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# Influence of a Transverse Electric Field on the Alternating Currents Rectification Effect in Superstructures with Non-additive Energy Spectrum

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It is investigated the effect of mutual rectification of alternating currents, induced by an electric field of two uniformly polarized electromagnetic waves with different frequencies in two-dimensional superlattice with non-additive energy spectrum under the influence of a constant transverse electric field. The possibility of control of constant component of electric current (amplification, change of sign, suppression) by the transverse electric field is shown. The abilities of the practical use of the results are discussed.

**Keywords**: Mutual rectification of alternating currents, Coherent wave mixing, Stark-photon resonance.

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## 1. INTRODUCTION

Nonlinear phenomena in solid states are widely used for information processing (modulation and heterodyning), for spectroscopic measurements, frequency multiplication, creation of different types of detectors etc [1]. Bulk semiconductors begin to show their nonlinear properties in very strong fields, therefore use of artificial structures for production of the information processing devices is more promising [2-4]. Currently multilayer structures, in particular, semiconductor superlattices (SL) [5, 6] got wide spread. Semiconductor SL is a significantly non-linear media (see, for example, [7, 8]). One of the interesting and important effects is an appearance of a direct current under the influence on the material of only alternating fields – so-called mutual rectification effect [9-22]. The physical cause of a current rectification in such effects is the presence of certain preferred direction in material. This preferred direction appears under the influence of one of the waves (this wave usually can be considered as a strong) [9, 10]. Mutual enhancement effects of electromagnetic waves in nonlinear media [23-26], as well as the effects of the so-called coherent wave mixing [27-30] are very similar in physical nature to mutual rectification effects.

The effect of a direct current appearance in the semiconductor SL under the influence of two electromagnetic waves polarized along the superlattice axis, which frequencies $\omega_1$ and $\omega_2$ satisfy the relation $\omega_1 / \omega_2 = 1/2$, is investigated in [13]. Electric field strength is defined by the next expression:

$$E = E_1 \cos \omega t + E_2 \cos(2\omega t + \varphi)$$

Calculations of the direct current in [12] are performed on the base of approach of the constant relaxation time $\tau$. The expression of current density was obtained in linear approximation on the strength of the electric field $E_2$ with a higher frequency. The dependence of the current density on electric field strength $E_1$ is investigated. Particularly, in the case of weak fields this dependence has a form:

$$j \sim E_1^2 E_2 \cos \varphi .$$

Proportionality of the current to a square of amplitude of electric field of the wave with a lower frequency and linearity on the field strength of the wave with greater frequency in the case of weak fields is a feature of the effects of mutual rectification. Similar dependence was obtained before for bulk semiconductors [9-12, 14-17]. This effect is known as the photostimulated photovoltaic effect [9, 11, 12].

Recently, graphene and superlattices on the base of graphene attract interest of researchers in the field of nonlinear optical effects [31-37]. In [38] photothermoelectric detector on the base of the effect of two-wave mixing in graphene is offered. One of the features of the energy spectrum of charge carriers in graphene is its non-additivity, that allows the manifestation of the effects of many-wave mixing even in the case when polarization planes of electromagnetic waves are directed perpendicularly each other [36-38]. Furthermore, non-additivity of the electron spectrum leads to the fact that perturbation in the motion of the electron along one direction results in a change of its dynamics in the perpendicular direction too. It might be interesting for creation of detectors based on two-wave mixing with the control field which is applied perpendicularly to the polarization direction of the waves. And, perhaps, a superlattice with non-additive energy spectrum (see, for example, [40]) is more suitable on the role of such detector than graphene due to a more strong influence of the transverse field on the dynamics of charge carriers in the longitudinal direction.

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## 2. PROBLEM STATEMENT

Let us to consider a problem of the influence of a constant electric field on the current, generated by two electromagnetic waves, incident normally on two-dimensional SL with non-additive energy spectrum, with frequencies, differing twice, the electric field strength vectors of which are parallel. Problem geometry is presented on Fig. 1, where $E_1 = E_{10}\cos\omega t$, $E_2 = E_{20}\cos(2\omega t)$ are the components of strength of alternating electric fields, $E_0$ is a constant component of electric field. Energy spectrum of charge carriers can be written as

$$\varepsilon(p_x, p_y) = \Delta\left(1 - \cos\left(\frac{p_x d}{\hbar}\right)\cos\left(\frac{p_y d}{\hbar}\right)\right) \qquad (1)$$

where $d$ is a SL period, $\Delta$ is a half-width of allowed miniband. Note that materials with spectrum (1) and ways of that production are described in [40].

We shall solve this problem in semiclassical approximation, so it should be assumed $eE_{0,1,2}d,\ \hbar\omega \ll \Delta$. A current density is calculated in a standard way:

$$j_y = e\sum_p \mathrm{v}_y(\mathbf{p}) f(\mathbf{p}, t) \qquad (2)$$

where

$$\mathbf{v} = \frac{\partial\varepsilon(\mathbf{p})}{\partial\mathbf{p}} = \left\{\frac{\Delta d}{\hbar}\sin\frac{p_x d}{\hbar}\cos\frac{p_y d}{\hbar},\ \frac{\Delta d}{\hbar}\cos\frac{p_x d}{\hbar}\sin\frac{p_y d}{\hbar}\right\} \qquad (3)$$

Non-equilibrium electron distribution function $f(\mathbf{p}, t)$ is a solution of Boltzmann kinetic equation, which collision term we'll take in an approximation of a constant collision frequency $\nu$. Thus, Boltzmann equation has a form

$$\frac{\partial f(\mathbf{p}, t)}{\partial t} + e\mathbf{E}\frac{\partial f(\mathbf{p}, t)}{\partial\mathbf{p}} = -\nu\left[f(\mathbf{p}, t) - f_0(\mathbf{p})\right], \qquad (4)$$

where $\mathbf{E} = \mathbf{E}(t) = \{E_0, E_1 + E_2, 0\}$, $f_0(\mathbf{p})$ – is the equilibrium distribution function.

## 3. CALCULATION OF A CURRENT DENSITY

A solution of equation (4) can be presented as:

$$f(p, t) = \upsilon\int_{-\infty}^{t} dt' \exp(-\upsilon(t - t')) f_0(\mathbf{p}'(t'; \mathbf{p}, t)) \qquad (5)$$

where $\mathbf{p}'(t'; \mathbf{p}, t)$ is a solution of classical equation of motion of electron

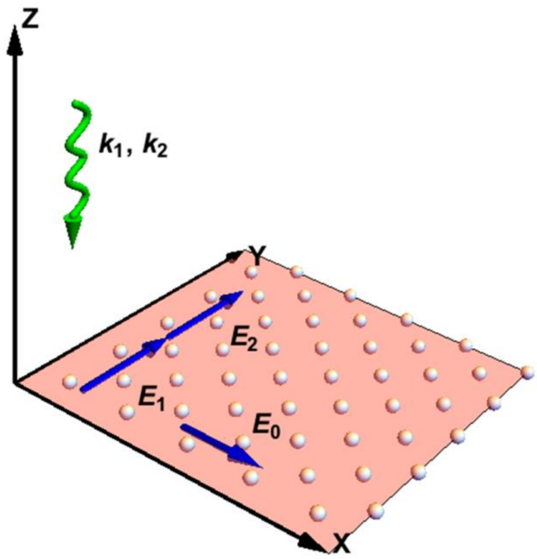


**Fig. 1** - Problem geometry

$$\frac{d\mathbf{p}'}{dt'} = e\mathbf{E}(t'), \qquad (6)$$

with initial condition $t' = t$, $\mathbf{p}' = \mathbf{p}$.

Substituting the solution of (6) in (2), (5) and taking $f_0(\mathbf{p})$ in a form of Boltzmann distribution function, we get next expression for current density:

$$j_y = j_0\left\langle\int_{-\infty}^{0} d\tau_1 \exp(\gamma\tau_1)\cos(\alpha_0\tau_1)\cdot\right.$$
$$\left.\cdot(\cos g_1 \sin g_2 - \sin g_1 \cos g_2)\right\rangle. \qquad (7)$$

Here $\gamma = \nu/\omega$, $\alpha_0 = eE_0 d/\hbar\omega$, $\alpha_1 = eE_{10}d/\hbar\omega$, $\alpha_2 = eE_{20}d/2\hbar\omega$, $g_1 = \alpha_1(\sin(\tau_1 + \tau) - \sin\tau)$, $g_2 = \alpha_2(\sin(2\tau_1 + 2\tau) - \sin 2\tau)$,

$$j_0 = en\frac{\Delta d\gamma}{\hbar}\cdot$$
$$\cdot\frac{\int_{-\pi}^{\pi} dp_x dp_y \cos\frac{p_x d}{\hbar}\cos\frac{p_y d}{\hbar}\exp\left(\frac{\Delta}{T}\cos\frac{p_x d}{\hbar}\cos\frac{p_y d}{\hbar}\right)}{\int_{-\pi}^{\pi} dp_x dp_y \exp\left(\frac{\Delta}{T}\cos\frac{p_x d}{\hbar}\cos\frac{p_y d}{\hbar}\right)}, \qquad (8)$$

$T$ – is a lattice temperature, expressed in energy values. Angle brackets in (7) mean the averaging on large in compare to period of waves time interval. After the expansion of factors like $\cos(\alpha\sin\tau)$ in Fourier series, averaging and integration on $\tau_1$ one obtains

$$j_y = j_0\sum_{n_1,n_2,n_3,n_4}^{\infty}\left[J_{2n_1}(\alpha_1)J_{2n_2}(\alpha_2)J_{2n_3}(\alpha_1)J_{2n_4+1}(\alpha_2)\delta_{2n_1+4n_2+2n_3+4n_4+2,0}I_1 +\right.$$
$$\left.+J_{2n_1}(\alpha_2)J_{2n_2+1}(\alpha_1)J_{2n_3+1}(\alpha_1)J_{2n_4+1}(\alpha_2)\delta_{4n_1+2n_2+2n_3+4n_4+4,0}I_2\right]. \qquad (8)$$

Here

$$I_1 = \frac{(-i\gamma + 4n_2 + 2n_3)(2 + 2n_3 - 2(2n_2 + n_3) + 4n_4)(\gamma + 2i(1 + n_3 + 2n_4))}{((\alpha_0 - i\gamma + 4n_2 + 2n_3)(\alpha_0 + i\gamma - 2(2n_2 + n_3))(2 + \alpha_3 - i\gamma + 2n_3 + 4n_4)(\alpha_0 + i\gamma - 2(1 + n_3 + 2n_4)))} +$$
$$+ \frac{\alpha_0^2(\gamma - i(-i\gamma + 4n_2 + 2n_3) + 2i(1 + n_3 + 2n_4))}{((\alpha_0 - i\gamma + 4n_2 + 2n_3)(\alpha_0 + i\gamma - 2(2n_2 + n_3))(2 + \alpha_3 - i\gamma + 2n_3 + 4n_4)(\alpha_0 + i\gamma - 2(1 + n_3 + 2n_4)))};$$

$$I_2 = \frac{(\alpha_0^2 + (\gamma + i(1 + 4n_1 + 2n_3))^2)(\gamma + i(3 + 2n_3 + 4n_4))}{((-1 + \alpha_0 + i\gamma - 4n_1 - 2n_3)(1 + \alpha_0 - i\gamma + 4n_1 + 2n_3)(-3 + \alpha_0 + i\gamma - 2n_3 - 4n_4)(3 + \alpha_0 - i\gamma + 2n_3 + 4n_4))} -$$
$$- \frac{(\gamma + i(1 + 4n_1 + 2n_3))(\alpha_0^2 + (\gamma + i(3 + 2n_3 + 4n_4))^2)}{((-1 + \alpha_0 + i\gamma - 4n_1 - 2n_3)(1 + \alpha_0 - i\gamma + 4n_1 + 2n_3)(-3 + \alpha_0 + i\gamma - 2n_3 - 4n_4)(3 + \alpha_0 - i\gamma + 2n_3 + 4n_4))}.$$

## 4. DISCUSSION OF THE RESULTS

Typical dependencies of $j_y$ on a strength of constant electric field $\alpha_0$ (in non-dimensional units) plotted by the formula (8) at different values of $\gamma$ are presented on Fig. 2.

From graphics on Fig. 2 one can see that at some values of $\alpha_0$ the current $j_y$ changes its sign. In general case of arbitrary values of $\alpha_1, \alpha_2$ the roots of equation $j_y(\alpha_0) = 0$ can be found numerically, but for $\alpha_1, \alpha_2 << 1$ they can be found analytically. After Taylor expansion of (8), taking into account only first non-vanishing terms, one obtains:

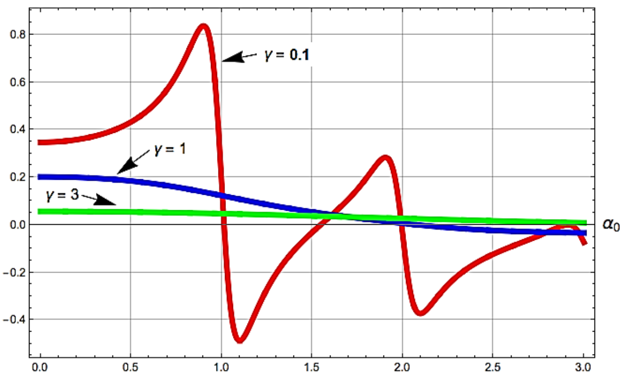


**Fig. 2** - Dependence of a current density $j_y$ on $\alpha_0$. All curves are plotted for $\alpha_1 = \alpha_2 = 1$

$$j_y = \frac{j_0 \alpha_1^2 \alpha_2}{2} \left( \frac{(\gamma^2 + 1 - \alpha_0^2)}{\alpha_0^4 + 2\alpha_0^2(\gamma^2 - 1) + (\gamma^2 + 1)^2} + \right.$$
$$\left. + \frac{(\gamma^2 + 4 - \alpha_0^2)}{\alpha_0^4 + 2\alpha_0^2(\gamma^2 - 4) + (\gamma^2 + 4)^2} \right) \quad (9)$$

Note, that the dependence of current density on amplitude of alternating fields in (9) has a form $j_y \sim \alpha_1^2 \alpha_2$ correspondingly a fact that in lowest on the amplitude of high-frequency fields approximation the rectified electric current is proportional to $E_2 E_1^2$ where $E_2$ is an amplitude of field with doubled frequency. This fact can be explained as follows. The dependence of current on the electric field strengths of high-frequency fields one can present in a form

$$j_y = \sum_{n,m=0}^{\infty} c_{n,m} E_{10}^n E_{20}^m \cos^n(\omega t) \cos^m(2\omega t), \quad (10)$$

where $c_{n,m}$ are some functions that depend on parameters of material and waves (expect electric field strengths of waves). This expression can be obtained by the expansion of (7) in Taylor series. After averaging of (10) the lowest powers of amplitudes of electric field strength, which gives non-zero contribution into current, are equal to $n = 1, m = 2$.

Solving the equation $j_y(\alpha_0) = 0$ where $j_y$ is defined by (9) one obtains the expression for its root in a form

$$\alpha_0 = \sqrt{\frac{5}{2} + \gamma^2}. \quad (11)$$

Note, that the current is zero for all values of alternating fields (of course, with the condition $\alpha_1, \alpha_2 << 1$). It is also important that in the case of high frequencies of waves ($\gamma \ll 1$) value of $\alpha_0$ at which $j_y = 0$ does not depend on any parameters of material and waves. Remarkable that the equality to zero of the current (9) is equitable with the same precision with which this formula was obtained (i.e. $\sim \alpha_1^2 \alpha_2$). However, the numerical analysis of the expression (8) confirms the conclusion about the existence of roots of the equation $j_y(\alpha_0)$ for arbitrary specified $\alpha_{1,2}$, so this result can be used in a compensation method of measuring of parameters of the material and the incident waves. Also it should be noted a resonant behavior of dependence $j_y(\alpha_0)$ at small values of $\gamma$, which can be used to enhance the effect of rectification at the two-wave mixing. This behavior is mathematically explained by the nature of the denominators in the expressions of $I_1, I_2$. The physical meaning of resonances is easiest to be traced in the case of $\alpha_1, \alpha_2 \ll 1$. Herewith the graph of dependence $j_y(\alpha_0)$ can be plotted by the formula (8). In fact, we are interested only in the dependence of the expression in brackets at (9) on $\alpha_0$. Let us to designate this bracket as

$$f = \frac{(\gamma^2 + 1 - \alpha_0^2)}{\alpha_0^4 + 2\alpha_0^2(\gamma^2 - 1) + (\gamma^2 + 1)^2} +$$
$$+ \frac{(\gamma^2 + 4 - \alpha_0^2)}{\alpha_0^4 + 2\alpha_0^2(\gamma^2 - 4) + (\gamma^2 + 4)^2}. \quad (12)$$

The dependence, plotted by formula (12) at $\gamma = 0.1$ is presented on Fig. 3.

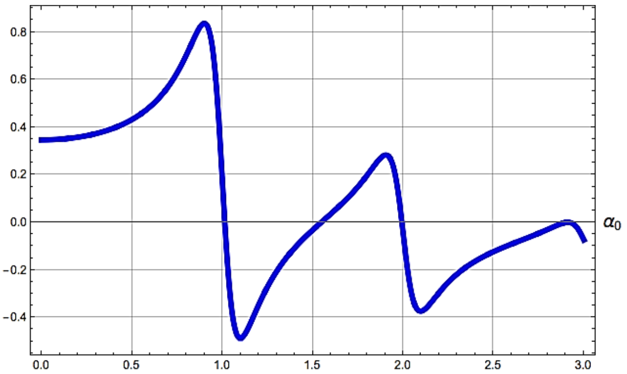


**Fig. 3** - Graph to illustrate the nature of the resonant behavior of the current

From the Fig. 3 one can see that in the dependence there are two local maxima shifted slightly to the left on the abscissa of the points $\alpha_0 = 1$ and $\alpha_0 = 2$. Minimums of denominators a placed at points $\alpha_0 = \sqrt{1-\gamma^2}$ and $\alpha_0 = \sqrt{4-\gamma^2}$ for first and second terms, respectively. At $\gamma = 0.1$ these points correspond to 0.995 и 1.998 which is close to the position of the maxima on the graphic. A slight discrepancy follows from the contribution made by the behavior of the numerators of each fractions in (12). Note that, when $\gamma \ge 1$, in the denominators in (12) only one minimum remains and a resonant behavior of the dependence $j_y(\alpha_0)$ disappears. In practice, this behavior disappears at $\gamma$ slightly less than 1 in connection with a strong broadening of the resonances. To clear a physical meaning of the resonances let us to present the expression for $\alpha_0$ in a form

$$\alpha_0 = \frac{eE_0 d}{\hbar\omega} = \frac{\Omega_{st}}{\omega} \quad (13)$$

The quantity $\Omega_{st} = eE_0 d/\hbar$ in (13) of the dimension of frequency is well known in the theory of quantum superlattices and is called the Stark frequency. From the quantum-mechanical point of view, a constant electric field is changing the spectrum of the carriers. It becomes equidistant so that the distance between the levels (also called Stark) equal to $\hbar\Omega_{st}$. Moreover, localization of the electron also depends on the level number, and the resonance condition $\Omega_{st} = n\omega$ means that the probability of transitions between levels of Stark ladder increases and at the same time the electron moves in the coordinate space, that is, it leads to an increase of the current. The resonance of this kind can be called Stark-photon and its analogs are already appeared in the solution of problems in which the superlattice was under the influence of both constant and alternating electric fields (see, for example, [41]). However, in contrast to the earlier work, where the fields were applied in the same direction, in the present problem a specificity of non-additive spectrum (1) is manifested. Non-additivity leads to mixing of transverse movements, resulting in a new type of resonance arise.

Let us, finally, to make numerical estimates. For typical values of material parameters ($n \approx 10^{10} cm^{-2}$, $\gamma \approx 0.1$, $\Delta \sim 0.01 eV$, $d \sim 10^{-6} cm$) $j_0 \approx 10^{-2} A/cm$. At these conditions semi-classical approximation can be applied at $\omega \sim 10^{12} s^{-1}$, $E_{0,10,20} \le 10$ $CGS$ units.

## 5. CONCLUSIONS

In a paper it is studied the influence of the transverse constant electric field on the effect of mutual rectification of alternating currents, induced by the field of two unidirectional electromagnetic waves of different frequencies in the two-dimensional superlattice with non-additive energy spectrum. It is demonstrated the possibility of controlling the rectified current by changing of the intensity of the constant transverse electric field. In particular,

1) It is possible to suppress the rectified current up to the vanishing and change its sign;

2) It is revealed a resonant type of the dependence of the rectified current on the constant field in the case of rare collisions between electrons and lattice irregularities. This resonance has a nature of the new Stark-photon resonance, a condition of existence of which contains the quantities determining the dynamics of carriers in mutually perpendicular directions. This resonant behavior of the current can be used to enhance the effect of the rectification at the two-wave mixing.


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## Воздействие поперечного электрического поля на эффект выпрямления переменных токов в сверхструктурах с неаддитивным энергетическим спектром

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Исследован эффект взаимного выпрямления переменного тока, индуцированного полем двух однонаправленных электромагнитных волн разной частоты в двумерной сверхрешетке с неаддитивным энергетическим спектром в условиях воздействия постоянного поперечного электрического поля. Показана возможность управления (усиления, смены знака, подавления) постоянной составляющей электрического тока поперечным полем. Обсуждается возможность практического использования результатов.

**Ключевые слова:** Взаимное выпрямление переменных токов, Когерентное смешивание волн, Штарк-фотонный резонанс

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