set naively to zero but are being integrated out of the conditional expectation. The definition of the GIRF for a time series $\{z_t\}$, is given below :

$$GIRF_z(n, \delta, \omega_{t-1}) = E(z_{t+n} \mid \delta, \omega_{t-1}) - E(z_{t+n} \mid \omega_{t-1})$$

where δ is a realisation of the random variable, ϵ_t, which denotes stochastic shocks to the system and ω_t denotes the history of the system up to time t and is a realisation of the random element Ω_t. This is conditional on realisations of ϵ_t and Ω_{t-1}. It is natural to consider conditioning on these random variables instead. This gives the unconditional GIRF defined as:

$$GIRF_z(n, \epsilon_t, \Omega_t) = E(z_{t+n} \mid \epsilon_t, \Omega_{t-1}) - E(z_{t+n} \mid \Omega_{t-1})$$

which is a random variable. Note that the GIRF, conditional on δ and ω_t, is a realisation of the unconditional GIRF.

The procedure followed in the construction of the GIRFs for this paper will now be outlined. The most fruitful approach for analysing the properties of the models is to condition the

GIRF on a history relating to a particular regime. In this paper, we pick a number of most representative periods for each regime and produce GIRFs using the histories pertaining to these observations. The most representative periods for the 'floor' regime are those when the negative distance between y_t and $\hat{y}_t^{\tau}$ is largest. The same holds for the 'ceiling' regime but with positive distance. Finally, the representative periods for the 'corridor' regime are those when the absolute distance between y_t and $\hat{y}_t^{\tau}$ is smallest. GIRFs, in the way explained below, are then constructed for each history and the average over a given regime is taken and plotted in Figures 2 and 3 at the end of the paper. The effect of four different shocks are considered. These shocks are set to $\pm 1, \pm 2$ standard deviations of the disturbance. Mathematically, the theoretical quantity we are trying to estimate is

$$\frac{1}{\#N^i}\sum_{j=1}^{\#N^i} GIRF_x(n,\delta,\omega_{N_j^i-1}) = \frac{1}{\#N^i}\sum_{j=1}^{\#N^i} E(x_{N_j^i+n}|\epsilon_t=\delta,\ \omega_{N_j^i-1}) - E(x_{N_j^i+n}|\omega_{N_j^i-1})$$

where $i =$ f,c,cor is an index indicating the regime, $N^i = \{N_1^i, \ldots, N_{\#N^i}^i\}$ is the set of indices of the observations defining the representative histories for each regime, $\#N^i$ denotes the number of elements[31] of N^i, $\delta = \pm 1, \pm 2$ and $n = 1, \ldots, 25$. The actual construction of each individual GIRF follows the lines proposed in Koop, Pesaran, and Potter (1996). Because of the nonlinearity of the models, the expectations in the above expression are approximated using simulation methods. The construction is as follows:

i. A 25×1 vector of standard normal variates, $\wp$, is generated using a random number generator.

ii. $\wp$, augmented by the given shock, δ, is used together with the nonlinear model under investigation and the estimated parameters to produce the forecasts[32], $x^k_{N_j^i|\delta}, \ldots, x^k_{N_j^i+25|\delta}$, for the shocked system. To minimise sampling variability[33], the same vector of standard normal random variables is augmented by another standard normal variable and used to obtain the 'baseline' forecasts, $x^k_{N_j^i}, \ldots, x^k_{N_j^i+25}$.

iii. The above procedure is repeated K times to produce[34] K sets of forecasts for the shocked systems and K sets of 'baseline' forecasts.

iv. The theoretical GIRF is then estimated by

$$\frac{1}{K}\sum_{k=1}^{K}(\hat{x}^k_{N_j^i+n|\delta} - \hat{x}^k_{N_j^i+n}) \tag{7}$$

The average of (7) over N_j^i given by

$$\frac{1}{\#N^i K}\sum_{j=1}^{\#N^i}\sum_{k=1}^{K}(\hat{x}^k_{N_j^i+n|\delta} - \hat{x}^k_{N_j^i+n})$$

[31] $\#N^i$ was set to 9 for the EDTAR models, 7 for the SETAR model in differences and 8 for the SETAR model in deviations. The same $\#N^i$ is used for all regimes in a given model.

[32] The superscript k indicates that these are values obtained through simulation.

[33] See Ripley (1987, pp. 138).

[34] K was set to 250.

for the three regimes and for all the models considered in this paper is plotted in Figures 2-3. Having given the necessary background concerning the computation of the GIRFs we may start the analysis of the results.

We first investigate EDTAR models. The first thing to note is that, in the 'floor' regime, negative shocks have no persistent effect on the economy. After the initial shock, the GIRFs for both deviations from the trend and differences bounce back to being positive, and counterbalance the initial negative shock. The larger the negative shock is, the larger the response of the system becomes. On the other hand, large positive shocks have a persistent positive effect, unlike smaller positive shocks which make little difference. In the 'ceiling' regime, positive and negative shocks have roughly the same effect, although negative shocks produce a slightly larger and more persistent effect. In the 'corridor' regime shocks are not dampened by nonlinear forces. As a result, effects are more persistent and larger in both directions. There is little evidence that output is I(2). All the graphs in levels indicate that the effects of past shocks die down sooner or later.

Results from SETAR models are similar. The main difference from EDTAR models is that the system does not bounce back as vigorously in the 'floor' regime when a negative shock arises. This is a feature of both the model in deviations and the model in differences. In both cases, responses to negative and positive shocks in the 'floor' are only slightly asymmetrical. For both the EDTAR and the SETAR models, the versions of the models using deviations from the trend, exhibit more persistent effects when shocked.

Overall, the results are in accordance with with our theoretical analysis. In recessions and expansions, nonlinear forces dampen the effects of shocks asymmetrically, depending on the direction of the shock. The most pronounced asymmetry occurs in the 'floor' regime where negative shocks and average shocks produce only slightly different effects. The models are able to reproduce the stylised fact that recessions are followed by strong recoveries discussed in a number of papers[35] since entry into the 'floor' regime produces a more powerful dampening effect, that entry into the 'ceiling' regime, thereby causing a rapid recovery.

9 Alternative trend specification

The models presented in this thesis investigate the properties of trended time series. The question of the construction of the trend has not been addressed in length in the theoretical investigation of the models. The focus of the theoretical discussion was the dynamics of the system around a given, suitably defined, trend. However, it is obvious that the construction of the trend is of crucial importance. The use of the Hodrick-Prescott filter was motivated by its ease of construction which was of importance in the investigation of predictive ability and impulse response analysis rather than by its theoretical suitability. Despite its widespread use, the HP filter has been criticised by a number of authors[36]. King and Rebelo (1993) indicate that the conditions needed for the HP are unlikely to be satisfied in practice. Harvey and Jaeger (1993) claim that the HP filter is too mechanistic and provide evidence suggesting that it can induce spurious cyclical behaviour. Other authors find it suitable for data smoothing

[35]See Sichel (1994), Balke and Wynne (1996) and Emery and Koenig (1992).

[36]However, some evidence in favour of the HP filter is provided by Canova (1994). In this paper the HP filter is found to be the most reliable tool in reproducing standard NBER business cycle dating.

but not for trend extraction as it may not remove from the detrended series frequencies which should belong to the trend (See Pollock (1997, pp. 343-344)). The authors propose alternatives. Harvey and Jaeger propose detrending through the use of a structural time series model. Pollock proposes the use of a square wave filter which induces a sharp separation of the high and low frequency components of a series. It is possible that the results presented in the previous Sections depend crucially on the use of the specific detrending procedure. Thus, we reestimate some of the models using the structural time series approach to detrending. Pollock's suggestion is not considered as the aim of this Section is to investigate model-based techniques as an alternative to heuristic detrending techniques which include the HP filter. As Harvey and Jaeger point out, the HP filter has the same effect as detrending using a structural time series approach in US GNP. Although we use GDP data, it should be expected that the results will not change significantly. Following Harvey and Jaeger we use a local linear trend model with a cycle. The results of the estimation of the nonlinear models, using the trend estimate obtained from the structural time series (STS) model, are presented in Tables 13 and 14. Additionally, in Figure 1 the two alternative detrended series based on the HP and structural time series trends are presented.

The results indicate that the models are robust to the specification of the trend using two alternative detrending procedures. All the main features of the estimation results of the models are retained. It should be noted that for the second version of the model and some information criteria a radically different value for the 'floor' threshold is found. It seems likely that the likelihood function with respect to the threshold parameters has multiple maximisation points. The important nonlinear effects identified previously are present in these results as well. All but one of the tests for nonlinearity reject the null of linearity. We do not attempt to repeat the predictive ability and impulse response analysis undertaken previously since the computational burden and programming requirements are prohibitive.

10 Model selection

In the previous Sections of the paper, tentative conclusions have been reached concerning the suitability of the models as specifications for the evolution of US GDP. Linearity and standard misspecification tests and predictive ability analysis have been used for reaching those conclusions. However, the issue of model selection has not been addressed using formal model evaluation tools. In this Section, an attempt to use such tools is made. The models under consideration will be a 2 regime and a 3 regime SETAR model, and the two versions of the EDTAR trend model. Following Monte Carlo evidence on the performance of information criteria in Kapetanios (1999), we will use three information criteria: Akaike (AIC), Schwarz (SC) and Hannan-Quinn (HQ). Estimation of the 2 regime SETAR model is carried out along the same lines as for the estimation of the 3 regime SETAR model described in Section 5. Note that the values of the information criteria for the models given in previous Tables are not comparable between classes of models since different numbers of observations are used for each class, following truncations of different length at the beginning of the sample. To correct for this effect all models have been reestimated at the baseline sample size of 134 observations used for the estimation of the EDTAR models. The results are given[37] in Table 11. Addi-

[37]The opposite of the objective function of each criterion is used. Therefore, larger values for the criteria indicate more suitable models.

tionally, in Table 12 we provide similar results for models estimated using the structural time series model trend analysed in the previous Section.

The results for the HP filter are ambiguous. Two criteria suggest that the 3-regime SETAR model is best. However, SC picks the 2-regime SETAR model. The EDTAR models perform badly and no criterion selects them as the preferred model. When a structural time series trend is used, the conclusions change radically. The EDTAR models are clearly preferred. According to the criteria, the fourth version of the EDTAR model should be preferred over the third. The choice between the two SETAR models remains ambiguous.

We next consider nonnested hypotheses testing. Two setups are considered The first considers the two SETAR models and the second considers the 3-regime SETAR model versus the first version of the EDTAR model. We use two bootstrap testing procedures, denoted by LB and LB_p, described in detail in Kapetanios (1998a). 199 bootstrap replications are carried out. For the first setup and under the null of a 2-regime SETAR model, the p-value of the LB test is 0.1909 and that of the LB_p test is 0.1608. For the null of a 3-regime SETAR model the respective values are 0.8140 and 0.1407. As we can see the null is accepted in both cases, albeit with a higher p-value in the case of LB and the null of a 3-regime SETAR model. This fact indicates the limitations of nonnested testing as a model evaluation method. No clear-cut selection can be made and therefore the issue of which model to choose is not resolved. For the second setup, the conclusion is more clear. Under the null of a 3-regime SETAR model the p-values of LB and LB_p are 1 and 0.9396 respectively. Under the null of an EDTAR trend model they are 0.0854 and 0.0050. These indicate a rejection of the EDTAR model in favour of the 3-regime SETAR model. The conclusions are in accordance with the evidence from information criteria. Although no testing is carried out for the models using a structural time series trend, it is valid to conjecture that the conclusion would be different, if the evidence from the information criteria analysis is taken under consideration.

11 Conclusion

In this paper we proposed empirical nonlinear reduced form threshold models based on a theoretical model of the business cycle developed by Hicks. Two classes of threshold models were used to provide the econometric specification of the empirical models. Issues concerning estimation and linearity testing have been extensively discussed.

The models developed may be viewed as nonlinear error correction models. To see that, we claim that there exists a cointegrating relation between y_t and y_t^τ. This is reasonable, given our specification of y_t^τ as the long-run growth process of y_t. However, unlike linear error correction models where the deviation from the trend would be modelled as a stationary series throughout its range, the nonlinear model we propose allows for nonstationary dynamics in the 'corridor' regime. The specification of the 'ceiling' and 'floor' provide the error correction mechanism, which, under certain conditions discussed in Appendix B of Kapetanios (1998a) for the EDTAR model, provides global stability for the deviation from the trend. In the EDTAR model, the feedback variables F_t and C_t may then be considered as error correction terms. A major advantage of the nonlinear error correction specification over its linear counterpart is the treatment of asymmetry as the 'floor', 'corridor' and 'ceiling' regimes obey different linear laws of motion. Further, the setup of the EDTAR model allows for a variety of alternative error correction specifications. This interpretation of the models links them with the relatively small but expanding literature on nonlinear error correction and cointegration (see for example Granger and Lee (1989), Escribano and Mira (1996) and Escribano and Pfann (1998)). The last paper proposes a variety of specifications for nonlinear error correction models. Among these specifications, the authors provide a piecewise linear specification which resembles the SETAR model presented here. However, no explicit connection to threshold models is made.

During the empirical presentation of the models significant nonlinear effects were found and linearity was rejected. Analysis of the predictive performance of the models indicated that the nonlinear models perform better than linear ones. Impulse response analysis provided further insight concerning asymmetries in the evolution of output during recessions and expansions. Finally, model selection in the context of threshold models was undertaken. Overall, evidence suggests that further work on the models presented may be beneficial for the analysis and prediction of a variety of business-cycle based macroeconomic series.

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Table 1: ADF tests for $y_t - \hat{y}_t^\tau$

	$y_t - \hat{y}_t^\tau$
ADF(1)	-3.5227
ADF(2)	-4.1529
ADF(3)	-3.9682
ADF(4)	-4.2408
95% Critical Value	-2.883

Table 2: Results for the SETAR model

	Information Criteria[a]				
	AIC[b]	SC[c]	GIC[d]	ICOMP[e]	HQ[f]
Model[g]	6	6	6	6	6
Criterion[h]	-12.2687	-25.4086	-12.3684	-5.5891	-17.6085
$\phi_{\mathrm{cor},0}$	$-0.0229_{(0.0622)}$	$-0.0229_{(0.0622)}$	$-0.0229_{(0.0622)}$	$-0.0229_{(0.0622)}$	$-0.0229_{(0.0622)}$
$\phi_{\mathrm{cor},1}$	$1.1508_{(0.0710)}$	$1.1508_{(0.0710)}$	$1.1508_{(0.0710)}$	$1.1508_{(0.0710)}$	$1.1508_{(0.0710)}$
$\phi_{\mathrm{cor},2}$	$0.1505_{(0.1070)}$	$0.1505_{(0.1070)}$	$0.1505_{(0.1070)}$	$0.1505_{(0.1070)}$	$0.1505_{(0.1070)}$
$\phi_{\mathrm{cor},3}$	$-0.3362_{(0.0724)}$	$-0.3362_{(0.0724)}$	$-0.3362_{(0.0724)}$	$-0.3362_{(0.0724)}$	$-0.3362_{(0.0724)}$
$\phi_{f,2}$	$-0.5303_{(0.0735)}$	$-0.5303_{(0.0735)}$	$-0.5303_{(0.0735)}$	$-0.5303_{(0.0735)}$	$-0.5303_{(0.0735)}$
$\phi_{c,2}$	$-0.1916_{(0.0909)}$	$-0.1916_{(0.0909)}$	$-0.1916_{(0.0909)}$	$-0.1916_{(0.0909)}$	$-0.1916_{(0.0909)}$
σ_f	$0.3893_{(0.1013)}$	$0.3893_{(0.1013)}$	$0.3893_{(0.1013)}$	$0.3893_{(0.1013)}$	$0.3893_{(0.1013)}$
σ_c	$0.6212_{(0.1068)}$	$0.6212_{(0.1068)}$	$0.6212_{(0.1068)}$	$0.6212_{(0.1068)}$	$0.6212_{(0.1068)}$
σ_{cor}	$0.6423_{(0.0431)}$	$0.6423_{(0.0431)}$	$0.6423_{(0.0431)}$	$0.6423_{(0.0431)}$	$0.6423_{(0.0431)}$
r_f	2.7800	2.7800	2.7800	2.7800	2.7800
r_c	1.6900	1.6900	1.6900	1.6900	1.6900
SUP	0.0000	0.0000	0.0000	0.0000	0.0000
AVE	0.0000	0.0000	0.0000	0.0000	0.0000
EXP	0.0000	0.0000	0.0000	0.0000	0.0000

[a] Standard errors of the parameter estimates are given in parentheses, where applicable
[b] Akaike's Information Criterion
[c] Schwarz's Information Criterion
[d] Generalised Information Criterion (See Kapetanios (1999) for details)
[e] Informational Complexity Criterion (See Kapetanios (1999) for details)
[f] Hannan-Quinn Information Criterion
[g] Model Specification selected
[h] Value of the opposite of the minimised objective function of the information criterion

Table 3: Results for the first version of the EDTAR model

	Information Criteria[a]				
	AIC	SC	GIC	ICOMP	HQ
Model	33	33	37	41	33
Criterion	-15.9058	-28.9461	-15.7031	-11.3493	-21.2049
ϕ_0	$-0.0098_{(0.0651)}$	$-0.0098_{(0.0651)}$	$-0.0098_{(0.0651)}$	$-0.0098_{(0.0652)}$	$-0.0098_{(0.0651)}$
ϕ_1	$1.1777_{(0.0844)}$	$1.1777_{(0.0844)}$	$1.1777_{(0.0844)}$	$1.1777_{(0.0844)}$	$1.1777_{(0.0844)}$
ϕ_2	$0.0784_{(0.1224)}$	$0.0784_{(0.1224)}$	$0.0784_{(0.1224)}$	$0.0784_{(0.1224)}$	$0.0784_{(0.1224)}$
ϕ_3	$-0.2981_{(0.0807)}$	$-0.2981_{(0.0807)}$	$-0.2981_{(0.0807)}$	$-0.2981_{(0.0807)}$	$-0.2981_{(0.0807)}$
θ_f	$1.0822_{(0.2758)}$	$1.0822_{(0.2758)}$	$1.0818_{(0.2757)}$	$1.0818_{(0.2757)}$	$1.0822_{(0.2758)}$
θ_c	$-0.1430_{(0.0815)}$	$-0.1430_{(0.0815)}$	$-0.1430_{(0.0815)}$	$-0.1430_{(0.0815)}$	$-0.1430_{(0.0815)}$
σ_f	$0.8323_{(0.1897)}$	$0.8323_{(0.1897)}$	$0.8324_{(0.1897)}$	$0.8324_{(0.1897)}$	$0.8323_{(0.1897)}$
σ_c	$0.5413_{(0.0713)}$	$0.5413_{(0.0713)}$	$0.5413_{(0.0713)}$	$0.5413_{(0.0713)}$	$0.5413_{(0.0713)}$
σ_{cor}	$0.6532_{(0.0475)}$	$0.6532_{(0.0475)}$	$0.6532_{(0.0475)}$	$0.6532_{(0.0475)}$	$0.6532_{(0.0475)}$
r_f	2.7500	2.7500	2.7500	2.7500	2.7500
r_c	1.1400	1.1400	1.1400	1.1400	1.1400
SUP	0.0610	0.0610	0.0650	0.0720	0.0610
AVE	0.0000	0.0000	0.0000	0.0000	0.0000
EXP	0.0000	0.0000	0.0010	0.0000	0.0000

[a]For an explanation of the notation used see notes in Table 2

Figure 1: Detrended GDP series based on the HP filter and the STS trend plus cycle model

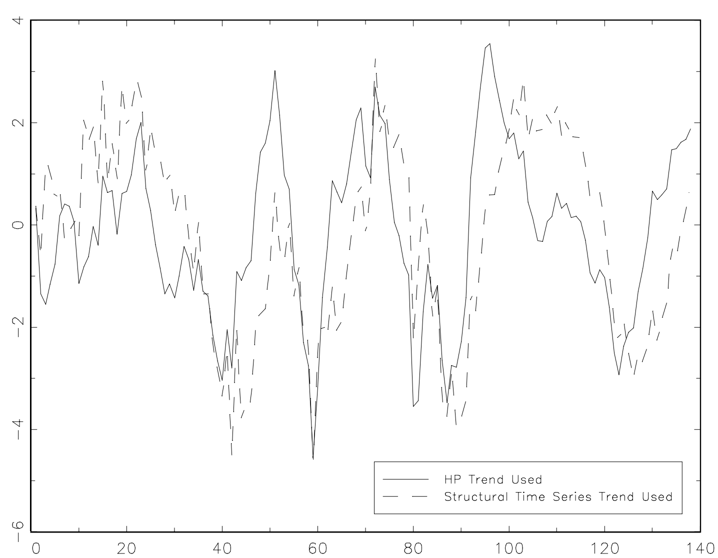

Table 4: Results for the second version of the EDTAR model

	Information Criteria[a]				
	AIC	SC	GIC	ICOMP	HQ
Model	34	34	34	37	34
Criterion	-13.8056	-26.8459	-13.5120	-9.2075	-19.1047
ϕ_0	$0.0019_{(0.0601)}$	$0.0019_{(0.0601)}$	$0.0019_{(0.0601)}$	$-0.0004_{(0.0599)}$	$0.0019_{(0.0601)}$
ϕ_1	$1.2039_{(0.0829)}$	$1.2039_{(0.0829)}$	$1.2039_{(0.0829)}$	$1.2184_{(0.0848)}$	$1.2039_{(0.0829)}$
ϕ_2	$0.0770_{(0.1198)}$	$0.0770_{(0.1198)}$	$0.0770_{(0.1198)}$	$0.0744_{(0.1199)}$	$0.0770_{(0.1198)}$
ϕ_3	$-0.3003_{(0.0801)}$	$-0.3003_{(0.0801)}$	$-0.3003_{(0.0801)}$	$-0.3154_{(0.0803)}$	$-0.3003_{(0.0801)}$
θ_f	$1.1300_{(0.2749)}$	$1.1300_{(0.2749)}$	$1.1300_{(0.2749)}$	$1.1414_{(0.2758)}$	$1.1300_{(0.2749)}$
θ_c	$-0.1823_{(0.0598)}$	$-0.1823_{(0.0598)}$	$-0.1823_{(0.0598)}$	$-0.2318_{(0.0765)}$	$-0.1823_{(0.0598)}$
σ_f	$0.8455_{(0.1931)}$	$0.8455_{(0.1931)}$	$0.8455_{(0.1931)}$	$0.8453_{(0.1933)}$	$0.8455_{(0.1931)}$
σ_f	$0.5669_{(0.1051)}$	$0.5669_{(0.1051)}$	$0.5669_{(0.1051)}$	$0.5686_{(0.1052)}$	$0.5669_{(0.1051)}$
σ_{cor}	$0.6206_{(0.0421)}$	$0.6206_{(0.0421)}$	$0.6206_{(0.0421)}$	$0.6205_{(0.0421)}$	$0.6206_{(0.0421)}$
r_f	2.7500	2.7500	2.7500	2.7500	2.7500
r_c	1.8300	1.8300	1.8300	1.8300	1.8300
SUP	0.0030	0.0030	0.0030	0.0040	0.0030
AVE	0.0000	0.0000	0.0000	0.0000	0.0000
EXP	0.0000	0.0000	0.0000	0.0000	0.0000

[a]For an explanation of the notation used see notes in Table 2

Table 5: Specification test results

Tests	SET V1[a]	ED V1	ED V2
Jarque-Bera[b]	18.3426	14.0015	24.6143
p-value	0.0001	0.0009	0.0000
Jarque-Bera[c]	1.4335	0.8135	3.2022
p-value	0.4883	0.6658	0.2017
Outliers	2	2	2
SC(LM)[d]	3.8589	4.1402	2.6894
p-value	0.4254	0.3874	0.6111
ARCH(LM)	4.2602	3.7160	5.2101
p-value	0.3719	0.4458	0.2664

[a]SET V1: SETAR model, ED V1: First version of the EDTAR model, ED V2: Second version of the EDTAR model.

[b]Normality test using all residuals

[c]Normality test when outliers have been dropped

[d]LM test for serial correlation

Table 6: In-sample prediction results

	AR(1)	AR(2)	AR(3)	ED V1[a]	ED V2	SET V1
RMSE 1-step (M)	0.7889	0.7474	0.7341	0.6655	0.6513	0.6363
RMSE 2-step (M)	1.2021	1.1291	1.1148	0.9778	0.9437	0.9716
RMSE 1-step (Var)	1.0922	1.0429	1.0234	0.9128	0.9095	0.8917
RMSE 2-step (Var)	1.9977	1.8229	1.8261	1.4888	1.4597	1.4562
ρ 1-step[b]	0.9000	0.9108	0.9141	0.9301	0.9331	0.9369
ρ 2-step	0.7470	0.7795	0.7856	0.8404	0.8525	0.8450
RMSE 1-step(M) ratio to AR(3)	107.4619	101.8021	100.0000	90.6557	88.7207	86.6737
RMSE 2-step(M) ratio to AR(3)	107.8327	101.2826	100.0000	87.7109	84.6515	87.1557
RMSE 1-step (Var) ratio to AR(3)	106.7213	101.9066	100.0000	89.1893	88.8744	87.1305
RMSE 2-step (Var) ratio to AR(3)	109.3973	99.8249	100.0000	81.5278	79.9346	79.7422

[a] For an explanation of the notation used see notes in Table 5

[b] ρ denotes the correlation coefficient between the predictions and the actual observations

Table 7: Tests for in-sample prediction accuracy

Models Tested against the AR(3) Model	Test Statistics [a]			
	DM[b] (1-step)	DM*[c]	DM (2-step)	DM*
ED V1[d]	−2.5690*	−2.5561*	−3.2326*	−3.1836*
ED V2	−2.9006*	−2.8861*	−3.8933*	−3.8342*
SET V1	−3.0740*	−3.0586*	−3.3739*	−3.3227*

[a] Starred entries indicate significance at the 5% significance level

[b] Diebold-Mariano test

[c] Diebold-Mariano corrected test

[d] See notes in Table 5

Table 8: In-sample prediction results for specific regimes

Regime	Statistic	ED V1[a]	AR(3)	ED V2	AR(3)	SET V1	AR(3)
Floor Regime	RMSE 1-step (M)	0.8323	1.1262	0.8455	1.1262	0.3893	1.0654
	RMSE 2-step (M)	1.0147	1.6027	0.9992	1.6027	0.9084	1.5587
	RMSE 1-step (Var)	0.8687	1.1955	0.8730	1.1955	0.1232	0.9457
	RMSE 2-step (Var)	1.6870	2.9011	1.5952	2.9011	1.4652	2.9650
	ρ 1-step	0.5974	0.3678	0.5917	0.3678	0.8903	0.8035
	ρ 2-step	0.3424	-0.2272	0.3559	-0.2272	0.1586	-0.2995
Corridor Regime	RMSE 1-step (M)	0.7002	0.7340	0.6499	0.6905	0.6727	0.7224
	RMSE 2-step (M)	0.9692	1.0031	0.9592	1.0066	0.9658	1.0056
	RMSE 1-step (Var)	1.0736	1.1391	0.9804	1.0464	1.0075	1.1053
	RMSE 2-step (Var)	1.5492	1.6562	1.5408	1.5166	1.5421	1.5385
	ρ 1-step	0.8630	0.8643	0.9036	0.9043	0.9048	0.9058
	ρ 2-step	0.4828	0.3915	0.6814	0.6330	0.6536	0.6108
Ceiling Regime	RMSE 1-step (M)	0.5025	0.5236	0.4983	0.6006	0.5695	0.5740
	RMSE 2-step (M)	0.8173	0.9106	0.8232	1.0706	0.9219	1.0262
	RMSE 1-step (Var)	0.3033	0.3199	0.3429	0.4004	0.3876	0.3848
	RMSE 2-step (Var)	0.8015	1.0020	0.8365	1.2385	0.9397	1.1892
	ρ 1-step	0.7477	0.7352	0.8116	0.6778	0.7130	0.7002
	ρ 2-step	0.4349	0.3448	0.5596	0.1128	0.3674	0.1894

[a]For an explanation of the notation used see notes in Tables 5 and 6

Table 9: Out-of-sample prediction results

	AR(1)	AR(2)	AR(3)	ED V1[a]	ED V2	SET V1
RMSE 1-step (M)	0.5035	0.4512	0.4585	0.4440	0.4635	0.4680
RMSE 2-step (M)	0.8720	0.8269	0.8072	0.7206	0.7699	0.7407
RMSE 1-step (Var)	0.3276	0.2414	0.2549	0.1990	0.2068	0.2201
RMSE 2-step (Var)	0.9525	0.9400	0.8991	0.6904	0.5830	0.7271
ρ 1-step	0.9118	0.9311	0.9285	0.9301	0.9279	0.9229
ρ 2-step	0.7126	0.7601	0.7780	0.8166	0.7993	0.8034
RMSE 1-step(M) ratio to AR(3)	109.8075	98.3968	100.0000	96.8408	101.0835	102.0723
RMSE 2-step(M) ratio to AR(3)	108.0245	102.4478	100.0000	89.2686	95.3820	91.7588
RMSE 1-step (Var) ratio to AR(3)	128.5093	94.7007	100.0000	78.0619	81.1217	86.3265
RMSE 2-step (Var) ratio to AR(3)	105.9420	104.5525	100.0000	76.7929	64.8405	80.8775

[a]For an explanation of the notation used see notes in Tables 5 and 6

Table 10: Tests for out-of-sample prediction accuracy

Models Tested[a] against the AR(3) Model	Test statistics			
	DM (1-step)	DM*	DM (2-step)	DM*
ED V1	-1.0425	-1.0072	-0.8138	-0.7260
ED V2	-0.7085	-0.6844	-0.2212	-0.1973
SET V1	0.3846	0.3715	0.4191	0.3739
ED V3	-0.3570	-0.3449	-2.7975^*	-2.4958^*
ED V4	0.0777	0.0751	-0.4056	-0.3618
SET V2	0.2212	0.2137	-1.4315	-1.2771

[a]For an explanation of the notation used see notes in Tables 5 and 7

Table 11: Model Selection Using Information Criteria for models using an HP filter trend

Model	Information Criteria		
	AIC	SC	HQ
2-regime SETAR	-14.2662	-26.1323	-19.2513
3-regime SETAR	-13.5770	-26.6172	-18.8761
EDTAR V3	-15.9058	-28.9461	-21.2049
EDTAR V4	-13.8056	-26.8459	-19.1047

Table 12: Model Selection Using Information Criteria for models using a structural time series trend

Model	Information Criteria		
	AIC	SC	HQ
2-regime SETAR	-51.5222	-62.2871	-56.2080
3-regime SETAR	-51.3355	-63.2301	-56.3846
EDTAR V3	-48.1729	-60.0919	-53.2464
EDTAR V4	-47.2658	-58.7970	-51.9515

Table 13: Results for the first version of the EDTAR model under a STS trend

	Information Criteria[a]				
	AIC	SC	GIC	ICOMP	HQ
Model	37	22	41	41	22
Criterion	-48.1729	-60.0919	-48.0048	-42.2746	-53.2464
ϕ_0	$0.0238_{(0.1032)}$	$0.0062_{(0.1023)}$	$0.0238_{(0.1031)}$	$0.0238_{(0.1031)}$	$0.0062_{(0.1023)}$
ϕ_1	$0.7207_{(0.0867)}$	$0.6567_{(0.0792)}$	$0.7207_{(0.0867)}$	$0.7207_{(0.0867)}$	$0.6567_{(0.0792)}$
ϕ_2	$0.4573_{(0.0845)}$	$0.3877_{(0.0755)}$	$0.4573_{(0.0845)}$	$0.4573_{(0.0845)}$	$0.3877_{(0.0755)}$
ϕ_3	$-0.1349_{(0.0736)}$		$-0.1349_{(0.0736)}$	$-0.1349_{(0.0736)}$	
θ_f	$0.7015_{(0.2042)}$	$0.6784_{(0.2005)}$	$0.7015_{(0.2042)}$	$0.7015_{(0.2042)}$	$0.6784_{(0.2005)}$
θ_c	$-0.1953_{(0.1139)}$	$-0.1222_{(0.0781)}$	$-0.1953_{(0.1139)}$	$-0.1953_{(0.1139)}$	$-0.1222_{(0.0781)}$
σ_f	$0.7287_{(0.1524)}$	$0.7144_{(0.1492)}$	$0.7287_{(0.1524)}$	$0.7287_{(0.1524)}$	$0.7144_{(0.1492)}$
σ_c	$0.5086_{(0.0588)}$	$0.5224_{(0.0605)}$	$0.5086_{(0.0588)}$	$0.5086_{(0.0588)}$	$0.5224_{(0.0605)}$
σ_{cor}	$1.0421_{(0.0823)}$	$1.0496_{(0.0830)}$	$1.0421_{(0.0823)}$	$1.0421_{(0.0823)}$	$1.0496_{(0.0830)}$
r_f	2.9000	2.9000	2.9000	2.9000	2.9000
r_c	1.1000	1.1000	1.1000	1.1000	1.1000
SUP	0.0030	0.0090	0.0060	0.0060	0.0090
AVE	0.0000	0.0000	0.0000	0.0000	0.0000
EXP	0.0010	0.0050	0.0030	0.0030	0.0050

[a]For an explanation of the notation used see notes in Table 2

Table 14: Results for the second version of the EDTAR model under a STS trend

	Information Criteria[a]				
	AIC	SC	GIC	ICOMP	HQ
Model	32	32	45	45	32
Criterion	-47.2658	-58.7970	-47.4755	-41.3989	-51.9515
ϕ_0	$-0.1687_{(0.1302)}$	$-0.1687_{(0.1302)}$	$0.0379_{(0.0971)}$	$0.0379_{(0.0971)}$	$-0.1687_{(0.1302)}$
ϕ_1	$0.6926_{(0.0790)}$	$0.6926_{(0.0790)}$	$0.7451_{(0.0871)}$	$0.7451_{(0.0871)}$	$0.6926_{(0.0790)}$
ϕ_2	$0.5618_{(0.0916)}$	$0.5618_{(0.0916)}$	$0.4616_{(0.0820)}$	$0.4616_{(0.0820)}$	$0.5618_{(0.0916)}$
ϕ_3			$-0.1597_{(0.0751)}$	$-0.1597_{(0.0751)}$	
θ_f	$0.1060_{(0.0346)}$	$0.1060_{(0.0346)}$	$0.7122_{(0.2002)}$	$0.7122_{(0.2002)}$	$0.1060_{(0.0346)}$
θ_c	$-0.1352_{(0.0497)}$	$-0.1352_{(0.0497)}$	$-0.1842_{(0.0834)}$	$-0.1842_{(0.0834)}$	$-0.1352_{(0.0497)}$
σ_f	$0.9102_{(0.0844)}$	$0.9102_{(0.0844)}$	$0.7318_{(0.1528)}$	$0.7318_{(0.1528)}$	$0.9102_{(0.0844)}$
σ_c	$0.4995_{(0.0571)}$	$0.4995_{(0.0571)}$	$0.4965_{(0.0574)}$	$0.4965_{(0.0574)}$	$0.4995_{(0.0571)}$
	$1.1891_{(0.1451)}$	$1.1891_{(0.1451)}$	$1.0420_{(0.0822)}$	$1.0420_{(0.0822)}$	$1.1891_{(0.1451)}$
r_f	0.3000	0.3000	2.9000	2.9000	0.3000
r_c	1.1000	1.1000	1.1000	1.1000	1.1000
SUP	0.0600	0.0600	0.1290	0.1290	0.0600
AVE	0.0000	0.0000	0.0000	0.0000	0.0000
EXP	0.0080	0.0080	0.0070	0.0070	0.0080

[a]For an explanation of the notation used see notes in Table 2

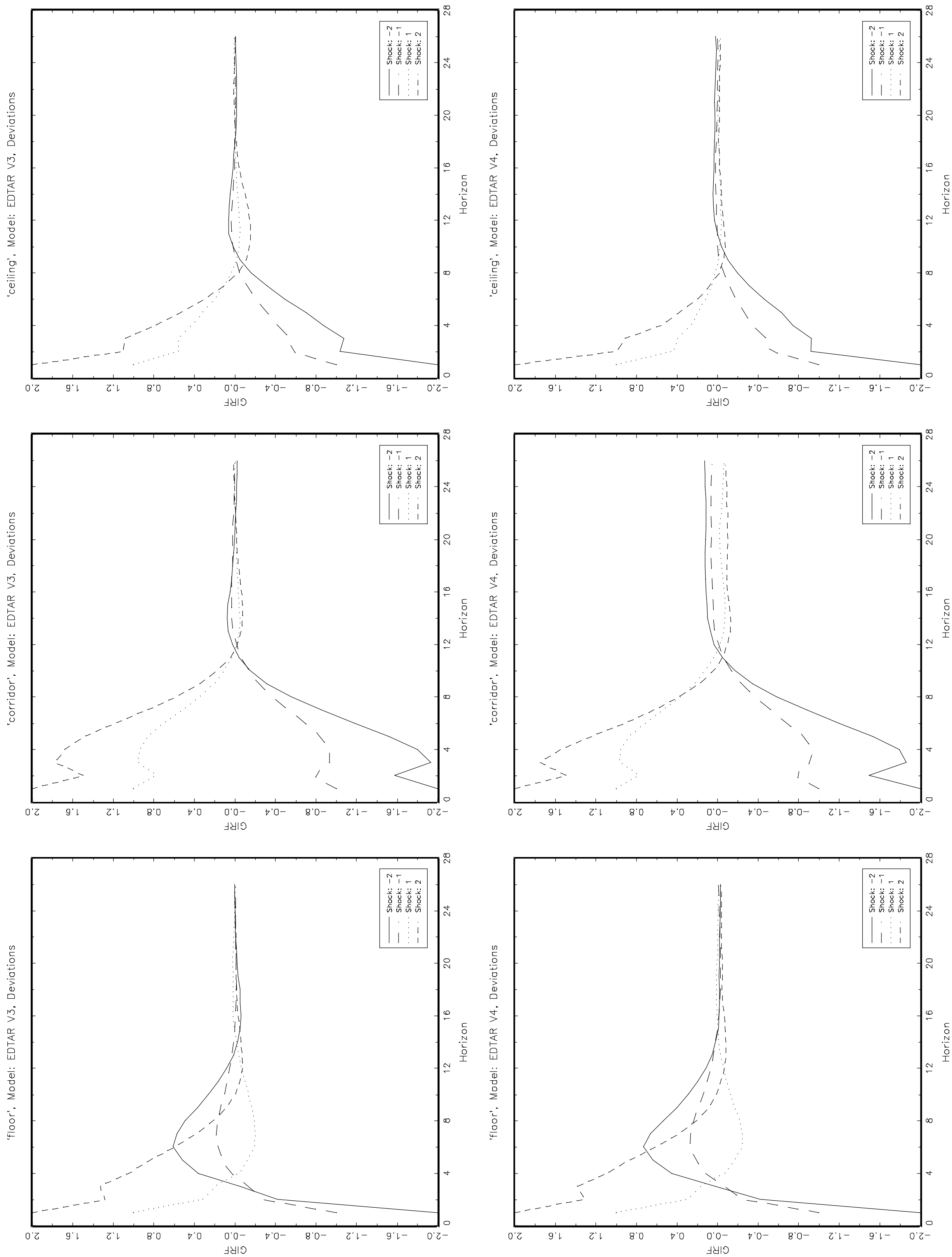

Figure 2: Generalised Impulse Response Functions for the first and second versions of the EDTAR model

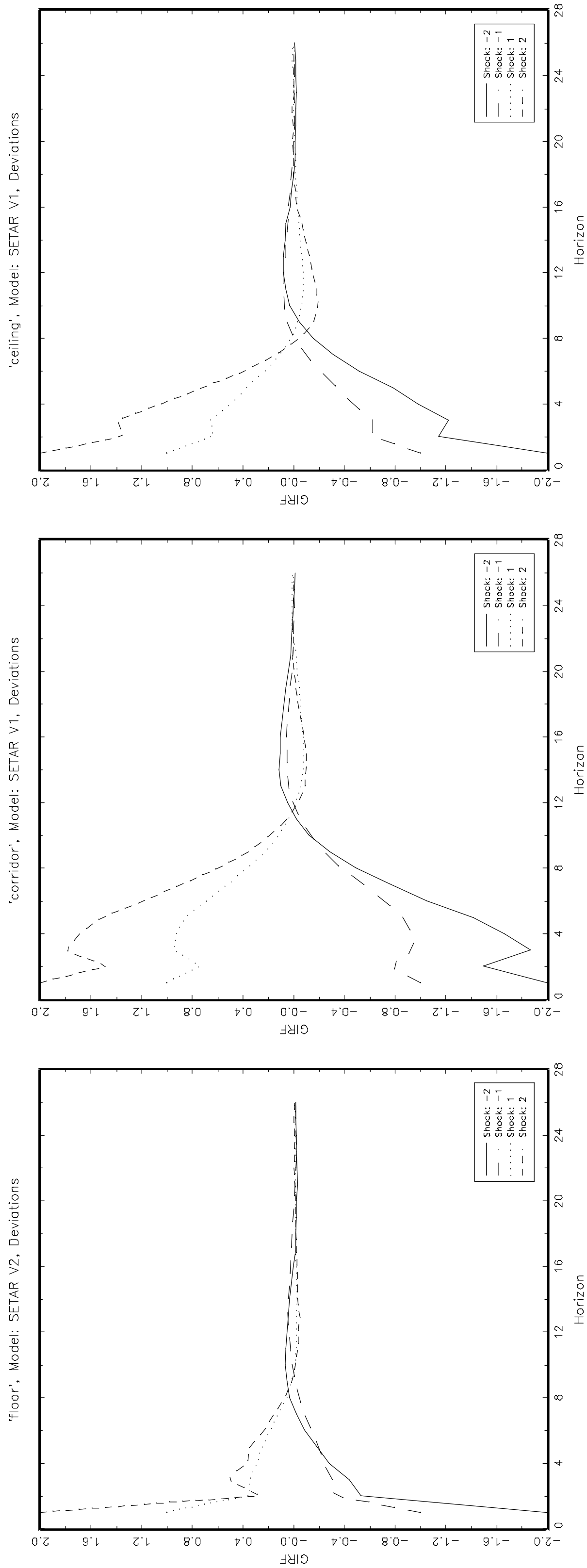

Figure 3: Generalised Impulse Response Functions for the SETAR model