Method of generating and measuring static small force using down-slope component of gravity

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A method of generating and measuring static small forces at the micro-Newton level is proposed. In the method, the down-slope component of gravity acting on a mass on an inclined plane is used as a static force. To realize a linear motion of the mass with a small friction, an aerostatic linear bearing is used. The forces acting on the mass, such as the down-slope component of gravity and the dynamic frictional force, are determined by the levitation mass method. In an experiment, a static small force of approximately 183 μN is generated and measured with a standard uncertainty of approximately 2 μN. © *2007 American Institute of Physics.* [DOI: 10.1063/1.2746823]

Recently, number of requirements for evaluating small force in the range of 1 nN to 1 mN has increased in various industrial and research applications. However, it is sometimes very difficult to generate and evaluate small forces accurately. The difficulties in measuring small forces are mainly due to the following facts.

(1) No methods of measuring micro-Newton level forces have been established. Some methods of supporting the direct realization of static micro-Newton level forces linked to the International System of Units (SI) below 1 mN (Refs. 1–6) are currently being developed in some institutes.
(2) Small forces to be generated or measured are usually varying forces, and no dynamic calibration techniques for force sensors have been established yet. Some techniques are currently being developed.[7–11] In other words, both the uncertainty evaluation of the measured small force and the uncertainty evaluation of measurement time are extremely difficult.

As methods of static small-force generation and measurement, the following have been proposed and are now under development.

(1) Use of electrostatic balance:[1,2] In this method, an electrostatic balance that has two modes of operation is used. One is for the measurement of capacitance gradient and the other is for force comparison. The difference in the stress distribution and attitude of the electrodes between the two modes should be carefully considered.
(2) Use of precision electric balance:[3,4] In this method, a static small force is generated and measured by pressing the pan of the commercially available precision electric balance. The difference in conditions between the calibration using static weighing and the actual use should be considered. The feedback control system for the pan positioning might cause an error when the weighing pan is pressed mechanically and displacement is restricted.

For dynamic small forces, the method based on the Levitation Mass Method (LMM) proposed by the author[5,6] is the only means capable of generating and measuring small dynamic forces traceable to SI. In this article, a method of static small-force generation and measurement is proposed. The method is based on the method of dynamic small-force generation and measurement.[5,6]

Figure 1 shows a schematic diagram of the experimental setup for generating and measuring static small forces at the micro-Newton level. In the method, the down-slope component of gravity acting on a mass on an inclined plane is used as a static force. An aerostatic linear bearing is used to realize linear motion with a small friction acting on the mass, i.e., the moving part of the bearing. The moving part is made of aluminum and with a square pole shape; its total mass M is approximately 21.03 g. The inertial force acting on the mass is measured highly accurately using an optical interferometer. An arm of a hard-disk drive (HDD) is used as a spring element to which the generated static small force is applied.

In the experiment, tilt angle is set to be 3 min (0.87 mrad). The velocity of the mass is measured as the Doppler shift frequency of the signal beam using an optical interferometer. The measurement procedure is as follows: First, the mass is released around the right side of the guide way, and then it moves leftward due to gravity, collides with the arm of the HDD, and bounces back from it. This movement is damped oscillation. Finally, the mass reaches a standstill, where the down-slope component of gravity acting on the mass balances the force generated by the spring element.

During the measurement, the total force F_{mass} is measured as the product of mass and acceleration. Acceleration is calculated from the velocity of the moving part. Velocity is calculated from the measured Doppler shift frequency of the signal beam of the laser interferometer, f_{Doppler}, which is expressed as

$$v = \lambda_{\text{air}}(f_{\text{Doppler}})/2, \tag{1}$$

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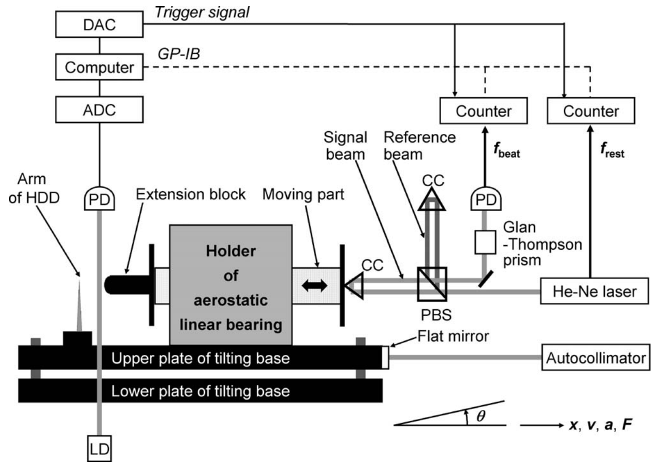


FIG. 1. Experimental setup.

$$f_{\mathrm{Doppler}} = -(f_{\mathrm{beat}} - f_{\mathrm{rest}}), \tag{2}$$

where λ_{air} is the wavelength of the signal beam under the experimental conditions, f_{beat} is the beat frequency, i.e., the frequency difference between the signal beam and the reference beam, f_{rest} is the rest frequency, which is equivalent to f_{beat} when the moving part is at a standstill.

The total force acting on the mass, F_{mass}, can be expressed as

$$F_{\mathrm{mass}} = Ma = F_{\mathrm{gravity}} + F_{\mathrm{air}} + F_{\mathrm{object}}, \tag{3}$$

where F_{gravity} is the down-slope component of gravity acting on the mass, F_{air} is the force component acting inside the air bearing such as dynamic friction, and F_{object} is the force acting from the HDD arm.

When the moving part is in a free fall motion along the slope apart from the arm of the HDD, the total force F_{mass} is expressed as

$$F_{\mathrm{mass}} = Ma = F_{\mathrm{gravity}} + F_{\mathrm{air}}. \tag{4}$$

By regression analysis, the down-slope component of gravity acting on the mass at the position after the damped oscillation is accurately determined.

A Zeeman-type two-frequency He–Ne laser is used as the light source. The frequency difference between the signal beam and the reference beam, i.e., the beat frequency f_{beat} is measured from the interference fringe that appears at the output port of the interferometer; it varies around the rest frequency f_{rest} of approximately 2.75 MHz, depending on the velocity of the movement.[6] Two electric frequency counters continuously measure the beat frequency f_{beat} and the rest frequency f_{rest} 4000 times and stores the values in memory. The sampling period of the counters are approximately 15 ms at a frequency of 2.75 MHz. The three-sample moving average is applied to the measured frequencies f_{beat} and f_{rest}.

In our experiment, one set of damped oscillation measurements is conducted at a tilt angle of approximately 3 min (0.87 mrad). Figure 2 shows the data processing procedure using the result of the experiment. The velocity, position, acceleration, and force are calculated from the measured frequency. The origin of position is set as the average of the beginning times of the impulses whose peak value is less than 1 mN.

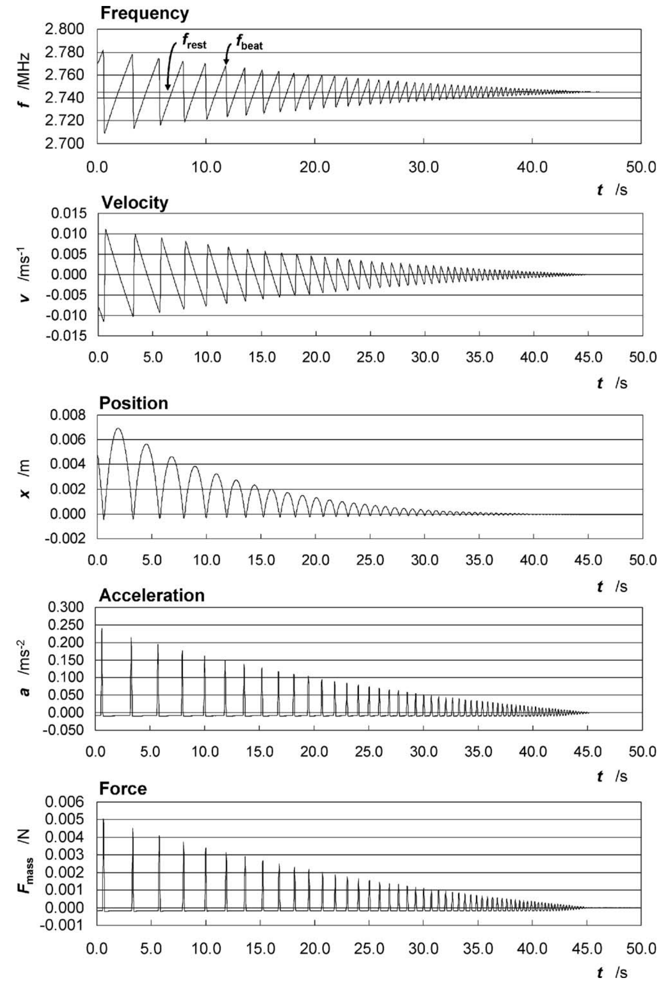


FIG. 2. Data processing procedure. Calculation of velocity, position, acceleration, and force from measured frequency.

Figure 3 shows the magnified figure of the change in force. There are 61 positive peaks of force during the measurement period of 50 s. Until the 50th peak of force at $t = 42.5$ s, the moving part moves away from the HDD arm after the collision; this region is indicated in Fig. 3 as "os-

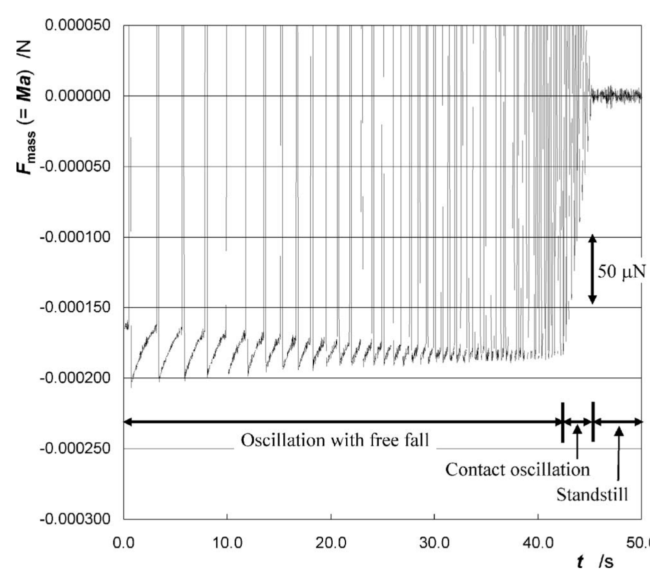


FIG. 3. Magnified figure of change in force.

TABLE I. Coefficients obtained experimentally and theoretically.

	Experimental results (results of regression)	Theoretical estimates
A_1 (N m^{-1} s)	-1.89×10^{-3}	-2.8×10^{-3}
A_2 (N m^{-1})	1.10×10^{-3}	0
A_3 (N)	-1.83×10^{-4}	-1.8×10^{-4}

cillation with free fall." After the 50th peak of acceleration, the moving part does not move away form the HDD arm during the oscillation; this region is indicated in Fig. 3 as "contact oscillation."

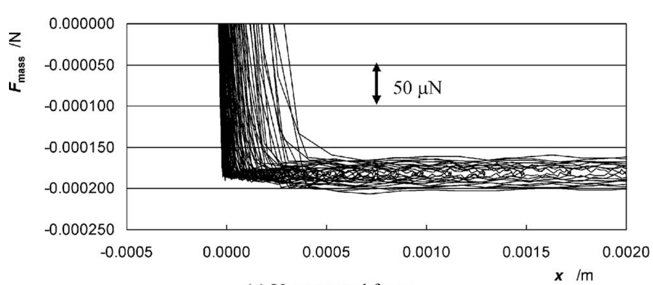
When the moving part is in the free fall motion along the slope apart from the arm of the HDD, the total force $F_{\text{mass}}=F_{\text{regression}}=F_{\text{gravity}}+F_{\text{air}}$ is supposed to be expressed as

$$F_{\text{regression}} = F_{\text{gravity}} + F_{\text{air}} = A_1 v + A_2 x + A_3. \quad (5)$$

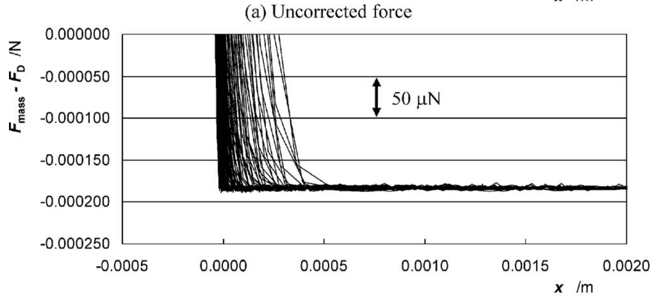
$F_{\text{regression}}=F_{\text{gravity}}+F_{\text{air}}$ can be considered as the force acting on the moving part due to the gravity and the airflow inside the bearing. Thus,-$F_{\text{regression}}$ when $v=0$ can be considered as the force acting on the object under test from the moving part when the moving part is at the standstill.

Using 574 sets of (v, x, F_{mass}) chosen under the condition that 0.5 mm$<x<$1.5 mm, the three coefficients A_1, A_2, and A_3 are determined by the least-squares method. Table I shows the coefficients obtained experimentally and theoretically. The experimental results are obtained by the least-squares method. On the other hand, the theoretical estimates are derived as described above.

Figure 4 shows the effect of the dynamic friction correction. In Fig. 4, there are 61 positive peaks of force measured during 50 s. Figure 4(a) shows the total force acting on the mass F_{mass} and Fig. 4(b) shows the value obtained by subtracting the frictional force $F_{\text{friction}}=A_1 v$ from the total force F_{mass}.



(a) Uncorrected force



(b) Corrected force

FIG. 4. Effect of dynamic friction correction.

The total force acting on the HDD arm from the mass at standstill is estimated by substituting $v=0.0$ ms^{-1} and $x=-38.6$ μm to Eq. (5).

$$F_{\text{mass}} = F_{\text{regression}} = A_1 v + A_2 x + A_3 = 0.0 + (-4.2\times10^{-8}) + (-1.83\times10^{-4}) = -1.83\times10^{-4}.$$

Therefore, the force acting on the HDD arm after the moving part stops is estimated to be -1.83×10^2 μN.

In the proposed method, the force acting on the HDD arm after the moving part stops was estimated by extrapolating the regression equation [Eq. (5)]. The uncertainty sources in the determination are as follows.

(1) Uncertainty of the regression equation [Eq. (5)]. The root mean square value (rms value) of the difference between F_{mass} and $F_{\text{regression}}$ is 2.0 μN. This discrepancy can be due to the inappropriateness of the form of the regression equation [Eq. (5)]. To be on the safe side, the whole amount is considered as the uncertainty due to the inappropriateness of the form of the regression equation [Eq. (5)].

(2) Uncertainty caused by extrapolation. The distance between the center of the sampling region 0.5 mm$<x$ $<$1.5 mm and the estimated point $x=-38.6$ μm is approximately 1.0 mm. The coefficient for position dependence is estimated to be $A_2=1.10\times10^{-3}$ N m^{-1}. The amount of correction for position is approximately 0.6 μN. To be on the safe side, the whole amount is considered as the uncertainty due to extrapolation.

Therefore, the standard uncertainty in determining the force acting on the HDD arm after the moving part stops is estimated to be 2.1 μN. This corresponds to 1×10^{-2} (1%) of the static force applied to the HDD arm of approximately -1.83×10^2 μN.

The proposed method of static small-force generation and measurement is based on the method of dynamic small-force generation and measurement.[5,6] The instrument described in Fig. 1 is capable of generating and measuring not only static small forces but also dynamic small forces. In this respect, the proposed method is the only method with such capability that has been proposed.

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