

Robust Aircraft Conceptual Design using AD in Matlab

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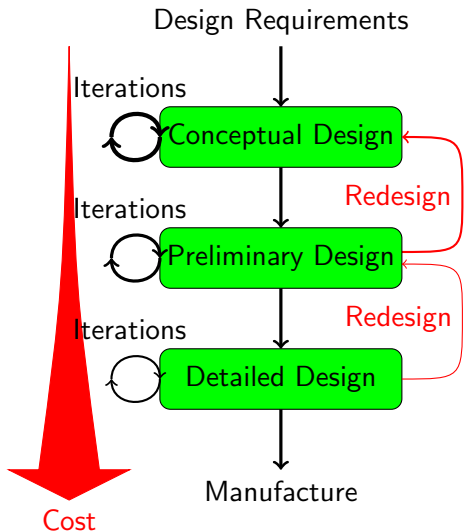
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Aeronautical Design



- **Design Requirements** - e.g., business jet, passengers 20, range 2000 km.
- **Conceptual Design** - rapid evaluation and optimisation of low fidelity models.
- **Preliminary Design** - define and validate design using CAD, CFD, FEA etc.
- **Detailed Design** - engineering data for tooling and manufacture.

Conceptual Design

- Design parameters e.g.,
 - ▶ A_{wing} - wing area
 - ▶ tuc - wing thickness-to-chord ratio.
 - ▶ N_{pax} - number of passengers
- Multidisciplinary, interacting components.
- e.g., Fuel capacity,

$$\text{Fuel} = (A * A_{wing} * tuc + B) * A_{wing} * tuc;$$

where A and B are propriety constants.

- Several hundred other algebraically simple relationships linking e.g., range to drag and fuel; lift to cruise altitude, speed, wing area etc.

Robust Design

- Seek designs with performance relatively insensitive to conceptual design.
- Reduces likelihood of expensive, large-scale design changes downstream and cycling between design phases [CA96].
- **Robust design** models the design parameters as taken from statistical distributions so increasing modelling complexity.

Robust Design (ctd.)

If the design task is to minimise some deterministic objective (e.g., fuel consumption) subject to multiple deterministic constraints (e.g., maximum wing span, range, ...) then we:

- 1 Assume design parameters are of known, independent, statistical distributions.
- 2 Estimate means and variances of the objective and constraints.
- 3 Form **robust objective** and **robust constraints** favouring designs with:
 - ▶ Low objective value and standard deviation.
 - ▶ High likelihood of satisfying deterministic constraints.
- 4 Solve numerically the robust optimisation problem [PLLH06].

Deterministic Design Optimization

Conceptual design analyses performed for:

- Objective functions $f(\mathbf{x})$.
- Constraint functions $g_i(\mathbf{x})$, $i = 1, 2, \dots, r$

with $\mathbf{x} \in \mathbb{R}^n$ the vector of the design variables.

Deterministic Design Optimization Problem

Find \mathbf{x} to minimise

$$y = f(\mathbf{x})$$

such that

$$g_i(\mathbf{x}) \leq 0, i = 1, 2, \dots, r, \quad \mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U. \quad (1)$$

Robust Design Optimisation

- \mathbf{x} 's components assumed stochastic and of known independent probability distributions with mean \mathbf{E}_x and variance \mathbf{V}_x .
- Simultaneously minimize objective's variance V_f and its expectation E_f via some robust objective $F(E_f(\mathbf{x}), V_f(\mathbf{x}))$.
- Ensure constraints, distributed with mean E_{g_i} and variance V_{g_i} , satisfied to some probabilistic satisfaction (e.g., 99% certainty) via robust constraint functions $G_i(E_{g_i}(\mathbf{x}), V_{g_i}(\mathbf{x}))$.

Robust Design Optimization Problem

$\min_{\mathbf{E}_x} F(E_f(\mathbf{x}), V_f(\mathbf{x})),$ such that:

$$G_i(E_{g_i}(\mathbf{x}), V_{g_i}(\mathbf{x})) \leq 0, i = 1, 2, \dots, r,$$

$$P(\mathbf{x}_L \leq \mathbf{E}_x \leq \mathbf{x}_U) \geq \mathbf{p}_{bounds},$$

where \mathbf{p}_{bounds} is the prescribed probability with which the mean of the design variables belongs to the original deterministic range.

Robust Design Optimisation (ctd.)

- If all variables are continuous,

$$E_f(\mathbf{x}) = \int_{-\infty}^{+\infty} f(\mathbf{t}) p_{\mathbf{x}}(\mathbf{t}) d\mathbf{t}$$

$$V_f(\mathbf{x}) = \int_{-\infty}^{+\infty} [f(\mathbf{t}) - E_f(\mathbf{x})]^2 p_{\mathbf{x}}(\mathbf{t}) d\mathbf{t},$$

in which $p_{\mathbf{x}}$ is \mathbf{x} 's joint probability density function.

- Closed-form expression for integrals of practical interest rarely exist.
- Numerical approximation techniques, termed **uncertainty propagation**, involve trade-offs between cost and accuracy.
 - ▶ **Monte Carlo** methods.
 - ▶ Taylor-based **method of moments**.
 - ▶ **Quadrature**-based techniques[PCG07].
 - ▶ **Polynomial chaos expansions**.

Method of Moments

- Statistical moments estimated from the moments of a truncated Taylor series expansion of y about the mean of \mathbf{x} .
- Third order approximation (IIIMM)

$$f(\mathbf{x}) =$$

$$\begin{aligned} & f(\mathbf{E}_x) + \sum_{p=1}^n \left(\frac{\partial f}{\partial x_p} \right) (x_p - E_{x_p}) \\ & + \frac{1}{2} \sum_{p=1}^n \sum_{q=1}^n \left(\frac{\partial^2 f}{\partial x_p \partial x_q} \right) (x_p - E_{x_p})(x_q - E_{x_q}) \\ & + \frac{1}{3!} \sum_{p=1}^n \sum_{q=1}^n \sum_{r=1}^n \left(\frac{\partial^3 f}{\partial x_p \partial x_q \partial x_r} \right) (x_p - E_{x_p})(x_q - E_{x_q})(x_r - E_{x_r}) \\ & + \dots \end{aligned}$$

Method of Moments - Mean Estimation

- Assume:

- ▶ The components of \mathbf{x} are independently distributed,

$$E((x_p - E_{x_p})(x_q - E_{x_q})) = 0 \text{ for } p \neq q,$$

- ▶ Each component x_p is symmetrically distributed about its mean E_{x_p} (zero skewness).

- Taking expectations and using $E(x_p) = E_{x_p}$, we obtain **third order Taylor estimate of mean** $E_{f_{\text{IIMM}}}$

$$E_{f_{\text{IIMM}}} = \overbrace{f(\mathbf{E}_x)}^{m_1} + \overbrace{\frac{1}{2} \sum_{p=1}^n \left(\frac{\partial^2 f}{\partial x_p^2} \right) V_{x_p}}^{m_2} + O(\mathbf{V}_x^2), \quad (2)$$

- Retaining just terms m_1 would give 1st order $E_{f_{\text{IMM}}}$ - 1st order accurate in variance.
- For symmetric distributions - no third order term for mean estimation.



Method of Moments - Variance Estimation

Similar approach for variance gives, with K_{x_p} the kurtosis of x_p .

$$\begin{aligned} V_{f_{\text{IIMM}}} &= \overbrace{\sum_{p=1}^n \left(\frac{\partial f}{\partial x_p} \right)^2}^{v_1} V_{x_p} \\ &+ \sum_{p=1}^n \left[\left(\frac{\partial^3 f}{\partial x_p^3} \right) \left(\frac{\partial f}{\partial x_p} \right) \frac{K_{x_p}}{3} + \left(\frac{\partial^2 f}{\partial x_p^2} \right)^2 \frac{K_{x_p} - 1}{4} \right] V_{x_p}^2 \\ &+ \sum_{p=1}^n \sum_{\substack{q=1 \\ q \neq p}}^n \left[\left(\frac{\partial^3 f}{\partial x_p^2 \partial x_q} \right) \left(\frac{\partial f}{\partial x_q} \right) \frac{1}{2} \left(\frac{\partial^2 f}{\partial x_p \partial x_q} \right)^2 \right] V_{x_p} V_{x_q}, \quad (3) \end{aligned}$$

which is second order accurate in variance.

Term v_1 alone gives first order approximation $V_{f_{\text{IIMM}}}$.

Sigma-Point (SP) Technique [PCG07]

- Based on numerical quadrature,

$$E_{f_{\text{SP}}} = W_0 f(\mathbf{x}_0) + \sum_{p=1}^n W_p [f(\mathbf{x}_{p^+}) + f(\mathbf{x}_{p^-})], \quad (4)$$

$$V_{f_{\text{SP}}} = \frac{1}{2} \sum_{p=1}^n \left\{ W_p [f(\mathbf{x}_{p^+}) - f(\mathbf{x}_{p^-})]^2 + (W_p - 2W_p^2) [f(\mathbf{x}_{p^+}) + f(\mathbf{x}_{p^-}) - 2f(\mathbf{x}_0)]^2 \right\}. \quad (5)$$

- Sampling points $\mathbf{x}_0 = \mathbf{E}_x$ and $\mathbf{x}_{p^\pm} = \mathbf{E}_x \pm \sqrt{V_{x_p} K_{x_p}} \mathbf{e}_p$.
- Weights are $W_0 = 1 - \sum_{p=1}^n \frac{1}{K_{x_p}}$ and $W_p = \frac{1}{2K_{x_p}}$.

Sigma-Point (SP) Technique (ctd.)

The SP technique:

- has a higher accuracy for the mean than IMM [PCG07].
- requires $2n + 1$ function evaluations for each analysis.
- Robust objective/constraints do not require derivatives of deterministic objective/constraints.
- Gradient of SP objective/constraints are weighted combination of deterministic objective/constraints.

AD of the Conceptual Design Package

- Aircraft conceptual design package was coded in Matlab so MAD[For06] package adopted.
- Forward mode AD:
 - ▶ Forward mode `fmad` class objects possess `value` and `deriv` components.
 - ▶ Overloading used to propagate derivatives, e.g., for $z = x \cdot y$,
`z.value = x.value.*y.value;`
`z.deriv = x.value.*y.deriv + y.value.*x.deriv;`
 - ▶ For single directional derivative all components are of Matlab's intrinsic class `double`.
- For multiple directional derivatives the highly optimised `derivvec` class is used to store and manipulate derivatives.

Calculating Higher Derivatives

- Rendered MAD's `fmad` and `derivvec` classes self-differentiable for forward-over-forward(-over-forward...) differentiation.
- Objects of `derivvec` class contain derivative components that themselves must be differentiated - add source code line, `superiorto('fmad')` to the `derivvec` class constructor.
- Ensures differentiation applied to components of `derivvec` objects.
- Calculate 2nd derivatives of $f(x)$ using:

```
xfmad = fmad(x,eye(length(x)));  
xfmad2 = fmad(xfmad,eye(length(x)));  
yfmad2 = f(xfmad2);  
y = getvalue(getvalue(yfmad2));  
Dy = getinternalderivs(getvalue(yfmad2));  
D2y=getinternalderivs(getinternalderivs(yfmad2));
```



Differentiating the Nonlinear Solve of `fsolve`

- Within our design model, some variables $\mathbf{w} \in \mathbb{R}^p$ are found in terms of some predecessors $\mathbf{v} \in \mathbb{R}^p$, as the solution of,

$$\mathbf{h}(\mathbf{w}, \mathbf{v}) = 0, \quad (6)$$

with nonlinear function $\mathbf{h} \in \mathbb{R}^p \times \mathbb{R}^p \rightarrow \mathbb{R}^p$ for some $p > 1$.

- Matlab's trust-region `fsolve` used to solve (6) using \mathbf{h} 's Jacobian.
- Our strategy to calculate derivatives of order d :
 - 1 Solve (6) for the value component of \mathbf{w} using a call to `fsolve` making use of only \mathbf{v} 's value
 - 2 Perform d Newton iterations,

$$\mathbf{w} \leftarrow \mathbf{w} - \left(\frac{\partial \mathbf{h}}{\partial \mathbf{w}}(\text{value}(\mathbf{w}), \text{value}(\mathbf{v})) \right)^{-1} \mathbf{h}(\mathbf{w}, \mathbf{v}), \quad (7)$$

with \mathbf{w} and \mathbf{v} manipulated as nested `fmad` objects storing derivatives up to and including those of order d .

- Jacobian matrix $\partial \mathbf{h} / \partial \mathbf{w}$ needed but not its higher derivatives.
- More efficient strategies not investigated ($p = 2$ in our test case).

Aircraft Sizing Test Case

- Robust optimization of a Matlab-implemented, industrially relevant, conceptual design test case.
- Determination of performance and sizing of a short-to-medium range commercial passenger aircraft.
- 96 sub-models and 126 variables.

Deterministic Problem

Deterministic Problem

- **Objective:** Minimize Maximum Take-Off Weight $MTOW$ with respect to the design variables \mathbf{x} (next slide).
- **Constraints:**
 - 1 Approach speed: $v_{app} < 120$ Kts $\Rightarrow g_1 = v_{app} - 120$;
 - 2 Take-off field length: $TOFL < 2000$ m $\Rightarrow g_2 = TOFL - 2000$;
 - 3 Percentage of total fuel stored in wing tanks: $K_F > 0.75$
 $\Rightarrow g_3 = 0.75 - K_F$;
 - 4 Percentage of sea-level thrust available during cruise: $K_T < 1$
 $\Rightarrow g_4 = K_T - 1$;
 - 5 Climb speed: $v_{zclimb} > 500$ ft/min $\Rightarrow g_5 = 500 - v_{zclimb}$;
 - 6 Range: $R > 5800$ Km $\Rightarrow g_6 = 5800 - R$.

Design Variables

Design Variable	Definition [units]	Bounds [min,max]
S	Wing area [m^2]	[140, 180]
BPR	Engine bypass ratio []	[5, 9]
b	Wing span [m]	[30, 40]
Λ	Wing sweep [deg]	[20, 30]
t/c	Wing thickness to chord ratio []	[0.07, 0.12]
T_{esL}	Engine sea level thrust [kN]	[100, 150]
FW	Fuel weight [Kg]	[12000, 20000]

Fixed Parameters

Parameter	Value
Number of passengers	150
Number of engines	2
Cruise Mach number	0.75
altitude [ft]	31000

Robust Optimisation

- Assume \mathbf{x} are independent Gaussian variables with $\mathbf{V}_{x_i}^{1/2} = 0.07\mathbf{E}_{x_i}$.
- Robust objective is taken as

$$MTOW_{\text{rob}} = E_{MTOW} + V_{MTOW}^{1/2}$$

- Constraints take the form,

$$G_i(\mathbf{x}) = E_{g_i}(\mathbf{x}) + kV_{g_i}^{1/2}(\mathbf{x}) \leq 0,$$

and

$$\mathbf{x}_L + k\mathbf{V}_x^{1/2} \leq \mathbf{E}_x \leq \mathbf{x}_U - k\mathbf{V}_x^{1/2}.$$

Coefficient $k = 1$ enforces constraint satisfaction with probability of about 84.1%.

Two different techniques for mean and variance

① **First-order method-of-moments (IMM):**

- ▶ MAD used to calculate first derivatives of deterministic objective and constraint to approximate E_{f,g_i} and V_{f,g_i} .
- ▶ MAD used to calculate second derivatives to form gradients $\nabla E_{f,g_i}$ and $\nabla V_{f,g_i}$ for optimisation.

② **Sigma-point (SP) Technique:**

- ▶ MAD is used to calculate the gradient of objectives $E_{f_{SP},g_{iSP}}$ and $V_{f_{SP},g_{iSP}}$ constraints for optimisation.

Both optimised using Matlab's gradient-based constrained [fmincon](#).

Results - Design Variables

Design variable	I MM	SP
$E_S [m^2]$	160.843	162.558
$E_{BPR} []$	8.580	8.580
$E_b [m]$	37.753	37.753
$E_\Lambda [deg]$	21.531	21.531
$E_{t/c} []$	0.095	0.094
$E_{T_{eSL}} [kN]$	122.553	123.224
$E_{FW} [Kg]$	18084.940	18171.282

Results - Objective and Constraints

		I MM	SP
Objective			
Max. take-off weight	$MTOW_{rob}$ [Kg]	86023.272	86207.016
Constraints			
Approach speed	G_1 [Kts]	0.000	0.000
Take-off field length	G_2 [m]	-161.568	-151.806
% fuel in wings	G_3 []	0.000	0.000
% thrust for cruise	G_4 []	-0.114	-0.107
Climb speed	G_5 [ft/min]	0.000	0.000
Range	G_6 [Km]	0.000	0.000

Post-Optimality Analysis

Relative error compared to Monte Carlo estimates of mean and variance at each optima.

	% Error Mean		% Error Variance	
	IMM	SP	IMM	SP
Objective <i>MTOW_{rob}</i>	-0.83×10^{-4}	0.27×10^{-6}	0.41	0.39
Constraint				
G_1	-0.12	0.25×10^{-3}	-0.71	0.24
G_2	-0.88	0.17×10^{-2}	-1.89	-0.31
G_3	-0.86	0.17×10^{-1}	-1.27	-0.48
G_4	-0.87	0.12×10^{-2}	-2.18	0.17
G_5	1.15	0.10×10^{-3}	0.23	0.73
G_6	0.23	-0.11×10^{-2}	0.56	0.01

Post-Optimality Analysis (ctd.)

- SP method attains increased accuracy in the mean estimate compared to IMM benefiting design optimisation (lower truncation error).
- Results from IIIMM analysis at the optimal design points are comparable to those obtained by the SP method (cross-derivatives are negligible [PCG07]).
- IIIMM more accurate than other methods and can be adopted to reduce cost of the post-optimality analysis:
 - ▶ IIIMM c.p.u. time ≈ 8 s.
 - ▶ MCS c.p.u. time ≈ 133 s.

Effect of using Finite-Differencing for Gradients

	SP		IMM	
	Iterations	c.p.u. time [s]	Iterations	c.p.u. time [s]
AD	10	351	10	45
FD	10	708	24	1212

Conclusions

- Performed robust optimisations of an industrially relevant, Matlab-implemented aircraft sizing problem using the AD tool MAD.
- Two robust design strategies:
 - ▶ **First Order Method of Moments (IMM)** - robust objective and constraints use AD-obtained first order derivatives; second order derivatives used for gradients.
 - ▶ **Sigma Point Method (SP)** - reduced quadrature for robust objective and constraints; AD for their gradients.
- For test case considered, a Monte-Carlo post-optimality analysis indicates that SP more accurate for estimation of the mean but IMM more efficient (with AD gradients).
- In both cases AD gradients significantly reduced optimisation c.p.u. time compared to finite-differencing.
- **AD may benefit robust optimisation for aircraft conceptual design.**

Further Work

- To date all Method of Moments mean and variance estimates built up from AD gradients by hand-coding - Marina Menshikova (poster) is developing an easy to use interface for arbitrary order Taylor estimates of arbitrary statistical moments.
- Closer coupling of the optimization algorithms with the uncertainty propagation methods.
- Integration with suitable visualization techniques.

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