Abstract

In the context of liberalisation of electricity markets world wide, the need for agreed protocols for electricity trades between systems with different charges poses a special challenge. System operators need to know how much a given trade uses the network, in order to allocate an appropriate portion of their costs to that trade. This paper discusses a technique, tracing, for determining how much each of a number of trades uses different parts of the electricity network. The scheme is based on the assumption that at any network node, inflows are shared proportionally between outflows (and vice versa). The paper outlines the technique and shows how it could be applied to the problem of charging cross-border trades. The paper goes on to demonstrate that the technique has a game theoretic rationale, in that it produces the Shapley value solution to a game equivalent to this allocation problem.

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A TRACING METHOD FOR PRICING INTER-AREA ELECTRICITY TRADES*

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1 INTRODUCTION

Around the world, electricity industries are being restructured and liberalised. Electricity is being treated as a commodity to be bought and sold by generators, retailers (suppliers) and other traders. Vertically integrated utilities are being broken up, which allows end users and distributors to buy power from more distant generators. No commodity can be traded, however, unless there are appropriate arrangements for its delivery. In the electricity industry, this is the responsibility of transmission companies, and the special nature of electricity poses a number of challenges. These make transmission charging a complex subject, and many different approaches have been adopted around the world. However, to allow electricity trades between systems with different charges it is necessary to agree protocols for cross-border trades. In particular, system operators need to know how much a given trade is making use of the network, in order to allocate an appropriate portion of their costs to that trade. This paper discusses a technique, tracing, for determining how much each of a number of trades is using different parts of the electricity network. It outlines the technique and shows how it could be applied to the problem of charging cross-border trades. The paper also demonstrates that the technique has a game theoretic rationale, in that it produces the Shapley value solution to a game equivalent to the problem we study.

Chile was the first country to introduce an electricity wholesale market, in 1978, but the world-wide trend towards deregulation can be dated from the start of the 1990s. In England and Wales, the nationalised electricity industry was divided into three generating companies, competing to sell power in a wholesale spot market, the Pool. Power was then carried to the dozen (pre-existing) distribution companies by a new transmission company, the National Grid Company. In Norway, existing trading arrangements developed into NordPool's formalised spot market, which gradually expanded to cover Sweden, Finland, and part of Denmark. New Zealand developed a spot market based on the marginal cost pricing ideas of Schweppe et al (1988). By the end of the decade, the US Federal Energy Regulatory Commission was requiring the electricity industry to form regional transmission organisations in order to better co-ordinate trading, and the European Union was working towards a Single European Market in electricity.

Electricity markets cannot be operated in isolation from transmission systems. It is essential to balance generation and demand on a second-by-second basis, or risk damage to the entire network. This risk, and the need to ensure that all customers receive their power at a satisfactory voltage, place limits on the amount of electricity that can flow through each part of the network. The amount of electricity that does flow through each part of the network depends upon its design, and the pattern of generation and demand, and is given by Kirchhoff's laws. In the short term,

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transmission operators can do little to change the network configuration, and must adjust generation or demand if power flows are in danger of breaching the safe limits. In an integrated system, transmission operators simply instructed generators to operate in a manner that could meet demand without breaching transmission limits, and were paid for generation and transmission together. In some market-based systems, the transmission operator still gives direct instructions to generators, but these must now be accompanied by payments, to ensure that the generators are willing to obey the instructions. In other systems, market prices do most of the work of signalling to generators. Green (1997) surveys some of the systems in use around the world; the key point for our purposes is their great variety.

The transmission company also needs to be paid. Generally, the cost of transmission consists of four main elements: (i) transmission losses; (ii) operation and maintenance of the network; (iii) return and depreciation of the capital equipment; (iv) the cost of resolving transmission constraints (congestion). Typically, transmission systems have separate charges for losses, for operation, maintenance and capital costs (use-of-system charges) and for resolving congestion. This paper concentrates on the case of losses, but the algorithm we propose, which assigns responsibility for the flows in each part of the network, could also be used to set use-of-system and congestion charges. The regulatory challenge is to devise an open access charging system that recovers common costs with fairness, while inducing efficient use of the grid by participants. The system must provide incentives for efficient use of the existing system as well as incentives for future development of the network, depending on the changing needs of the market.[1] At the same time, the charging system must be practical enough for real time application and transparent enough to be politically acceptable (Green, 1997). Different countries placed different emphasis on each of these requirements and, as the result, there are hardly two countries in the world with identical transmission pricing regimes.

This creates a problem when we come to cross-border trades. How should these be charged for? If two adjacent systems shared a common transmission pricing methodology, it would be straightforward (economically, if not always politically) to think of them as a common system and obtain the relevant prices. Cadwalader *et al* (1999) show how this could be done for a group of system operators using spot prices. When the systems have chosen to implement very different transmission prices, this approach is not feasible. A specific methodology for cross-border trades is required. For example, the European Transmission System Operators' association, ETSO, has proposed a simple methodology (ETSO, 2000a,b), in which operators would charge 2 Euro/MWh on all cross-border flows. The level has been calculated to recover the share of the costs of the main ("horizontal") grid imposed by cross-border trades. This charge might be appropriate for cross-border trades that went straight between two neighbouring countries. In practice, however, some trades are designed to cover long distances, and it would be more appropriate for these trades to pay a higher share of the costs of the grid. Furthermore, "loop flows" imply that a high proportion of the power will flow through third countries, which are not parties to the trade, but will have to accommodate the electricity flowing through their grids. The ETSO methodology appears to imply that these countries would be charged for their "exports".

Economic efficiency is encouraged when agents face the consequences of their actions. In this case, the cross-border charges should be paid by the original exporter, not the country suffering from the loop flow. Conventional wisdom says that it is impossible to trace the flow of power from individual generators to individual loads in meshed electricity networks, so that it would be impossible to identify the agents causing any particular cross-border flow. Bialek (1996) has

[1] Few systems rely on market mechanisms for investments in transmission. Most transmission investment is still based on decision-making by the incumbent transmission company, and the point we are making is that the transmission charging system should not create perverse incentives that discourage efficient investments.

suggested a tracing-based methodology that does assign flows to individual generators and loads. The underlying assumption behind this methodology is that the flow of power in the network can be represented by a directed graph in which the flows are mixed proportionally at every node. This means that the costs of loop flows could be allocated to the agents causing them. It would also allow transit charges to be related to the costs involved in each route. Bialek (2000) shows how the methodology could be applied to set charges for cross-border flows.

The methodology does not require any information about flows within each country. Nor is it linked to transmission charges within a country. This means that intra-national transmission charging can be left to each country, following the principle of subsidiarity. Furthermore, these national charges should recover the bulk of each country's transmission costs, with inter-national transit charges only meeting the incremental costs of cross-border trade. If international charges are set too high, cross-country transactions are subject to "pancaking", paying several sets of transmission charges in turn. These charges are likely to greatly exceed the true costs of transmission, and wrongly make many transactions uneconomic. At the same time, cross-border transactions should face charges reflecting the *incremental* costs they impose, or they will be artificially favoured. ETSO has suggested that the proportion of network costs to be met from transit charges should equal the ratio of transit flows to transit flows plus home country consumption. This appears to be a reasonable rule of thumb, and the detailed design of a tariff is not the aim of this paper. Instead, we wish to show how the tracing-based methodology can assign responsibility for flows, and illustrate its theoretical links to the Shapley value, a well-known solution concept in cost allocation problems.

In the next section of the paper, we show why charges based on marginal losses may not be appropriate for cross-border trades. In section 3, we introduce the tracing methodology. Section 4 discusses the game-theoretic justification for the tracing methodology. In section 5, we show how it could be applied to cross-border trades. Section 6 concludes.

2 TRANSMISSION PRICING OF INTER-SYSTEM TRADES

Transmission pricing is one of the most important problems to be solved when introducing market principles to the trading of electricity. In economical terms, transmission is an externality which has to be internalised into energy trading.

The theory of optimal pricing of electricity has been developed by Schweppe *et al.* (1988). In this approach, an optimal short-run price for electricity is determined at every node from the submitted bids (or cost curves) of every generator and every load. This price is a combination of the system marginal cost of generation at the swing bus p^*, the impact on losses, and the impact on congestion.

In this paper the loss charge is of particular interest and therefore we will start by assuming that the network is not congested. Then the optimal price of electricity at node k can be seen as the value of electricity at the swing bus, p^*, modified by the cost of transporting it from the swing bus to that node:

$$p_k = p^* \left[1 + \frac{\partial \text{ losses}}{\partial d_k} \right] \qquad (1)$$

where d_k is the demand at node k. The derivative $\partial \text{losses} / \partial d_k$ is often referred to as the marginal Transmission Loss Factor (TLF) at node k.

In a pool with the true optimal nodal prices, there is no need to decompose the nodal price into generation and losses as the transmission price is contained in the overall nodal price p_k (Rivier and Perez-Arriaga, 1993). However, in order to facilitate transaction-based trading in the system, loss charges are often calculated and charged separately from the main energy auction. A bilateral transaction from node *i* to node *j* would then pay a loss charge equal to:

$$(\text{loss charge})_{i\text{-}j} = p^* \left[\frac{\partial \text{ losses}}{\partial d_j} - \frac{\partial \text{ losses}}{\partial d_i} \right] \tag{2}$$

Marginal pricing for losses sends the optimal pricing signals to all the users but its implementation, especially for inter-area trades, can be very difficult. Firstly, calculation of marginal loss factors may be cumbersome for a large interconnected system. It requires a lot of data exchange between the areas and a number of additional assumptions have to be made. Also the computational time for a system containing thousands of nodes may be excessive. Secondly, the methodology itself is non-transparent and this opaqueness increases with the system size. Non-transparent and difficult to verify charges are bound to be met with suspicion from the market participants. Markets that are opaque will discourage new entry and lessen price competition. Thirdly, the marginal loss factors are known to be volatile hence requiring of some additional hedging mechanisms. The cost of hedges increases the transaction costs. Fourthly, due to convexity of the loss function (transmission losses are approximately proportional to the square of the power flow), the sum of collected loss charges is higher than the actual sum of losses. Hence an additional mechanism would be necessary in order to distribute the surplus. And finally, anomalies and opportunities for arbitrage could occur if marginal loss charging was applied for inter-area trades but the losses inside each area were not charged on the marginal basis. This however would be very difficult to achieve as each area usually follows their own internal pricing methodology. Making them adopt the same methodology, whether marginal or any other, would probably meet very strong resistance.

These problems could be increased if marginal pricing was also used to recover the costs of congestion. At times, the desired pattern of generation and demand implies that too much power would flow down some lines. In extreme cases, the desired flows would be sufficient to overload the line, but the problem is more usually that the system would have no security margin. If a line or generator fails, the flows around the network will instantly redistribute themselves, increasing the flows on some lines. The system controllers must ensure that there is enough spare capacity on all of these lines that they would not be overloaded after a fault. The calculations and judgements involved are inevitably opaque to most observers.

The end result, however, is that some generators will be required to produce more, and others to produce less. The generators required to produce more (“constrained on”) will almost inevitably have relatively high costs (or they should have been running anyway), while those required to produce less will have relatively low costs. At nodes with a constrained generator, the appropriate nodal spot price is generally equal to the marginal cost of that generator. At other nodes, spot prices are derived from the impact of demand at the node on the flow across each congested link, and the cost of dealing with that congestion:

$$p_k = p^* \left[1 + \frac{\partial \text{ losses}}{\partial d_k} \right] + \sum_i \mu_i \frac{\partial z_i}{\partial d_k} \tag{3}$$

where z_i is the flow along the congested line i, and μ_i is the shadow price of the constraint – the cost of rescheduling generation or demand to reduce flows along the link by 1 MW. Note that demand at some nodes will reduce the flow along a congested link, and if the cost of that congestion is high, the price at that node could easily become negative. Nodal prices that incorporate congestion effects are particularly volatile and can be hard to predict. This has discouraged their practical application in many, though not all, markets.

The two simple alternatives to marginal pricing, which are already widely used, are the postage stamp methodology and contract path pricing. The postage stamp methodology is the simplest as it allocates a uniform transmission price per MWh to all the transactions without regard to the location of the buyer or the seller. As such, the methodology completely neglects any transmission effects and sends no locational signal. The grid operator typically deals with congestion by counter-trading, buying power from high-cost constrained-on generators, and selling it back to low-cost constrained-off generators, and smears the costs across all consumers.

The contract path methodology works as follows. If there is a contract from country A to country B, the two countries determine arbitrarily the physical path along which the electricity is deemed to flow. They then pay the owner(s) of this path for using it – the payment might be negotiated, or according to a tariff. In fact, however, electricity flows on a number of physical paths, which are rarely properly reflected in the contract path agreement. This phenomenon is called *loop* or *parallel flow*. Some of the power will probably flow along lines owned by companies that are not parties to the transmission contract, but are required to bear the costs imposed by it. In some cases, if these flows run the risk of over-loading circuits, these costs can be severe. Thus, although very attractive from the point of view of simplicity and ease of application, the contract path methodology is widely believed to be a fiction endangering the secure operation of the power system.

3 THE TRACING METHODOLOGY

3.1 The principle

To overcome the problems related to charging based on marginal losses, while providing better signals than the postage stamp or contract path methodologies, Bialek (1996) suggested the tracing-based methodology. Conventional wisdom says that it is impossible to trace the flow of power from individual generators to individual loads in meshed transmission networks. Assuming, however, that at any network node the inflows are distributed proportionally between the outflows, it is possible - by following the acyclic directed graph (digraph) of flows in the network - to trace how real and reactive power flows in the network from individual sources to individual sinks. In other words, the tracing methodology allows us to "tag MWs", i.e. establish physical paths linking generators and the loads. Consequently transmission charges can be calculated, as with the traditional contract path approach. It should be stressed that the tracing paths are based on the physical power flows in the network while the contract paths are based on the financial contracts which usually do not reflect the physics of transmission.

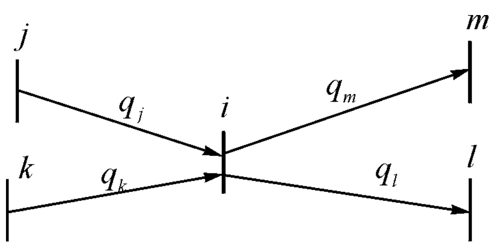

Figure 1: Proportional sharing rule

Electricity tracing is based on the proportional sharing rule illustrated in figure 1 which shows node *i* connected with neighbouring nodes *j, k, m,* and *l* by four lines: *j-i, k-i, i-m,* and *i-l*. Real power flowing into node *i* is denoted by q_j and q_k, respectively, while power flowing out of node *i* is denoted by q_m and q_l, respectively. Obviously $q_j + q_k = q_m + q_l$. Nodes *j* and *k* can be either some other nodes in the system or local generators supplying node *i*. Nodes *m* and *l* can be either some other nodes in the system or local demands supplied from node *i*. The question is how the inflowing power is distributed among the outflows. In the absence of any additional information, the most logical assumption is that the network node is a perfect "mixer" of incoming flows so that nodal inflows are shared proportionally between the outflows. This would give the following result:

- q_m is assumed to consist of two components: $\frac{q_j}{q_j + q_k} q_m$ coming from q_j and $\frac{q_k}{q_j + q_k} q_m$ coming from q_k
- similarly q_l is assumed to consist of $\frac{q_j}{q_j + q_k} q_l$ coming from q_j and $\frac{q_k}{q_j + q_k} q_l$ coming from q_k
- q_j is assumed to consist of $\frac{q_m}{q_m + q_l} q_j$ going to q_m and $\frac{q_l}{q_m + q_l} q_j$ going to q_l
- q_k is assumed to consist of $\frac{q_m}{q_m + q_l} q_k$ going to q_m and $\frac{q_l}{q_m + q_l} q_k$ going to q_l

The proportional sharing principle can be extended to all the network nodes and allows electricity to be traced in the network by a series of recursive calculations. A more general matrix-based solution is presented in the Appendix, while the next section demonstrates an intuitive graph-based approach due to Kirschen *et al* (1997).

3.2 Using the Tracing-based Methodology to Allocate Transmission Charges

To use the graph-based approach to the tracing-based methodology, the network nodes need to be re-ordered so that pure source node of the directed graph is first and the pure sink node is the last[2]. We will initially assume that the directed graph of network flows is *acyclic*, i.e. contains no cycles. Consider for example a simple network shown in figure 2a. The numbers on the diagram correspond to real power flows. Re-ordering the nodes by following the flows will result in a network shown in figure 2b. All the flows are now seen to flow from the left to the right. We will trace the flow of power in the network downstream (from the left to the right) which will allow us to calculate the usage of all the lines by all the generators and allocate the losses to all the loads.

The tracing algorithm will be introduced by tracing the flow of power downstream, i.e. from the pure source node of the digraph (node #1) to the pure sink node (#3). A dual tracing upstream is obviously also possible but will not be shown here due to lack of space. For downstream tracing let us introduce the concept of the gross power that would flow in the network if it were fed with the actual generation and the network was lossless. A gross flow is equal to the sum of the actual flow and the transmission losses accumulated in all lines supplying a given line or node. As the gross flows are lossless, the value of the flow at the sending and the receiving end is the same. The downstream algorithm applied to the real power flows shown in figure 2b would then give the gross flows shown in figure 2c obtained in the following manner:

[2] In the pure source node, power flows out from the node in all the connecting lines. In the pure sink node, power flows into the node in all the connecting lines.

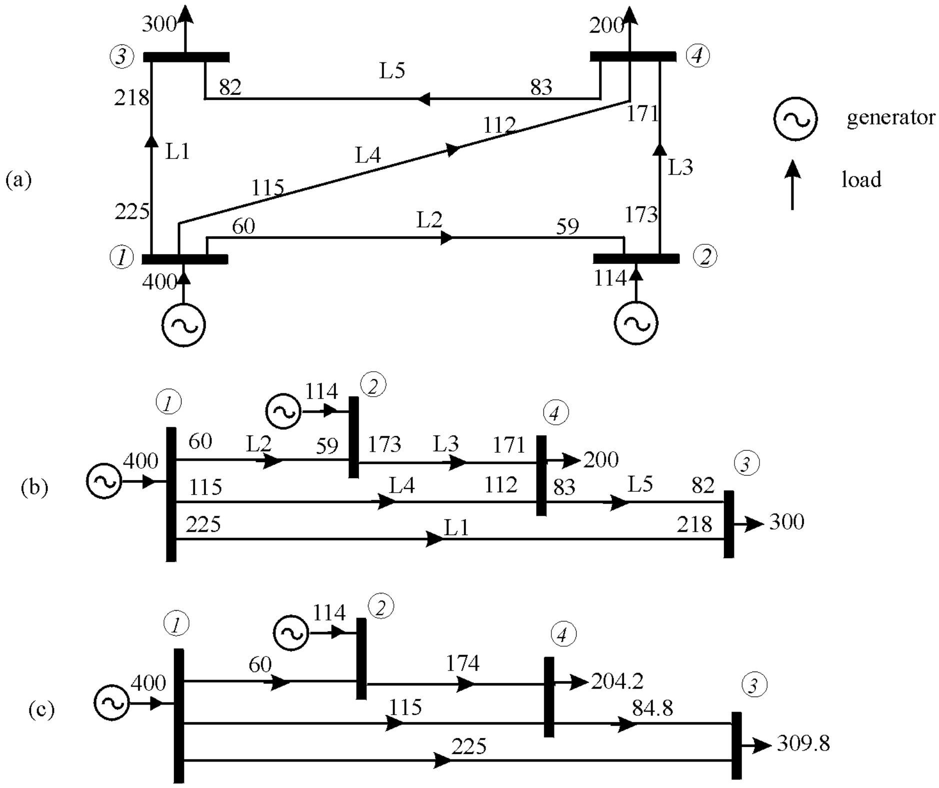

Figure 2: Real power flows in the example network: (a) actual network; (b) network corresponding to the acyclic graph of flows; (c) gross flows in the network.

Node 1: As this is the pure source node, power outflowing in lines L1, L2 and L4 comes exclusively from G1 and is equal to the power at the sending end of the line.
Node 2: The gross nodal power is equal to 60 + 114 = 174 supplied from G1 via L2 and from G2. The power outflowing through L3 has to be scaled up proportionally so that it is equal to the gross nodal power of 174. The composition of power outflowing through L3 can be calculated using the proportionality principle as $\frac{174}{174}60 = 60$ from G1 and $\frac{174}{174}114 = 114$ from G2.
Node 4: The gross nodal power is equal to (115 supplied from G1 via L4) + (174 supplied via L3) = 289. Adding the components originating from G1 gives the following composition of the gross nodal power: (115 + 60) = 175 supplied from G1 and 114 from G2. The gross nodal outflows have to be now scaled up proportionally as their sum must be equal to the gross nodal power. This gives the gross flow in L5 equal to $83\frac{289}{83+200} = 84.8$ and the gross demand D4 equal to $200\frac{289}{83+200} = 204.2$. The composition of the inflows in line L3 has been calculated above. The composition of the outflows is then:

D4: $\frac{175}{289}204.2 = 123.7$ from G1 and $\frac{114}{289}204.2 = 80.5$ from G2.

L5: $\frac{175}{289}84.8 = 51.3$ from G1 and $\frac{114}{289}84.8 = 33.5$ from G2.

Node 3: The gross nodal power is equal to the sum of gross nodal inflows, i.e. 84.8 + 225 = 309.8. Adding the inflowing components originating from the same generators gives the following decomposition: D3 can be obtained by adding the shares supplied by lines L5 and L1 as: (51.3 + 225) = 276.3 from G1 and (33.5 + 0) = 33.5 from G2. As D4 is the only nodal outflow, these are also the components of D4. Note that gross demand D4 has also to be scaled up ($300\frac{309.8}{300} = 309.8$) to be equal to the nodal inflows

Table 1: Results of the downstream tracking of gross flows.

From	To						
	D3	D4	L1	L2	L3	L4	L5
G1	276.3	123.7	225	60	60	115	51.3
G2	33.5	80.5	-	-	114	-	33.5
Total	309.8	204.2	225	60	174	115	84.8
Loss allocated	9.8	4.2					

Table 1 summarises the results of the calculations. The sum of the contributions of individual generators gives the actual generation (276.3 +123.7 = 400 for G1 and 33.5 + 80.5 = 114 for G2) while the sum of contributions for individual loads does not give the actual load demand. The last row in table 1 shows the difference between the gross and the actual demand. As the gross flow was defined as the sum of the actual flow and the loss accumulated in all the lines supplying a given element, the last row in table 1 gives the transmission loss allocated to individual loads. The sum of the allocations is equal to 14, that is the total transmission loss. In effect, the losses are passed down the network starting from the pure source node and finishing at the pure sink node. At each intermediate network node the losses are distributed proportionally between the outflows.

Table 1 also allows us to determine the contributions of individual generators to line flows. If a particular line is congested, the contributions may be used to allocate congestion costs among the generators. It is also possible to calculate the "usage" of the system as a whole by multiplying contributions of a particular generator by the line cost and adding up all the components for all the lines[3] (Bialek, 1997). This allows us to allocate the use-of-system, or access, charge to individual generators.

In a similar manner the upstream tracing of flows, i.e. from the right to the left in figure 2, can also be conducted (Bialek and Kattuman, 1999). The main difference is that the net flows (i.e. with the transmission loss taken away) rather than the gross flows will be considered. The results are summarised in table 2. The sum of the contributions to individual loads gives the actual demand (267.4 +32.6 = 300 for D3 and 120.3 + 79.7 = 200 for D4) while the sum of contributions of individual generators does not give the actual generation. The last row in table 2 shows the difference between the net and the actual generation. As the net flows were defined as the difference between the actual flow and the loss accumulated in all the lines supplied by a given element, the last column in table 2 gives the transmission loss allocated to individual generators. The sum of the allocations is equal to 14, that is the total transmission loss.

Table 2 also allows us to determine the contributions of individual loads to line flows. This can be used to allocate the congestion costs and use-of-system charges to individual loads.

[3] It is also possible to calculate in a similar manner usage of node-based elements such as e.g. substations.

Table 2: Results of upstream tracing of net flows.

To	From						
	G1	G2	L1	L2	L3	L4	L5
D3	267.4	32.6	218	16.9	49.5	32.5	82
D4	120.3	79.7	-	41.2	120.9	79.1	-
Total	387.7	112.3	218	58.1	170.4	111.6	82
Loss allocated	12.3	1.7					

One of the problems connected with the practical implementation of the tracing algorithm is the problem of circular flows. These circular flows may be present when the sub-systems of the interconnected network are represented by single supernodes. A simple example is given in figure 3 which shows circular flows in a network. Country A is a net exporter while countries B and C are net importers. The total loss is 10 MW. There is no pure source node to start the calculations from or a pure sink node to end them. Hence the graph-based algorithm outlined above would fail to trace the flows. However the matrix-based algorithm outlined in the Appendix would provide a fair allocation of losses (Bialek & Kattuman, 1999; Bialek, 2000).

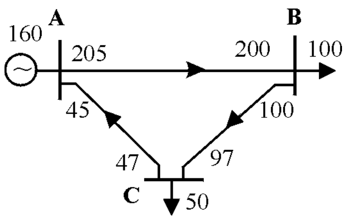

Figure 3: Example of circular flows.

4 GAME THEORETIC RATIONALE OF TRACING

4.1 Cost Allocation Games

The principle behind the tracing methodology is not a physical principle, and we need to examine its rationale. Transmission loss recovery is a cost allocation problem. A transmission network connects a certain number of nodes where electricity is either generated and/or consumed, through branches or lines. Some of the nodes are source nodes (power flows out of these nodes in all connected branches) and some are sink nodes (power flows into these nodes in all connected branches), the rest are intermediate nodes where incoming flow(s) are shared out to outgoing flow(s). Given the vectors of nodal generation and demands, the pattern of network connections, and the impedances of lines, Kirchhoff's laws determine power flows in the lines of the network. What is the best way to allocate the total transmission loss in the network between generators? In economics, problems of this type are usefully stated in the form of dividing the cost of a jointly used facility among participants in a co-operative venture.

We may start by considering the allocation among the generators. In those terms, generators i=1,2, ..., n use the transmission grid to supply their generated power, q_i, to users, and it is necessary to divide up the total transmission loss $c(q_1, ..., q_n)$ among the generators in an efficiency-inducing, fair, and individually as well as jointly acceptable way. This cost allocation process can be carried out line by line, since total transmission loss is the sum of transmission losses in the lines.[4] Because power flows are additive in any line, the cost function for any line takes a simple symmetric form: $c(\Sigma q_i) = r(\Sigma q_i)^2$, where r is resistance and i indexes the set of supplying generators[5] . The first question is: how should this amount be recovered from each generator? To proceed line by line, it is necessary to "trace", for each generator, the distributed flow of its power over all the lines.

The natural framework for the study of cost allocation problems is game theory (Shubik, 1962; Young, 1994). This framework takes note of strategic aspects of the situation and also enable an examination of the properties of allocation rules. Willing co-operation in the joint enterprise is the essence of cost sharing, and the appropriate framework is the theory of co-operative games. We show that for an appropriately defined transmission loss allocation game, the proportional sharing assumption leads to a cost allocation solution that satisfies all desirable properties one would look for in a solution.

Consider the proportional sharing principle shown in figure 1. What is the best way to allocate the total transmission loss sustained in the lines upstream from the node in question between the node outflows? In order to answer this question first consider figure 4 below which shows an elementary segment of the network where three lines are connected to a single node; this is the building block for any network of whatever size or complexity in the sense that the design of a satisfactory loss allocation rule is preserved when segments of this type are put together. The node is denoted i; two lines carry inflows (q_j in line j-i, from node indexed by j, and q_k in line k-i, from node indexed by k) and one line carries outflow to some other node. The total power flow through the node is the sum of the power flows in j-i and k-i, and this is the power flow in i-m: $q = q_j + q_k$. The transmission loss in line i-m is given by: $A(q) = rq^2$. The loss should be allocated to nodes j and k, since they are responsible for the flow. What apportioning rule should be applied to $A(q)$ in this case?

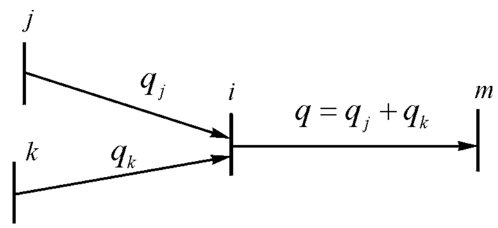

Figure 4: Network segment with single outflow line.

Figure 1 presented earlier extends the above to include more than one outflow. There are two lines carrying outflows from i (q_m in line i-m and q_l in line i-l). Total power flow through the node is still q. How much of the outflow in each line comes from each of the inflows? The answer is of importance in cost allocation. With a convex cost function, if the power flow in i-m is greater than that in i-l, then loss in i-m is much greater than in line i-l. If k, for instance, is assumed to supply most its output to i-l, it will be expected to bear a much lower loss levy than if it is assumed to supply most of its output to line i-m. What proportion of k's output do we attribute to i-l, and what proportion to i-m?

[4] See section 3.2 for a discussion of how the tracing methodology can be used to allocate congestion and use-of-system charges. The game theoretic analysis here deals only with the issue of transmission loss allocation but the results are equally applicable to the allocation of other transmission costs.

[5] The function is strictly quadratic when q represents the line current. When power flows are considered, the relationship is approximately quadratic. The argument below is not restricted to the quadratic functional form. It applies to any function $f(.)$ that is symmetric in its arguments.

4.2 The Game

The key task is to define the appropriate game. It is relatively simple to choose and apply the appropriate solution concept from the set of well understood solution concepts to any game once defined. A general cost allocation game, is fully specified in terms of the (finite) set of participants or players, N:={1,2, ..., n}, their demands to supply through the grid, represented by the vector q:=(q_1, q_2,…, q_n) and $c = c(q_1, q_2,…, q_n)$, the cost function (here the transmission loss) or the characteristic function. This is the data of the allocation problem.

4.2.1 Players

The context of transmission loss allocation is that of a fixed number of generators supplying a set of lines[6]. The levies to recover transmission losses must ultimately fall on generators, and it is natural to think of N as the set of generators. However, transmission loss is due to the flow of power, and it is more useful to identify the set of players as the set of units of power (e.g., MWs) flowing through the network.[7] The cost allocation suggested by the equilibrium of this co-operative game would specify a levy for each player; i.e., each unit of power. The allocation of each generator can be obtained by adding up the levies upon the units of power generated by it. In the general cost allocation context, the characteristic function, c, specifies the minimal cost that will be incurred by each *coalition* of players arranging matters to suit its members. The notion of a coalition requires some interpretation to fit this context.

4.2.2 Coalitions

Coalitions capture the essential strategic element in co-operative games. Rational players may be expected to take advantage of possibilities of coalition formation. For example, in a stable equilibrium each participant will have compared any proposed allocation with what it is able to get by "working alone" to the extent that is possible. Further, any group of players who find that they can do better for themselves by co-operating only among themselves and excluding others from their arrangement, could form a coalition and hold out for their worth, in the formation of any other coalition. The equilibrium must respect all prospects of such coalition formation. It follows that the "worth" of any player, the share the player can be expected to get in the game as a whole, must be related to her worth to all possible coalitions. In the canonical formulation, each subset of {1, ..., R} is a potential coalition; there are 2^R coalitions. The characteristic function, c, attaches a real number, denoting the minimal cost that will be incurred by it, to each one of 2^R coalitions possible. If an allocation is such that none of all possible coalitions can do better for themselves, it is a good candidate for the equilibrium of the game. Such allocations are said to be in the core of the game, and denote solutions acceptable to all players. This general notion of the coalition can be interpreted to fit the problem of transmission loss allocation, as follows.

[6] Allocation to the loads (demands) introduces no conceptual difference.

[7] See Littlechild and Owen (1993) for an early example of this type of abstract game. They analyze optimal airport landing charges, and the players in their game are aircrafts of different types. Costano-Pardo and Garcia-Dias (1995) treat a highway cost allocation problem in terms of a game where each passage made on a highway is a player in a game. The standard cost allocation games have firms as players, for example, Billera and Heath (1982) and Linhart et. al. (1995) on sharing revenues from caller id services, where telecommunication companies are players, and Aadland and Kolpin (1995) on sharing joint irrigation costs where farmers located along the ditch border are the players.

4.2.3 Single Outflow Line Case

Consider the network segment with only one outflow line, as in figure 4. How is the loss in that line to be allocated? To explicitly represent coalition formation in this context, it is useful to have a labelling system to refer to the players; for the moment, abstracting away the identity of the originating generator. Consider a one-to-one map from the set of MWs flowing through the node to the set of natural numbers, running from 1 through *R*, where *R* is the total number of MWs in the nodal flow. The precise nature of this mapping does not matter. The numbers are labels and have no sequential interpretation relating to power flow. From a purely accounting point of view, we could consider the flow from the node to the outflow line as a process whereby players are treated as fed to the line, one at a time, in the order in which they have been labelled, 1, ..., *R*.

One *unsuitable* loss allocation rule presents itself immediately. Each MW could be charged with the incremental transmission loss when it joins its “predecessors” (from an accounting point of view) in the outflow line. With a convex cost function, the incremental loss attached to a MW will be higher, the larger the number of its predecessors in the outflow. This loss allocation rule is, of course, grossly unfair. The charge depends critically on the order in which players are considered to enter the line, and this is based on the arbitrary labelling procedure. Note however, that this serial or incremental cost-sharing rule recovers actual cost exactly. Moreover, the “incremental” cost recovery has the efficiency inducing marginal principle built into it, albeit in an unfair way. Can this procedure be modified to ensure fair treatment to all players?

4.2.4 Shapley Value:

We can proceed by constructing a co-operative game using the above framework. Let π denote one permutation of the set {1, ..., *R*}, with the players accounted as flowing out in the sequence $\pi(1)$, $\pi(2)$, ..., $\pi(R)$. Each $i \in \{1, ..., R\}$, can be thought of as determining its worth relative to permutation π, based on the incremental cost when the accounting is done according to this order. The incremental, or marginal cost vector relating to permutation π is given by:

$$m_i^{\pi}(c) \equiv c(P(\pi, i) \cup \{i\}) - c(P(\pi, i)) \tag{4}$$

where $P(.)$ denotes the set of “predecessors” of i with respect to π, $P(\pi, i) \equiv \{j \in \{1,..., R\} | \pi(j) < \pi(i)\}$. Incremental cost is increasing in |P|, the number of predecessors:

$$m_i^{\pi}(c) = r((|P+1|)^2 - (|P|)^2) \tag{5}$$

Obviously, *i* places highest value on that permutation where it is the first to be “accounted” to flow out, leaving it with the smallest incremental transmission loss allocation. Denote the set of all possible permutations of {1, ..., *R*} by ΠR .

Each $\pi \in \Pi R$ can be considered a coalition. This equivalence can be established by noting that each coalition $S \subset \{1, ..., R\}$ has the power to orchestrate a permutation π such that only members of *S* are permuted. For each coalition $S \in 2^R$, the cost $c(S)$ of *S* is defined to be the minimum of the sum of the allocations of all the players in *S*, taken over all the permutations that can be orchestrated by *S*.

$$c(S) = \min_{\pi \in \Pi_R} m_c^{\pi}(S), \text{ for all } S \in 2\text{R} \text{ where } m_c^{\pi}(S) = \sum_{i \in S} m_i^{\pi}(c) \tag{6}$$

In the canonical formulation, the Shapley value allocation $\phi(c) = \{\phi_i(c)\}_{i=1,\dots,R}$ is given by:

$$\phi_i(c) = \sum_{S\subset\{1,\dots,R\}\setminus\{i\}} \frac{|S|!(R-|S|)!}{R!}(c(S \cup \{i\}) - c(S)) \tag{7}$$

When permutations are interpreted as coalitions, the Shapley value allocation is the average, taken over all permutations, of the marginal vectors of the game:

$$\begin{aligned}\phi(c) &\equiv \left\{\frac{1}{R!}\sum_{\pi\in\Pi_R} m_i^{\pi}(c)\right\}_{i=1,\dots,R} \\ &= \left\{\frac{1}{R!}\sum_{\pi\in\Pi_R}(c(P(\pi,i)\cup\{i\}) - c(P(\pi,i)))\right\}_{i=1,\dots,R}\end{aligned} \tag{8}$$

The axiomatic characterisation of the Shapley value is well known. The Shapley value is a suitable solution concept because it satisfies all the desirable properties we may demand of a cost allocation rule. It lies in the core of the game[8]; the implication is that no coalition can do better, and so the Shapley value allocation will be acceptable to all players. The Shapley value allocation is monotonic, additive and consistent. The monotonicity property guarantees that the charges will be non-negative, and the system will not lead to any player subsidising another. The additivity property is useful if we were considering other types of charges, such as use of system charges, added on to transmission charges. It guarantees that if charges were decomposable into components, then the order in which the component-wise allocation is done will not make a difference to the cost allocation. Consistency guarantees symmetric treatment of players.

Symmetric treatment takes a simple form in this context. The Shapley value captures the idea that the worth of an individual player is the average of her worth in all possible coalitions. Each coalition is one permutation of the ordered set of player labels, denoting one possible order in which players can be *accounted* to have increased the flow in the line. From the point of view of a player, the set of permutations Π*R*, signifies the set of all possible incremental (marginal) costs due to transmission loss, $m_{\bullet}^{\pi}(c); \pi \in \Pi_R$, that she could be potentially charged with. This set is the same for all players in this game because the power flow in the line, and the associated transmission loss, depends only on the total number of units of power flowing through the line; and not on their provenance, or identity. The implication, then, of considering all permutations equally likely is that the players are treated fairly in the face of any symmetric cost function, and moreover, the Shapley value allocation, $\phi_{\bullet}(c)$, is the same for all players. In other words, the transmission loss allocated to each MW of power flowing in the line is the same, regardless of its provenance.

It follows that the loss allocation for each generator that supplies a single outflow line is proportional to the share of its generated output in that line. Proportional sharing follows directly from accepting the Shapley value as the solution concept.[9]

[8] In any convex game, the Shapley value lies in the core of the game.

[9] This is complementary to an information theoretic rationalization. With no information, and yet faced with the need to find an allocation in this logically indeterminate situation, the probability distribution that imposes no unwarranted structure is the one that maximizes entropy; the equi-probable or uniform probability distribution. This assigns to each MW, the same probability of flowing to either *i-m* or *i-l*; so the node is a perfect "mixer". This is the proportional sharing principle.

4.2.5 Allocation of Inflows to Outflows

The logic employed in defining the game presented above applies in the context of the network segment shown in figure 1. The question is: if there were more than one outflow line, how are the inflows to be distributed among the outflows lines from the point of view of sharing out total transmission loss? The essential point is that since $c(q)$ is convex, attributing a disproportionate share of the power flow in a line that carries more power than the others to one generator (or load, if the loss is allocated to loads) will attribute to it a much larger share of transmission losses; that will be unfair.

From an accounting point of view, one may consider the outflow from the node to the different lines to be toted up, MW by MW, in some order, for example, in the order in which the units of power have been labelled, 1 through R.[10] Given the accounting procedure, the Shapley value allocation is based on equal consideration of all possible permutations of the set $\{1, ..., R\}$. If each $\pi \in \Pi_R$ has the same probability ($1/R!$), this implies that each MW has equal probability of being allocated to any of the outflow lines. In other words, the proportional sharing rule is implicit in the determination of the Shapley value of this cost allocation game.[11]

Cost allocation over the whole network follows in a straightforward additive way from cost allocation in each of the lines in the network. Thus proportional sharing rule extends to the entire game.

5 CROSS-BORDER TRANSMISSION CHARGING

The tracing-based methodology is likely to be at its most useful in the context of cross-border transmission charging. A group of transmission operators, such as ETSO, needs to be able to allocate cross-border flows to the agents causing them, so that appropriate charges can be imposed. The examples discussed here concentrate on losses, but the same principles can be applied to operating costs recovered through use of system charges, and the costs of counter-trades to resolve congestion.

For tracing purposes, each area (country) in the interconnected network will be represented by a single node, referred to as the supernode, with a net import or export. The net import/export is equal to the balance of internal generation and demand (including losses), or in other words, a balance of all the imports and exports. This means that only the area's net import/export will be allocated any losses, which can be justified the following way. The inter-area tracing algorithm allocates the costs due to the inter-area power exchanges. If any area is internally balanced (generation covers internal demand plus internal losses), but its transmission network is used for the inter-area trades, it should not be allocated any extra costs due to inter-area trades. Even more, the Transmission System Operator (TSO) in such a transit network should be compensated for making their network available for inter-area trades. This means that only the net area export/import should be allocated any costs.

Now real power flows in the interconnected network will be represented by a directed graph (digraph) in which the net country exports/imports are connected by the tie-lines linking the

[10] We need make no assumptions about the precise nature of this disbursement procedure. For instance, one convention might be that successively numbered MWs are accounted to be fed to different lines, till all inflows have been disbursed. Another might be that successively numbered MWs are accounted to be fed through in blocks to a line till its power flow "target" is met, before the next line is fed, and so on, till all inflows are accounted for.

[11] The assumption that the set of players is a finite set - of units of power (e.g. MW) - can be relaxed without loss. If player size goes to zero, the non-trivial generalization of the Shapley value to atomic games by Aumann and Shapley (1974) preserves the validity of the proportional sharing principle.

countries. Flows in the tie-lines are usually known as they are tightly monitored for other purposes (e.g. Automatic Generation Control). In order to illustrate how the digraph is formed consider figure 5a. Country A is a net exporter while countries B and C are net importers. First let us concentrate on country B as, apart from being a net importer, its network is used also for a transit of power from A to C.

The import to country B is 149 MW while the export from it is 49 MW. The type of methodology used for the internal loss allocation within the country is irrelevant for tracing, but let us assume that in this case the marginal loss allocation has been used and, as the result, the loss allocated to the import is ΔP_{imp} = -3 MW. In other words, in this example, the import creates a counterflow, reducing losses in B, and the TSO in B is prepared to pay the equivalent of 3 MW to the users causing it. The loss allocated to the export is ΔP_{exp} = 2 MW (i.e. the users responsible for this export have to pay the equivalent of 2 MW to the TSO in B). Obviously generally there may be many import/export nodes in a country, each with its own loss allocation. The country's net export/import is equal to the balance of imports and exports plus the balance of the loss charges allocated to the imports and exports, i.e. 49 – 149 – 3 + 2 = -101. The minus sign means that it is a net import.

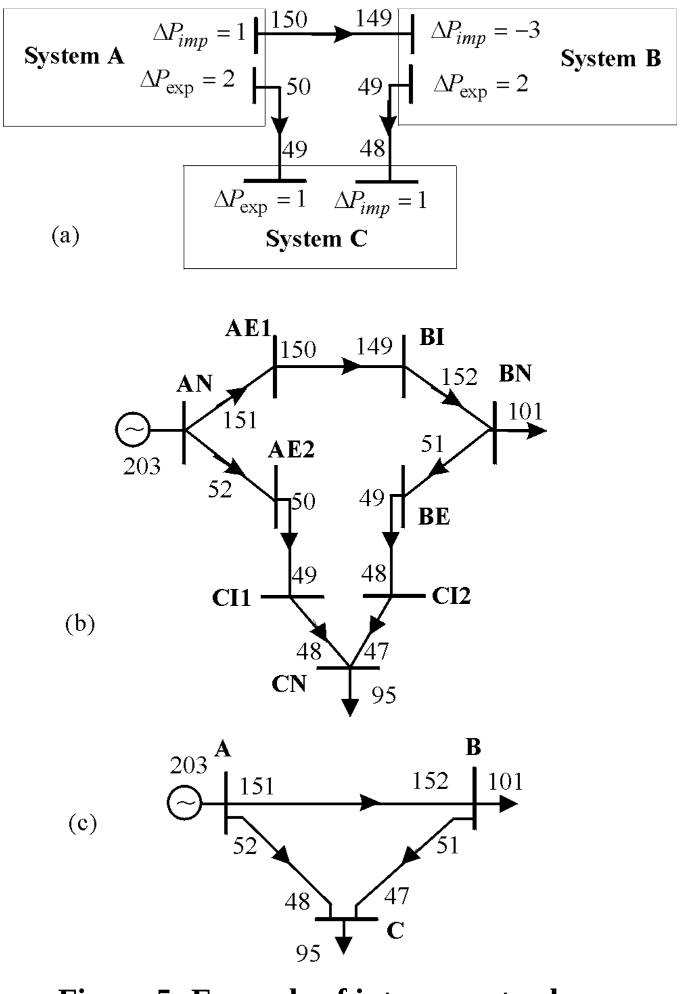

Figure 5: Example of inter-area trades

The digraph corresponding to the inter-area flows is shown in figure 5b. Nodes BI (import in B) and BE (export in B) are the actual border nodes while node BN (net B) is the fictitious node responsible for the net import/export. With many export/import nodes in the country, there would be many export/import branches connected to node BN. The power loss in the fictitious branch BI-

BN is equal to the loss allocated to the import (note that it is negative in this case) while the power loss in the fictitious branch BN-BE is equal to the loss allocated to the export.

Country A is a net exporter with two export nodes, while country C is a net importer with two import lines. Their graphs are created in the same way as for country B. For country A, the losses allocated to the exports are shown as losses in the fictitious lines AN-AE1 (where 1 MW is allocated) and AN-AE2 (where 2 MW are allocated). For country C, 1 MW of losses is allocated to each of the fictitious lines CI1-CN and CI2-CN.

The graph shown in figure 5b contains the actual tie-lines (AE1-BI, AE2-CI1, BE-CI2) and a number of fictitious lines. This graph can be simplified by combining the series-connected branches (eliminating the border nodes) as shown in figure 5c. The total loss to be allocated is 7 MW: 4MW in A-C, 4MW in B-C, and –1 MW in branch A-B, which benefits from the saving in losses allocated to the imports within country B.

Country A is the only net exporter in this system, and so if 50% of the losses are charged to exports, country A will have to pay for 3.5 MW. If the other 50% are charged to imports, they must be divided between countries B and C. We can use the graph-based tracing outlined in Section 3 to allocate the losses. The only difference is that each of the nodes in the digraph is a supernode representing a country. Country B imports directly from country A via tie-line A-B and country C imports via two parallel paths: tie-line A-C and the path A-B-C. Thus, looking at the allocation to the net imports, -0.5 MW of loss in line A-B (50% of the total) is shared in proportions (101/152) and (51/152) between net imports in countries B and C. On the other hand 2 MW of loss in line A-C, and 2 MW of loss in line B-C (in both cases 50% of the total losses), are allocated to the net import in country C. Adding these allocations gives –0.33 MW allocated to country B and 3.83 MW allocated to country C. The total comes to 3.5 MW, 50% of the overall losses, as required.

Note that the exporter in A would not have to pay as much if the "loop flow" through B were not taken into account. If the flow between B and C was treated as a bilateral transaction between the two countries, an "exporter" in B would have to pay for 2 MW of losses on line B-C. The exporter in A has to pay for half of the losses on line A-C (2 MW), but gains from the negative charge on line A-B (– 0.5 MW). It no longer has to pay anything for the losses on line B-C. The importer in country C is no longer credited with any of the losses saved on line A-B, even though some of its imports travelled down that line, and has to pay more. Table 3 summarises these results. Clearly tracing provides a fairer allocation of charges.

Table 3: Loss charges due to inter-area trades.

	Exp A	Imp B	Exp B	Imp C	Total
Loss charge from tracing	3.5	-0.33		3.83	7
Loss charge from bilateral flows	1.5	-0.5	2.0	4.00	7

Tracing allows allocation of losses to countries that cause them. It does not dictate the way in which they are paid for. One approach, "self-provision", implies that each country would have to increase its generation to compensate for the losses it causes. If the additional generation (or reduction for countries that are due to receive compensation) equalled the losses due, no cash need to change hands. In practice, however, demands and generation levels fluctuate and rarely match their contracted levels, so that the imbalances must be converted to cash payments. Given this, it might be best to charge for losses in cash terms from the beginning.

Thus a price is needed, and a simple solution is to take a (weighted) average price of energy from all the countries. This could be unfair to countries with relatively high energy costs and prices. It would be better to set charges based on the price of electricity at each point on the network. This implies multiplying the losses allocated to imports/exports within each country by the unit price of energy in that country and multiplying the losses in tie-lines by the average price of energy between the connected countries.

So far, this tracing-based allocation scheme has been interpreted as a scheme for inter-TSO compensation payments. That is, each TSO handles transmission pricing inside their systems and allocates charges to the border nodes (imports and exports). Then the tracing scheme is used to re-allocate these import/export charges to individual TSOs and reimburse the transiting TSOs for their costs. Therefore the whole scheme is executed on the TSO level. How should these compensation charges be apportioned to individual loads and generators within each country? This question is really beyond the scope of this paper. One possibility, however, is to charge only the loads/generators with import/export contracts while another is to allocate the charges evenly to all the users within a given country. The former is probably more cost-reflective but, on the other hand, it may be seen as creating barriers impeding development of the common market. Evenly distributing the import/export charges to all the users is less cost-reflective but promotes competition and development of inter-area trades. This may be seen as more important than the cost-reflectiveness, especially during the initial stages of market opening.

It may be argued that, as the proposed scheme is not based on the marginal pricing methodology, it is not optimal. However it should be noted that the societal welfare is hardly affected by the non-optimal transmission prices. As shown by Green (1998), moving from uniform to optimal transmission prices in England and Wales (including congestion pricing) would increase social welfare by only 0.6% of the generators' revenue. This would suggest that a practical, but not necessarily theoretically optimal, methodology facilitating the trades is preferable to a theoretically optimal but complicated one. The tracing methodology seems to provide a sensible compromise: it is simple and practical yet provides some locational signals. By comparison the marginal transmission pricing methodology, which is theoretically optimal, is too complicated and volatile while the postage stamp is not cost-reflective.

The important advantage of the proposed approach is that no information is required about individual transactions as the charges are exit/entry based (i.e. a transaction from A to B will pay the charge for injecting power at A and withdrawing at B). The only information required are the flows in the tie-lines linking the countries and the loss charges on imports/exports. This limits the amount of information to be shared between individual TSOs and also prevents disclosure of commercially sensitive information about individual contracts. The methodology retains the subsidiarity principle, since each country is allowed to use whatever methodology they wish for internal loss allocation.

Implementation of the tracing methodology requires knowledge of all the tie-line flows in the interconnected system. This information is usually available because tie-line flows are monitored for AGC purposes. However the scheme still requires someone to collect all the information and do the calculations. In other words a central TSO, if only for transmission pricing purposes, would be necessary. Although one could argue that for system security purposes such a central TSO is necessary anyway, the individual TSOs may be opposed to this idea (mostly for political reasons). In order to overcome this obstacle, a possibility of implementing the tracing scheme that does not require existence of the central TSO would be advantageous. In order to do that one should notice that the graph-based scheme satisfies this requirement. The calculations are performed step by step with the information being passed down from the pure source node of the digraph downstream until it reaches the pure sink node of the digraph. At each step communication only between neighbouring utilities is required. Hence the whole calculation can be executed in a completely decentralised way. In the presence of circular flows the solution has to be obtained using the matrix-based algorithm. The decentralised solution can no longer be employed but, if the central TSO is not desired, the cost allocation can be performed by an Internet-based program.

The tracing methodology can also be applied by the intermediate countries to charge transiting flows for the use of their networks (Bialek, 2000). To understand how this is done note that table 1 shows the usage of particular lines by particular generators (i.e. net exports) while table 2 shows the usage by particular demands (i.e. net imports). In a similar way the usage of entire networks (i.e. supernodes) can be determined.

6 CONCLUSION

In this paper assumptions underlying a methodology for transmission pricing in a meshed network (and applicable to inter-system trades) have been analysed. According to this methodology there is a simple method - by following the acyclic directed graph (digraph) of flows in the network - to notionally trace how real and reactive power flows in the lines from individual sources to individual sinks. In other words the tracing methodology establishes paths linking the sources and the sinks. Consequently transmission prices can be calculated in a similar manner to in the traditional contract path approach. It should be stressed, however, that the tracing paths are based on the actual power flows in the network while the contract paths are based on financial contracts which do not reflect the physics of transmission.

The tracing methodology is based on the assumption that, at any network node, the incoming flows are proportionally distributed among the outgoing flows. One aim of this paper was to critically examine this assumption, which can be neither proved nor disproved physically. The analysis used the loss allocation problem as an example, and rationalised the proportional sharing principle using co-operative game theory. We showed that the Shapley value, the solution concept that has all desirable properties one may demand of a loss allocation scheme, justifies the proportional sharing rule.

The Shapley value justification for the sharing rule implies that it is based on average of marginal, rather than pure marginal, costs. This means that locational signals will be muted, compared to those that nodal prices would produce. However, since nodal prices seem unlikely to be implemented, we believe that it is better to send muted signals than none at all.

In the final part of the paper, we have shown how the tracing methodology could be applied to the problem of charging for inter-country electricity flows. The tracing methodology allows us to assign responsibility for flows anywhere on the system to countries, generators, or loads. While we have concentrated on charging for losses in our examples, the methodology could easily be used to ensure that the costs of using the transmission system, and particularly those of resolving congestion, could also be apportioned more fairly than they are at present. At present, the European transmission system operators are trying to find a pricing methodology which is simple enough to be easily implemented, but also sends appropriate cost messages. We believe that a scheme based on tracing is a good compromise between these potentially conflicting objectives.

REFERENCES

Aadland D and Kolpin Van (1998) Shared Irrigation Costs: an Empirical and Axiomatic Analysis, *Mathematical Social Sciences*, 849, 203-218.

Aumann, RJ, Shapley, LS: *Values of Non-atomic Games*, Princeton University Press, 1974.

Billera L Heath D and Raanan, J (1978) Internal Telephone Billing Rates - A Novel Application of Nonatomic Game Theory, Operations Research, 26, 956-965.

Bialek. J.: "Tracing the flow of electricity" *IEE Proc.-Gen. Transm. Distrib.*, Vol. 143, July 1996, pp. 313-320.

Bialek, J.: "Topological generation and load distribution factors for supplement charge allocation in transmission open access" *IEEE Trans. on Power Systems,* Vol. 12, pp. 1185-1193, August 1997.

Bialek, J, Kattuman, P.: "Real and Reactive Power Tracking: Proof of Concept and Feasibility Study" Electric Power Research Institute, report TR112416, Feb. 1999.

Bialek, J. "Tracing-based unifying framework for transmission pricing of cross-border trades in Europe" Int. Conf. on Electric Utility Deregulation and Restructuring, and Power Technologies DRPT'2000, London, April 2000.

Cadwalader, M.D., S.M. Harvey, W.W.Hogan, and S.L. Pope (1999), *Coordinating Congestion Relief Across Multiple Regions* mimeo, Harvard Electricity Policy Group.

Calviou, M.C., Dunnet, R.M., Plumptree, P.H.:"Charging for Use of a Transmission System by Marginal Cost Methods" *Proc. Power System Computation Conference,* Avignon 1993.

Costano-Pardo, A and Garcia-Dias, A (1995) Highway Cost Allocation: An Application of the Theory of Nonatomic Games, Transportation Research - A 29A, 3, 187-203.

ETSO (2000a) *Cross-border Tariffs for the Internal Market of Electricity in Europe* European Transmission System Operators, 19 March 2000

ETSO (2000b) *Proposal for the Implementation of the Cross-border Tariffs for the year 2001*, European Transmission System Operators, 27 March 2000

Green, R.: "Electricity transmission pricing: an international comparison", *Utilities Policy*, Vol. 6, No. 3, pp. 177-184, 1997.

Green, R.: "Electricity Transmission Pricing: How much does it cost to get it wrong?", University of California Energy Institute, report PWP 58, April 1998.

Kirschen, D., Allan, R., Strbac, G.: "Contributions of individual generators to loads and flows" *IEEE Trans. on Power Systems*, Vol. 12, February 1997, pp. 52-60.

Linhart P, Radner R, Ramakrishnan KG, and Steinberg R (1995) The Allocation of Value of Jointly Provided Services, Telecommunication Systems 4, 151-175.

Littlechild SC and Owen G (1973) A Simple Expression for the Shapley Value in a Special Case, *Management Science*, 20, 3, 370-372.

Rivier, M., Pérez-Arriaga, I. J.: "Computation and decomposition of spot prices for transmission pricing" *Proc. 11th PSCC*, pp. 371-378, Avignon, 1993.

Schweppe, F.C., Caramanis, M.C., Tabors, M.D., and Bohn, R.E.: *Spot Pricing of Electricity*. Boston: Kluwer, 1988.

Shapley LS (1953) A Value for n-person Games in Kuhn HE and Tucker, AW, (eds.) *Contributions to the Theory of Games*, Vol 2, Princeton University Press, Princeton, NJ, pp 307-317.

Shubik, M. (1962) Incentives, Decentralized Control, The Assignment of Joint Costs and Internal Pricing, *Management Science*, 8, 325-343.

Young, HP (1994) Cost Allocation, in Aumann RJ and Hart S (eds.) *Handbook of Game Theory with Economic Applications*, Volume 2, Amsterdam; London and New York: Elsevier, North-Holland, pp. 1193-1235.

APPENDIX: MATRIX ANALYSIS OF THE TRACING METHODOLOGY

The tracing algorithm can be executed using either an intuitive graph-based approach, proposed by Kirschen *et al* (1997) or a more general matrix analysis proposed by Bialek (1996). Explanations in Section 3 were based on the graphs as they are more easily understood. Here we will outline the matrix-based approach as it is more useful when dealing with circular flows.

Let us define an unknown *gross nodal power*, $P_i^{(gross)}$, as a total power flow through node *i* which satisfies the Kirchhoff's Current Law and which would flow if the network was fed with the actual generation and no power was lost in the network. Similarly, let $P_{i-j}^{(gross)}$ be an unknown *gross flow* in line *i-j* which would flow if no power was lost. Obviously $\left|P_{i-j}^{(gross)}\right| = \left|P_{j-i}^{(gross)}\right|$.
The gross nodal power, when looking at the inflows, can be expressed as

$$P_i^{(gross)} = \sum_{j \in \alpha_i^{(u)}} \left|P_{i-j}^{(gross)}\right| + P_{Gi} \quad \text{for } i = 1,2,...,n \tag{9}$$

where $\alpha_i^{(u)}$ is the set of upstream nodes supplying directly node *i* (i.e. power must flow towards node *i* in the relevant lines), and P_{Gi} is the generation at node *i*. As $\left|P_{i-j}^{(gross)}\right| = \left|P_{j-i}^{(gross)}\right|$, the flow $P_{i-j}^{(gross)}$ can be replaced by $\left(\left|P_{j-i}^{(gross)}\right| / P_j^{(gross)}\right) P_j^{(gross)}$. Normally the transmission losses are small so that it can be assumed that $\left|P_{j-i}^{(gross)}\right| / P_j^{(gross)} \cong \left|P_{j-i}\right| / P_j$, where P_{j-i} is the actual flow from node *j* in line *j-i* and P_j is the actual total flow through node *j*. This corresponds to assuming that the distribution of gross flows at any node is the same as the distribution of actual flows. This is the only approximating assumption of the method. Under this assumption eq. (9) can be re-written as

$$P_i^{(gross)} - \sum_{j \in \alpha_i^{(u)}} \frac{\left|P_{j-i}\right|}{P_j} P_j^{(gross)} = P_{Gi} \quad \text{or} \quad \mathbf{A_u P_{gross}} = \mathbf{P_G} \tag{10}$$

where $\mathbf{P_{gross}}$ is the unknown vector of gross nodal flows and $\mathbf{A_u}$ is the upstream distribution matrix calculated from the actual, not modified, flows. Its elements are equal to:

$$[\mathbf{A_u}]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -\left|P_{j-i}\right| / P_j & \text{for } j \in \alpha_i^{(u)} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

As $\mathbf{A_u}$ and $\mathbf{P_G}$ are known, the solution of (10) will give the unknown gross nodal flows.

Once the gross nodal flows have been determined, the gross line flows and gross demands can also be found using the proportional sharing principle. The gross outflow from node *i* in line *i-l* is

$$\left|P_{i-l}^{(gross)}\right| = \frac{\left|P_{i-l}^{(gross)}\right|}{P_i^{(gross)}} P_i^{(gross)} \cong \frac{\left|P_{i-l}\right|}{P_i} \sum_{k=1}^{n} \left[\mathbf{A_u^{-1}}\right]_{ik} P_{Gk} \quad \text{for all } l \in \alpha_i^{(d)} \tag{12}$$

where α_i^d is the set of downstream nodes supplied directly from node *i* (that is power flows from those nodes to node *i* in the relevant lines). The gross demand at node *i* can be calculated as

$$P_{Di}^{(gross)} = \frac{P_{Di}^{(gross)}}{P_i^{(gross)}} P_i^{(gross)} \cong \frac{P_{Di}}{P_i} P_i^{(gross)} = \frac{P_{Di}}{P_i} \sum_{k=1}^{n} \left[\mathbf{A_u^{-1}}\right]_{ik} P_{Gk} \qquad (13)$$

where P_{Di} is the actual demand at node *i*. This equation is especially important as it shows what would be the load demand at a given node if a lossless network was fed with the actual generation. Hence the difference between the gross demand and the actual demand

$$\Delta P_{Di} = P_{Di}^{(gross)} - P_{Di} \qquad (14)$$

gives the loss which is attracted by power flowing from all the generators to a particular load.

Let us apply this algorithm to the real power flow shown in figure 2. Eq. (10) gives:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -60/400 & 1 & 0 & 0 \\ -225/400 & 0 & 1 & -83/283 \\ -115/400 & -173/173 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1^{(gross)} \\ P_2^{(gross)} \\ P_3^{(gross)} \\ P_4^{(gross)} \end{bmatrix} = \begin{bmatrix} P_{G1} = 400 \\ P_{G2} = 114 \\ 0 \\ 0 \end{bmatrix} \qquad (15)$$

Solving this equation gives the following values of gross nodal powers: $\mathbf{P_{gross}}$ = [400 174 309.8 289]^T. The gross load demands are $D_3^{(gross)}$ = (300/300)×309.8 = 309.8 and $D_4^{(gross)}$ = 289x(200/283) = 204.2. Hence the loss apportioned to D_1 is equal to 9.8 while the loss apportioned to D_4 is 4.2 which matches the values shown in table 1.

The share of the generation used to supply each of the loads and each of the lines can be calculated from eq. (12) and (13). All these calculations give the same result as that obtained using the graph-based algorithm, table 1.

In a similar manner matrix-based tracing of net power flows can be conducted, which gives allocation of the losses to the generators, but it will not be shown here due to the lack of space. Details can be found in Bialek (1996).

At first sight it might seem that the graph-based approach is simpler than the matrix approach as the former does not require matrix manipulations. A closer investigation, discussed by Bialek (1997 and 1999), reveals that both approaches are computationally equivalent because the calculation of the inverse matrices can be performed recursively. This is due to a well-known graph theory theorem saying that it is possible to order nodes in an acyclic directed graph in such a way that the adjacency matrix is upper triangular. Consequently $\mathbf{A}_u$ is lower triangular and $\mathbf{A}_u^{-1}$ can be obtained by recursive forward substitution. It is easy to prove that the computational steps involved are then identical to those when the graph-based algorithm was used. The required node ordering can be done by a simple graph search technique determining the length of the longest path to each node starting from each head node. The nodes are then allocated to a level equal to the length of the longest path reaching that node. When a node belongs to two or more domains it is allocated to the highest of its levels. Obviously the same graph search technique must be used to form the directed graph when the graph-based approach is used.

We have shown in Section 5 that the graph-based tracing lends itself to decentralised calculations, suitable for inter-area trades. Decentralised calculations can also be executed when the matrix approach is used. First of all note that the matrix equations involved are sparse and their structure is similar to that of the admittance matrix. Thus it can be formed in a decentralised manner, as each TSO needs to know only the flows in its tie-lines. When the nodes are ordered according to their order in the directed graph of flows, the resulting matrices are either upper or lower triangular (Bialek and Kattuman, 1998). Hence they can be solved in a decentralised manner by recursive backsubstitution. The computational steps are then identical to those of the graph-based algorithm.

Let us consider again the circular flow example in figure 3, where country A is a net exporter while countries B and C are net importers. The total loss is 10 MW. There is no pure source node to start the calculations from or the pure sink node to finish them. Hence the graph-based algorithm outlined above would fail to trace the flows and the matrix-based algorithm cannot be executed recursively as the matrices are no longer triangular.
The tracing based algorithm outlined above would provide a fair loss allocation as follows: $\mathbf{P_G}$ = [160 0 0]', $\mathbf{P_D}$ =[0 100 50]', $\mathbf{P}$ = [205 200 97]' with the downstream and upstream distribution matrices equal to:

$$\mathbf{A}_u = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 0 & \frac{-100}{200} & 1 \end{bmatrix} \qquad \mathbf{A}_d = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \frac{-45}{205} & 0 & 1 \end{bmatrix}$$

If all the loss is to be allocated to the net imports, application of equation (14) would allocate 5.6 MW to the net import in B and 4.4 MW to the net import in C. Similarly, if all the loss is to be allocated to the net exports, all the loss of 10 MW would be allocated to the net export in A. This is logical as A is the only net exporter so all the loss must be allocated to it. Thus the matrix-based algorithm treats the circular flows without any problem even when the matrices are not triangular.