Microforce materials tester

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A material tester for evaluating the mechanical response of general objects against micro-Newton level impact forces has been proposed. In the tester, an inertial mass is made to collide with the material under test. The impact force acting on the material is measured as the inertial force of the mass. A pneumatic linear bearing is used to realize linear motion with small friction acting on the mass that is the moving part of the bearing. The velocity of the mass is measured highly accurately by means of an optical interferometer. The acceleration, the inertial force, and the position of the mass is numerically calculated from the velocity after the collision measurement. The other force, which is mainly consisted of the friction acting inside the bearing, is determined using the same instrumentations. Using the estimated friction inside the bearing, the measured force is corrected. The force acting on the mass from the material under test of approximately 0.1 mN is measured with the standard uncertainty of approximately 1.4 μN. © 2005 American Institute of Physics.
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I. INTRODUCTION

Recently, the requirements for evaluating small force in the range of 1 μN to 1 N have increased in various industrial and research applications. However, it is sometimes very difficult to generate and evaluate small force properly.

The difficulties in measuring small force mainly come from the following two facts:

(1) Any method for measuring micro-Newton level forces has not yet been established even for static forces. Some methods for supporting a direct realization of micro-Newton level forces linked to the International System of Units (SI) below 1 N (Refs. 1–4) are currently being developed by some national standard laboratories.
(2) Small force to be generated and/or measured is usually a varying force and any dynamic calibration technique for force sensors has not yet been established. Some methods are now being developed.[5–12] In other words, this fact means that both the uncertainty evaluation for the measured value of the small force and the uncertainty evaluation for the time of the measurement are very difficult.

Force can be defined as the product of mass and acceleration as

$$\mathbf{F} = \mathbf{Ma},$$

where $\mathbf{F}$ is the force acting on an object, $\mathbf{M}$ is the mass of the object, and $\mathbf{a}$ is the acceleration of the center of the gravity of the object. This means that a well-defined acceleration is required to generate forces directly based on the above definition. Acceleration due to gravity $\mathbf{g}$ is convenient and usually used for generating and/or measuring constant force.

There is an attempt[1,2] for developing a method for generating and measuring static micro-Newton level forces directly linked to SI using the electrostatic force measured in terms of the quantum standard of electrical quantities: in other words, in terms of the recommended value of the Planck constant. The method is a modification of the Watt Balance Method,[13] in which the Planck constant is measured by means of comparing the electric power measured in terms of the Josephson effect and the quantum Hall effect with the mechanical power measured in terms of the mass of the international prototype of kilogram and the acceleration of gravity $\mathbf{g}$. The method[1,2] will be very useful for generating and measuring the static micro-Newton level forces if the unknown uncertainty sources come from the balance mechanism, such as friction and irregularity of alignment of the electrode, are sufficiently eliminated. There is another attempt for developing a method for generating and measuring microconstant force, in which a combination of an electric balance and a lever mechanism is used as a scale.[4] To summarize the state of the art of the microforce measurement technology, its importance has been widely recognized and some attempts for establishing them are being pursued.

As for the dynamic calibration method for force transducers, there are no established methods for calibrating force transducers under dynamic conditions. Only static methods, in which transducers are calibrated by static weighting, are widely available at present. Therefore, it is very difficult to determine the uncertainty in measuring a varying force or dynamic force using force transducers. Force transducers are usually used without any knowledge on the difference between the dynamic characteristics against the static characteristics. Usually it is widely believed that force transducers with higher resonant frequency are suitable for measuring the dynamic force with higher frequency. However, that is irrational, of course. Although the methods of dynamic calibra-

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tion of force transducers are not yet well established, there have been a number of trials aiming at establishing the dynamic calibration methods for force transducers. The dynamic calibration methods can be divided into three categories: methods for calibrating force transducers using an impact force,[5–7] methods for calibrating force transducers using a step force,[8] and methods for calibrating force transducers using an oscillation force.[9–12]

The author has proposed methods for all three categories.[5,6,8,12] In the methods, the inertial force of a mass levitated using a pneumatic linear bearing is used as the reference force applied to the force transducers. The inertial force of the levitated mass is measured using an optical interferometer. All these methods are variations of a method named as the Levitation Mass Method. The author has applied the Levitation Mass Method for evaluating the frictional characteristics of pneumatic linear bearings.[14] The Levitation Mass Method was developed as a ground test of a mass-measuring instrument for use under microgravity conditions.[15] The Levitation Mass Method has also been applied as methods for material testing, such as a method for evaluating material viscoelasticity under oscillating load[16] and under impact load.[17]

As for microforce measurement, the Levitation Mass Method has been applied as a method for generating and measuring the micro-Newton level forces.[18] Using the method, the mechanical response of a piece of paper has been evaluated with an impact force whose maximum value was approximately 2 mN. However, in the paper,[18] the force acting inside the linear bearing has been ignored and any uncertainty evaluation has not been made.

In this article, an *in situ* observation technique for estimating the force acting inside the pneumatic linear bearing is developed. The method, in which the force applied to the material under test is accurately evaluated by means of subtracting the force acting inside the bearing from the total inertial force of the levitation mass, is developed. Using the method, the mechanical response of a human hair has been evaluated with an impact force whose maximum value is approximately 0.1 mN with the standard uncertainty of approximately 1.6 μN.

II. EXPERIMENTAL SETUP

Figure 1 shows a schematic diagram of the experimental setup for the cantilever bending test of materials against small impact force. In the method, the inertial force of a mass is used as the reference force acting on the material under test. A pneumatic linear bearing is used to realize linear motion with small friction acting on the mass, i.e., the piston-shaped moving part of the bearing. Impact force is generated and applied to the material by colliding the mass. An initial velocity is given to the moving part manually. A cube-corner prism (CC) for interferometer and a metal block with a round-shaped tip for adjusting the collision position are attached to the moving part made in aluminum with square pole shape; its total mass $\mathbf{M}$ is approximately 21.17 g.
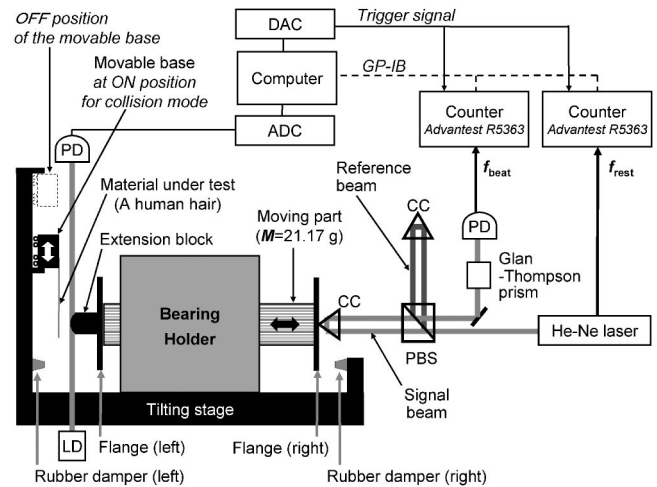

FIG. 1. Experimental setup.

The inertial force acting on the mass is measured highly accurately using an optical interferometer. A human hair is used as the material for test.

The material under test is attached to a movable base, which enables the two modes of measurement, i.e., the mode of collision measurement and the mode of free-sliding measurement. In Fig. 1, the movable base is set to be at the "ON position" for the mode of collision measurement. The movable base at the upper position or the "OFF position" is also shown by the dotted line in Fig. 1. The movable base is set to be at the "OFF position" for the mode of free-sliding measurement, which is the *in situ* measurement for evaluating the force acting inside the bearing. At the "OFF position," the material under test is set back from the moving path of the piston-shaped mass and the mass can be traveled back and forth between the side dampers.

The total force acting on the moving part $\mathbf{F}_{\text{mass}}$ is the product of its mass $\mathbf{M}$ and its acceleration $\mathbf{a}$

$$\mathbf{F}_{\text{mass}} = \mathbf{Ma}.$$

The total force acting on the moving part $\mathbf{F}_{\text{mass}}$ is divided into two components, i.e., the force acting from the material under test $\mathbf{F}_{\text{material}}$ and the other force $\mathbf{F}_{\text{bearing}}$.

$$\mathbf{F}_{\text{mass}} = \mathbf{F}_{\text{material}} + \mathbf{F}_{\text{bearing}}.$$

If the other force $\mathbf{F}_{\text{bearing}}$ can be ignored, the force acting on the material from the moving part is equal to the inertial force of the moving part—$\mathbf{M}\,\mathbf{a}$, according to the law of inertia. If it cannot be ignored $\mathbf{F}_{\text{bearing}}$ should be taken into account. The other force $\mathbf{F}_{\text{bearing}}$ is thought to mainly consist of three components

$$\mathbf{F}_{\text{bearing}} = \mathbf{F}_{\text{friction}} + \mathbf{F}_{\text{airflow}} + \mathbf{F}_{\text{gravity}},$$

where $\mathbf{F}_{\text{friction}}$ is the frictional force inside the air film of the bearing due to the relative motion between the moving part and the bearing holder, $\mathbf{F}_{\text{airflow}}$ is the frictional force inside the air film of the bearing due to the asymmetrical flow of the air, and $\mathbf{F}_{\text{gravity}}$ is the component of the gravitational force due to the inclination of the bearing holder against the hori
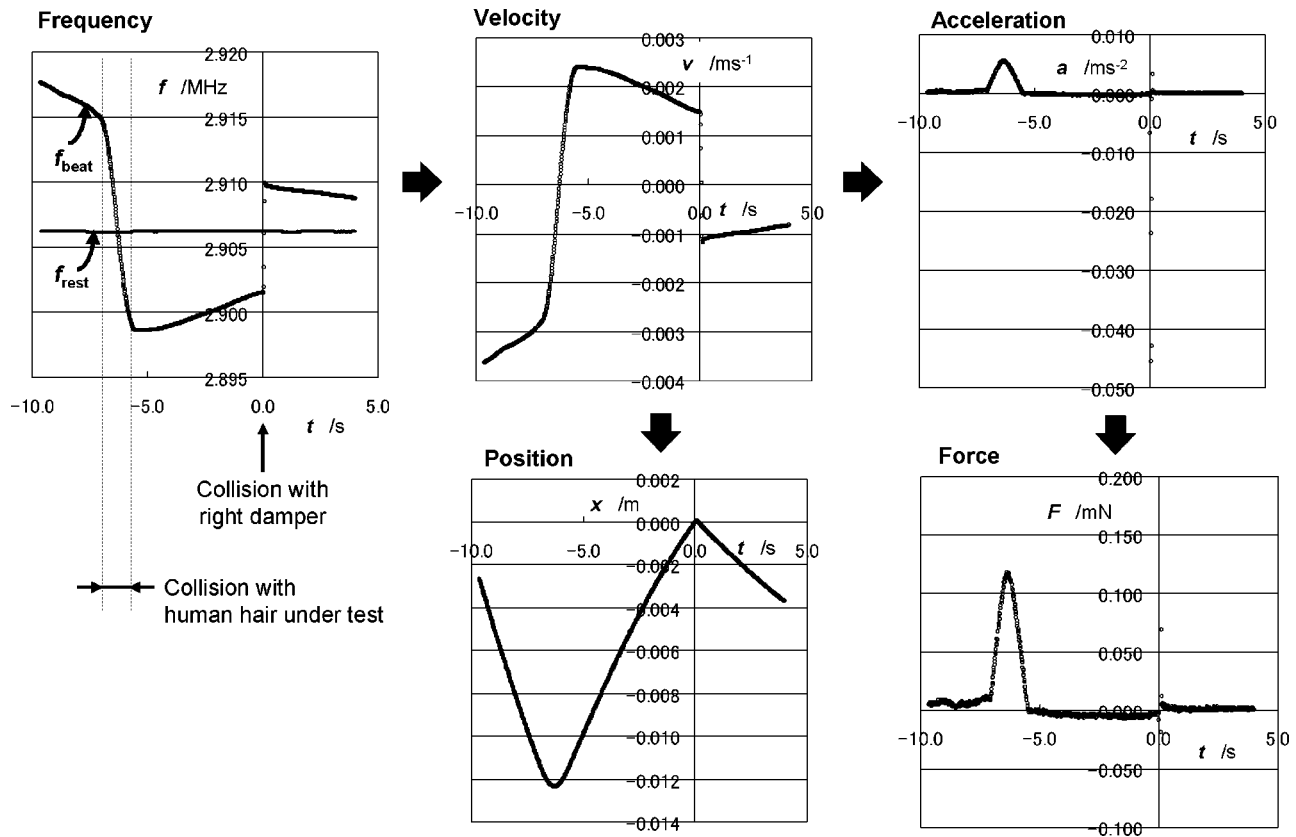
FIG. 2. Data processing procedures: Calculation of velocity, position, acceleration, and force from frequency.

zontal plane. This hypothesis is derived from the basic knowledge obtained from the experiments using a pneumatic linear bearing with larger size.[14]

In the measurement, the total force $\mathbf{F}_{mass}$ is measured as the product of the mass and the acceleration. The acceleration is calculated from the velocity of the moving part. The velocity is calculated from the measured value of the Doppler shift frequency of the signal beam of a laser interferometer $\mathbf{f}_{Doppler}$, which can be expressed as

$$\mathbf{v} = \lambda_{air}(\mathbf{f}_{Doppler})/2,$$

$$\mathbf{f}_{Doppler} = -(\mathbf{f}_{beat} - \mathbf{f}_{rest}),$$

where λ_{air} is the wavelength of the signal beam under the experimental conditions, $\mathbf{f}_{beat}$ is the beat frequency, i.e., the frequency difference between the signal beam and the reference beam, $\mathbf{f}_{rest}$ is the rest frequency which is the value of $\mathbf{f}_{beat}$ when the moving part is at a standstill. The direction of the coordinate system for the velocity, the acceleration, and the force acting on the moving part is toward the right in Fig. 1.

A Zeeman-type two-frequency He–Ne laser is used as the light source. The frequency difference between the signal beam and the reference beam, i.e., the beat frequency $\mathbf{f}_{beat}$ is measured from an interference fringe which appears at the output port of the interferometer; it varies around $\mathbf{f}_{rest}$, approximately 2.9 MHz, depending on the velocity of movement. An electric frequency counter (model: R5363; manufactured by Advantest Corp., Japan) continuously measures and records the beat frequency $\mathbf{f}_{beat}$ 14 000 times with a sam-

pling interval of $T = 40\,000/\mathbf{f}_{beat}$, and stores the values in memory. This counter continuously measures the interval time of every 40 000 periods without dead time. The sampling period of the counter is approximately 14 ms at a frequency of 2.9 MHz. Another same-model electric counter measures the rest frequency $\mathbf{f}_{rest}$, using the electric signal supplied by a photodiode embedded inside the He–Ne laser.

The pneumatic linear bearing, "GLS08A50/25-2571" (NSK Co., Ltd., Japan), is attached to an adjustable tilting stage. The tilt angle of the tilting stage can be adjusted by means of rotating three compression bolts and three tension bolts. The mechanism of the tilting stage is not shown in Fig. 1. According to the design specifications, the maximum value of additional mass that can be attached to the moving part is approximately 1 kg, the stroke of the movement is approximately 25 mm, and the nominal thickness of the air film is approximately 10 μm. The tilting angle of the upper surface of the bearing holder can be roughly adjusted horizontally with the uncertainty of approximately 0.1 mrad using a bubble level. The slope angle of approximately 0.1 mrad corresponds to the slope component of the gravitational force acting on the moving part $\mathbf{F}_{gravity}$ of approximately 0.02 mN (20 μN). In the experiment, the fine adjustment of the tilting stage is conducted so that the piston-shaped mass can be at a standstill at the "OFF position" where the mass and the material under test are in contact at the "ON position."

Measurements using the two electric counters are triggered by means of a sharp trigger signal generated using a
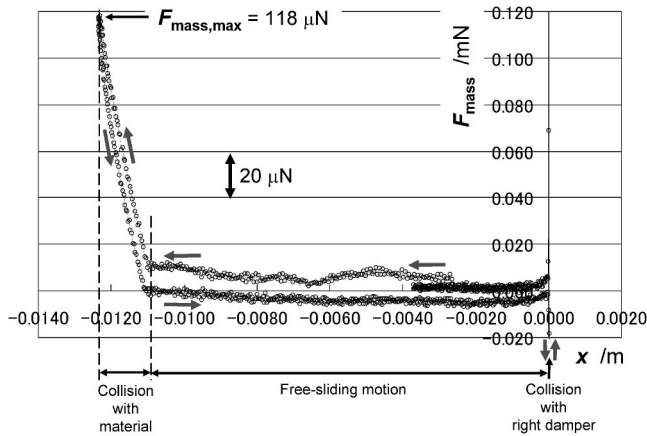
FIG. 3. Measured force against position.

FIG. 4. Measured force against position during free-sliding motion.

digital to analog converter. This signal is initiated by means of a light switch, a combination of a laser-diode and a photodiode.

In the mode of collision measurement, the movable base is set to be at the "ON position" (see Fig. 1) and the initial velocity is manually given to the mass, then it collides with the material under test. In the mode of free-sliding measurement, the movable base is set to be at the "OFF position" (see Fig. 1), the initial velocity is manually given to the mass, then the mass begins to move back and forth between the two dampers.

In the experiment, one set of collision measurement and one set of free-sliding measurement are conducted.

III. MEASUREMENT

A. Collision measurement

In the collision measurement, the mass is made to collide to the material under test and the total force acting on the mass is measured as the product of mass and acceleration. Figure 2 shows the data processing procedures. During the collision measurement, only the time-varying beat frequency f_{beat}, and the rest frequency f_{rest} are highly accurately measured using an optical interferometer. The Doppler frequency shift is measured as the difference between the beat frequency and the rest frequency. The velocity, the position, the acceleration, and the force are calculated from the measured motion-induced time-varying Doppler frequency. The origins of the time and position axes are set to be the time and the position where the reaction force from right damper is detected, respectively.

Figure 3 shows the change in the total force acting on the mass against position. The maximum value of the impact force $F_{mass,max}$ is approximately 0.12 mN (1.2×10^2 μN). As seen in the figure, the total force acting on the mass F_{mass} is not small when the moving part is apart from the material under test. In other words, the other force $F_{bearing}$, which is thought to consist of $F_{friction}$, $F_{airflow}$, and $F_{gravity}$, cannot be ignored in this measurement.
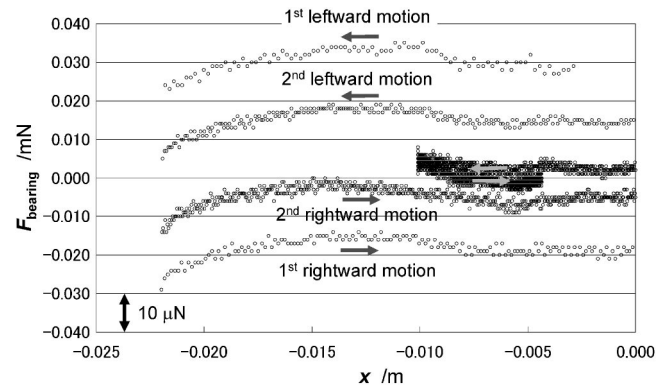
B. Free-sliding measurement

To evaluate the other force $F_{bearing}$, which is thought to consist of $F_{friction}$, $F_{airflow}$, and $F_{gravity}$, the free-sliding measurement is conducted. In the experiment, the movable base is set to be at "OFF position" and the moving part is made to have a sliding motion back and forth between both rubber dampers. The total force acting on the moving part F_{mass} $=F_{material}+F_{bearing}$, is measured in the same way as the collision measurement. Then the data in which the moving part is in contact with one of the side dampers is removed and only the data during the free-sliding motion is selected. During the free-sliding motion, the total force F_{mass} is equivalent to the other force $F_{bearing}$, according to its definition. In the experiment, 2902 sets of $(t, f_{Doppler}, v, x, a, F_{bearing})$ are obtained in the range of -22 mm$< x < 0$ mm, where the moving part is apart from both the side dampers.

Figure 4 shows change in the force $F_{bearing}$ against the position x during the free-sliding motion of the moving part. The position dependency of the force $F_{bearing}$ during free-sliding motion is clearly observed.

Figure 5 shows change in the force $F_{bearing}$ against the velocity v during the free-sliding motion of the moving part. The velocity dependency of the force $F_{bearing}$ during free-sliding motion is clearly observed.

IV. ANALYSIS

Under the assumption that the force $F_{bearing}$ is the sum of the component proportional to the velocity v and the component expressed as the polynomial expression of degree 3 for the position x the following is derived:
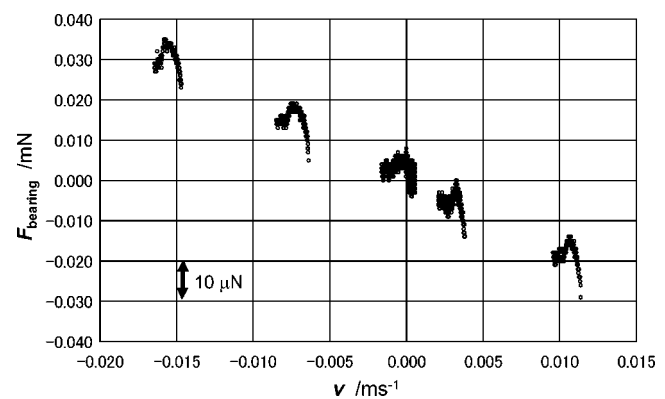

FIG. 5. Measured force against velocity during free-sliding motion.

TABLE I. The coefficients obtained experimentally and theoretically.

	Experimental results (results of regression)	Theoretical estimates
A_1 (N/ms^{-1})	-1.9×10^{-3}	-2.8×10^{-3}
A_2 (N/m^3)	7.0×10^0	0
A_3 (N/m^2)	1.8×10^{-1}	0
A_4 (N/m)	7.3×10^{-4}	0
A_5 (N)	-3.0×10^{-7}	0

$$\mathbf{F}_{\text{bearing}} = A_1\mathbf{v} + A_2\mathbf{x}^3 + A_3\mathbf{x}^2 + A_4\mathbf{x}^1 + A_5.$$

Using the 2902 sets of ($\mathbf{v}$, $\mathbf{x}$, $\mathbf{F}_{\text{bearing}}$) shown in Figs. 4 and 5, the five coefficients of the equation A_1, A_2, A_3, A_4, and A_5, are determined by means of the least-squares method. Table I shows the coefficients obtained experimentally and theoretically. The experimental results are obtained by means of the least-squares method. On the other hand, the theoretical estimates are derived as follows:

As for the coefficient for velocity dependency A_1, the force component dependent on velocity can be theoretically estimated as the fluid friction due to the airflow inside the thin air film which is formed between the inner surface of the bearing holder and the outer surface of the moving part. Due to the relative motion between the inner surface of the bearing holder and the outer surface of the moving part, the dynamic frictional force $\mathbf{F}_{\text{friction}}$ is generated. Under the assumption that the dynamic frictional force is equal to the frictional drag of Couette flow in the air film, it is expressed as

$$\mathbf{F}_{\text{friction}} = A_1\mathbf{v} = -\mu_{\text{air}}(S/h)\mathbf{v},$$

where μ_{air} is the coefficient of viscosity of air, S is the surface area of the inside wall of the bearing holder, and h is the thickness of the air film. Then the dynamic frictional force $\mathbf{F}_{\text{friction}}$ is proportional to the velocity of the moving part $\mathbf{v}$. The coefficient A_1 is calculated to be -2.8×10^{-3} (Nm^{-1} s) using the nominal value of the thickness $\mathbf{h}$ of approximately 10 μm. The relatively large difference between the experimental results and the theoretical estimates on A_1 is thought to come from the form tolerance of the air film formed by the outer shape of the moving part and the inner shape of the bearing holder.

As for the other coefficients, A_2, A_3, A_4 and A_5, these coefficients are related to the steady aerodynamic force inside the air film of the bearing due to the asymmetrical flow of the air $\mathbf{F}_{\text{airflow}}$. The last coefficient A_5 is also related to the component of the gravitational force due to the inclination of the bearing holder against the horizontal plane $\mathbf{F}_{\text{gravity}}$.

$$\mathbf{F}_{\text{airflow}} + \mathbf{F}_{\text{gravity}} = A_2\mathbf{x}^3 + A_3\mathbf{x}^2 + A_4\mathbf{x}^1 + A_5.$$

However, there is not any information either on the asymmetry of airflow or on the inclination of the bearing holder. Therefore, the theoretically estimated values for these coefficients are zero.

Figure 6 shows the relationship between the measured value of the force acting on the moving part during the free-sliding measurement $\mathbf{F}_{\text{bearing,meas}}$ and the calculated value using the regression equation $\mathbf{F}_{\text{bearing,cal}}$. The data shown in Fig. 6 is the same 2902 sets of data as shown in Figs. 4 and 5. The root mean square value (rms value) of the difference
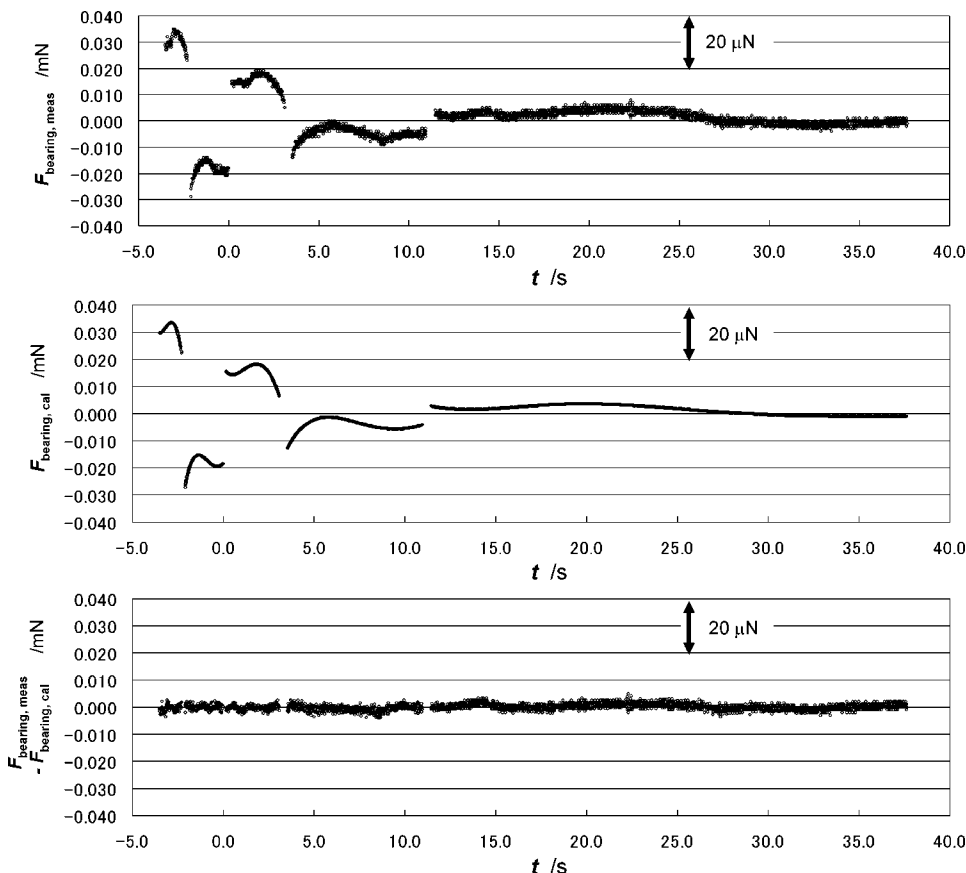

FIG. 6. Relationship between the measured force and the values calculated using the regression equation (during free-sliding motion).
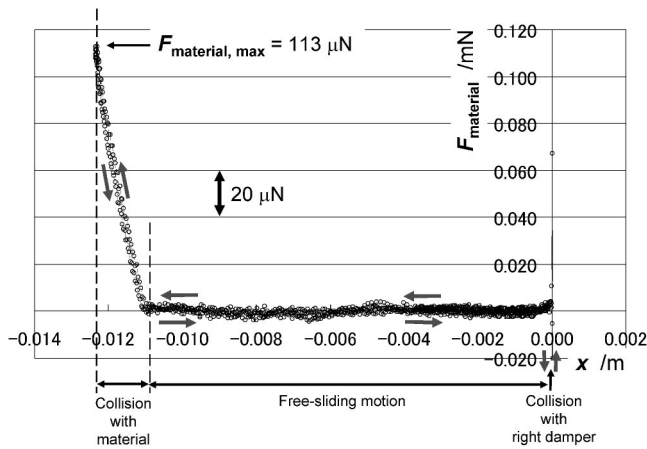
FIG. 7. Corrected force against position.

between $\mathbf{F}_{bearing,meas}$ and $\mathbf{F}_{bearing,cal}$ is 0.0012 mN (1.2 μN). This indicates both the accuracy of the force measurement and the validity of the supposed form of regression equation.

Figure 7 shows the change in the estimated force component acting on the moving part from the material under test $\mathbf{F}_{material}$ against the position $\mathbf{x}$. The force acting on the moving part from the material $\mathbf{F}_{material}$ is calculated by means of subtracting the other force calculated using the regression equation $\mathbf{F}_{bearing,cal}$ from the total force acting on the mass $\mathbf{F}_{mass}$ as follows:

$$\mathbf{F}_{material} = \mathbf{F}_{mass} - \mathbf{F}_{bearing, cal}.$$

The mean value and the root mean square value (rms value) of the force acting on the moving part from the material $\mathbf{F}_{material}$ in the region of the free-sliding of -11 mm $< \mathbf{x} < 0$ mm are approximately 0.3 and 1.3 μN, respectively.

V. DISCUSSION

The uncertainty components in the determination of the instantaneous value of the force acting on the test specimen—$\mathbf{F}_{material}$ are as follows:

(1) Determination of the total force acting on the moving part $\mathbf{F}_{mass}$.
(2) Electric counter (R5363). The uncertainty originating from the electric counter R5363 with the sampling interval of $\mathbf{dt}=40\,000/\mathbf{f}_{beat}$ (s) is estimated to be approximately 1 Hz. This uncertainty of the beat frequency corresponds to the uncertainty of the velocity of the moving part of approximately 5×10^{-7} m/s, according to the relational expression $\mathbf{v}=-\lambda_{air}(\mathbf{f}_{beat}-\mathbf{f}_{rest})/2$. This corresponds to the uncertainty of the acceleration and force of approximately 3×10^{-5} ms^{-2} and 7×10^{-7} N(0.7 μN), respectively.
(3) Optical alignment. The major source of uncertainty in the optical alignment is the inclination of the signal beam of 1 mrad, and it results in a relative uncertainty in the velocity of approximately 5×10^{-7}, which is negligible.
(4) Mass. Mass of the moving part is calibrated with a standard uncertainty of approximately 0.01 g, which corre-

sponds to the relative standard uncertainty in force determination of approximately 5×10^{-4}. This is negligible.
(5) Determination of the other force acting on the moving part $\mathbf{F}_{bearing}$. As explained above concerning Fig. 6, the root mean square value (rms value) of the difference between $\mathbf{F}_{bearing, meas}$ and $\mathbf{F}_{bearing, cal}$ is 0.0012 mN (1.2 μN). This corresponds to the uncertainty in determining the other force acting on the moving part $\mathbf{F}_{bearing}$ using the regression equation.

Therefore, the standard uncertainty in the determination of the force acting on the material is estimated to be 1.4 μN. This corresponds to 1.2×10^{-2} (1.2%) of the maximum force applied to the material under test of approximately 113 μN in the experiments.

The component of the gravitational force due to the inclination of the bearing holder against the horizontal plane $\mathbf{F}_{gravity}$ is very sensitive to the tilt angle of the tilting stage. The tilt angle of approximately 0.01 mrad (approximately 2 s) corresponds to the slope component of the gravitational force acting on the moving part $\mathbf{F}_{gravity}$ of approximately 0.002 mN (2 μN). This value is not negligible comparing with the measurement uncertainty of approximately 1.4 μN. Therefore, the slope angle of the bearing holder must be carefully monitored during the experiment. This can be monitored by means of monitoring the corrected force, during the free-sliding motion before and after the collision with the material as shown in Fig. 7. If the corrected force during the free-sliding motion greatly changes, this means that the component of the gravitational force due to the inclination of the bearing holder greatly changes and recalibration is required. Recalibration can be easily conducted by means of one more set of free-sliding measurement and one more regression analysis. However, the stability of the experimental setup described here was so high that recalibration was not necessary during several hours.

Using the developed method, the mechanical response of any material or structure, which can be attached to the movable base against impact microforce can be accurately evaluated. The method can be applied for various kinds of mechanical tests of materials using microforce, such as the viscoelasticity evaluation test against impact load and against oscillation load, friction evaluation test and strength evaluation test.

The developed method will also be very valuable both as a dynamic microforce calibration method and as a static microforce calibration method for force transducers. As for the static microforce calibration of force transducer, the developed method will be valuable since it is based on a unique principle that is different from the other methods developing. In general, comparing different methods is a very effective way to find out overlooked uncertainty sources.

ACKNOWLEDGMENTS

[1] D. B. Newell, J. A. Kramar, J. R. Pratt, D. T. Smith, and E. R. Williams, IEEE Trans. Instrum. Meas. **52**, 508 (2003).

[2] J. R. Pratt, D. T. Smith, D. B. Newell, J. A. Kramar, and E. Whitenton-rogress, J. Mater. Res. **19**, 366 (2004).

[3] P. J. Cumpson and J. Hedley, Nanotechnology **14**, 1279 (2003).

[4] S. Niehe, *Proceedings of XVII IMEKO World Congress*, Dubrovnik, Croatia, June 2003, p. 335.

[5] Y. Fujii and H. Fujimoto, Meas. Sci. Technol. **10**, N31 (1999).

[6] Y. Fujii, Rev. Sci. Instrum. **72**, 3108 (2001).

[7] T. Bruns, R. Kumme, M. Kobusch, and M. Peters, Measurement **32**, 85 (2002).

[8] Y. Fujii, Meas. Sci. Technol. **14**, 1741 (2003).

[9] R. Kumme, Measurement **23**, 239 (1998).

[10] Y-K. Park, R. Kumme, and D-I. Kang, Meas. Sci. Technol. **13**, 654 (2002).

[11] Y-K. Park, R. Kumme, and D-I. Kang, Meas. Sci. Technol. **13**, 1311 (2002).

[12] Y. Fujii, Meas. Sci. Technol. **14**, 1259 (2003).

[13] E. R. Williams, R. L. Steiner, D. B. Newell, and P. T. Olsen, Phys. Rev. Lett. **81**, 2404 (1998).

[14] Y. Fujii, Rev. Sci. Instrum. **74**, 3137 (2003).

[15] Y. Fujii, H. Fujimoto, and S. Namioka, Rev. Sci. Instrum. **70**, 111 (1999).

[16] Y. Fujii and T. Yamaguchi, Rev. Sci. Instrum. **75**, 119 (2004).

[17] Y. Fujii and T. Yamaguchi, J. Mater. Sci. (to be published).

[18] Y. Fujii, Mech. Syst. Signal Process. (to be published).