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Quantifying Uncertainties in Navigation and Orbit Propagation Analyses

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Quantifying Uncertainties in Navigation and Orbit Propagation Analyses

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Abstract

A tool used to calculate dilution of precision (DOP) was created in order to assist the Space Communications and Navigation (SCaN) program to analyze current and future user missions. The SCaN Center for Engineering, Networks, Integration, and Communication (SCENIC) is developing a new user interface (UI) to augment and replace the capabilities of currently used commercial software, such as Systems Tool Kit (STK). The DOP tool will be integrated in the SCENIC UI and will be used analyze the accuracy of navigation solutions. This tool was developed using MATLAB® (The MathWorks, Inc.) and free and open-source tools to save cost and to use already existing orbital software libraries. GPS DOP data was collected and used for validation purposes. The similarities between the DOP tool results and GPS data show that the DOP tool is performing correctly. Additional improvements can be made in the DOP tool to improve its accuracy and performance in analyzing navigation solutions.

1.0 Introduction

Currently SCaN uses programs such as STK to plan and assess current and future user missions. STK allows users to create their own satellite mission scenarios and perform calculations such as satellite orbits, trajectories, line of sight analysis, and DOP. There is a demand for augmenting the capabilities of STK and replacing it with free and open-source tools. STK licensing fees limit the number of people that can use STK at a single moment and which computers the software can be installed on. There are also many areas where STK does not have the functionality to model specific mission scenarios and calculations. SCENIC is currently developing a new and easy to use UI to augment and replace the capabilities of STK. The SCENIC UI is being built with open-source tools so SCaN can avoid STK licensing fees and add their own custom capabilities. One future capability of the SCENIC UI is to calculate DOP metrics for SCaN user missions. This paper explains the theory behind dilution of precision and the development of the DOP tool for the SCENIC UI.

DOP describes the geometric strength of the satellite's configuration on the accuracy of a navigation solution (Ref. 1). DOP is also weighted by the accuracy of the error performance in the navigation measurements. In a GPS system, DOP uses the estimate of GPS receiver's position, the position of the visible satellites, and the user range error of the satellites in its calculation. The calculated DOP value describes the level of confidence in the measurement of the receiver's position. Figure 1 shows two GPS satellites and two colored rings surrounding each satellite. These satellites are trying to determine the location of the GPS receiver, represented as the orange circle. The width of these colored rings represent the error in the satellites' range measurements. The area where these rings intersect is colored green and represents the area where the GPS receiver must be located. In Figure 1, the green area of uncertainty is

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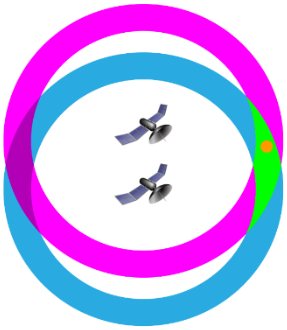

Figure 1.—Two GPS Satellites with Weak Geometric Configuration

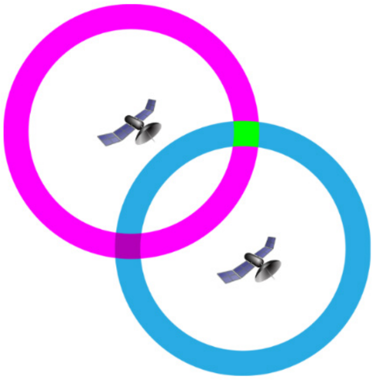

Figure 2.—Two GPS Satellites with Strong Geometric Configuration

large. In Figure 2, the two satellites have the same error in their range measurements and a different configuration. This configuration minimizes the green area of uncertainty in their navigation estimation. Figure 1 has a high dilution of precision because the satellites have small angular separation. Figure 2 has a much lower dilution of precision because the angle between the satellites and receiver minimizes the overlapping regions of error in their measurements.

Calculating DOP for a navigation measurement has many applications. The position dilution of precision (PDOP) measurement uses the x, y, and z positions of the satellites and is commonly used to evaluate the accuracy of navigation systems (Ref. 1). A lower DOP value means that there is a high level of confidence in the accuracy of the navigation measurement, and a higher DOP value means there is a lower amount of confidence in that measurement. In the case of multiple GPS satellites and a single GPS receiver, the PDOP value is used to determine how accurate the calculated location of the GPS receiver is. GPS receivers use PDOP to determine if the calculated position of the GPS receiver should be used as an accurate position or not. If the PDOP value is above a specified threshold (too much dilution of precision), the GPS receiver will not use that position measurement because there is too much uncertainty. DOP can also be used to analyze the effects of adding or removing satellites from the GPS constellation. In general, adding more sources to a navigation solution increases its accuracy and decreases the DOP. Using DOP measurements, one can determine how much more accurate GPS would be if one, two, or three satellites were added to the system.

In addition to analyzing GPS, DOP measurements can be used to evaluate ground to space, space to space, and deep space network (DSN) systems. The position of the International Space Station (ISS) can be determined by using navigation signals from multiple ground-based tracking stations. Depending on where the ground-based receiver are located and how many receivers are currently visible from the ISS, the DOP indicates the amount of uncertainty there is in the calculated position of the ISS. Space navigation systems can also be analyzed using DOP. A space-based navigation system to calculate the position of satellites orbiting Earth, Moon, and Mars could be desirable in the near future. Ground-based DSN receivers are currently used to determine the position of satellites that are orbiting Mars and satellites outside of the orbit of terrestrial planets such as *New Horizons* and *Voyager 1*. There would be a significant difference between the DOP for a GPS and a DSN navigation solution. DOP would indicate the geometric configuration of the GPS satellites is much better than the configuration of DSN receivers on Earth. The signals sent by the DSN receivers travel almost parallel to each other because the relative distance between DSN receiver is miniscule compared to the distance between the DSN receivers and satellites in the outer solar system. Using DOP, one can determine how to improve current navigation systems and how to arrange satellites and ground-based receivers in future systems.

2.0 Background

The pseudorange between a satellite and a navigation receiver can be calculated by multiplying the speed of light by the time the signal has taken from the satellite to the receiver (Ref. 1). The clocks on satellites and receivers are not perfectly synchronized so there are time bias errors within the pseudorange measurement. Calculating the dilution of precision involves computing the ratio between the standard deviations of a specified parameter, such as the x, y, z coordinates and time, and the pseudoranges between the satellites and receiver.

There are a few different types of DOP measurements that are commonly used in analyzing navigation systems. Geometric DOP (GDOP) uses the x, y, and z pseudoranges and the time bias in its calculation (Ref. 2). This means GDOP is sensitive to the position of the satellites in each of the three dimensions, and takes into account the time bias between the satellites' and receiver's clock. At least four navigation signals are required to perform the GDOP calculation. GDOP encompasses the most information in its calculation and it is used to determine the overall confidence level in the 3D position of the satellites and their time bias. Position DOP (PDOP) ignores the time bias component and only uses the position of the satellites in the x, y, and z axis. The PDOP value of a navigation solution would change if any of the satellites change position and is not affected by a change in time bias. PDOP is used when there is no time bias in the navigation solution, or when only the arrangement of the satellites are being measured. This measurement requires at least three navigation signals to calculate.

Horizontal DOP (HDOP) is only used for ground based receivers and provides an estimation in the accuracy of a navigation solution in the x and y coordinates in the local East North Up system (Ref. 2). The measurement is only affected by the horizontal position of the satellites and requires only two navigation signals. HDOP is used with ground-based GPS receivers because the altitude of the satellites is relatively constant and the horizontal position of the satellites would have the largest impact on the position estimation. Vertical DOP (VDOP) only takes into account the z component and time bias. VDOP would be affected by a change in altitude of the satellites, but not a change in their horizontal position. Time DOP (TDOP) ignores the position of the satellites and only uses the satellite time bias in its measurement. This is useful for measuring how different satellite time bias would affect estimating the receiver's position. Lastly, horizontal/time DOP (HTDOP) is a combination of HDOP and TDOP. This measurement takes into account the horizontal position of the satellites and their time bias and requires three navigation signals.

3.0 Methodology

3.1 Software Development and Tools

Already existing MATLAB® code that calculated DOP was used to construct the foundation of the DOP tool for the new SCENIC UI. This already existing code analyzed the effectiveness of a Lunar Network (LN) of navigation satellites (Ref. 2). DOP was used to measure system availability for multiple different satellite constellations. The original code used to perform these measurements was not up to software standards and could only analyze LN navigation systems. In order to expand the functionality of this code, heavy modifications were made and in some cases complete reconstruction was necessary. The original datasets and results from the unmodified code were used to verify that the modified code was performing the DOP calculations correctly.

During the course of developing the updated DOP tool, many improvements were made. The first step was to improve readability. The original code had little to no comments or documentation, and was difficult to follow. Headers that explain exactly what each function computes, the expected inputs and outputs, and descriptive variable names and comments throughout the functions were added. This work made the code much easier to work with. The improved readability increased the productivity of the DOP tool development. This also allows future users of the DOP tool to understand its functionality. The second step was to make the code more modular. The original code was mostly contained in one large function that had many repeated steps and unnecessary calculations. Splitting the single complicated function into multiple smaller and simpler functions resulted in many improvements. This reduced the amount of repeated lines of code and made the calculations easier to follow. Each function was tested individually so their performance and flexibility could be improved at a much faster pace. The individual functions can also be used by other tools within the SCENIC UI. This makes the development of future tools much easier.

The next step was to replace computationally heavy and complicated functions with already existing functions in the Orbit Determination Toolbox (ODTBX) (Ref. 3) and other SCENIC UI tools. ODTBX is an orbit determination analysis tool that is written primarily in MATLAB® and Java. This is a free and open-source software library was developed by the Navigation and Mission Design branch at the NASA Goddard Space Flight Center. ODTBX is currently supported by NASA developers and is updated with bug fixes and new functionality. Replacing computationally intensive calculations with functions from ODTBX improved performance and usability. The DOP tool also uses functions from Christian Gilbertson's N-Body Orbit Propagation tool. Collaborating with other tool developers enabled the DOP tool to have more functionality and future usability within the SCENIC UI.

3.2 Verification

Software verification is the process of checking that a software system performs its intended purpose. The simplified purpose of the DOP tool was to calculate DOP measurements from the position of a receiver, multiple navigation transmitters, and their range errors. The original code used to develop the DOP tool was built by a trustworthy team and assumed to be performing accurate DOP calculations. During the development of the DOP tool, the intermediate calculations and results were compared to the results of the original code. This verification process assured that the DOP tool was performing the same underlying calculations as the original code.

The DOP tool is able to calculate GDOP, PDOP, HDOP, and HTDOP from the position of a receiver, multiple navigation transmitters, and their range errors. The DOP tool can also be used in ground to space, space to ground, and space to space scenarios. The capabilities of the DOP tool meet the original purpose and performs the same as the original code. The DOP tool also has a basic test suite that tests the different paths within the code. These tests confirm the DOP tool works in the nominal case and should be extended for further verification purposes.

3.3 Validation

Software validation is the process of checking that a software system is performing correctly and returns accurate results. In the case of the DOP tool, the validation process involved gathering real world DOP data and comparing that data with the results of the DOP tool. Using a GPS receiver, DOP values were gathered over a 40 minute time period. The DOP data included timestamped measurements with the PRN's (pseudo-random noise, a unique integer assigned to each GPS satellite) of the currently visible satellites and PDOP, HDOP, VDOP, and TDOP values. Publically available Two-Line Element Set (TLE) data and Christian Gilbertson's N-Body Orbit Propagation tool were used to calculate the position of the GPS satellites over the 40 minute time period.

The DOP tool first read the GPS receiver data and determined the period of time when the data was collected. Then The DOP tool read the TLE data and used that data to calculate the positions of the satellites over the 40 minute time period. Each of these positions have a matching timestamp that specifies when the satellite should be at that position. Next, each timestamp from the GPS receiver data was matched with the timestamp from the calculated position data. The PDOP and HDOP values were then calculated from the positions of the satellites at each timestamp. The calculated DOP values were then compared to the GPS receiver data. The process of using real world data as inputs to the DOP tool and comparing the results to expected DOP values is validating the performance of the tool.

4.0 Results and Discussion

The PDOP and HDOP values from the GPS receiver were compared with the PDOP and HDOP results from the DOP tool.

Figure 3 shows the PDOP values collected by the GPS receiver and the DOP tool. The DOP tool used satellite range error of 7.8 meters (Ref. 4) and the positions of the satellites were calculated using orbit propagation tools. The DOP tool results were within 0.5 meters of the expected PDOP value throughout the whole 40 minute time period. The general pattern of the results of the DOP tools also tool closely match the data from the GPS receiver. This means that DOP tool is returning relatively accurate PDOP results. The specific algorithm that the GPS receiver used to calculate PDOP is also unknown. It could be possible that the GPS receiver is using time integration techniques in its calculations. The decrease of PDOP from time 14:03 to 14:30 in the GPS receiver data shows that the GPS receiver is performing different techniques than the DOP Tool. The difference in PDOP values could be attributed to the GPS receiver using different satellite range errors and the GPS receiver using different PDOP algorithms in its calculations.

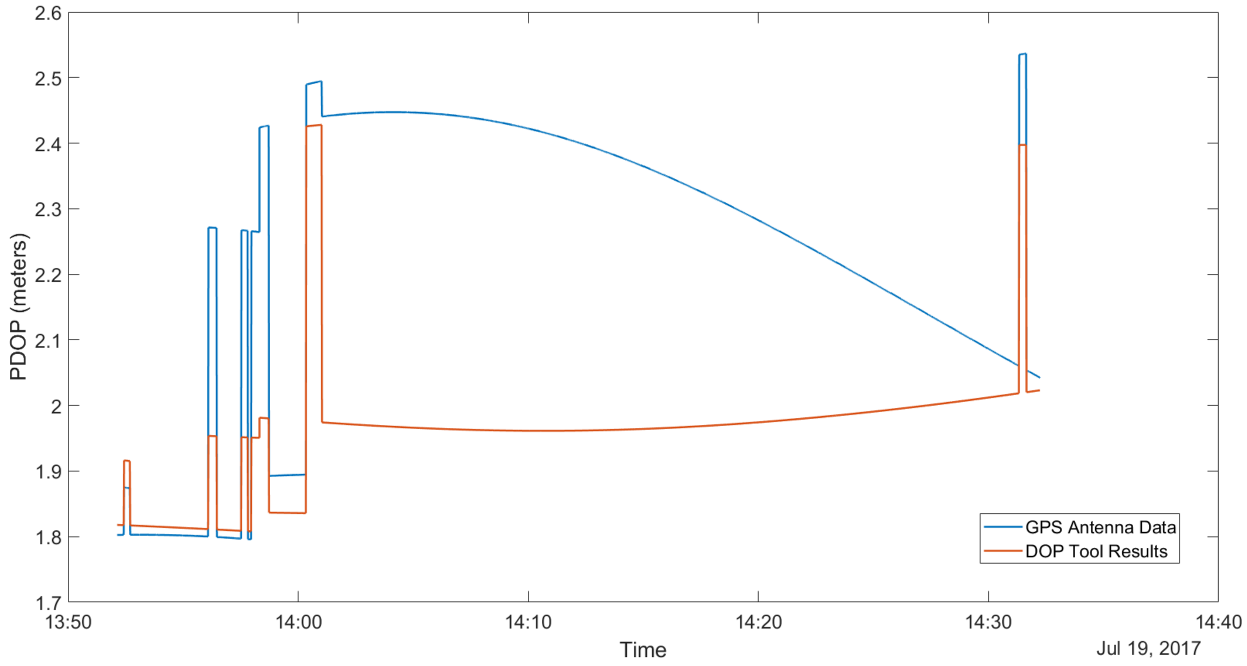

Figure 3.—Comparing PDOP between GPS antenna data and DOP tool results

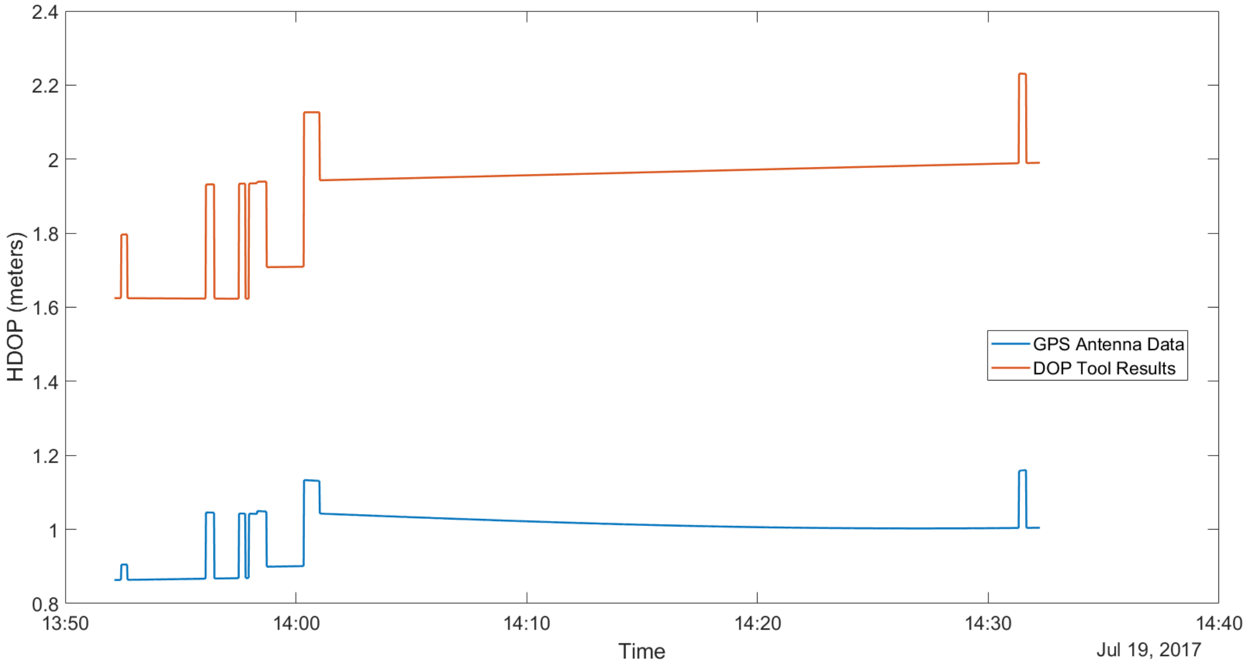

Figure 4.—Comparing HDOP between GPS antenna data and DOP tool results

Figure 4 shows the HDOP values collected by the GPS receiver and DOP tool. The differences in HDOP results is larger than the differences in PDOP results. The difference between HDOP values is about 0.8 throughout the 40 minute time period. The DOP tool consistently overestimated the HDOP values. The difference in HDOP values could be attributed to the GPS receiver using different satellite range errors and the GPS receiver using different HDOP algorithms in its calculations. More work can be done with fine tuning the DOP algorithms to more closely match the data from the GPS receiver. Adjustments in the satellite range error and methods of calculating DOP from the information matrices could decreases the differences in PDOP and HDOP.

5.0 Conclusions

The ability to perform DOP measurements is needed to analyze the effects of satellite geometry and time bias in current and future navigation systems. The DOP tool was developed to replace and augment the DOP capabilities used in STK and other commercial software. By using free and open-source tools such as ODTBX, SCaN saves money on license fees and gains the ability to add their own features and capabilities. The DOP tool was built using software verification and validation techniques, and includes a basic testing suite. This test suite was used to verify that the tool performed DOP calculations correctly and returned accurate results. The results of the DOP tool were compared with the data from the GPS receiver. The PDOP results were fairly accurate and these results show the algorithms used in the GPS receiver are slightly different than the algorithms used in the DOP tool. The HDOP results show a significant difference in the GPS receiver’s data and the results of the DOP tool.

Future work includes continuing verification and validation processes and integrating the DOP tool in the SCENIC UI. The DOP tool should be reviewed by multiple developers to confirm that the algorithms and calculations it is using are correct. Validation techniques should also be used to improve the accuracy of the tool's results. Currently, the DOP tool's results show that it is not using the same techniques as the GPS receiver in the PDOP and HDOP calculation. The DOP tool can be improved by modifying the algorithms used on the information matrices and using different range error values. In the future, this DOP tool will be integrated into the SCENIC UI and will be used with the other SCENIC UI tools to analyze current and future user missions.

References

1. Yuen, M. F., "Dilution of Precision (DOP) Calculation for Mission Planning Purposes," Mar. 2009.
2. Sands, Obed S., et al. "Dilution of Precision-Based Lunar Navigation Assessment for Dynamic Position Fixing," NASA/TM—2006-214247, 2006. Available https://ntrs.nasa.gov/
3. "Orbit Determination Toolbox (ODTBX)," *SourceForge.net* Available: http://odtbx.sourceforge.net/.
4. "Performance Standards and Specifications," *GPS.gov* Available: http://www.gps.gov/technical/ps/.