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**HUNTSVILLE, ALABAMA**

LUNAR FLIGHT STUDY SERIES: VOLUME 8  
EARTH-MOON TRANSIT STUDIES BASED ON EPHEMERIS DATA  
AND USING BEST AVAILABLE COMPUTER PROGRAM  
PART III: ANALYSIS OF SOME LUNAR LANDING SITE PROBLEMS  
UTILIZING TWO FUNDAMENTAL PRINCIPLES

By

W. B. Tucker and H. L. Hooper



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ABSTRACT

This report presents two fundamental properties of lunar trajectories and makes use of these properties to solve various lunar landing site problems. Not only are various problems treated and solved but the properties and methods are established for use in the solution of other problems.

This report presents an analysis of lunar landing site problems utilizing the direct mission mode as well as the orbital mission mode. A particular landing site is then specified and different flight profiles are analysed for getting an exploration vehicle to that landing site. Rendezvous compatible lunar orbits for various stay-times at the landing site are treated. Launch opportunities are discussed for establishing rendezvous compatible lunar orbits without powered plane changes. Then, the minimum required plane changes for rendezvous in the lunar orbit are discussed for launching from earth on any day. On days that afford rendezvous compatible opportunities, there are no powered plane change requirements in the operations from launch at AMR through the rendezvous in lunar orbit, after the stay at the lunar site.

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FUTURE PROJECTS BRANCH  
AEROBALLISTICS DIVISION

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#### SUMMARY

This report presents two fundamental properties of lunar trajectories and makes use of these properties to solve various lunar landing site problems. Not only are various problems treated and solved but the properties and methods are established for use in the solution of other problems.

Various lunar landing site problems are resolved by using two fundamental properties of earth-to-moon trajectories as basis for the analysis. The problems treated here represent only a sampling of the problems that may be treated using these two fundamental properties and methods similar to the ones used in this report.

The accessibility of lunar locations via either the direct mission mode or the orbital mission mode is investigated. For the direct mission mode, and various constraints, locations are determined that are never accessible for use as a landing site, as well as those that are always accessible for use as a landing site. For the orbital mission mode, it is shown that all locations are accessible on every day. If rendezvous compatibility is required, and the stay-time at the landing site is specified, there are sites that are forever ruled out as possible landing sites. Such sites are determined for reasonable values of stay-time at a site.

A landing site is picked in the Sea of Tranquility and the orbital mission mode is assumed as basis for a study of some lunar rendezvous problems. Launch opportunities for establishing rendezvous compatible orbits are pointed out. This means that if these opportunities are used, there is no powered plane change required in rendezvous of the exploration vehicle and parent vehicle after the stay on the moon. Finally, assuming that launch is on any day, regardless of whether rendezvous compatibility is afforded on that day or not, it is shown that the minimum required plane change after a one day stay on the moon varies from zero to about  $4^\circ$  for the rendezvous in orbit. The velocity required to accomplish this plane change is estimated to vary from zero to 110 (m/s) as the minimum required plane change varies up to  $4^\circ$ .

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## SECTION I. INTRODUCTION

Reference 1 presents some principles about earth-to-moon trajectories that are proving valuable in the solution of lunar trajectory analysis problems. This report presents some of these problems and their resolution through the use of the principles or properties that were developed in Reference 1.

No trajectory computations are involved in generating the results presented in this report. All these results are based on an analysis using the results of Reference 1 and spherical trigonometry relationships. Great circle projections are used to represent the trajectory projections on the lunar surface.

## SECTION II. THE PRINCIPLES TO BE USED AND THEIR CHARACTERISTICS

The detailed development of the principles to be used in this report is presented in Reference 1. A few figures from Reference 1 are included in this report to make it as independent as is reasonably possible.

We emphasize that these principles are not brought about by simplifying assumptions. The study that resulted in the principles to be used here is subject to the following constraints:

1. Launch is from AMR at an azimuth between  $70^{\circ}$  and  $110^{\circ}$ .
2. A near-circular parking orbit is used at an 6647.8 (km) initial radius; central arc from launch to parking orbit is  $18.5^{\circ}$ .
3. Injection is accomplished by an S-IVB stage using thrust vector control from the horizon.
4. No powered plane changes are made.
5. Variations in launch time, coast time, and S-IVB burn time are used to generate trajectories having desired arrival conditions at the moon. Desired arrival conditions are expressed in terms of flight time, altitude, and inclination.



All of these constraints are realistic. Furthermore, the model used for the trajectory computations is as accurate as is available for our use. Celestial body ephemeris data is taken from a prepared tape. The influence of the oblate earth, sun, and triaxial moon are included in the model. Jupiter's influence is included if it becomes significant. The computational error is less than 100 (m) in arrival position and less than .01 (m/s) in arrival velocity.

Notice that launch azimuths from  $70^\circ$  through  $110^\circ$  were investigated in the development of the principles to be used in this report. It was shown there, in Reference 1, that launch azimuth is a strong parameter. Hence, for the analysis in this report, we assume a due east launch from AMR. Any other launch azimuth could be used here. Carrying the analysis through with one azimuth rather than another would produce a corresponding shift in the results.

A few results are pointed out assuming that launch azimuth is variable between  $70^\circ$  and  $110^\circ$ . In such cases, a specific statement is included at that point in the analysis.

#### A. THE ARRIVAL VERTEX PRINCIPLE

"The Arrival Vertex Principle" says that all the trajectories that arrive at the moon at about the same time, with the same flight time from injection to periselenium, pass over a common point, a VERTEX. The trajectory may have any inclination that's possible or any reasonable altitude but it still passes over the vertex. It was pointed out in Reference 1 that the inclinations that can be attained are limited by the latitude of the vertex point (latitude and inclination being referenced to the same plane).

It means that variations up to about 15 minutes in launch time, up to about 50 seconds in coast time, and of some fractional part of a second in burn time are sufficient to generate all the various combinations of possible inclinations and reasonable altitudes at arrival, flight time being held constant for all the various combinations.

There are other important implications that will be brought out in the solution of various problems.

## B. PERISELENUM CIRCLES ABOUT THE VERTEX

It is shown in Reference 1 that the locus of periselenena (as arrival inclination takes on all possible values) is nearly a circle about the vertex point if arrival altitude is kept constant. The radius of the periselenium circle increases as altitude increases; the radius decreases as flight time increases (in the 66 to 90 hours range).

More details will be given in the solution of various problems.

## SECTION III. LUNAR LANDING SITE STUDIES AS CONSTRAINTS ARE ADDED

Many problems associated with the availability of lunar locations for use as landing sites are currently unresolved. One major concern is associated with the determination of exactly which lunar locations may be considered as possible landing sites for a given flight profile.

In the following, the flight constraints are changed and, for each profile, the lunar locations that are available for consideration as landing sites are determined. The analysis is based upon the arrival vertex principle, the periselenium circle about a vertex and great circle projections for the trajectory representations on the lunar surface.

### A. LUNAR LOCATIONS NOT ACCESSIBLE VIA DIRECT MODE

Performing the mission via the direct mode implies that a vehicle will arrive at its landing site without ever having orbited the target body. In fact, this analysis is made as though the vehicle would travel ballistically to hard-impact. The various procedures for achieving the soft-landing can be superimposed onto the results presented here.

#### 1. FOR A SPECIFIC FLIGHT TIME AND ARRIVAL TIME

Notice from Figure 1 that the perpendicular impact trajectory arrives opposite the VERTEX point, pierce point of the  $\bar{S}$  on the lunar surface. As the

magnitude of  $\bar{B}$ ,  $b$ , increases from zero, impact trajectories are defined in approximately concentric circular patterns about the perpendicular impact point. All trajectories making up one of the impact circles arrive with approximately the same path angle. The arrival path angle approaches the local horizontal as "b" increases until the circle of "grazer" trajectories (horizontal path angle at moon radius) is defined. This impact circle for "grazer" trajectories is approximated in Figure 2 by the locus of periselena for zero altitude. Hence, for that flight time and arrival time associated with the VERTEX of Figure 2, the locations inside the zero altitude circle are not accessible via direct flights. The central angle for soft-landing maneuvering must be superimposed onto this circle to actually complete the analysis for a specified soft-landing procedure.

## 2. For 66 Hour Flights Arriving At Any Time

Before one can determine which sites are accessible for all arrival times, two questions must be answered. First, what are the bounds for the vertex motion for all arrival times? Figure 3 presents the vertex motion for 66 hour flights during November - December 1964, October 1966, and March - April 1969. It is reasonable to assume that the vertex motion is bounded in selenographic latitude between  $\pm 15^\circ$  and in selenographic longitude between  $125^\circ$  and  $145^\circ$  for any and all arrival times. Second, how do the radii of the zero altitude periselenium circles behave as arrival times varies? These radii were examined for 66 hour flights arriving in the three months mentioned previously. The radii magnitudes vary between about  $47^\circ$  and  $51^\circ$  as arrival times varies within a month. Each month shows about the same variation.

In Figure 4 the vertex motion is bounded by the rectangle  $(15^\circ, 125^\circ)$ ,  $(15^\circ, 145^\circ)$ ,  $(-15^\circ, 145^\circ)$ , and  $(-15^\circ, 125^\circ)$  and is labeled Area I. Using the  $47^\circ$  and  $51^\circ$  radii appropriately the following facts are exhibited in Figure 4 (66 hour flights):

a. Areas I and II are absolutely inaccessible via direct mode.

b. All sites outside Areas I, II, and III are accessible at any time.

c. Area III is made up of limited access sites. About all that can readily be stated about these sites is that at sometime, possibly only once in 18.6 years, most of them are accessible. One can't say that all of this area will be accessible at some time because the periselenium circle radii were used appropriately to determine definite "no access" and "complete access" areas.

### 3. Sites Available for Perpendicular Impact Flights of 66 to 90 Hours Duration

Each vertex point on the lunar surface shown in Figure 3 lies just opposite its perpendicular impact point. Consequently, if we assume as before that the vertex will at some time be at each point in Area I of Figure 4, the following is true:

d. The perpendicular impact area for all arrival times is bounded by the rectangle  $(15^\circ, 305^\circ)$ ,  $(-15^\circ, 305^\circ)$ ,  $(-15^\circ, 325^\circ)$ , and  $(15^\circ, 325^\circ)$ . Recall that this is for 66 hour flights via the direct mode.

Figure 5 shows that as flight time increases the vertex area (Area I) shifts in longitude considerably but very little in latitude. As a rough approximation, one may consider increases in flight time to decrease longitude of the vertex point by about  $1^\circ$  per hour. In other words:

e. For flight times between 66 and 90 hours,

$$\frac{\partial \lambda_V}{\partial T_F} = -1 \text{ (}^\circ/\text{Hour)}$$

is roughly correct where  $\lambda_V$  is the longitude of the vertex point and  $T_F$  is the flight time.

Using Figure 5, then, one can approximate the perpendicular impact area for flight times between 66 and 90 hours for all arrival times.

### 4. For 66 Hour Through 90 Hour Flights Arriving At Any Time

Having already shown the sites accessible via 66 hour flights, we need only to show the 90 hour flight results. The periselenium circle radii for 90 hour flights

vary from about  $37^\circ$  through  $42^\circ$  (zero altitude) as arrival time varies throughout a month. Various months are not appreciably different.

Figure 5 exhibits the 66 hour and 90 hour data. By correlating these data a rough approximation is available for the intermediate flight times.

The various locations are separated by boundaries and are classified as follows:

- Area I     Vertex area
- Area II    Absolutely inaccessible via the direct mode.
- Area III   Limited access sites, you might get there some time or you might not.

All other locations are fully accessible via the direct mode.

#### B. SITES ACCESSIBLE VIA THE ORBITAL MISSION MODE

The orbital mission mode as used here implies that the vehicle will establish an orbit about the target body. For this analysis we assume that the vehicle may exit from the orbit at our discretion.

f. All locations on the moon are accessible via the orbital mode.

Figure 6 exhibits a sampling of the dense trajectory coverage that can be generated. It is seen that, upon selecting a landing site, the arrival trajectory inclination is determined for passage over that landing site. The arrival trajectory plane contains the vertex, center of the moon, and the landing site. The arrival altitude and direction of motion (direct or retrograde) are still open parameters. Also, one can conclude the following from Figure 6.

g. The "latitude" of a vertex point is the value of the minimum inclination that can be established relative to the same plane to which latitude was referenced.

### C. PROBLEMS ANALYSIS FOR A PARTICULAR LANDING SITE VIA THE ORBITAL MISSION MODE

A particular landing area has been designated as "The Sea of Tranquility" on the lunar surface. We arbitrarily specify, within this area, the landing site to be at  $2^{\circ}$  north latitude and  $28^{\circ}$  east longitude. The true equator of the moon is the basic reference plane and a mean earth-moon line is the longitude reference direction. The landing site lies  $2^{\circ}$  north of the equator and  $28^{\circ}$  toward the trailing edge of the moon.

In the following sections when azimuth is mentioned it is the azimuth of a direct orbit. The azimuth of a retrograde flight in the same plane would be the stated azimuth plus  $180^{\circ}$ .

#### 1. Bounds For The Variation In Arrival Azimuth

The fact that we have been able to establish bounds for the vertex motion for all arrival times leads to the establishment of bounds for other parameters. Having designated a landing site, we may use the extreme vertex points and determine bounds for the azimuth at the instant the vehicle passes over the landing site. Figure 7 exhibits the geometry of the problem and the azimuth bounds are computed to be as follows:

h(1). The extreme right vertex points for 66 hour flights are at  $(\pm 15^{\circ}, 145^{\circ})$ . These yield azimuths at the landing site of  $72.3^{\circ}$  and  $105.8^{\circ}$ .

h(2). The extreme left vertex points for 90 hour flights are at  $(\pm 16^{\circ}, 102^{\circ})$  and they yield azimuths at the landing site of  $73.9^{\circ}$  and  $107.1^{\circ}$ .

Note that the larger vertex area on Figure 7 admits both launch opportunities per day. The area for the vertex motion can be restricted to  $(\pm 8^{\circ}, 102^{\circ})$  and  $(\pm 8^{\circ}, 145^{\circ})$  by picking the appropriate one of the pair of launch opportunities that occur each day. Computations yield the following azimuth bounds for this restricted vertex area:

i(1). For vertex points at  $(\pm 8^{\circ}, 145^{\circ})$  the azimuth bounds at the landing site are  $80.1^{\circ}$  and  $98.0^{\circ}$ .

i(2). For vertex points at ( $\pm 8^\circ$ ,  $102^\circ$ ) the azimuth bounds at the landing site are  $82.2^\circ$  and  $98.9^\circ$ .

## 2. The Rendezvous Compatible Orbit For Various Stay Times On The Moon

Consider the situation where a parent vehicle is to be in a lunar orbit such that an exploration vehicle can be sent to the lunar surface, remain there a designated stay-time, and be in position to return to the parent vehicle without making powered plane changes. This type of orbit is termed a RENDEZVOUS COMPATIBLE ORBIT. Now, when the landing site and stay-time are specified, there is only one corresponding rendezvous compatible orbit. The center of the moon, the landing site at arrival time, and the landing site at departure time determine the lunar surface projection of that orbit.

We approach this trajectory problem in terms of azimuth at the landing site. One can ascertain that the rendezvous compatible azimuth at the landing site is independent of site longitude. Figure 8 exhibits a plot of rendezvous compatible azimuth at the landing site for various site latitudes and stay-times on the moon. From the plot one can determine the desired azimuths to be as follows:

j. For a landing site at  $2^\circ$  north latitude the rendezvous compatible orbit must pass over the landing site at an azimuth of  $89.89^\circ$  for  $1/2$  day stay-time;  $89.84^\circ$  for  $3/4$  day stay-time, and  $89.78^\circ$  for 1 day stay-time.

It was pointed out previously from Figure 7 that as arrival time varies the arrival azimuths at this landing site will be between about  $72.3^\circ$  and  $107.1^\circ$ . We conclude immediately that one cannot establish a rendezvous compatible orbit on every day without making powered plane changes. In fact, Figure 9 illustrates the following for stay-times of about one to three days:

k. There are in general 4 launch opportunities each month for establishing rendezvous compatible orbits without making powered plane changes.

Notice that the rendezvous compatible orbit through the landing site cuts each of the vertex patterns in 2 places. This means that each one of those vertex points is suitable for establishing the desired orbit.

Also from Figure 9, which has as an example the vertex motion approximated for March - April 1969, we conclude as follows for stay-times of about one to three days:

1. Trajectories with flight times between 66 and 90 hours have very nearly the same launch time opportunities for establishing the rendezvous compatible orbit. There are normally four opportunities, two when the moon is near maximum declination at arrival and two when near its minimum declination at arrival.

We have seen that the launch time opportunities are few for establishing rendezvous compatible orbits; also, the azimuth spread at passage over the site is shown to be very small for stay-times up to one day.

One possibility for alleviating these conditions is an extension in stay time on the moon. It has been shown that azimuths at the landing site are between  $72.3^\circ$  and  $107.1^\circ$  for all launch opportunities. One might entertain the possibility of staying until the azimuth he came in on is the one at which he should depart. Figure 10 exhibits the rendezvous compatible azimuths associated with various stay-times and various site latitudes (longitude has no influence on these azimuths). From Figure 10 we conclude the following for the site at  $2^\circ$  north latitude:

m. Stay-times on the moon up to about 12 days afford rendezvous compatible azimuths at the site from near  $90^\circ$  for short stay-times to about  $80^\circ$  for the 12 day stay time.

n. For 66 to 90 hour flights, we note that

(1) Rendezvous compatible orbits can be established for stay times from near zero up to about 13 days. Notice from Figure 9 that the launch time opportunities go from 4 per month to only one as stay-time increases.

(2) For stay-times from about 13 to 15 days, there are no possibilities for rendezvous compatible orbits without powered plane changes.

(3) For stay-times of 15 days on up to the assumed lunar period opportunities again exist for establishing rendezvous compatible orbits without making powered plane changes. Launch time opportunities start at one per month at about 15 days, go to two per month, and work on up to four per month as stay-time approaches the lunar period.



(4) Observe from Figure 9 that for short stay-times the vertex points are near the lunar equator for establishing rendezvous compatible orbits. The corresponding launch opportunities occur when the moon is in the maximum or minimum declination region.

The statements about launch opportunities are made assuming a due east launch azimuth. We now consider the influence of having the freedom to pick launch azimuth to be any value between  $70^\circ$  and  $110^\circ$ . It was shown in Reference 1 that azimuth variations of  $\pm 20^\circ$  about  $90^\circ$  shifts the vertex latitude by about  $3^\circ$  on days when the moon is near maximum or minimum declination. This means that variable launch azimuth may increase the rendezvous compatible launch opportunities. For example, consider a situation having four  $90^\circ$  launch opportunities. It may be that, besides the four  $90^\circ$  launch opportunities resulting in rendezvous compatible orbits, the neighboring opportunities are good also if launch is at the correct azimuth. This depends upon the change in latitude of the vertex from day to day and azimuth to azimuth.

### 3. Sites That Afford No Rendezvous Compatible Orbit For Specified Stay-Times On The Moon

We should recall that the center of attraction and any two points on a sphere determine a unique great circle containing those points. The direction of motion along that great circle, or orbit, is still not specified but the orbit projection on the surface of the sphere is uniquely specified.

With this in mind, then, consider the lunar rendezvous compatible orbit. The moon rotates about 13 (deg/day). The site location at the beginning and end of the stay-time on the moon and the center of the moon determine the space-fixed rendezvous compatible orbit. We know already that flights satisfying the constraints already discussed, arriving on any day, and having injection to periselenium flight time between 66 and 90 hours will pass over a vertex in the area depicted in Figure 7. The objective here, then, is to determine which sites afford no rendezvous compatible orbit passing through the vertex area. These sites are thereby ruled out for flights that must establish rendezvous compatible lunar orbits and still satisfy the other constraints of this study.

Figure 11 exhibits a Mercator projection of the lunar surface. We have depicted there sites that afford no opportunity for establishing rendezvous compatible orbits for one day and three day stay-times at the site. All perturbations are neglected during the stay on the moon.

A polar site must be analysed apart from Figure 11. We simply point out that a polar site is rendezvous compatible for all launch opportunities and for any stay-time. Just place the parent vehicle in a polar orbit and, since the moon rotates about its pole, the nodal shift has no effect at all on the passage of the orbit over the site. Hence, we emphasize as follows:

o. Sites at the poles are the only sites that are rendezvous compatible on every day and for any stay-time on the moon (no perturbations considered).

The following conclusion may be drawn from Figures 3, 9, and 11 for stay-times up to about three days:

p. Sites at latitudes between  $\pm 7^\circ$  offer more rendezvous compatible opportunities in the long run - say, over a number of months, or years - than any other sites. This is seen by the fact that both curves representing the vertex motion as arrival time varies pass over these latitudes.

q. Sites in the dotted areas of Figure 11 never afford an opportunity for rendezvous compatible orbits (for that associated stay-time).

#### 4. Surface To Lunar Orbit Rendezvous Via Plane Changes

Assume that the parent vehicle's trajectory is established so that it passes over a vertex point and the landing site. The azimuths at the landing site for all such trajectories having flight times between 66 and 90 hours have already been determined to be between  $72.3^\circ$  and  $107.1^\circ$ , the site being at  $2^\circ$  north latitude and  $28^\circ$  toward the trailing edge of the moon. These azimuths, then, afford a convenient parameter for characterizing the motion of the parent vehicle for any and all arrival dates and flight times between 66 and 90 hours.

The primary objective here is to determine the plane change that an exploration vehicle must make to rendezvous with the parent vehicle after having spent various time periods on the moon. If the moon didn't

rotate on its axis, there would be no required plane change. The exploration vehicle would be free to stay as long as desired, under our assumptions, and return to the parent vehicle in the same plane in which it landed. Assume now that the moon rotates on its axis at 13 (deg/day), and that the exploration vehicle may leave the moon at any azimuth desired and perform the plane change at any desired time.

The minimum plane change required for making the rendezvous is exhibited in Figure 12 for various stay-times on the moon and for various parent vehicle orbits. The exploration vehicle leaves the site at an azimuth that minimizes the relative inclination of the return plane to the parent vehicle's plane of motion. This minimized relative inclination is the minimum plane change required for making the rendezvous. Notice the following from Figure 12:

r. For stay-times up to one day and the assumptions just mentioned, the minimum required plane change ranges from  $0^{\circ}$  to  $4^{\circ}$ . The  $0^{\circ}$  points correspond to the rendezvous compatible orbits that were discussed earlier in this report.

A rough estimate of the velocity required to make these minimum plane changes is available after a few simplifying assumptions. The velocity required to shift a circular velocity vector through the required angle, the resultant again being circular, is a rough estimate of the velocity the exploration vehicle requires to perform its corresponding plane change. Optimized plane change studies at the earth have shown that this type estimate of velocity requirements is considerably higher than that velocity actually required in an optimized maneuver. Figure 13 exhibits the velocity estimate for stay-times up to one day and various parent vehicle orbits. The parent vehicle's orbit is assumed to be circular at a 100 (n.m.) altitude. The following is evident from Figure 13:

s. A rough estimate of the velocity increments needed to make the minimum plane changes required for lunar orbit rendezvous after stay-times up to one day ranges from 0 to 110 (m/s). This admits all launch opportunities and flight times between 66 and 90 hours.

## SECTION IV. CONCLUSIONS

Various lunar landing site problems are resolved by using two fundamental properties of earth-to-moon trajectories as basis for the analysis. The problems treated here represent only a sampling of the problems that may be treated using these two fundamental properties and methods similar to the ones used in this report.

The accessibility of lunar locations via either the direct mission mode or the orbital mission mode is investigated. For the direct mission mode, and various constraints, locations are determined that are never accessible for use as a landing site, as well as those that are always accessible for use as a landing site. For the orbital mission mode, it is shown that all locations are accessible on every day. If rendezvous compatibility is required, and the stay-time at the landing site is specified, there are sites that are forever ruled out as possible landing sites. Such sites are determined for reasonable values of stay-time at a site.

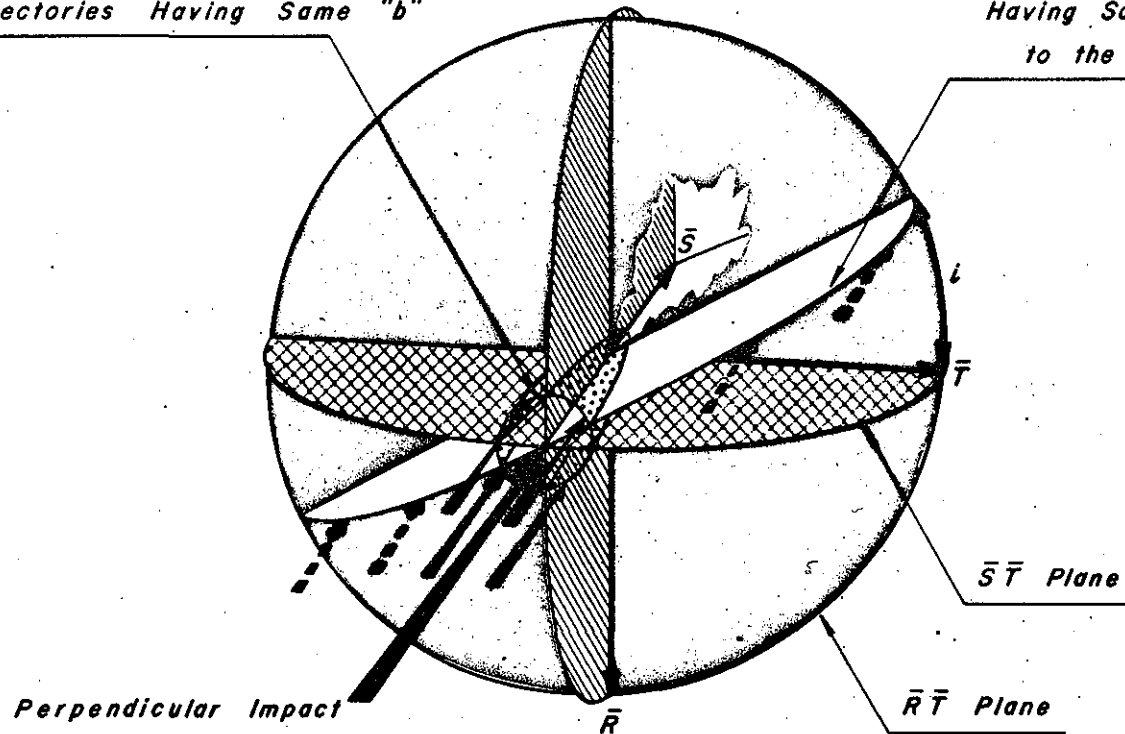
A landing site is picked in the Sea of Tranquility and the orbital mission mode is assumed as basis for a study of some lunar rendezvous problems. Launch opportunities for establishing rendezvous compatible orbits are pointed out. This means that if these opportunities are used, there is no powered plane change required in rendezvous of the exploration vehicle and parent vehicle after the stay on the moon. Finally, assuming that launch is on any day, regardless of whether rendezvous compatibility is afforded on that day or not, it is shown that the minimum required plane change after a one day stay on the moon varies from zero to about  $4^\circ$  for the rendezvous in orbit. The velocity required to accomplish this plane change is estimated to vary from zero to 110 (m/s) as the minimum required plane change varies up to  $4^\circ$ .

## REFERENCE

1. Tucker, W. B., and Hooper, H. L., "Lunar Flight Study Series, Volume 7. Earth-Moon Transit Studies Based on Ephemeris Data and Using Best Available Computer Program. Part II: Principles for Reducing Earth-Moon Trajectory Analysis to Fundamentals," MTP-AERO-63-25, April 16, 1963.

*Circle of Possible Impact Points  
for Trajectories Having Same "b"*

*Band of Impact Points  
for Trajectories  
Having Same Inclination  
to the  $\bar{S}\bar{T}$  Plane*



**FIG. 1. SCHEMATIC OF LUNAR APPROACH PATTERN FOR  
CONSTANT "b" AND CONSTANT "l" TRAJECTORY FAMILIES**

- 66 hr Flight Time
- Arrival Time Is 30 Nov 1964

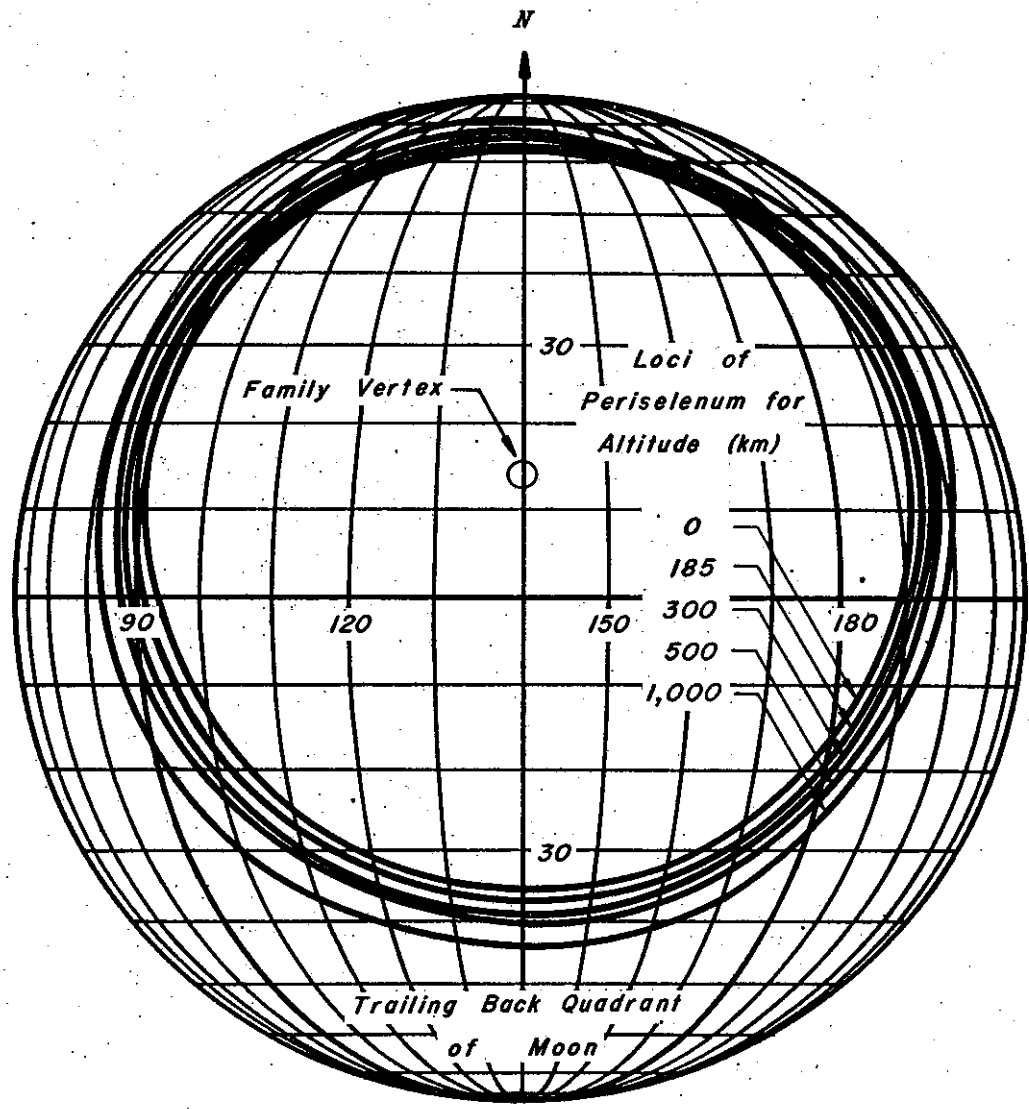
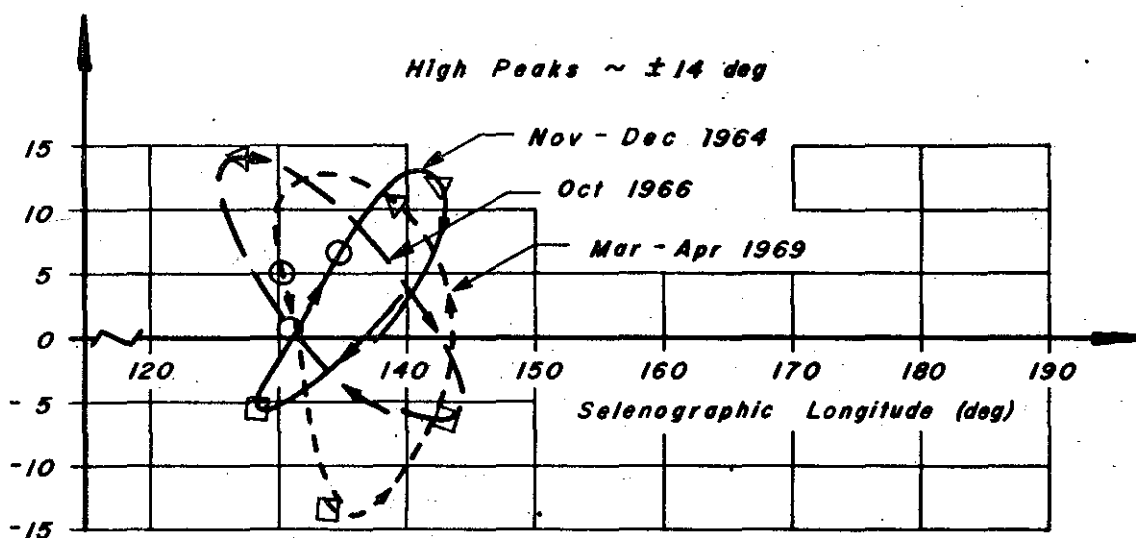


FIG. 2. PERISELENA CONTOURS ABOUT THE VERTEX FOR VARIOUS ARRIVAL ALTITUDES

Selenographic Latitude (deg)



Selenographic Latitude (deg)

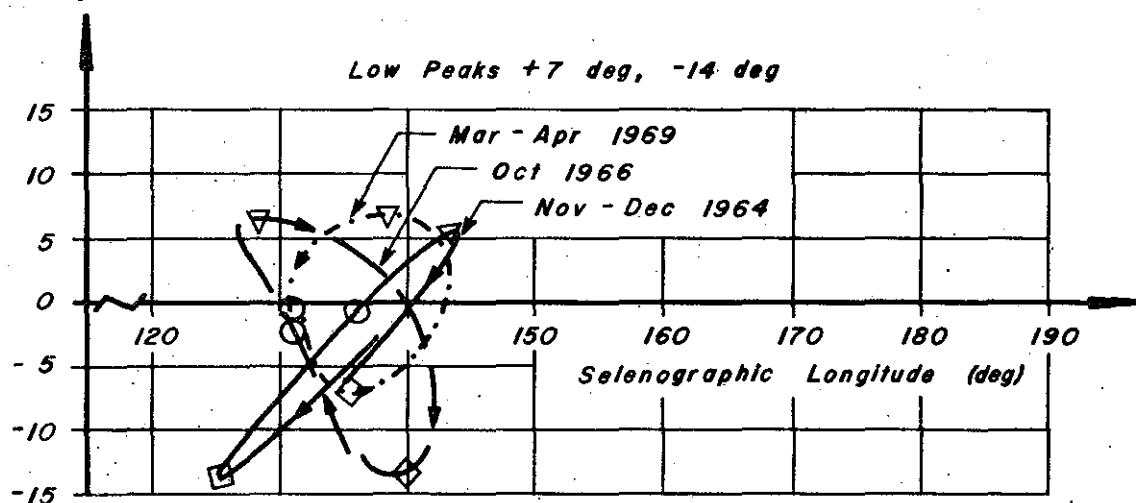


FIG. 3. CONSOLIDATED PLOT OF  
66 HR FLIGHT VERTEX POINTS  
HAVING HIGH AND LOW PEAKS  
IN VARIOUS MONTHS AND YEARS

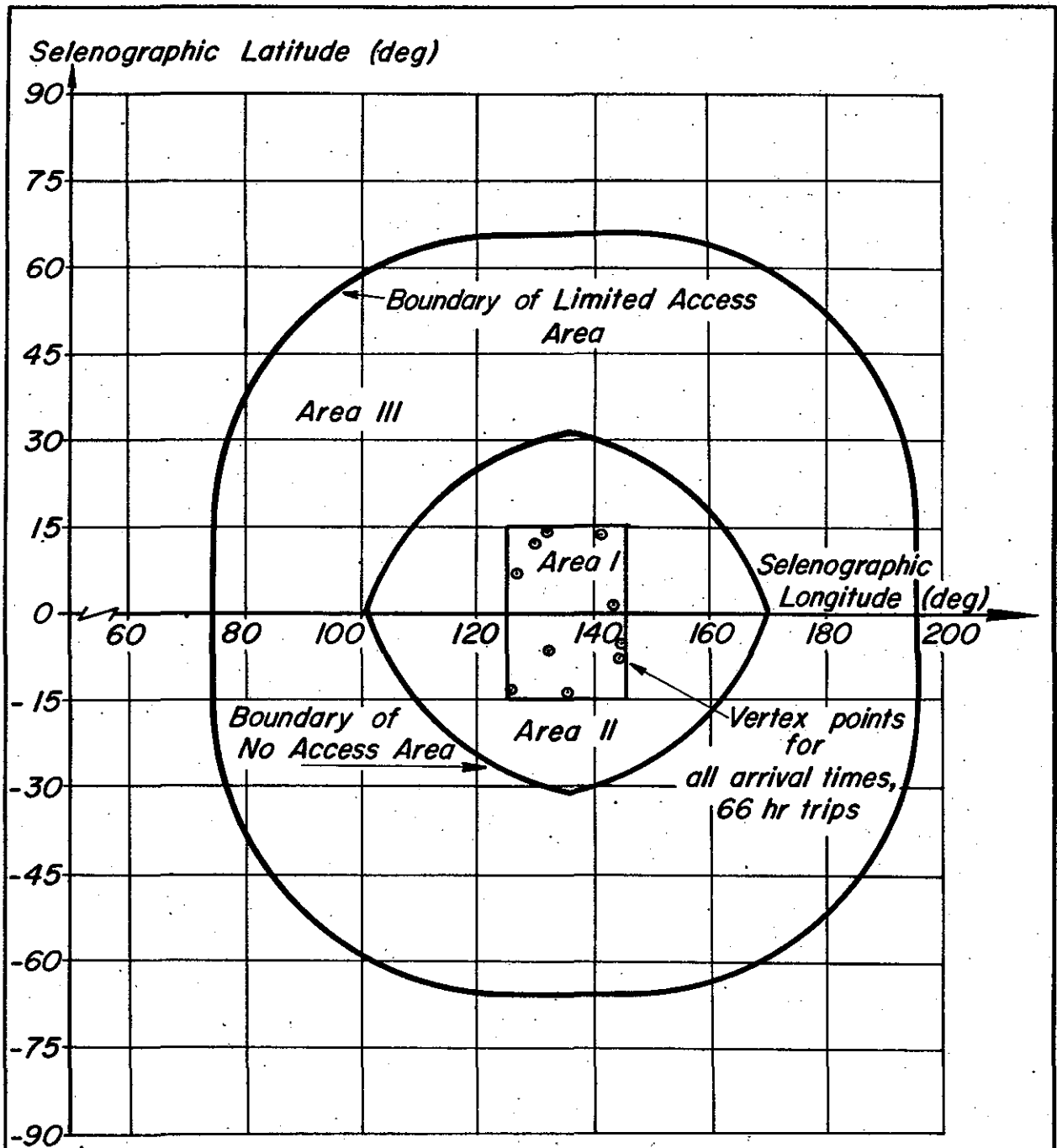
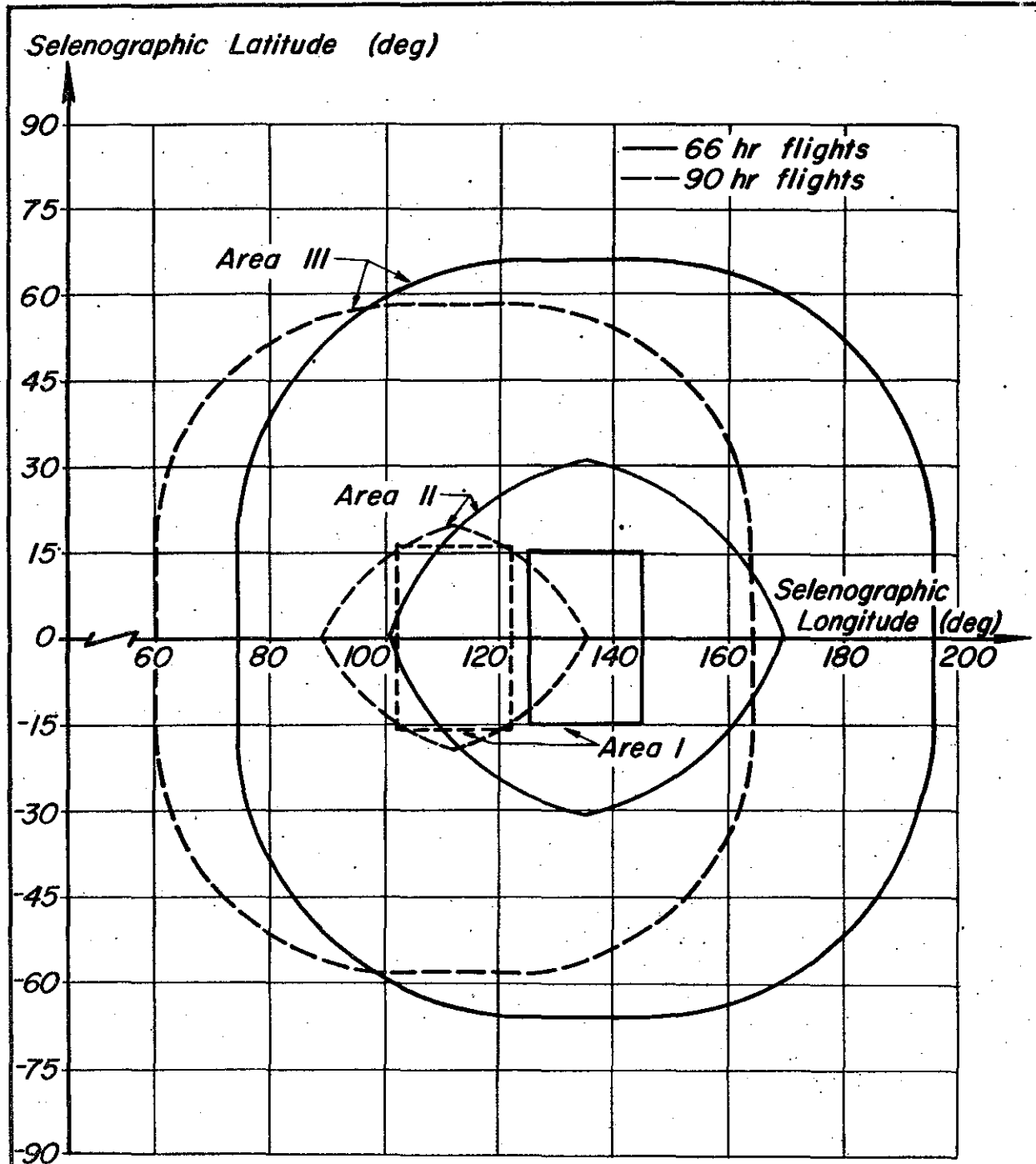
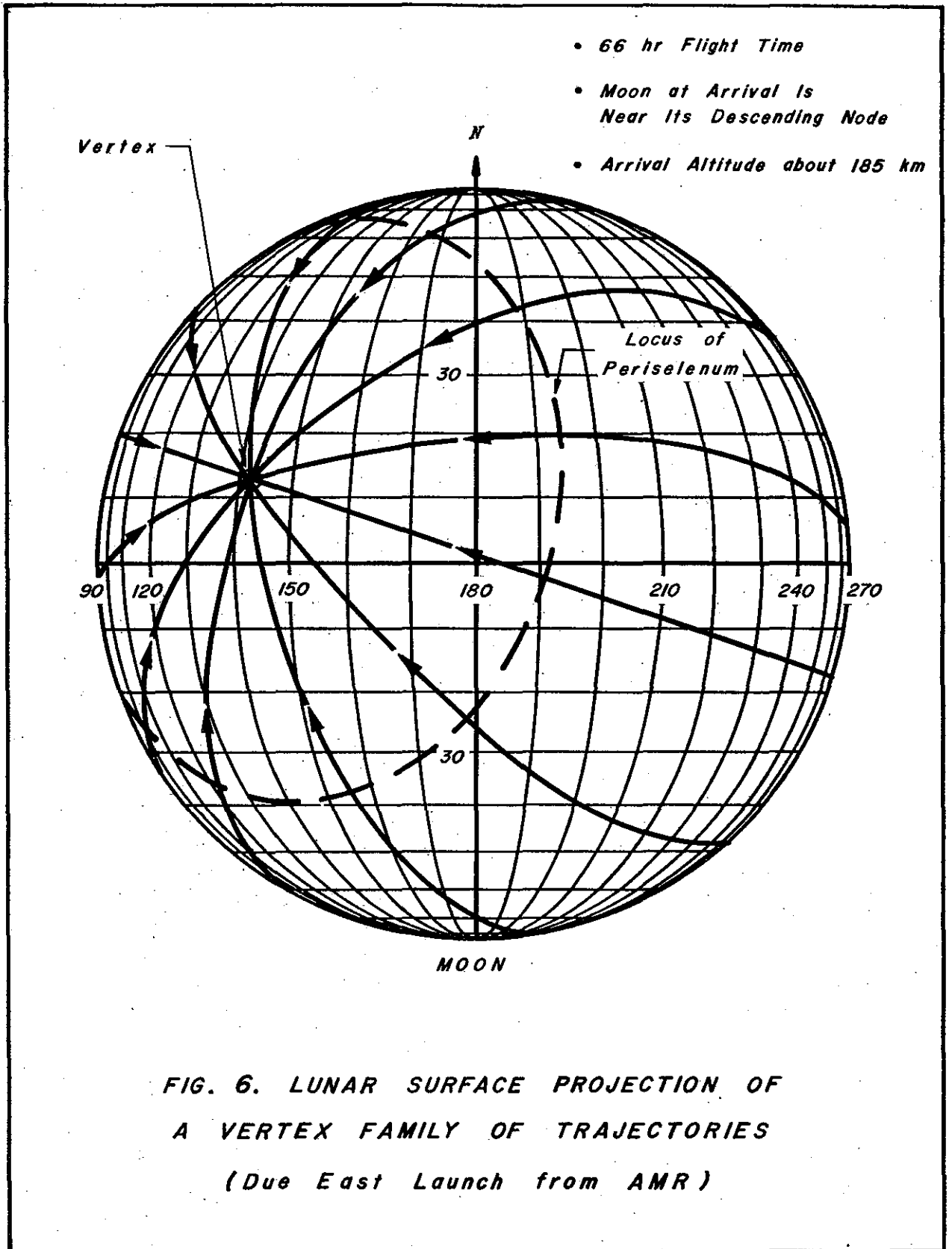


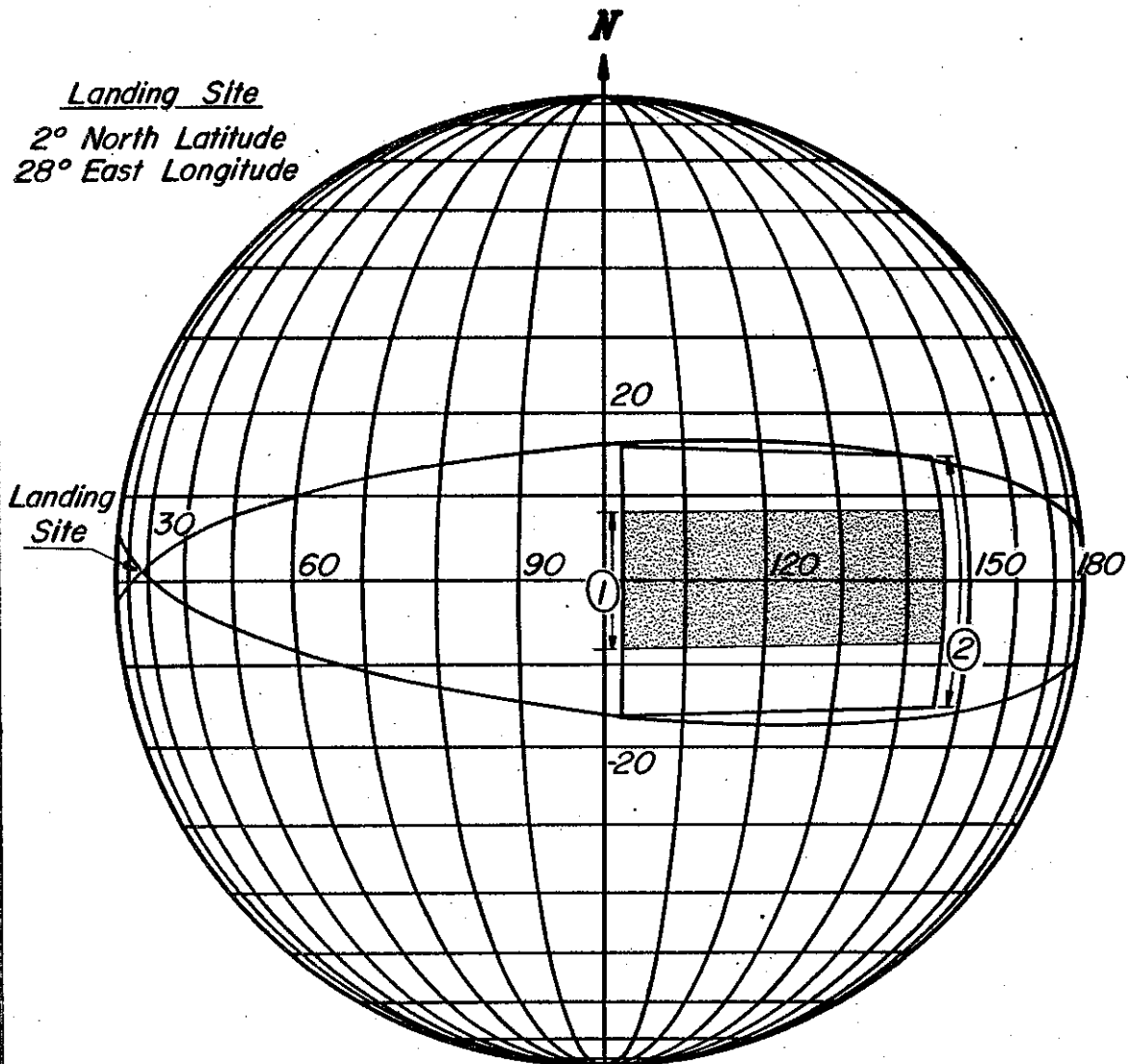
FIG. 4. LANDING SITE ACCESSIBILITY FOR  
66 HOUR FLIGHTS VIA THE DIRECT MISSION MODE  
FOR ALL ARRIVAL TIMES





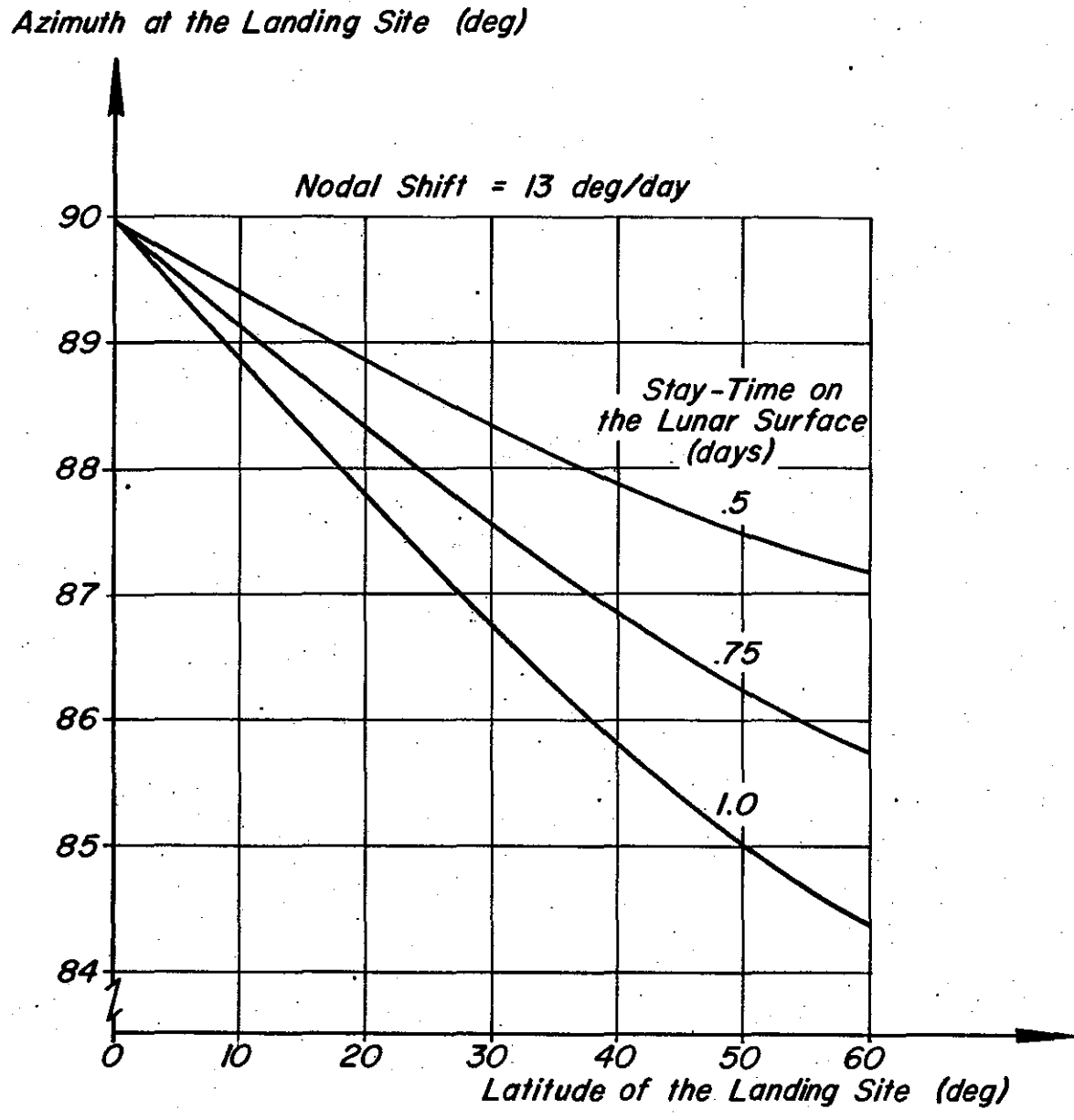
**FIG 5. LANDING SITE ACCESSIBILITY FOR  
66 HOUR AND 90 HOUR FLIGHTS VIA THE DIRECT MISSION MODE  
FOR ALL ARRIVAL TIMES**





- ① Area of vertex motion if the appropriate one of the two launch opportunities of each day is chosen.
- ② Area of motion of vertex points for any arrival time and flight times in the 66 to 90 hr range for both launch opportunities of each day.

FIG. 7. ARRIVAL AZIMUTH BOUNDS FOR  
ALL ARRIVAL TIMES AND 66 THROUGH 90 HOUR FLIGHTS



**FIG. 8. RENDEZVOUS COMPATIBLE AZIMUTHS AT TOUCHDOWN FOR VARIOUS SITES AND STAY-TIMES ON THE MOON**

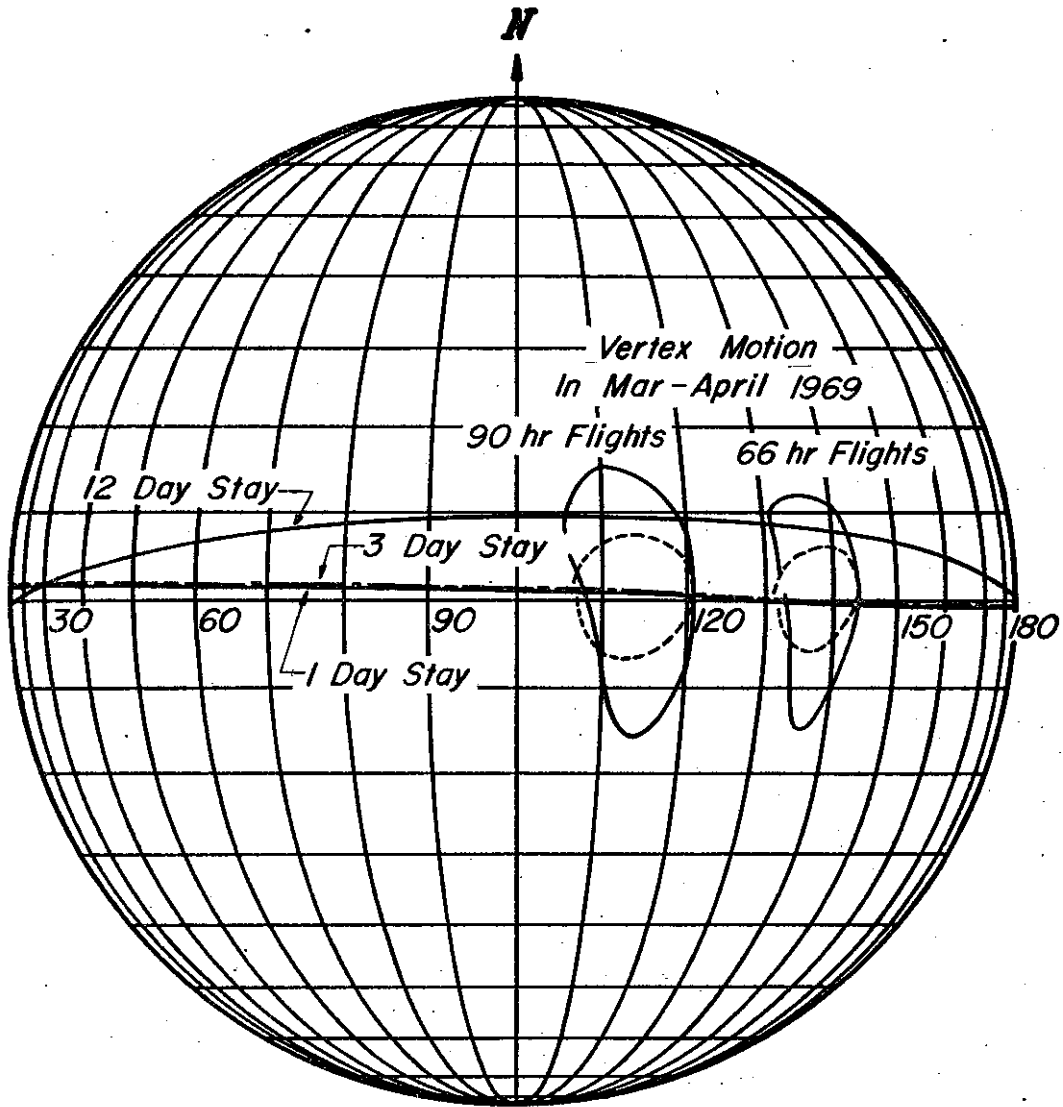
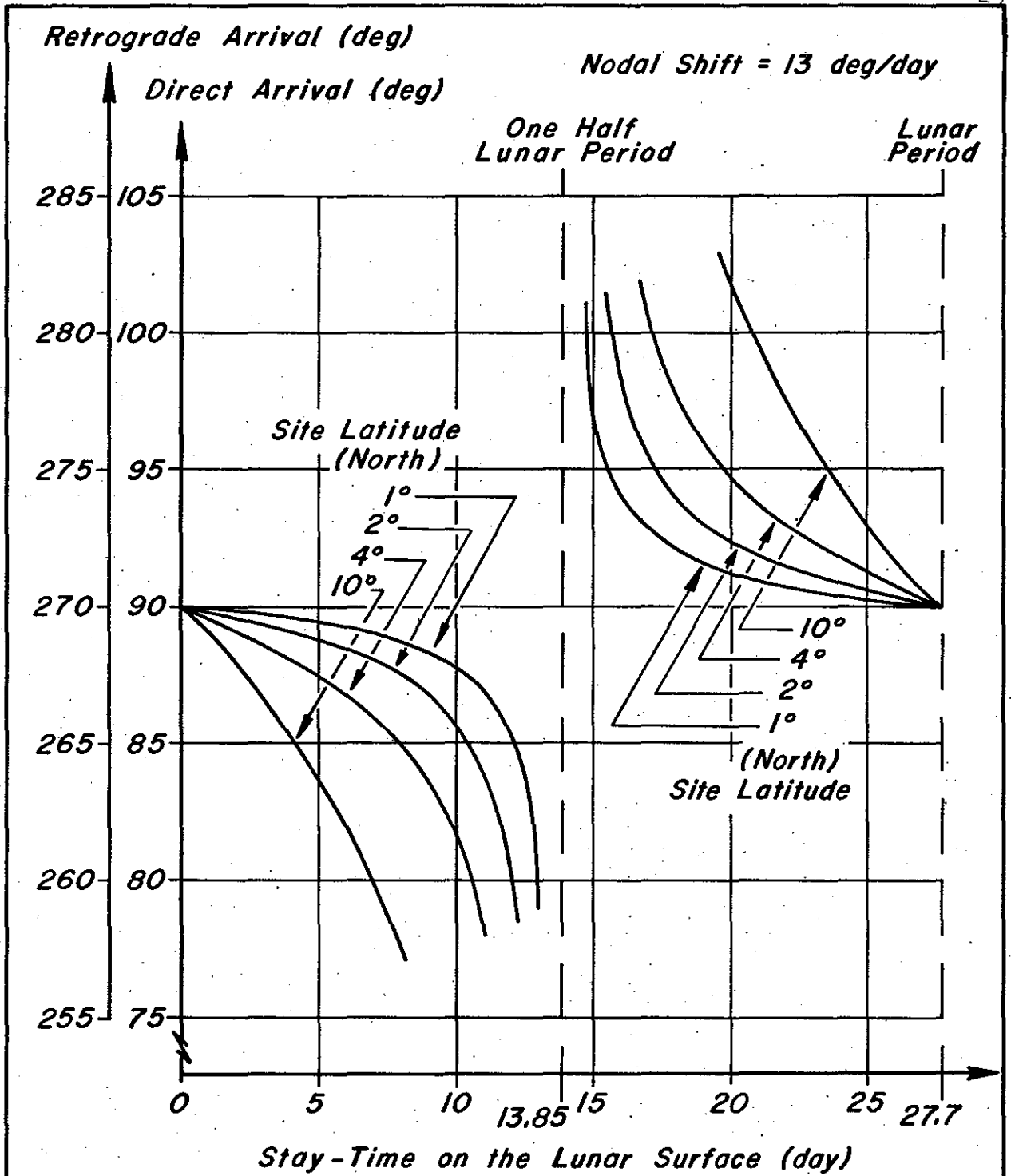
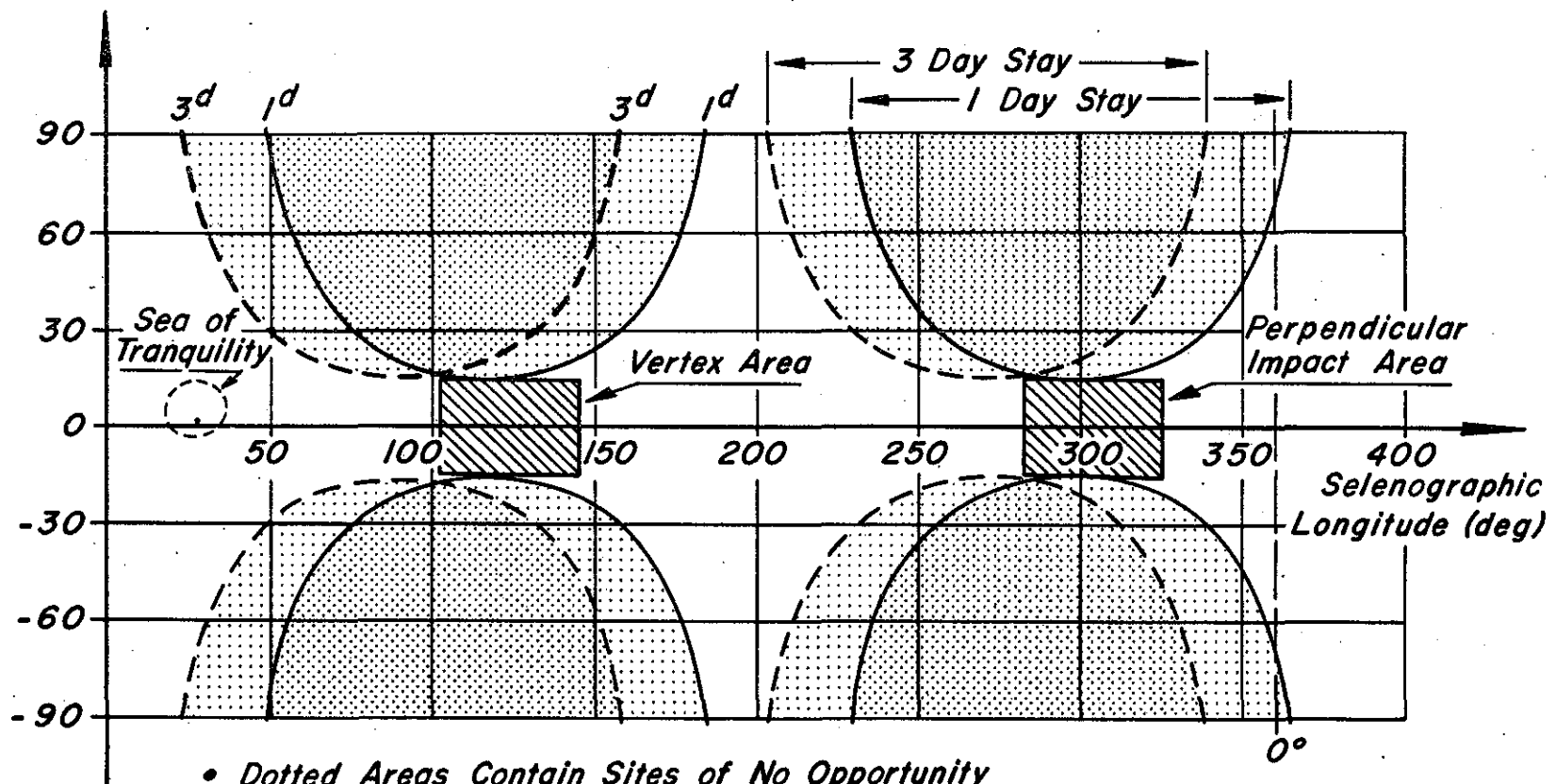


FIG. 9. RENDEZVOUS COMPATIBLE ORBITS FOR PARTICULAR STAY-TIMES AT THE SEA OF TRANQUILITY



**FIG. 10. RENDEZVOUS COMPATIBLE AZIMUTHS AT TOUCHDOWN FOR EXTENDED STAY-TIMES ON THE LUNAR SURFACE AT VARIOUS LATITUDE CIRCLES**

Selenographic Latitude (deg)



- Dotted Areas Contain Sites of No Opportunity
- Mercator Projection of the Lunar Surface
- Nodal Shift = 13 deg/day

FIG. II. SITES OF NO OPPORTUNITY FOR RENDEZVOUS CAPABILITY ORBITS FOR ONE AND THREE DAY STAY-TIMES AT A SITE

Minimum Required Plane Change (deg)

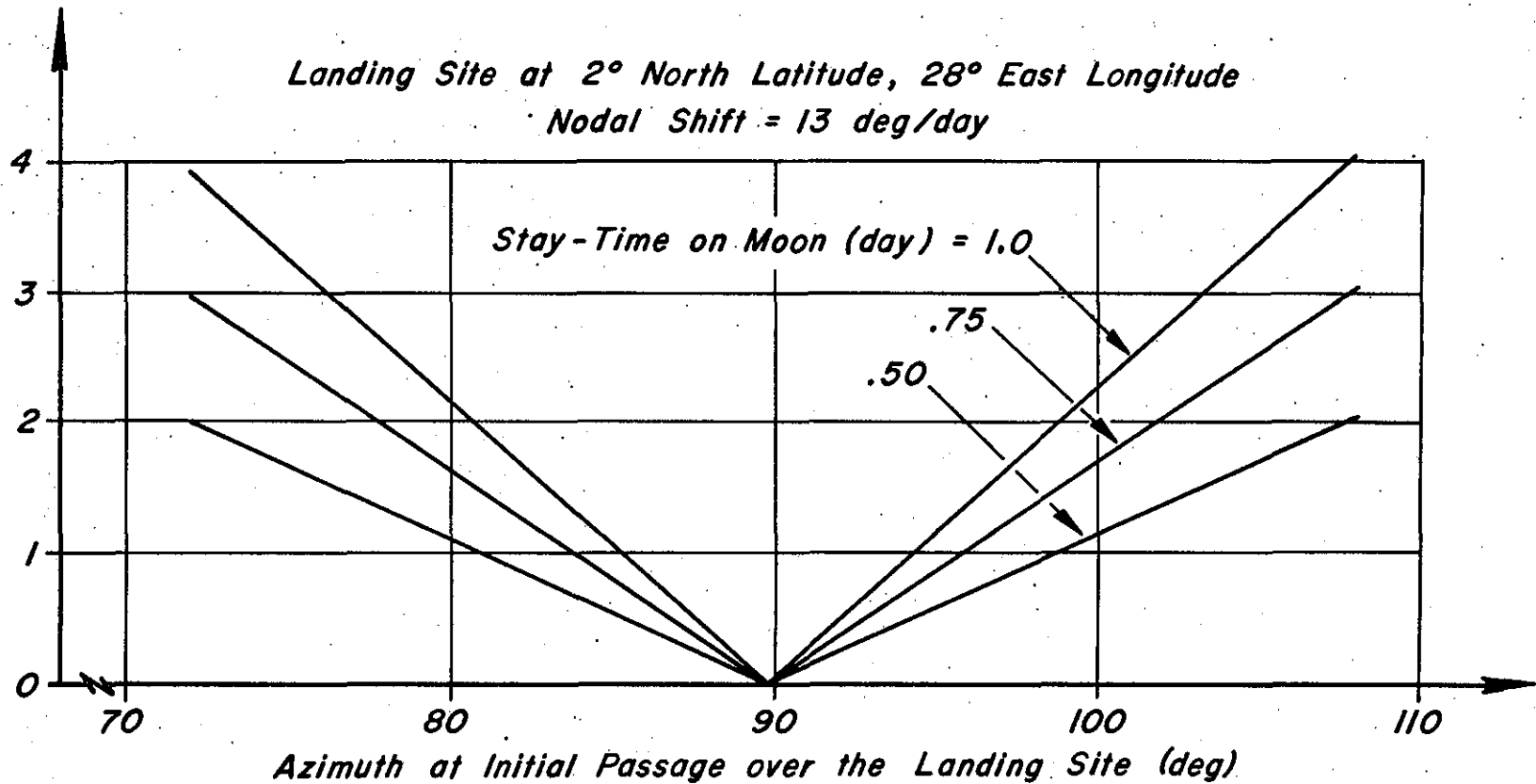
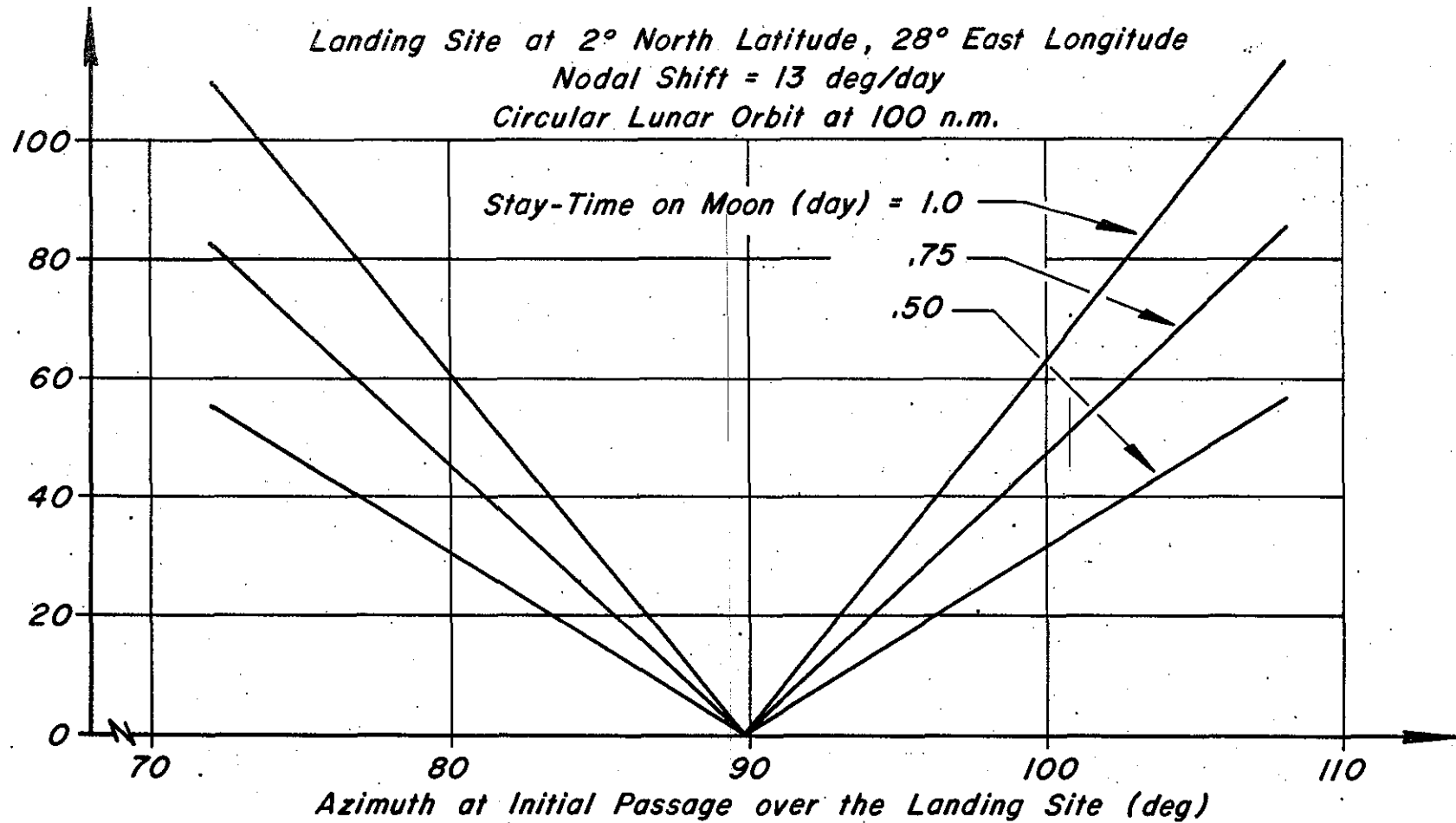


FIG. 12. MINIMUM PLANE CHANGE REQUIRED FOR LUNAR ORBIT RENDEZVOUS FOR VARIOUS STAY-TIMES AND ARRIVAL TRAJECTORIES



*Velocity for Making Minimum Required Plane Change (m/s)*



**FIG. 13. ESTIMATE BY VECTOR ADDITION FOR THE VELOCITY REQUIRED TO MAKE MINIMUM PLANE CHANGES FOR LUNAR ORBIT RENDEZVOUS**

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By

W. B. Tucker and H. L. Hooper

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