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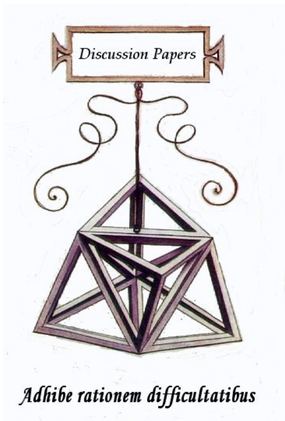

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Communicating the uncertainty of synthetic indicators: a reassessment of the HDI ranking

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Abstract

Composite indicators convert information about different facets of a given phenomenon into a single figure. Unavoidably, the "conversion process" involves a high level of arbitrariness, which, in general, makes the results not robust. The approach to composite indicators used in this paper aims at mitigating this problem and makes final users more aware of the unavoidable uncertainty of the results (e.g. rankings) based on a given composite. We illustrate our approach by applying it to the Human Development Index.

Keywords: Human Development Index, complexity, composite indicators, robustness, uncertainty analysis.

JEL: C430 Index Numbers and Aggregation Leading indicators; Q010 Sustainable Development; I310 General Welfare

1. Introduction

Rankings have become very popular not only in sports but in many domains of social and economic life. The most used approach is to build rankings by using composite indicators. The question why composites and rankings became so widespread is less trivial than it might appear. Obviously, rankings serve the purpose of evaluating objects that are not immediately comparable to one another due to their multidimensional nature. In some cases, e.g. sport competitions, composites easily accomplish this task and the rules used to calculate the composite indicator are a substantial part of the game. Why using composites beyond a context of games is less obvious. Why do we build composites for questions "Which is the best city to live in?" "How does a given university perform?". A common answer is benchmarking, i.e., comparing performances to one another in order to stimulate progress. To this purpose, however, it is necessary to go deep into each aspect of the synthetic performance as measured by the composite. This points out the core issue regarding composites, which is the loss of relevant information when heterogeneous pieces of information are combined together. It is true that our capability of processing information is limited, as, for instance, the work of Herbert Simon has emphasised (e.g. Simon, 1978). This limitation, however, does not imply the necessity of getting to the extreme level of synthesis, that is, to condense all the information into a single number. In fact, this is against rationality and against everyday experience. When buying a smartphone, a car, a laptop, nobody asks the expert for a single number to describe each alternative, we are rather interested in the specific features of the alternatives. Similarly, when looking for a hotel on the Internet, we give a glimpse at the global ranking, but we commonly choose the accommodation on the basis of those criteria that are most relevant to our personal needs and tastes.

Surprisingly, when the evaluation falls within the sphere of public debate, the reduction to a single figure is a very common practice, as in the case of Cost-Benefit analysis where the different aspects of each alternative project are summarised into a single figure. This is surprising since more robust methodological approaches are available, such as multi-criteria evaluation methods (see, e.g., Munda 2008). Paradoxically, also the theoretical debate suggests that is not a good idea to build a ranking by reducing all the information to a single figure. Actually, rank building can be easily seen as the social choice problem of aggregating individual preferences into a social ordering. The debate on this dates back at least to the end of the 18th century, that is, to the Borda-Condorcet controversy (see e.g. Brian, 2008). After Kenneth Arrow's impossibility theorem, one can safely affirm that no method for establishing a complete order is perfect.

The well-known Index of Human Development[1] (HDI) is an example of this "mania" for a single number. As for any composite indicator, its extremely synthetic nature is a major reason both for its success (see, e.g., Paruolo et al., 2013) and for well-founded criticism. By using HDI as an example, this paper aims to show that composites can be used in a wider perspective than the traditional one so as to (i) reduce the arbitrariness related to the choice among alternative normalization and combination rules, and (ii) avoid a simplistic view of the phenomenon under inquiry by communicating the intrinsic[2] uncertainty of the outcome involved by a single composite. To this purpose, similarly to Saisana and Munda (2008), Floridi et al. (2011), and Luzzati and Gucciardi (2015), we put uncertainty analysis at the centre of both the methodological approach and the substantive analysis. First we calculated many different composites, and their related rankings, according to different normalization and aggregation rules. Then we computed the frequency distribution of the different ranks got by each Country and calculated a plausible range for them. We also performed a clustering analysis, again under different standardisation rules, and checked if it could help in improving the uncertainty analysis, in particular whether it could be useful for testing the validity of the cut-off points currently used to classify different level of Human Development.

Not many papers have addressed the issue of the robustness of the HDI. Cahill (2005), Herrero et al. (2012), Klugman et al. (2011), Morse 2014, and Zambrano (2011) are among them. The purpose of these papers was to contribute to the debate on the methodological assumptions at the basis of the calculation of the HDI. The paper by García and Kovacevic (2010) is particularly relevant here since they followed the same approach that inspired the work presented in this paper, that is, the approach set forth in the guidelines for constructing composite indicators elaborated by the OECD and JRC (Nardo et al., 2008) and in several other works by members of composite indicators research group of the JRC of the European Commission. Our work, however, however has a more limited scope than that by García and Kovacevic (2010). We will discuss similarities and differences when appropriate and relevant.

[1] The HDI has been published since 1990 by the United Nations Development Program (UNDP) in its yearly Human Development Report. As well known, the purpose of HDI is to capture the development of a country on the basis of three dimensions - "health", "knowledge" and "standard of living". After some changes in the methodology that have been made in the past years, HDI is calculated as the geometric mean of normalized indices for each of its three dimensions.

[2] Such uncertainty derives from the above-mentioned arbitrariness related to the choice among alternative normalization and combination rules that are used to construct the composite. It also depends on other issues that we do not discuss here, such as the choice of the relevant set of information.

The paper is organised as follows. Section 2 illustrates the methods that we used, section 3 reports and shortly discusses the results of the uncertainty analysis, section 4 contains the cluster analysis, section 5 concludes.

2. Methods

Uncertainty analysis aims at understanding the effects of "non essential" changes in the method of calculating the index. The term "non essential" refers to assumptions that can be justified neither by some data properties nor by theoretical reasons. García and Kovacevic (2010) developed their analysis by considering as non essential (i) the functional form of life expectancy (whether log transformed or not), (ii) the minimum goalposts for income and life expectancy, and (iii) the weighting system. They kept the geometric aggregation assumption and the rule of data normalization[3]. As a result they obtained many possible indexes, which are, however, comparable among them due to the homogeneity of their aggregation and normalization rule.

The uncertainty analysis that is performed here is wider for two reasons. Firstly, although we are in favour of aggregation rules that reward balanced achievement in all dimension and limit substitutability, such as the geometric one, we also acknowledge (i) the theoretical reasons that justified using the linear aggregation for 20 years and (ii) the existence of other are interesting aggregation rules that are in between the geometric and the linear one. Secondly, we used also other normalization rules that do not contrast with the spirit of the HDI. Hence, our uncertainty analysis is based on varying the normalisation rule, the aggregation rule, and the weighting system.

We retained all the other methodological assumptions, in particular

1) the diminishing returns from income for human development, which involves using the logarithm of Gross National Income;
2) the existence of some goalposts, the 'natural zeros' and 'aspirational goals' (HDI 2015, p.2), which are, respectively, 20 and 85 years for Life Expectancy, 0 and 16.5 years[4] for Education, and 100$ and 75,000$ (at 2011 PPP) for GNI.

To keep those assumptions, we transformed the raw data available from to UNDP website as follows:

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[3] HDI normalisation is a sort of "distance from the leader" (Nardo et al, 2008) calculated after rescaling data for considering the minimum and maximum goalposts.

[4] Education is made of two components, the years of schooling for adults aged 25 years and more and the expected years of schooling for children of school entering age. The first indicator is capped at 15 years, the second at 18. Since they are linearly aggregated with equal weight, we made an average of their respective goalposts, i.e. 0 and 16,5, and rescaled them accordingly.

(i) we subtracted 20 years from Life Expectancy,

(ii) we capped income to 75,000$ and used the difference between the logarithm of income and subtracted the logarithm of 100$

(iv) we rescaled the education indicators as explained in footnote 3.

As emphasised in the mentioned OECD and JRC guidelines for composite indicators (Nardo et al., 2008), both standardisation and aggregation rules are very important. As we stated above, we combined different normalization and aggregation rules, as well as different weighting systems, in order to calculate many alternative composites and performed an uncertainty analysis of the HDI.
We started by considering three aggregation rules, that is, the linear, the geometric, and the concave one[5], as defined by Casadio, Tarabusi and Palazzi (2004). The concave one is a kind of compromise between the linear and the geometric one. It is close to the linear one when performances are high, while strongly punishes low performances. In practice, while the geometric aggregation punishes unbalanced performances, the concave one does it only when performances are poor. The aggregation rules are defined in Table 1.
Concerning the normalization rules, we chose[6] the z-score, the min-max, and the normalization that is currently in use for the HDI. We rescaled the z-score and the min-max as indicated in Table 2. We did so for two reasons, firstly in order to exclude negative and zero values so that they are suitable for the concave and the geometric aggregation rules, secondly in order to have ranges of variation, means and standard deviations similar to the ones got with the HDI normalization[7].

[5] An interesting rule is contained in the Mazziotta-Pareto Index (de Muro et al. 2011). We did not use it since with this particular dataset it would have given results too similar to the linear aggregation case.
[6] The OECD and JRC guidelines to the construction of composite indicators consider three more normalization rules that we did not use in this context. They are the so-called Borda Count, the Distance from the leader and the Distance from the average. The Borda Count was not used here because it rescales the performances according to equal intervals, which is contrary to the purpose of using partially non-compensatory aggregation rules, which leads to punish performances that are bad in absolute terms. The “distance from the leader” (the ratio between the indicator and the best performance) was not used since in this case it would be is almost identical to the HDI normalization. The same occurs when the “distance from the average” is used with geometric aggregation.
[7] We report below the averages over the three dimensions of the statistics of the data normalised with the three rules that we used.

	HDI	*z-score*	*min-max*
mean	0,70	0,68	0,68
st dev	0,16	0,18	0,19
min	0,30	0,23	0,20
max	0,97	0,98	1,00

Table 1: Aggregation rules

Name	*Rule*
Linear:	$CI_i = \sum_{q=1}^{Q} w^q I_i^q$
Geometric:	$CI_i = \prod_{q=1}^{Q} (I_i^q)^{w_q}$
Concave:	$CI_i = \sum_{q=1}^{Q} w^q \left(I_i^q - he^{-kI_i^q}\right)$

Where I_i^q is the normalised indicator for variable q and Country i, w stands for the weight, and h and k are parameters that we set here equal to 1.

Table 2: Normalization rules

Name	*Rule*	*Range*
z-score	$I_i^q = \dfrac{a + b(x_i^q - \bar{x}^q)}{\sigma^q}$	av(I)=$a/10$ $\sigma(I)$=$b/10$
HDI normalization	$I_i^q = \dfrac{x_i^q}{AspG_i}$	[0.2;1]
Min-max	$I_i^q = \left[\dfrac{x_c^q - \min(x^q)}{\max(x^q) - \max(x^q)} + \beta\right] * \alpha$	[0.2;1]

Where
I_i^q is the normalised indicator for variable q and Country i
AspG is the aspirational goal set in the HDI
$\bar{x}$ is the average, σ the standard deviation, *min* and *max* are respectively the highest and the lowest value of the indicator q across Countries.
Notice that the x data are not raw. They are already rescaled in order to take into account the HDI 'goalposts' and the diminishing returns from income.
We set a=68, b=18, α=0.8 and β=0.25

The HDI is meant not only to compare countries but also to assess changes of a single Country by comparing its HDI values over time. Only looking at a Country's position might be misleading since we could observe a decrease in the ranking (or vice versa), even though its absolute performances improve (or vice versa).

As for the HDI, we gave equal weight to each of the three dimensions. However, we also considered a kind of Benefit-of-the-Doubt approach[8] (see, e.g., Melyn and Moesen, 1991) by assigning a 0.2 weight to the worst of the three component indices and 0.4 to the other two. Hence, using the dataset available from the HDI website, we built 1,701 composites, three composite per each aggregation rule plus other nine for each of the 188 countries.

3. Results

It has to preliminary be highlighted that, the choice of using several normalization and aggregation rules makes both the range of variations and the meaning of our different indexes slightly different from each other, differently from García and Kovacevic (2010). For this reason, we prefer to focus only on ranks rather than on the values taken by the different indexes. The main results are illustrated by referring to Figures 1, 2 and 3. Usually, graphs are built by putting the HDI values or ranks on the x-axis and the distribution resulting from the uncertainty analysis on the y-axis. This however, does not allow to easily reading the name of each country. For this reason, we preferred to invert the axis and list on the y-axis the 188 countries, ordered according to their place in the HDI ranking, from the first to the last. The horizontal axis displays all the possible ranks from 1 to 188. In correspondence of any Country we have a horizontal string of one or more coloured squares, each of them telling also the frequency of the rank of the uncertainty exercise. This can be seen by looking at Figure 1 that focuses on the situation of the first 15 Countries.

The first column contains the HDI rank, the second one contains the median of the frequency distribution, the third one, labelled with "high", contains the 10th percentile and the fourth one, labelled with "low", contains the 90th percentile. We can see that Norway is always at the first place of the ranking whichever is the composite we look at; Australia is at the 2nd place according to some composites (for the 68% of them) and at the 3rd according to other ones (for the 32% of them), and so on. Each row represents the frequency distribution of the ranks for the corresponding Country, where the darker the square, the higher the number of composites that determine that particular rank. In general, one notices that the picture is rather different from the one observed by looking at the HDI only. For instance, the Netherlands, that ranks 5th according to the HDI (first column), has a median rank of 7 in our uncertainty exercise.

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[8] This is made, for instance, to consider that historical reasons could make very difficult for a Country to change its performances in one of the dimensions.

HDI	median	H - L		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	**1**	1 - 1	Norway	100																				
2	**2**	2 - 3	Australia		68	32																		
3	**3**	2 - 6	Switzerland		32	31		13	25															
4	**4**	3 - 4	Denmark			25	71	4																
5	**7**	5 - 11	Netherlands				4	38		21		13		17	8									
6	**6**	6 - 8	Germany					4	48	36	13													
6	**10**	8 - 13	Ireland							0	30	11	21	4	17	17								
8	**9**	8 - 14	United States								32	30			0		29	8						
9	**13**	7 - 17	Canada							30		0	0	11		13		21	4	18	4			
9	**10**	9 - 13	New Zealand									34	39	7	7	13								
11	**11**	5 - 16	Singapore					18	15				13	13	3				29	8				
12	**11**	3 - 18	Hong Kong			13	25			0			7	10	10		3		7		25			
13	**11**	5 - 15	Liechtenstein					25	13					21		7	17	14	3					
14	**12**	7 - 21	Sweden							13	25		7		10	14	0						11	21
14	**12**	9 - 16	United Kingdom									13	13	8	18	11	7	4	27					

Figure 1. Frequency distributions of the rankings for the 15 countries with highest HDI values.

Figure 2 zooms out to all Countries in order to provide an overview of the overall effect of the uncertainty exercise. As explained before, the frequency distribution of the 1,701 ranks for each country is drawn on its corresponding raw. For some country the distribution is concentrated and centred around the main diagonal. This indicates that the HDI rank of that Country is robust, i.e., it does not change much when the method of building the composite changes. In other cases, the distribution is still rather centred around the diagonal, but with more dispersion. In these cases the HDI rank is still close to the median, and therefore representative of the uncertainty exercise, but not very robust to change. Finally, we have countries for which the frequencies are dispersed and the HDI rank is quite close to one extreme of the frequency distribution. In the latter case the HDI rank is not robust.

Figure 3 gives a visualisation of the partial effects of using separately only the 9 basic indicators (left picture) and only the HDI with the Benefit of the Doubt weighting (189 composite, right picture). By looking at the horizontal positions of the coloured squares, one can guess that the range of the possible ranks becomes wide in both partial cases. To have a quantitative indication, for each Country we calculated the range of the ranks between the 10th and the 90th percentile. They sum to 666 when considering the nine basic composites, to 1540 for HDI with BOD weighting, and to 1904 when including all composites.

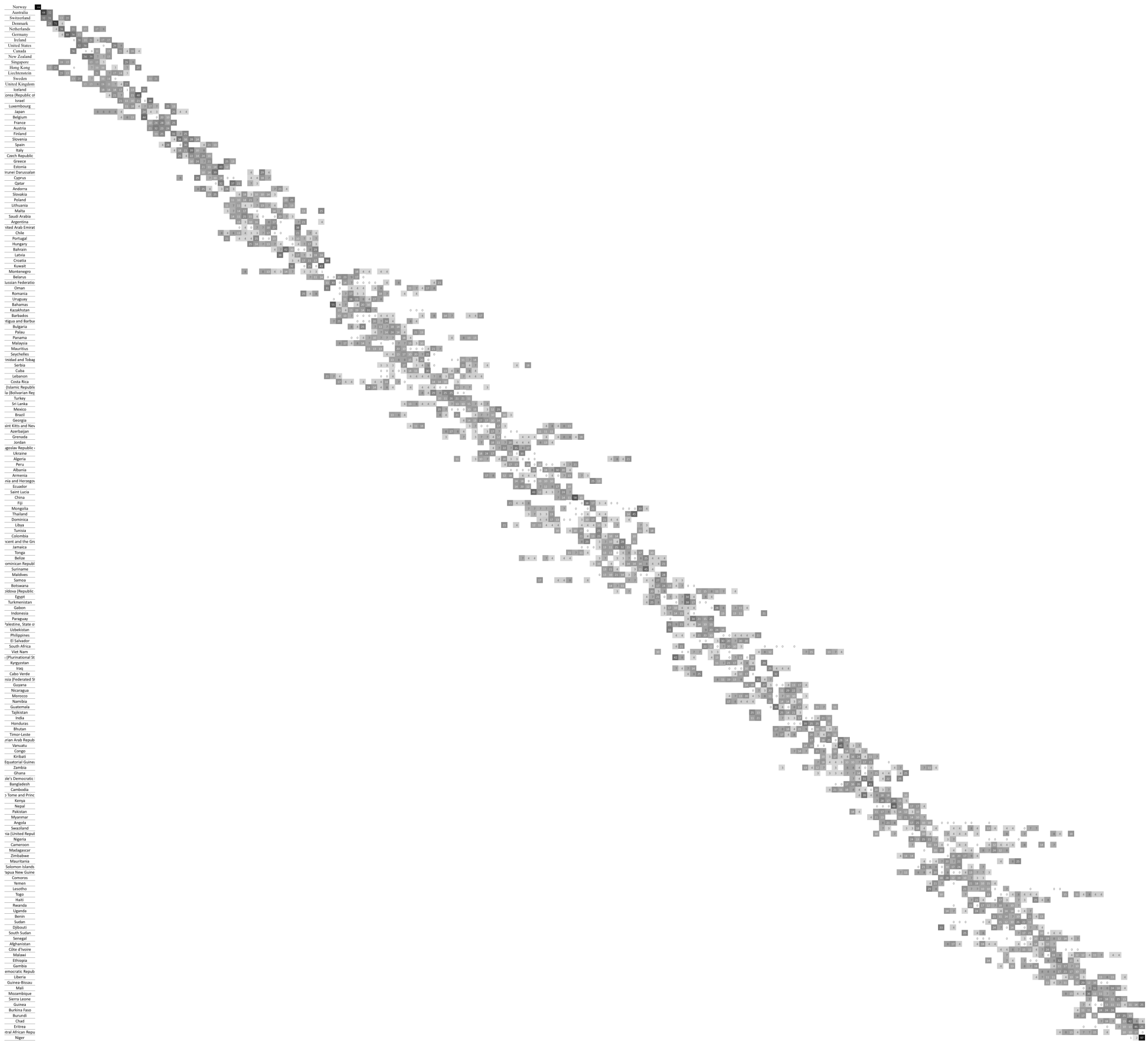

Figure 2. Frequency distributions of the rankings for each country

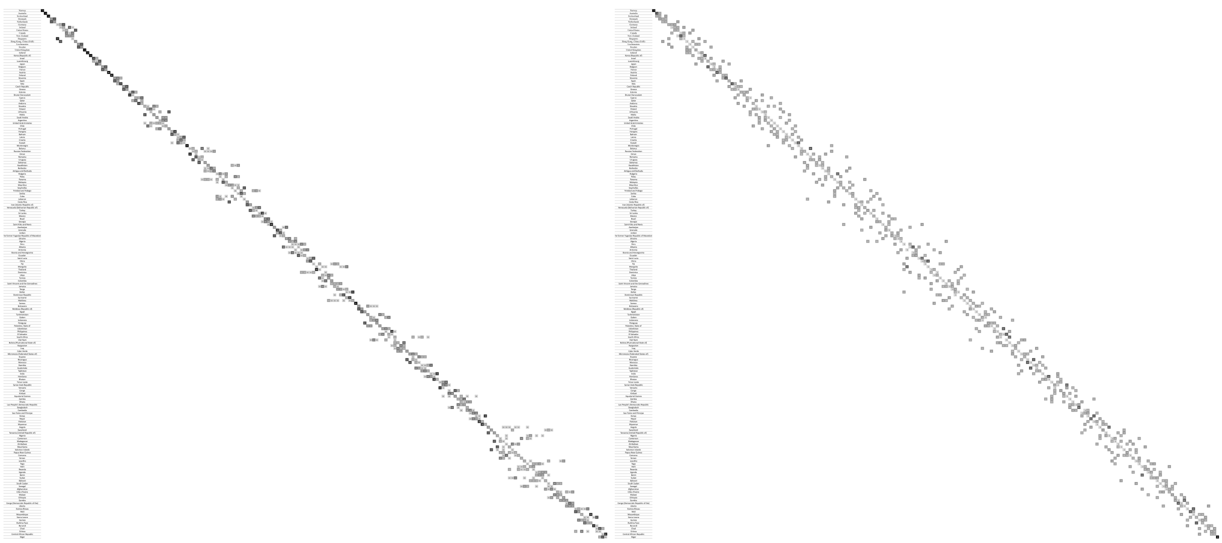

Figure 3. Frequency distributions of the ranks for the 9 basic indicators (left) and for HDI with BOD only (right)

In our opinion, only a range of plausible ranks should be communicated, for instance the ranks between the 10th and the 90th percentile as shown in Table 3, where Countries are listed according to the median of the frequency distribution of their ranks.

Table 3: Range of plausible ranks for the first 15 HDI ranked Countries

	H		L
Norway	1	-	1
Australia	2	-	3
Switzerland	2	-	6
Denmark	3	-	4
Germany	6	-	8
Netherlands	5	-	11
United States	8	-	14
Ireland	8	-	13
New Zealand	9	-	13
Hong Kong	3	-	18
Liechtenstein	5	-	15
Singapore	5	-	16
Sweden	7	-	21
United Kingdom	9	-	16
Canada	7	-	17

In the appendix, Table A1 reports the median ranks, and the 10^{th} and the 19^{th} percentiles for all countries.

Finally, one could ask which of the 9 basic composite indicators that we used best represents the ranking indicated by the medians of our uncertainty analysis. To do this we calculated the average rank shift[9] when moving from the rank of a particular composite to our median rank, that is,

$$\frac{1}{188}\sum_{i=1}^{188}|m_i - r_i|$$

where i indicates the i-th Country, m_i its median rank and r_i its rank according to one particular composite.

As a result, the composite that is closest to the ranking involved by our analysis are those based on HDI normalization, also in the concave and linear aggregation. Table 4 shows for each of the basic composites the distance of their rankings with respect to our median ranking, measured both by the Euclidean Distance and Average rank change.

[9] This makes possible a comparison with Garcia and Kovacevic (2010, p. 22). For them the average rank shift is from 2 to 4 depending on the group of Countries.

Table 4. The distance between the median ranking and each ranking from the 9 basic indicators

Index:	*HDI*	*HDI concave*	*HDI linear*	*z-score linear*	*z-score concave*	*z-score geom*	*min-max linear*	*min-max concave*	*min-max geom*
Av. rank change:	2.26	2.51	2.71	3.50	3.63	3.67	3.52	3.59	3.40

Table 5 gives more details than table 4. For the first 15 Countries it focuses on the ranks from respectively "HDI", "HDI-Concave" and "Minmax-Geom". We used the symbols ✓, ☝ and ☟ to visualize the difference between the rank according to each single composite and the median rank. More precisely, ☝ indicates that the composite assigns the country a better rank than our median, while ☟ a worse rank. A ✓ indicates that the absolute difference is 0 or 1, whereas a ☝ or a ☟ means that the absolute difference is 2 or 3. Finally we use ☝☝ or ☟☟ when the absolute difference is higher than 3.

Table 5: A focus on the comparison between the median rankings and each of the ranking from the 9 basic indicators; first 15 Countries

	median rank	*HDI*	*HDI-concave*	*Minmax-geom*
Norway	**1**	1 ✓	1 ✓	1 ✓
Australia	**2**	2 ✓	2 ✓	2 ✓
Switzerland	**3**	3 ✓	3 ✓	3 ✓
Denmark	**4**	4 ✓	4 ✓	5 ☟
Netherlands	**7**	5 ☝☝	5 ☝☝	4 ☝☝
Germany	**6**	6 ✓	7 ☟	8 ☟
Ireland	**10**	6 ☝☝	8 ☝☝	10 ✓
United States	**9**	8 ☝	9 ✓	12 ☟☟
Canada	**13**	9 ☝☝	11 ☝☝	9 ☝☝
New Zealand	**10**	9 ☝	10 ✓	11 ☟
Singapore	**11**	11 ✓	6 ☝☝	6 ☝☝
Hong Kong, CHN	**11**	12 ☟	12 ☟	7 ☝☝
Liechtenstein	**11**	13 ☟☟	13 ☟☟	14 ☟☟
Sweden	**12**	14 ☟☟	14 ☟☟	13 ☟
United Kingdom	**12**	14 ☟☟	15 ☟☟	15 ☟☟

4. Clustering the Countries

As García and Kovacevic (2010) did, we conducted a cluster analysis on our dataset in order to identify the similarities and the differences across the performances of the 188 analysed Countries. Since we have not a precise theory to provide an *ex-ante* number of clusters to be tested, we

conduct a hierarchical cluster analysis as recommended by Nardo et al. (2008). We first set the metric of the clustering (i.e., the distance between pairs of observations) as the Euclidean distance.
The analysis is then conducted on normalised data so as to avoid the “difference in scale” bias. Consistently with our uncertainty analysis, we considered three different normalization rules, z-score, min-max, and HDI normalizations. We also checked the outcome of the “distance from the mean” normalization.
We adopted the Ward’s method (Ward, 1963) as linkage criterion, in order to calculate the distance between sets of observations. According to this iterative process, at each step clusters or observations are combined in such a way as to minimize the increase in the variance within the groups. In order to establish the number of clusters, we used the Duda and Hart’s (1972) stopping rule[10]. Such rule, is based on the ratio Je(2)/Je(1), where Je(2) is the sum of squared errors within cluster when the data are partitioned into two clusters, and Je(1) gives the squared errors when only one cluster is present. Larger values of the Je(2)/Je(1) ratio indicate more distinct clustering. In Table 5, the Je(2)/Je(1) ratios are reported for the four different cluster analysis. The maximum value that determines the number of clusters is emphasised in bold.
As expected, the standardisation rule used for the cluster analysis affects the outcome. As shown in Table 6, according to the different standardisation, countries are grouped in five clusters (for the min-max and the HDI rule), in four clusters (for the z-score), and in eight clusters (for the distance from the mean).

Table 5. Values of the Je(2)/Je(1) ratio in four distinct cluster analysis

		N. of clusters														
	Standardization rule	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Je(2)/Je(1)	Min-Max	0.41	0.49	0.50	0.66	**0.72**	0.66	0.63	0.49	0.66	0.47	0.61	0.43	0.65	0.63	0.39
	Z-Score	0.41	0.49	0.50	**0.70**	0.66	0.66	0.63	0.49	0.65	0.47	0.61	0.44	0.65	0.59	0.40
	HDI	0.43	0.44	0.51	0.67	**0.72**	0.53	0.69	0.49	0.70	0.67	0.50	0.38	0.55	0.57	0.54
	Distance from the average	0.36	0.51	0.52	0.65	0.66	0.52	0.54	**0.71**	0.65	0.59	0.44	0.55	0.61	0.64	0.68

10 There exist several other "cluster validation" techniques including the choice minimizing the pseudo T-Square, the use of "internal measures" such as cohesion and separation or a mix of them also known as the “average silhouette width” (Rousseeuw, 1987). The choice of a different technique usually leads to different results in terms of number of clusters (and their composition) and may have relevant implications in the descriptive analysis of the data (Jain and Dubes, 1988). However, for the purposes of this work, the adoption of a different validation technique would not have changed its substantial results.

Table 6. Number and sizes of clusters for each cluster analysis

N. of Clusters	min-max	z-score	HDI stand.	Ratio to the mean
1	29	29	41	34
2	68	69	44	8
3	49	48	40	26
4	29	42	18	44
5	13		45	9
6				22
7				8
8				37

The dimensions of the clusters differ from one another, also when the number of clusters is the same. For instance, according to the min-max cluster analysis we get 5 clusters composed respectively of 29, 68, 49, 29 and 13 countries, while according to the HDI rule the 5 clusters are composed of 41, 44, 40, 18 and 45 countries. Notice also that not only the total number of countries by cluster, but also the composition of the single cluster changes. Overall, it is evident that a seemingly innocuous choice, as the adoption of a particular technique of data normalization, can strongly interfere with the outcome of the analysis of the data.

Comparing clustering resulting from different normalisations different, however, can give important specific indications, as in this case.

A first point to emphasise is that there is a very strong correlation between clusters and their performances in terms of HDI, particularly with the z-score and min-max standardisation. This allows using the cluster analysis to set the cut-off points for different levels of human development achievements.

Secondly, the min max and the z-score rules give almost identical clustering since the only relevant difference is that min-max splits into two groups the fourth group of the z-score rule so that they can be thought of as a subgroup of the last one. This is confirmed by considering that the last group built with the HDI standardisation is almost the same as the last group under the z-score rule. Finally also group 3 and 4 built with HDI standardisation are rather similar and could be re-grouped together. As a result, we suggest keeping the same number of groups chosen in the Human Development Reports, i.e. 4 groups, but changing the respective cut-off points.

Table 7 shows the number of countries and the cut-off points according to (i) the classification used in the 2014 and 2015 Human Development Reports, and those implied by (ii) z-score, (iii) min-max, and (iv) HDI normalisation rules. The cut-off points involved by the z-score and the min-max are the more or less the same and labelled with “*A*”, while the ones involved by the HDI standardisation are labelled with “*B*”. The resulting suggestion is to raise the cut-off points of the Very High and High classes and to lower the cut-off of the Medium human development class.

Table 7. Number of countries in the groups under the current UNRO classification and alternatives ones suggested by our cluster analysis

Group	Current HDI classification	*cut-offs points*	z-score norm.	min-max norm.	*cut-offs points "A"*	HDI norm.	*cut-offs points "B"*
1	49	*0.80*	29	29	*0.865*	41	*0.835*
2	56	*0.70*	69	68	*0.720*	44	*0.724*
3	39	*0.55*	48	49	*0.54*	3a) 40 3b) 18	*0.512*
4	44		42	4a) 29 4b) 13		45	

5. Conclusion

Building a composite index involves an unavoidable uncertainty, similarly and more than adding apples and oranges. The researcher has to make many arbitrary choices, concerning relevant indicators, normalization and aggregation rules, and weighting of the component variables. Hence uncertainty grows with the efforts of synthesising the data, moving away who reads the composite from the intrinsic content of the original data.

The HDI ranking, as shown in this paper and also by the paper that tackle this issue, is rather robust. This is not surprising since only four indicators, which are also rather correlated to one another, are used to calculate it. Still, the HDI is sensitive to small changes in its assumptions.

The difference between our analysis and previous one is epistemological[11]. We do not assume that the true picture of human development exists. Therefore, we are not interested in assessing whether HDI is biased or not. Biased with respect to what? A synthetic single picture cannot describe multifaceted and complex objects. Some faces can be described by what Georgescu Roegen (1971)

[11] For a detailed discussion of what we argue in the next few sentences see Funtowicz and Ravetz (1990) and Giampietro (2003).

named arithmomorphic notions, other by dialectical ones, e.g. quality indicators on an ordinal scale. In any case, different units of measurement involve an unavoidably incommensurability. This, however, does not imply that “nothing can be said”. Uncertainty analysis helps saying something, that is, that a plausible rank range for each country exists. This is sufficiently simple to be understood by the general public, but enough complicated to communicate that no measure of “the true human development” can exist.

Hence, we claim that rather than the HDI rankings, a range of plausible ranks should be communicated together with the HDI figure. This seems to us a reasonable compromise between the need of synthesis when the analysis involves many variables and the loss of relevant information due to composing different indicators into a single index. This would also help the comparison across time of a single country performance, which might reveal an improvement even if the Country does not surpass any other. As a personal comment, we find that making the results more uncertain would also mitigate the idea that building composites is just a nice game to satisfy our passion/obsession for competition.

A second issue that was investigated in this paper is the relationship between the HDI ranking and possible clusters of the Countries. As for the uncertainty analysis, we used different standardisation rules, which involves, as expected, different clustering. Nonetheless, quite a clear picture emerges. First, there is a strict relation between levels of development and clusters. Second, the grouping of the Countries made by the clustering analysis is different from the one currently used in the Human Development Report. The most evident difference is that some of the Countries classified as very highly developed are much more similar to the Countries classified as highly developed. The cluster analysis suggests keeping the division into four groups, but changing their cut-off points.

References

Brian E., 2008, Condorcet and Borda in 1784. Misfits and Documents, *Journal Electronique d´Histoire des Probabiltés et de la Statistique, 4 (1)*.

Cahill M. B., 2005, Is the human development index redundant? *Eastern Economic Journal*, *31(1),* 1-5.

Casadio Tarabusi E., Palazzi P., 2004, An index for sustainable development, *Banca Nazionale del Lavoro Quarterly Review, LVII, no. 229*

De Muro P., Mazziotta M., Pareto A., 2011, Composite indices of development and poverty: An application to MDGs. *Social indicators research 104 (1),* 1-18.

Duda, R. O., and Hart P. E., 1972, "Use of the Hough transformation to detect lines and curves in pictures." *Communications of the ACM* 15.1, 11-15.

Floridi M., Pagni S., Falorni S., Luzzati T , 2011, An exercise in composite indicators construction: Assessing the sustainability of Italian regions, *Ecological Economics, 70,* 1440-1447.

Funtowicz S.O. and Ravetz J.R. , 1990, *Uncertainty and Quality in Science for Policy*, Kluwer Academic Publishers, the Netherlands.

Georgescu Roegen N., 1971, The entropy law and the economic process. *Harvard University Press*.

Giampietro M., 2003, Multi-Scale Integrated Analysis of Agroecosystems, CRC Press.

Herrero, C. R. Martinez, and Villar A., 2012, A newer human development index. *Journal of Human Development and Capabilities, 13(2)*, 247–268.

Jain, Anil K., and Richard C. Dubes, 1988, *Algorithms for clustering data*. Prentice-Hall, Inc.

Klugman, J., Rodríguez, F., & Choi, H. J. , 2011, The HDI 2010: new controversies, old critiques. *The Journal of Economic Inequality*, *9*(2), 249-288.

Kovacevic M., Aguña, C.G., 2010. *Uncertainty and sensitivity analysis of the Human Development Index* (No. HDRP-2010-47). Human Development Report Office (HDRO), United Nations Development Programme (UNDP).

Luzzati T., Gucciardi G., 2015, A non-simplistic approach to composite indicators and rankings: an illustration by comparing the sustainability of the EU Countries, *Ecological Economics, 113*, 25-38.

Melyn W. and Moesen W.W., 1991, Towards a synthetic indicator of macroeconomic performance: unequal weighting when limited information is available, *Public Economic research Paper 17, CES*, KU Leuven, Belgium.

Morse S., 2014, Stirring the pot: Influence of changes in methodology of the human development index on reporting by the press. *Ecological Indicators, 45(October)*, 245–54.

Munda G., 2008, *Social Multi-Criteria Evaluation for a Sustainable Economy*, Springer.

Nardo M., Saisana M., Saltelli A., Tarantola S., Hoffmann A., Giovannini E., 2008, *Handbook on Constructing Composite Indicators – methodology and user guide*. OECD publishing

Paruolo P., Saisana M., Saltelli A., 2013, Ratings and rankings: Voodoo or Science?, *Journal of the Royal Statistical Society A, 176 (2)*, 1-26.

Rousseeuw P. J., 1987,"Silhouettes: a graphical aid to the interpretation and validation of cluster analysis." *Journal of computational and applied mathematics 20*, 53-65.

Saisana M., Munda G., 2008, Knowledge Economy: Measures and drivers. *EUR Reports 23486, EN, European Commission*, JRC-IPSC.

Simon H. A., 1978, Rationality as process and as product of thought. *The American economic review*, 1-16.

Ward Jr, Joe H. 1963, "Hierarchical grouping to optimize an objective function." *Journal of the American statistical association* 58.301, 236-244.

Zambrano, E., 2011, Functioning, capabilities and the 2010 human development index, controversies, old critiques. *Human Development Research Paper 2011/11, Human Development Report Office, UNDP-HDRO. Occasional Papers.*

Appendix

Table A1. HDI, median rank, 10th and 90th percentile of the uncertainty analysis

	HDI	median	high		low		HDI	median	high		low
Norway	1	**1**	1	-	1	Malta	37	**41**	34	-	49
Australia	2	**2**	2	-	3	Saudi Arabia	39	**36**	34	-	43
Switzerland	3	**3**	2	-	6	Argentina	40	**40**	35	-	46
Denmark	4	**4**	3	-	4	United Arab Emirates	41	**41**	38	-	45
Netherlands	5	**7**	5	-	11	Chile	42	**39**	33	-	47
Germany	6	**6**	6	-	8	Portugal	43	**38**	33	-	47
Ireland	6	**10**	8	-	13	Hungary	44	**40**	37	-	47
United States	8	**9**	8	-	14	Bahrain	45	**43**	42	-	48
Canada	9	**13**	7	-	17	Latvia	46	**45**	42	-	49
New Zealand	9	**10**	9	-	13	Croatia	47	**48**	46	-	50
Singapore	11	**11**	5	-	16	Kuwait	48	**47**	44	-	49
Hong Kong	12	**11**	3	-	18	Montenegro	49	**43**	39	-	57
Liechtenstein	13	**11**	5	-	15	Belarus	50	**52**	48	-	55
Sweden	14	**12**	7	-	21	Russian Federation	50	**53**	50	-	69
United Kingdom	14	**12**	9	-	16	Oman	52	**59**	50	-	67
Iceland	16	**15**	12	-	19	Romania	52	**54**	46	-	60
R. of Korea	17	**17**	14	-	18	Uruguay	52	**55**	53	-	58
Israel	18	**18**	15	-	20	Bahamas	55	**51**	51	-	57
Luxembourg	19	**20**	16	-	24	Kazakhstan	56	**56**	52	-	57
Japan	20	**19**	12	-	24	Barbados	57	**68**	52	-	76
Belgium	21	**19**	16	-	23	Antigua and Barbuda	58	**58**	52	-	64
France	22	**22**	20	-	24	Bulgaria	59	**59**	55	-	62
Austria	23	**21**	20	-	23	Palau	60	**62**	59	-	66
Finland	24	**24**	21	-	26	Panama	60	**63**	57	-	75
Slovenia	25	**26**	25	-	28	Malaysia	62	**56**	53	-	66
Spain	26	**26**	23	-	31	Mauritius	63	**62**	57	-	68
Italy	27	**27**	25	-	28	Seychelles	64	**65**	62	-	67
Czech Republic	28	**27**	25	-	30	Trinidad and Tobago	64	**66**	61	-	75
Greece	29	**30**	27	-	34	Serbia	66	**73**	61	-	84
Estonia	30	**32**	29	-	33	Cuba	67	**67**	64	-	74
Brunei Darussalam	31	**31**	29	-	39	Lebanon	67	**67**	50	-	74
Cyprus	32	**31**	28	-	39	Costa Rica	69	**60**	52	-	70
Qatar	32	**34**	32	-	37	IR of Iran	69	**60**	57	-	74
Andorra	34	**33**	29	-	42	Venezuela	71	**70**	67	-	71
Slovakia	35	**36**	30	-	40	Turkey	72	**72**	69	-	74
Poland	36	**37**	34	-	44	Sri Lanka	73	**72**	64	-	76
Lithuania	37	**39**	33	-	44	Mexico	74	**75**	69	-	79

Table A1. continued

	HDI	median	high		low		HDI	median	high		low
Brazil	75	**72**	61	-	80	Paraguay	112	**113**	112	-	115
Georgia	76	**76**	74	-	79	Palestine, State of	113	**110**	108	-	115
Saint Kitts and Nevis	77	**79**	65	-	91	Uzbekistan	114	**115**	108	-	117
Azerbaijan	78	**78**	71	-	88	Philippines	115	**114**	112	-	123
Grenada	79	**84**	73	-	93	El Salvador	116	**118**	117	-	121
Jordan	80	**81**	78	-	87	South Africa	116	**115**	110	-	119
Republic of Macedonia	81	**82**	79	-	84	Viet Nam	116	**125**	106	-	136
Ukraine	81	**78**	76	-	83	Bolivia	119	**112**	109	-	122
Algeria	83	**80**	72	-	101	Kyrgyzstan	120	**119**	116	-	124
Peru	84	**84**	81	-	92	Iraq	121	**122**	110	-	126
Albania	85	**89**	85	-	90	Cabo Verde	122	**120**	112	-	126
Armenia	85	**81**	77	-	93	Micronesia	123	**119**	117	-	124
Bosnia and Herzegovina	85	**87**	82	-	96	Guyana	124	**124**	121	-	130
Ecuador	88	**86**	82	-	91	Nicaragua	125	**128**	124	-	129
Saint Lucia	89	**86**	85	-	90	Morocco	126	**123**	119	-	129
China	90	**92**	90	-	93	Namibia	126	**125**	118	-	130
Fiji	90	**94**	81	-	95	Guatemala	128	**129**	126	-	136
Mongolia	90	**95**	85	-	103	Tajikistan	129	**127**	122	-	129
Thailand	93	**101**	85	-	102	India	130	**130**	122	-	135
Dominica	94	**94**	88	-	98	Honduras	131	**132**	131	-	135
Libya	94	**87**	80	-	103	Bhutan	132	**130**	126	-	135
Tunisia	96	**96**	89	-	105	Timor-Leste	133	**132**	127	-	136
Colombia	97	**96**	93	-	102	Syrian Arab Republic	134	**135**	132	-	138
/incent and the Grenadines	97	**98**	94	-	102	Vanuatu	134	**137**	131	-	139
Jamaica	99	**99**	97	-	101	Congo	136	**134**	130	-	140
Tonga	100	**98**	91	-	105	Kiribati	137	**139**	134	-	142
Belize	101	**101**	84	-	105	Equatorial Guinea	138	**139**	134	-	142
Dominican Republic	101	**103**	96	-	107	Zambia	139	**138**	131	-	152
Suriname	103	**103**	97	-	104	Ghana	140	**143**	136	-	148
Maldives	104	**100**	97	-	107	Lao PDR	141	**144**	140	-	147
Samoa	105	**102**	86	-	107	Bangladesh	142	**140**	138	-	142
Botswana	106	**106**	98	-	109	Cambodia	143	**138**	136	-	144
Moldova (Republic of)	107	**113**	101	-	117	Sao Tome and Principe	143	**143**	141	-	148
Egypt	108	**111**	105	-	114	Kenya	145	**145**	144	-	147
Turkmenistan	109	**111**	105	-	112	Nepal	145	**147**	146	-	150
Gabon	110	**116**	108	-	120	Pakistan	147	**145**	139	-	150
Indonesia	110	**117**	108	-	124	Myanmar	148	**148**	143	-	150

Table A1. continued

	HDI	**median**	high		low		HDI	**median**	high		low
Angola	149	**149**	145	-	152	Eritrea	186	**186**	185	-	187
Swaziland	150	**157**	145	-	169	Central African Republic	187	**180**	175	-	186
U.R of Tanzania	151	**160**	143	-	176	Niger	188	**187**	187	-	187
Nigeria	152	**151**	149	-	155						
Cameroon	153	**163**	152	-	171						
Madagascar	154	**162**	153	-	164						
Zimbabwe	155	**156**	148	-	159						
Mauritania	156	**157**	154	-	167						
Solomon Islands	156	**156**	151	-	159						
Papua New Guinea	158	**153**	148	-	161						
Comoros	159	**156**	154	-	159						
Yemen	160	**159**	153	-	162						
Lesotho	161	**158**	152	-	162						
Togo	162	**169**	154	-	179						
Haiti	163	**165**	160	-	171						
Rwanda	163	**162**	159	-	166						
Uganda	163	**161**	155	-	168						
Benin	166	**166**	163	-	171						
Sudan	167	**167**	164	-	169						
Djibouti	168	**163**	154	-	170						
South Sudan	169	**168**	158	-	171						
Senegal	170	**172**	170	-	176						
Afghanistan	171	**161**	156	-	172						
Côte d'Ivoire	172	**169**	166	-	173						
Malawi	173	**177**	170	-	181						
Ethiopia	174	**174**	162	-	176						
Gambia	175	**170**	166	-	177						
D.R. of Congo	176	**175**	172	-	177						
Liberia	177	**177**	171	-	183						
Guinea-Bissau	178	**178**	172	-	180						
Mali	179	**182**	180	-	184						
Mozambique	180	**179**	176	-	182						
Sierra Leone	181	**182**	181	-	185						
Guinea	182	**185**	179	-	187						
Burkina Faso	183	**181**	178	-	183						
Burundi	184	**184**	178	-	186						
Chad	185	**186**	182	-	186						