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The meaning of Beta: background and applicability of the target reliability index for normal conditions to structural fire engineering

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Abstract

Uncommon structural fire engineering designs must demonstrate adequate safety, in principle, through a balancing of the uncertain future costs and benefits of safety investments. In structural design for normal load conditions ('ambient design') this level of detail is commonly avoided through the application of reliability targets. In order to inform the development of reliability targets for structural fire design, the background of the ambient reliability targets is discussed. It is found that different common ambient reliability targets are broadly comparable when taking into account differences in assumptions and applications. As recent reliability targets have been informed by simplified cost-optimizations, the derivation of such a model is presented. The derivation allows identification of possible pitfalls when extending ambient reliability targets to structural fire design. It is concluded that ambient safety targets cannot readily be scaled as a function of the fire occurrence rate for application to structural fire engineering problems. The underlying cost-optimization model is, however, applicable as a concept. A number of issues requiring further attention are identified.

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Keywords: reliability index; cost-optimization; optimum safety level; structural fire engineering

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1. Introduction

In traditional structural fire safety design, an ‘adequate’ level of safety is typically assumed to result from the application of prescriptive design guidance and/or legislation [1]. These prescriptive recommendations have been developed over time, often in response to fire disasters [2], and represent the collective experience of the profession [1]. Due to their reliance on experience, prescriptive design rules are, however, unable to proactively adapt to new developments, resulting in a trend towards Performance Based Design (PBD) of structures for fire [2].

PBD in structural fire engineering is commonly deterministic in nature, meaning that uncertainties associated with the fire scenario, input parameters and material parameters are rarely explicitly considered. More precisely, although onerous inputs may be considered through engineering judgement, an explicit safety evaluation is largely absent [1,3]. For these deterministic evaluations, the attainment of an ‘adequate safety level’ is again fundamentally based on the collective experience of the profession [1], limiting its field of application to common situations [3].

For uncommon fire engineering designs, the attainment of adequate safety is necessarily demonstrated [1], in principle, through a Probabilistic Risk Analysis (PRA). As part of the PRA, the tolerability of the proposed design solution is assessed, and its compliance with the ALARP requirement demonstrated [1]. An explicit evaluation of the ALARP requirement entails balancing the uncertain future costs and benefits of further safety investments, thus demonstrating that the residual risk level is As Low As Reasonably Practicable (ALARP) [4]. The valuation of costs and benefits, however, quickly becomes challenging, and can be considered onerous by fire safety engineers.

In ambient structural engineering design, the challenge of valuing future costs and benefits of safety investments is regularly avoided by the direct application of a target reliability index, β_t [5,6,7]. The target reliability index corresponds with the *accepted maximum (target) failure probability*, $P_{f,t}$ [6], through Eq. (1), with Φ the standard cumulative normal distribution function.

$$P_f = \Phi\left(-\beta\right) \qquad (1)$$

Since no commonly accepted target reliability indices exist for the appraisal of structures exposed to fire, the possible application of ambient target values has received some attention [3,8]. Before applying ambient safety targets to structural fire engineering design, however, it is important to study their interpretation and background. The following sections provide background to the reliability targets applied in ambient structural design, and introduce a simplified cost-optimization model capable of making order-of-magnitude assessments of target reliability indices. The simplified model is then used to identify possible pitfalls in the application of ambient safety targets to structural fire design. All examples and derivations are concerned with ultimate limit state design (i.e. loss of structural stability).

2. Reliability targets in ambient structural design

Reliability-based design has found wide application in structural engineering [9]; for example as the basis of the partial safety factors applied in the Structural Eurocodes [10], the target reliability index, β_t, governs everyday structural engineering practice. Different (recent) target values are, however, available from several sources.

2.1. ISO 2394:1998 [11]

ISO 2394:1998 lists ‘example’ lifetime target reliabilities as a function of the failure consequence and the relative costs of safety measures [11], Table 1. Based on the formulation in ISO 2394:1998, these values have been informed by cost-optimization and calibrated against existing practice. The standard further recommends the values 3.1, 3.8 and 4.3 to be used in ultimate limit state design (Table 1) based on both consequence of failure and cost of safety measures. Considering the general content of the standard, these values are considered applicable at an element level. ISO 2394:1998 also notes that the cause and mode of failure should inform the assumed target reliability index, with a higher reliability required in case of failures without pre-warning, and that the target reliabilities should be considered as formal numbers intended for developing consistent design rules.

Table 1. Target β-values for elements (lifetime), ISO 2394:1998 [11]

Relative costs of safety measures	Consequences of failure			
	small	some	moderate	great
High	0	1.5	2.3	3.1
Moderate	1.3	2.3	3.1	3.8
Low	2.3	3.1	3.8	4.3

2.2. EN 1990 [5]

Target reliability indices specified in EN 1990 as a function of the 'reliability class' [5] are given below in Table 2. The reliability classes can be associated with the consequence classes (i.e. high, medium, low) [5]. As also noted in ISO 2394:1998, considerations such as brittle or ductile failure may influence the chosen target.

The Eurocode target reliability indices are specified both for a 1-year reference period and a 50-year reference period (where 50 years equals the indicative design working life for common structures [5]). Both sets, however, correspond with the same target reliability level, considering independence of yearly failure probabilities [10], i.e. irrespective of how long a structure has been standing, it is assumed that the per annum failure likelihood is constant. There is thus close agreement between $\beta_{t,50}$ in Table 2 and the lifetime targets in ISO 2394:1998.

The material-specific Eurocodes apply the 50-year reliability index of 3.8 on an element basis for the definition of partial safety factors. In case of additional redundancy in the system, this will result in a higher system reliability index. In a series system, however, applying the target reliability index to each of the members, will result in a system reliability index below the element target (if the members are to some degree independent).

Table 2. Target reliability index for structural elements in accordance with EN 1990 [5]

Reliability class	Consequences	Target reliability index $\beta_{t,tref}$		Examples of buildings
		t_{ref} = 1 year	t_{ref} = 50 years	
3 – high	High	5.2	4.3	Bridges, public buildings
2 – normal	Medium	4.7	3.8	Residential, office
1 – low	Low	4.2	3.3	Agricultural

2.3. Rackwitz [9]

Rackwitz proposed target (1 year) reliability indices for structural components (Table 3) based on simplified lifetime cost-optimization calculations. Further details of Rackwitz's model are given in Section 3. Unfortunately, no details on the input variables underlying Table 3 are available.

Table 3. Target β-values for structural elements based on cost-optimization (1 year), Rackwitz [9]

Relative costs of safety measures	Consequences of failure		
	insignificant	normal	large
High	2.3	3.1	3.7
Moderate	3.1	3.7	4.3
Low	3.7	4.3	4.7

2.4. Probabilistic Model Code [6] and ISO 2394:2015 [7]

Target values for a 1-year reference period are given in the Probabilistic Model Code [6] developed by the Joint Committee on Structural Safety (JCSS), see Table 4. These recommended values were derived from a calibration process with respect to existing practice [12] and are considered compatible with cost-benefit analyses [12], with explicit reference to the analysis by Rackwitz [9].

Table 4 is applicable to structural systems [12]. In case of a single element and failure mode dominating system failure, these targets are directly applicable to the structural element. Higher reliabilities are required when multiple equally important failure modes exist [6]. The target values are given as a function of the ratio ξ of the failure plus reconstruction cost to the construction cost, and consider an obsolescence rate on the order of 3%. For very large consequences ($\xi > 10$) an explicit cost-benefit analysis is recommended [6, 12].

JCSS clarifies that failure probabilities in structural design should be interpreted from a Bayesian perspective [6,12], i.e. corresponding with the best possible expression of belief. The target reliabilities in Table 4 have been incorporated in ISO 2394:2015 [7], with reference to both [6] and [9]. It is noteworthy that the targets for minor and large consequence classes in Table 4 correspond well with those for normal and large consequences proposed by Rackwitz (Table 3).

Table 4. Target β-values for structural systems (1 year), JCSS [6], and adopted in ISO 2394:2015 [7]

Relative costs of safety measures	Consequences of failure		
	minor ($\xi < 2$)	moderate ($2 < \xi < 5$)	large ($5 < \xi < 10$)
High	3.1	3.3	3.7
Moderate	3.7	4.2	4.4
Low	4.2	4.4	4.7

2.5. Comparison

Based on the above, two sets of target values can be identified: the Eurocode target values of EN 1990 [5] informed by ISO 2394:1998 [11], and the JCSS and ISO 2394:2015 targets [6,7] informed by Rackwitz's study [9].

The 1-year target reliabilities of EN 1990 [5] exceed the targets recommended in [6,7]. Vrouwenvelder, however, notes that the Eurocode independence assumption for yearly failures is unrealistic, and that $\beta_{t,50} = 3.8$ (which is the basis of Eurocode's partial factors) corresponds better with $\beta_{t,1} = 4.5$ [12], thus reducing the actual difference. Furthermore, the Eurocode target reliability index is applied in practice to structural elements, while the JCSS targets are applicable to structural systems. Vrouwenvelder clarifies that generally the element target reliability index must be larger than the system target reliability index, except for highly redundant structures [12]. Based on the above, it is tentatively concluded that the Eurocode and JCSS targets are broadly compatible.

3. Derivation of target reliability indices from cost-optimization

Rackwitz's cost-optimization procedure has strongly influenced recent thinking on appropriate target safety levels for structures. Rackwitz's simplified cost-optimization is therefore revisited here based on [9,13]. The analysis gives insight into the underlying parameters and assumptions, and forms the basis for discussing possible implications for structural fire design targets in Section 4 of this paper.

The underlying presumption in Rackwitz's analysis is that design codes should be optimized so as to minimize the total lifetime cost of structures [13]. The obtained societal reliability targets then function as a lower bound to possible private considerations, as recommended in ISO 2394:2015 [7].

Eq. (2) specifies the basic objective function (lifetime utility) to be maximized, with p being the optimization parameter and other terms as given in Table 5. As costs and benefits accrue over the lifetime of the structure, a present value evaluation must be made. The present value assessment considers systematic renewal after failure or obsolescence, assuming a continued need for similar structures [13]. Consequently, an infinite time horizon is considered, as in [9,13,14], see [13]. It is noteworthy that the time horizon is not directly linked to the reference period used e.g. as in Table 2. The reference period defines the timeframe over which variable parameters (e.g. imposed loads) are assessed, while the time horizon relates to the time interval considered in the decision.

Present value assessments for the (considered) constituent terms are given in Table 5, see [13,15] for details. Relevant parameters are given in Table 6.

$$Z(p) = B - C(p) - A(p) - D(p) - U(p) - M(p) - I(p) \tag{2}$$

Table 5. Constituent terms in the cost-optimization.

Symbol	Notes	Present value assessment
B, benefit from the structure's existence	Independent of *p*	Not considered for optimization
C, construction cost	Linear model as a function of *p*	$C(p) = C_0 + C_1 p = C_0(1+\varepsilon p)$
A, obsolescence cost	Defined by obsolescence rate *ω* [13]	$A(p) = \left[C(p) + A_0\right]\frac{\omega}{\gamma}$
D, ultimate limit state failure cost	Direct and indirect failure costs, including cost of reconstruction	$D(p) = \left[C(p) + H_0\right]\frac{\lambda P_f(p)}{\gamma} = C_0 \xi \frac{\lambda P_f(p)}{\gamma}$
U, serviceability limit state failure cost	Neglected	Not considered for optimization
M, ageing failure cost	Neglected	Not considered for optimization
I, inspection and maintenance cost	Included in the construction cost	Included in the construction cost

Table 6. Parameters in the cost-optimization

Symbol	Interpretation	Equation or default value
p	Design central safety factor (optimized parameter)	$p = \frac{\mu_R}{\mu_E}$
C_0	Base construction cost	-
C_1	Marginal safety cost	-
ε	Relative marginal safety cost	$\varepsilon = \frac{C_1}{C_0}$
A_0	Obsolescence cost (demolition), excl. reconstruction	-
ω	Obsolescence rate	0.02 / year
γ	Continuous discount rate	0.02 / year
λ	Adverse event occurrence rate [1/year]	λ = 1/year for ambient design
H_0	Direct and indirect failure costs, excl. reconstruction	-
ξ	Relative failure costs, incl. reconstruction	$\xi = \frac{C_0 + C_1 p + H_0}{C_0} \approx \frac{C_0 + H_0}{C_0}$
ε_ω	Obsolescence-corrected relative marginal safety cost	$\varepsilon_\omega = \varepsilon\left(1 + \frac{\omega}{\gamma}\right)$
η	Damage parameter	$\eta = \frac{\xi\lambda}{\gamma}$
DII	Damage to Investment Indicator	$DII = \frac{\eta}{\varepsilon_\omega} = \frac{\xi\lambda}{\varepsilon(\gamma+\omega)}$

The benefit term B is considered independent of the safety parameter p, and thus does not influence the optimum safety investment. Consequently, the utility maximization corresponds with a minimization of costs. Normalizing the lifetime cost by division with C_0 and subtracting constant terms, gives the normalized lifetime cost Y, Eq. (3), introducing the 'obsolescence-corrected relative marginal safety cost', ε_ω, and the damage-parameter, η (Table 6).

The number of parameters in the above can be reduced further by division by ε_ω, resulting in Eq. (4) which introduces *DII* as the 'Damage to Investment Indicator'. Minimizing the normalized cost of Eq. (4), results in the optimum design criterion of Eq. (5).

$$Y = \varepsilon\left(1+\frac{\omega}{\gamma}\right)p + \frac{\xi\lambda}{\gamma}P_f\left(p\right) = \varepsilon_\omega p + \eta P_f\left(p\right) \tag{3}$$

$$Y_0 = \frac{Y}{\varepsilon_\omega} = p + \frac{\eta}{\varepsilon_\omega}P_f\left(p\right) = p + DII\cdot P_f\left(p\right) \tag{4}$$

$$\min_p\left\{Y_0\left(p\right)\right\} \Rightarrow \frac{dY_0\left(p\right)}{dp} = 0 \Rightarrow \frac{1}{DII} = -\frac{dP_f\left(p\right)}{dp} \tag{5}$$

In accordance with the derivations in [9,13], and for illustrative purposes, the decision parameter *p* corresponds with the central safety factor of the design (Table 5). This central safety factor relates to the general limit state $g = R - E$, where *R* is the resistance effect and *E* is the load effect. Both parameters are assumed to be described by a lognormal distribution, with a mean value of μ and a coefficient of variation *V*, in accordance with the simplified model in [9,13]. Consequently, the probability of failure $P_f(p)$ is given by Eq. (6). Failure probabilities as a function of *p* are visualized in Fig. 1a for different V_R and V_E. The bold red curve relates P_f to the reliability index β, as defined by Eq. (1). Fig. 1a further illustrates how $p = 4.12$ results in $\beta = 4.7$, when $V_R = 0.1$ and $V_E = 0.3$.

$$P_f\left(p\right) = P\left[g = R - E < 0\right] = \Phi\left(\frac{\ln\left(\frac{1}{p}\sqrt{\frac{V_R^2+1}{V_E^2+1}}\right)}{\sqrt{\ln\left(\left(V_R^2+1\right)\left(V_E^2+1\right)\right)}}\right) \tag{6}$$

Considering $P_f(p)$, Eqs. (4) and (5) are evaluated in Fig. 1b for different 'Damage to Investment Indicators', *DII*, with *p* in the range 0 to 8. For $DII = 1$, no costs are associated with failure (apart from reconstruction cost), and Eq. (4) results in a local minimum (with the real optimum being zero investment in safety). For $DII = 1000$, Fig. 1b indicates $p_{opt} = 2.85$, corresponding with $\beta_{opt} = 3.5$.

Generalizing Fig. 1b, optimum reliabilities, β_{opt}, as functions of *DII* are given in Fig. 2a for different V_R and V_E. Fig 2b shows the same results, however given as optimum failure probabilities, $P_{f,opt}$.

The simplified model can be used to develop a table of optimum reliabilities, similar to Table 4. The relative cost of safety measures is assumed to be $\varepsilon = 0.1/0.01/0.001$, after Fischer [13]. Furthermore, $\omega = \gamma = 0.02$ (Table 6), and the consequences of failure are given by $\xi = 2/4/8$, after Vrouwenvelder [12]. Results are given in Table 7 for $V_R = V_E = 0.2$, with the bracketed values being the associated *DII* (considering $\lambda = 1$/year [9]).

Although the obtained optimum reliabilities in Table 7 deviate from the recommended values in Table 4, this simplified assessment gives a good order of magnitude approximation. It is concluded that the simplified model can be considered as, at minimum, informative with respect to ambient reliability targets.

4. Implication for structural fire safety design

The application of the ambient reliability targets to structural fire design has received considerable research attention [3,8]. In the Natural Fire Safety Concept (NFSC) [8], the Eurocode target reliability index of 3.8 (50-year reference), i.e. 4.7 for 1-year reference, was adopted as a starting point [8]. By further considering fire-induced structural failures to be conditional on the occurrence of a 'significant' fire, the NFSC derives a target reliability index, $\beta_{t,fi}$, for structural fire design through Eq. (7), with λ_{fi} being the annual occurrence rate of a significant fire.

$$\Phi\left(-\beta_{t,fi}\right)=P_{f,t,fi}=\frac{P_{f,t,EN1990}}{\lambda_{fi}}=\frac{\Phi\left(-\beta_{t,EN1990}\right)}{\lambda_{fi}}\Rightarrow\lambda_{fi}P_{f,t,fi}=P_{f,t,EN1990} \tag{7}$$

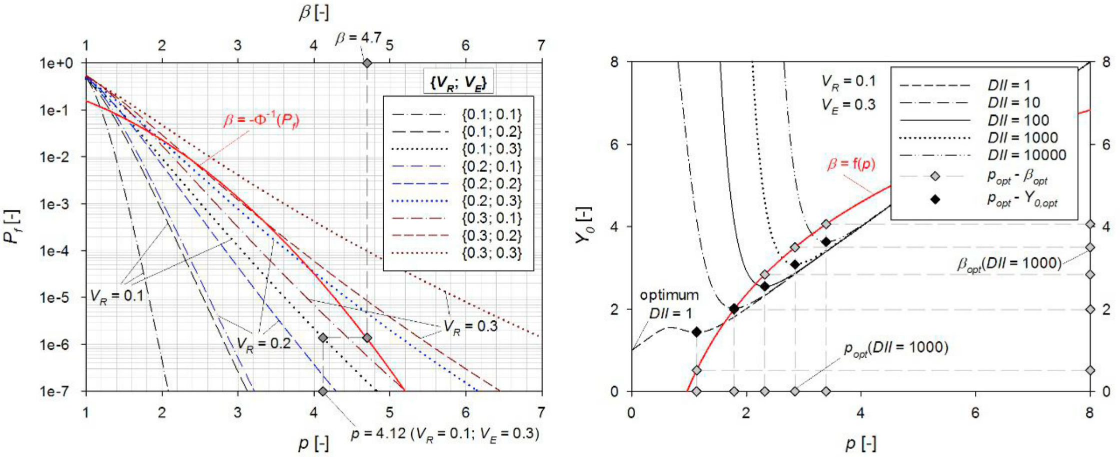

Fig. 1. (a) Evaluation of $P_f(p)$ (Eq. (6)) and corresponding β (Eq. (1)); (b) Y_0 as a function of p, and optimum design p_{opt}, for different *DII*.

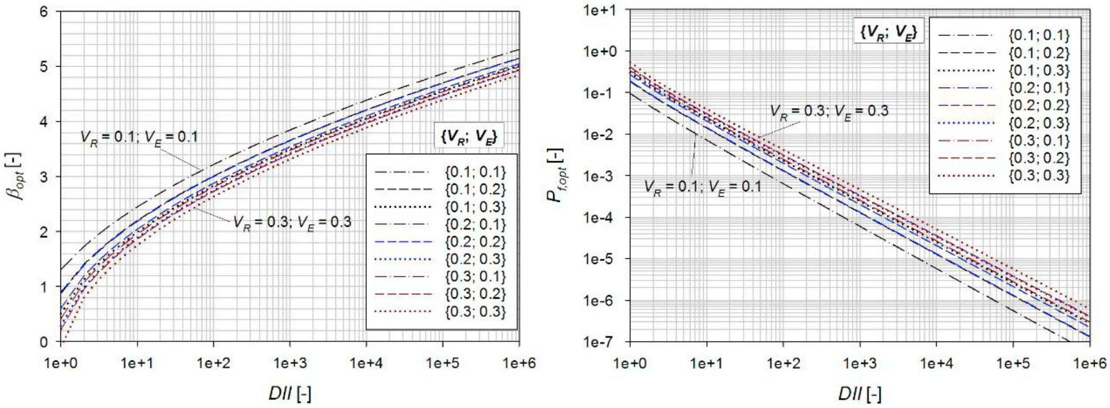

Fig. 2. (a) β_{opt} as a function of *DII*, considering Eqs. (6,7) and different V_R and V_E. (b) $P_{f,opt}$ as a function of *DII*, same results as in Fig 2(a).

Table 7. Optimum β-values in accordance with Eq. (6,7). Bracketed values for considered *DII*-value.

Relative costs of safety measures	Consequences of failure		
	minor ($\xi = 2$)	moderate ($\xi = 4$)	large ($\xi = 8$)
High ($\varepsilon = 0.1$)	3.4 (500)	3.5 (1000)	3.7 (2000)
Moderate ($\varepsilon = 0.01$)	3.9 (5000)	4.1 (10000)	4.3 (20000)
Low ($\varepsilon = 0.001$)	4.5 (50000)	4.6 (100000)	4.7 (200000)

However, based on the general derivation of target reliability indices in Section 3, the appropriateness of Eq. (7) can be questioned. Fig. 2b shows that the logarithm of the optimum failure probability $P_{f,opt}$ is approximately linear with respect the logarithm of *DII*. As *DII* is directly proportional to the adverse event occurrence rate λ (see Table 6), $P_{f,opt}$ is clearly not linear with respect to λ as implicitly assumed by Eq. (7).

The derivations in Section 3 have been made in general terms, and thus the results in Figs. 2a and 2b can, in principle, inform reliability targets for structural fire design. The general cost-optimization model of Section 3 can be extended to inform reliability targets for structural fire design, notwithstanding a number of potential pitfalls that require further attention before generalized reliability targets can be proposed (see Table 8).

Table 8. Parameter-wise discussion of application of Section 3 to inform reliability targets for structural fire design

Symbol	Discussion	Pitfall
$P_f(p)$	The limit state $g = R - E$ remains applicable	
p	Central safety factor as decision variable can in principle be applied (difficulty in valuation ε)	
R	Lognormal model for resistance effect less readily applicable in fire design, see e.g. [14]. Fire-induced restraining forces should (possibly) be taken into account.	X
V_R	Generally increases for fire, e.g. [14]. Appropriate values to be established for fire design.	X
E	Load effect not affected by fire, if fire-induced restraining forces are considered as directly affecting R. Lognormal simplifying assumption is as relevant as for ambient conditions.	X
V_E	Not affected, under above condition of fire-induced restraining forces directly affecting R.	
DII	Damage to Investment Indicator fundamentally different to ambient case (see def. Table 6)	X
ω, γ	Not affected – economic parameters	
λ	Directly influenced as occurrence rate relates to that of a significant fire [8,13,15]	
ε	Marginal safety cost can be different, as a function of application. Material-specific differences can be expected.	X
ξ	Relative failure costs can be different. Consequences of fire-induced structural failure not necessarily comparable with consequences of ambient structural failure.	X

5. Conclusions

Target reliabilities for ambient structural design have been compared. While some sources' recommendation is based on a calibration procedure, others refer to simplified cost-optimization. Considering differences in assumptions and applications, all proposed ambient reliability targets are found to be broadly comparable. It is thus concluded that simplified cost-optimization can inform the definition of appropriate reliability targets.

A general simplified cost-optimization model has been presented, based on well-established work by Rackwitz [9]. Optimum reliabilities are calculated as a function of cost and investment parameters, and the optimum reliabilities are found to be comparable to existing recommended values for ambient target reliabilities.

The application of the simplified model to inform reliability targets for structural fire design is briefly discussed. It is concluded that a direct scaling of ambient target reliabilities as a function of the occurrence rate of significant fires (as in the Natural Fire Safety Concept) is not strictly appropriate. Furthermore, potential pitfalls in the application of the simplified cost-optimization model for structural fire design are identified. It is concluded that the simplified model can, in principle, inform structural fire design, provided that the different cost-parameters are appropriately identified and that due consideration is given to the probabilistic description of the resistance effect.

References

[1] R. Van Coile, D. Hopkin, D. Lange, G. Jomaas, L. Bisby. Safety foundation of fire safety engineering. *In preparation.*

[2] G. Spinardi, L. Bisby, J. Torero, A Review of Sociological Issues in Fire Safety Regulation. Fire Technology 53 (2017) 1011-1037.

[3] D. Hopkin, S. Anastasov, K. Swinburne, S. Lay, D. Rush, R. Van Coile, Application of ambient temperature reliability targets for appraising structures exposed to fire. Proceedings of CONFAB 2017, 11-12/09 (2017), London, UK.

[4] BSI. PD 7974-7:2003: Application of fire safety engineering principles to the design of buildings – Part 7: Probabilistic risk assessment, 2003.

[5] CEN. EN 1990: Eurocode 0: Basis of structural design. European Standard, 2002.

[6] JCSS. Probabilistic Model Code: Part 1 – Basis of Design. The Joint Committee on Structural Safety, 2001.

[7] ISO. ISO 2394:2015 – General principles on reliability of structures. International Standard, 2015.

[8] EC. Competitive steel buildings through natural fire safety concepts. Office for Official Publications of the European Union, 2002.

[9] R. Rackwitz. Optimization – the basis of code-making and reliability verification. Structural Safety 22 (2000) 27-60.

[10] M. Holicky, A. Materna, G. Sedlacek, A. Arteaga, L. Sanpaolesi, T. Vrouwenvelder, I. Kovse, H. Gulvanessian. Implementation of Eurocodes: Handbook 2: Reliability Backgrounds. Leonardo Da Vinci Pilot Project CZ/02/B/F/PP-134007.
[11] ISO. ISO 2394:1998 – General principles on reliability of structures. International Standard, 1998.
[12] T. Vrouwenvelder. Developments towards full probabilistic design codes. Structural Safety 24 (2002) 417-432.
[13] K. Fischer. Societal Decision-Making for Optimal Fire Safety. Doctoral dissertation. ETH Zurich, 2014.
[14] R. Van Coile. Reliability-Based Decision Making for Concrete Elements Exposed to Fire. Doctoral dissertation. Ghent Univeristy, 2015.