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Estimating the Variability of Tillage Forces on a Chisel Plough Shank by Modeling the Variability of Tillage System Parameters

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Abstract

In this paper, a probabilistic approach is proposed for quantifying the variability of the tillage forces for the shank of a chisel plough with narrow tines and to estimate the failure probability. An existing three-dimensional analytical model of tool forces from McKyes was used to model the interaction between the tillage tools and the soil. The variability of tillage forces was modeled, taking into account the variability of soil engineering properties, tool design parameters and operational conditions. The variability of the soil engineering properties was modeled by means of experimental observations. The dispersion effect of each tillage system parameter on the tillage forces was determined by a sensitivity analysis. The results show that the variability of the horizontal and vertical forces follows a lognormal distribution ($(\mu = 0.872, \xi = 0.449)$; $(\mu = 0.004, \xi = 0.447)$) and the relationship between these forces is positive and quasi-linear ($\rho(P_H, P_V) = 0.93$). This lognormal variability was integrated into the estimation of the failure probability for the shank by using Monte Carlo simulation (MCS) and the first-order reliability method (FORM). The results obtained by these two methods, with the assumption of non-correlation between the horizontal and vertical forces, were almost identical. However, the FORM method was faster and simpler, compared to the MCS technique. Furthermore, the correlation between the horizontal and vertical forces has no significant effect on the failure probability, regardless of the correlation strength. Therefore, it is concluded that the FORM method can be used to estimate the failure probability without considering the correlation between horizontal and vertical forces.

Keywords: soil engineering properties; tillage forces; failure probability; chisel plough.

1. Introduction

Accurate prediction of the forces of tillage implements is of great value to both implement designers and farmers (Desbiolles et al., 1997). There are many available soil cutting models that can be used to predict the forces acting on a tillage tool (Zhang and Kushwaha, 1995). Analytical and numerical modeling methods are differently used approaches to achieve this goal. In the analytical methods, soil-tool forces are considered as functions of three categories of variables, namely soil engineering properties, tool design parameters and operational conditions. Soil engineering properties are conventionally considered to be constant, reflecting a homogeneous soil profile, and tillage forces are calculated for assigned tool design parameters and operational conditions (Godwin, 2007; Godwin and O'Dogherty, 2007; Godwin et al., 2007). When numerical methods, such as the finite element method (FEM), are adopted to model the soil-tillage tool interaction, two different theoretical approaches can be introduced, namely, the curve-fitting technique and the elastic-perfectly plastic assumption (Mouazen and Neményi, 1998). The elastic-perfectly plastic method considers Young's modulus of elasticity and Poisson's ratio as constants, while the curve-fitting method only accounts for a variable Young’s modulus as a function of load history (Chi and Kushwaha, 1991). For both of these FEM methods, the soil is treated as a homogeneous body during the FEM analysis, with very few exceptions. Mouazen and Neményi (1999) developed a three-dimensional FEM model for cutting non-homogeneous (vertically) sandy loam soil by a subsoiler. The non-homogeneity in the soil was proposed to simulate the differences in soil strength among different soil layers. However, they considered Young's modulus of elasticity and Poisson's ratio as constant in the FEM analysis. Moreover, Fielke (1999) studied the effect of a variable Poisson's ratio on tillage forces and soil movement around the cutting edge. Recently, the discrete element method (DEM) has been used to model the interactions between the soil and tillage tools. This method is based on a promising approach for constructing a high-fidelity model to describe the soil-tillage tool interaction (Shmulevich, 2010). However, the determination of model parameters to control the soil void ratio and the shape of particles, as well as, the modeling of breakage and the formation of agglomerates are still of great challenges and limit the DEM application for practical engineering problems.

In reality, soil is neither a continuous nor a homogeneous mass, but a three-phase medium consisting of solid, liquid and gaseous particles (Klenin et al., 1985; McKyes, 1989). Consequently, soil engineering properties are variable in both vertical and horizontal directions (Kai et al., 2007). Estimating tillage forces using analytical or numerical methods with the assumption that soil engineering properties are constant does not reflect the nature of soil. Therefore, a new approach is needed for quantifying the variability of tillage forces due to variability of tillage system parameters that is associated with probability of soil failure.

This study aims to overcome the drawbacks of classical design approaches, by explicitly taking into account the variability of design variables, and to calculate the failure probability for passive tillage tools. The objectives of this work are to: 1) propose a method for accurate modeling the variability of soil engineering properties (soil weight density, cohesion, internal friction angle, soil-tool friction angle and soil-tool adhesion), 2) develop a simple method for determining the dispersion effects of soil engineering properties, tool design parameters (tool width and rake angle) and operational conditions (tool working depth, surcharge pressure and tool speed) on tillage forces, 3) propose a methodology for quantifying the variability of tillage forces and 4) estimate the failure probability using the Monte Carlo simulation (MCS) technique and the first-order reliability method (FORM).

3. Materials and methods

3.1 Estimating tillage forces

McKyes and Ali's model (1977) was used to estimate the forces acting on a tillage tool. We selected this model because it is simple and accurate (Zhang and Kushwaha 1995), and has shown good agreement with experimental results, especially at low speeds (Grisso and Perumpral, 1985). For improving the estimation of tillage forces, the effects of soil-tool adhesion (McKyes, 1985) and tool speed (Onwualu and Watts, 1998) were taken into account. The total force can be written according to the general earth pressure model as:

$$P = \left(\gamma d^2 N_\gamma + \ c d N_c + c_a d N_{ca} + \ q d N_q + \ \gamma v^2 d N_a\right) w \tag{1}$$

where P is the total force in kN, γ is the soil specific weight in $\mathrm{kN.m^{-3}}$, d is the tool working depth in m, N_γ is the gravity coefficient (dimensionless), c is the soil cohesion in kPa, N_c is the cohesion coefficient (dimensionless), c_a is the soil-tool adhesion in kPa, N_{ca} is the adhesion coefficient (dimensionless), q is the surface surcharge pressure in kPa, N_q is the surcharge pressure coefficient (dimensionless), v is the tool speed in $\mathrm{m.s^{-1}}$, N_a is the inertial coefficient (dimensionless) and w is the tool width in m.

Dimensionless coefficients $(N_\gamma, N_c, N_{ca}, N_q, N_a)$ can be determined with respect to the soil failure pattern proposed by McKyes and Ali (1979), and a simplified form of the total force can be given by Equation (2):

$$P = \left[\frac{1}{2}\gamma r\left(1+\frac{2s}{3w}\right) + c\left(1+\frac{s}{w}\right)\frac{\cos(\phi)}{\sin(\beta_r)\sin(\beta_r+\phi)} - c_a\frac{\cos(\alpha+\beta_r+\phi)}{\sin(\alpha)\sin(\beta_r+\phi)} + q\left(1+\frac{s}{w}\right)\frac{r}{d} + \gamma v^2\left(1+\frac{s}{w}\right)\left(\tan(\alpha)+\frac{\cot(\beta_r+\phi)}{\tan(\beta_r)\cot(\alpha)}\right)\right]\frac{dw}{\cos(\alpha+\delta)+\sin(\alpha+\delta)\cot(\beta_r+\phi)} \tag{2}$$

where r is the distance from the tool to the forward failure plan in m, s is the width of the side crescent in m, ϕ is the angle of internal friction in deg, β_r is the rupture angle in deg, α is the rake angle of the tool from the horizontal in deg and δ is the angle of soil-tool friction in deg.

Furthermore, the width of the side crescent was calculated using an empirical regression equation recommended by Kuczewski and Piotrowska (1998), and the rupture angle β_r was obtained by minimizing the total force (Grisso et al., 1980; Zhang and Kushwaha, 1995). The horizontal and vertical forces were calculated using the following two equations, respectively (McKyes, 1985):

$$P_H = P\sin(\alpha+\delta) + c_a dw\cot(\alpha) \tag{3}$$

$$P_V = P\cos(\alpha+\delta) - c_a dw \tag{4}$$

where P_H is the horizontal force in kN and P_V is the vertical force in kN.

According to Equations (2), (3) and (4), the tillage system parameters considered for the calculation of the horizontal and vertical forces can be grouped into three main categories: soil engineering properties, tool design parameters and operational conditions.

3.2 Modeling the variability of tillage system parameters

Over the years, many methods and techniques have been developed for modeling the variability of a random variable depending on the number of data points and assumptions about the shape of the underlying distribution (Siegel and Castellan, 1988; Nikolaidis et al., 2005). In this work, a combination of graphical and quantitative techniques for modeling the variability of soil engineering properties is proposed. An illustration of these techniques is shown in Fig. (1). This approach provides an accurate estimation for the variability of soil engineering properties and allows one to select the best probability distributions that can simulate the variability of these properties. An empirical relationship for determining the number of intervals of the histograms of soil engineering properties was used (Haldar and Mahadevan, 2000a). Two statistical tests were implemented for selecting the probability distributions of these properties, namely the chi-square test and the Kolmogorov-Smirnov test (Ang and Tang, 1975). A total of 57 variations of soil engineering properties, representing 57 different soil samples collected from the literature (Appendix I), were considered for implementing our mixed technique approach (Abo Al Kheer et al., 2007). These data represent different soil texture types, namely sandy loam, clay loam, sandy clay loam, clay, and sand.

The variability of tool design parameters and operational conditions were modeled after proposing the following two assumption: 1) the tool width and rake angle have uniform distributions with lower and upper bounds, based on the manufacturing accuracy and 2) the tool working depth, surcharge pressure and tool speed have normal distributions with standard deviations equal to 5% of their mean values. Usually, a uniform distribution is used to model the variability associated with manufacturing processes, and a normal distribution is used to model the variability of a random variable when no data are available (Haldar and Mahadevan, 2000a; Fox, 2005).

3.3 Sensitivity analysis

Sensitivity analysis aims at studying the relationships between the output and input variables. Differential sensitivity analysis is considered to be the most commonly employed method in sensitivity analysis (Irving, 1992). This method deals with local sensitivity analysis by focusing on the evaluation of the partial derivatives $\partial f/\partial y$ of the function *f*. Many approximation methods are used to calculate the partial derivatives of f. Forward, backward and central differences are the three most common forms. The central difference method requires more computing time, but it yields a more accurate approximation. Therefore, this method was used in this work to calculate the partial derivatives of the horizontal and vertical forces for the mean values of the tillage system parameters, and for a constant change equal to $\Delta y_i = 0.001 y_i$ where y_i is a tillage system parameter.

However, differential sensitivity analysis leads to a local sensitivity analysis at mean values of the input random variables and does not take into account the dispersion effects of these variables. Therefore, we propose a new sensitivity method to overcome this limitation. The main advantage of the proposed method is its simplicity, compared to other available methods, such as the variance-based sensitivity. Its main drawback is that it cannot take into account correlations between random variables. However, the proposed method provides more accurate estimations for the dispersion effects of tillage system parameters than the classical differential sensitivity methods.

This method, which consists of two main steps, is shown in Fig. (2). In the first step, the confidence interval bounds (y_{max}, y_{min}) were computed for each tillage system parameter according to Equation (5) and (6). The values of the confidence interval bounds depend on the probabilistic characteristics (distribution type and distribution parameters) of each parameter. The higher the dispersion of a parameter, the greater the difference between the confidence interval bounds.

$$\mathbf{P_r}[y \leq y_{min}] = \alpha/2 \quad (5)$$

$$\mathbf{P_r}[y \geq y_{max}] = 1 - \alpha/2 \quad (6)$$

where $\mathbf{P_r}[\cdot]$ is the probability operator, y_{max} is the upper confidence interval bound, y_{min} is the lower confidence interval bound and $100(1-\alpha)\%$ represents the confidence interval.

In the second step, the differences between the maximum and minimum values of the tillage forces were calculated in the confidence interval of each tillage system parameter. These differences indicate the dispersion effects of the tillage system parameters on the tillage forces. The greater the difference between the maximum and minimum values of the tillage forces, the greater the influence of the variability of the tillage system parameters on the tillage forces.

The relationships between the tillage forces and the tillage system parameters show that $P_H(\mathbf{y}_i)$ and $P_V(\mathbf{y}_i)$ are either increasing or decreasing functions (Appendix II). Therefore, the dispersion effects of the tillage system parameters were estimated by computing the differences between the tillage forces at the maximum and minimum value for each tillage system parameter (y_{max}, y_{min}). The confidence interval was selected to be 95%. For the bounded probability distributions (uniform distribution …), y_{max} and y_{min} represent the two limits of the random variable.

3.4 Quantifying the variability of tillage forces

We propose a methodology, shown in Fig. (3), for quantifying the variability of tillage forces based on the MCS technique. The number of generated values (*n*) was chosen to obtain an accurate correlation coefficient between the horizontal and vertical forces. This methodology consists of the following steps:

1- Generate *n* values for each tillage system parameter according to its probabilistic characteristics.

2- Compute the total force P according to Equation (2) for different values of $\beta_r(\beta_r \in [0-90^{\circ}])$, for the set of tillage system parameters obtained in step 1. This is followed by the selection of the minimum value of P to respect the passive earth pressure theory and the corresponding value of β_r.

3- Calculate the horizontal and vertical forces according to Equations (3) and (4), respectively.

4- Repeat Steps 1, 2 and 3 for each set of tillage system parameters.

5- Calculate the mean and variance values for the horizontal and vertical forces, and then apply the goodness-of-fit tests to select the distribution that can best model the variability of these forces.

6- Compute the correlation coefficient between the horizontal and vertical forces, required to calculate the failure probability, according to Equation (7).

$$\mathrm{Corr}(P_H, P_V) = \rho(P_H, P_V) = \frac{\mathrm{Cov}(P_H, P_V)}{\sqrt{\mathrm{Var}(P_V)\mathrm{Var}(P_V)}} \qquad (7)$$

where $\mathrm{Corr}(P_H, P_V)$ is the correlation coefficient between the horizontal and vertical forces, $\mathrm{Cov}(P_H, P_V)$is the covariance between the two forces and $\mathrm{Var}(\cdot)$ is the variance of a random variable.

These steps were applied to quantify the variability of tillage forces for the shank of a chisel plough, as shown in Fig. (4). In fact, the relative positions of tines on a tool frame both laterally and in the direction of motion have a significant effect on tine forces (Godwin and O'Dogherty, 2007). For simplicity, the variability of tillage forces for only one shank was quantified, without considering the effects of tine interactions.

3.5 Failure probability

Failure probability is always associated with a particular performance criterion that defines a certain limit state function $\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\}) = 0$ in physical space, where $\{\mathbf{x}\}$ is a vector of deterministic variables and $\{\mathbf{y}\}$ is a vector of random variables. The limit state function represents the surface between the safe region $\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\}) > 0$ and the failure region $\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\}) < 0$. Conventionally, failure probability can be calculated by using the following integral:

$$P_f = \mathbf{P_r}[\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\}) < 0] = \int \cdots \int_{\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\})<0} f_{\{\mathbf{y}\}}\left(\mathbf{y}_1,\cdots,\mathbf{y}_\mathrm{n}\right)\mathrm{d}\mathbf{y}_1\cdots\mathrm{d}\mathbf{y}_\mathrm{n} \qquad (8)$$

where P_f is the failure probability, $f_{\{\mathbf{y}\}}(\mathbf{y}_1,\cdots,\mathbf{y}_\mathrm{n})$ is the joint probability density function for the random variables $\{\mathbf{y}\}$ and $\mathbf{P_r}[.]$ is the probability operator when the integral is performed over the failure region $\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\}) < 0$.

In general, evaluating the integral in Equation (8) is not simple because it represents a very small quantity and all of the necessary information for the joint density function is not available. Even if this information is available, evaluating the multiple integral is extremely complicated (Haldar and Mahadevan, 2000b; Radi and El Hami, 2007). Therefore, several analytical approximations of this integral are used to evaluate failure probability, namely, the FORM and the second-order reliability method (SORM), which are considered to be reliable computational methods (Zhao and Ono, 1999; Kharmanda et al. 2004). These methods are based on the determination of the design point P^* and a calculation of the reliability index β in normalized space (Fig. 5). The design point P^*, also called the most probable point of failure, represents the worst combination of the random variables and the search of design point in normalized space is an optimization problem. The reliability index β is the minimum distance from the origin of the axes in the normalized space to the limit state surface. The failure probability can be calculated, according to the FORM method by $P_{f\,FORM} = \Phi(-\beta)$, where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal distribution. However, analytical approximation methods require a background in probability and statistics. Other simulation techniques can be used to evaluate failure probability with only a minimal background in probability and statistics, but these methods require more computing time as compared to analytical approximation ones. The method commonly used for this purpose is the MCS technique.

Correlations between some or all random variables $\{\mathbf{y}\}$ in the limit state function $\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\})$ may modify the failure probability of a structure (Haldar and Mahadevan, 2000b). To estimate the failure probability when taking into account correlations between random variables, the correlated random variables should be converted into non-correlated normalized variables, and the original limit state function, which is expressed in terms of correlated random variables, must be rewritten in terms of non-correlated normalized variables. Two transformations were used for this purpose (Der Kiureghian and Liu, 1986; Liu and Der Kiureghian, 1986). The first one transforms the correlated random variables to correlate reduced variables and the second one transforms the correlated reduced variables to uncorrelated reduced variables.

A structure should be designed so that its strength is greater than the effects of the applied forces. Therefore, the limit state function of the studied shank, shown in Fig. (6), can be written analytically as:

$$\mathrm{G}(\{\mathbf{x}\},\{\mathbf{y}\}) = \sigma_{ad} - \frac{6}{bh^2}\left[(L_2 + L_4)P_H + \frac{L_4}{\tan(\alpha)}P_V\right] + \frac{1}{bh}P_H \geq 0 \qquad (9)$$

where σ_{ad} is the allowable stress in MPa, b and h are the dimensions of a shank section in mm, L_2 is the shank length in mm, P_H and P_V are the horizontal and vertical forces in kN, L_4 is the distance from the horizontal force to the tool side in mm and α is the rake angle in deg.

The same method used in Section **3**.3 was used here to determine the dispersion effects of the input random variables $(\sigma_{ad}, P_H, P_V, b, h, L_2, L_4, \alpha)$ on the limit state function. The allowable stress was considered as constant ($\sigma_{ad} = 235$ MPa). The probability distributions of b, h and L_2 were defined as uniform distributions with lower and upper bounds, based on the manufacturing accuracy, of ± 0.1mm. We assume that L_4 has a normal distribution with a coefficient of variation equal to 0.05. The variability of the rake angle was considered during the modeling of the variability of tillage forces (Section **3**.4), so it is considered here as a deterministic variable.

The results obtained from the sensitivity analysis study show that only L_2 and α can be considered as deterministic variables, so the other variables were taken as random variables during the reliability analysis. The vectors of deterministic and random variables are given by Equations (10) and (11).

$$\{\mathbf{x}\} = (\sigma_{ad}, L_4, \alpha) \tag{10}$$

$$\{\mathbf{y}\} = (P_H, P_V, b, h, L_2) \tag{11}$$

To evaluate failure probability for the studied shank, one million simulation cycles were used to perform the MCS technique. Meanwhile, the sequential quadratic programming (SQP) algorithm was used to determine the design point and to compute the reliability index according to the FORM method.

4. Results and discussion

4.1 Probabilistic characteristics of soil engineering properties

Histograms and PDFs of soil engineering properties are shown in Fig. (7), and their probabilistic characteristics are given in Table (1). It is worth noting that the soil engineering properties do not have the same probability distributions and that only the internal friction angle has a normal distribution. In addition, it is noted that the histogram shapes are non-homogeneous, particularly the histograms of the external friction angle and soil-tool adhesion. This is most likely due to the following: 1) an insufficient sample size is considered in this work, 2) the samples are not representatives of real soil textures or 3) there are inter-correlations between the soil engineering properties. However, from a statistical point of view, 57 samples are sufficient to model the variability of a random variable. As mentioned in the report of Fox (2005), a set of 25 samples or more is sufficient to obtain an accurate estimation of the variability of a random variable. In order to improve the estimation of the variability of soil engineering properties, a larger number of samples should be employed and the inter-correlations between these properties should be investigated.

4.2 Effects of the variability of tillage system parameters on tillage forces

The effects of the variability of soil engineering properties, tool design parameters and operational conditions on tillage forces, using differential sensitivity analysis and the proposed method, are shown in Table (2). According to the results of differential sensitivity analysis, we observe that the influence of the variability of the rake angle on the horizontal force is larger than the influence of the variability of the other variables, whereas the vertical force is most influenced by the variability of the internal friction angle. The influences of the variability of soil-tool adhesion and surcharge pressure are very small as compared to the influences of the variability of the other variables. These results are in agreement with many works reported in the literature (McKyes and Ali, 1977; Godwin and O'Dogherty, 2007).

In contrast, the proposed method shows that the effect of the variability of soil cohesion on both the vertical and horizontal forces is the largest as compared with the effects of the variability of the other variables. This is caused by the high dispersion of the soil-tool adhesion values around the mean value. Furthermore, only the variability of the surcharge pressure has no significant effect on either the horizontal or vertical forces. We conclude that only the surcharge pressure can be considered as a deterministic variable and the variability of the soil-tool adhesion and the other variables must be integrated into the probabilistic analysis of tillage forces.

4.3Quantifying the variability of tillage forces for the shank of a chisel plough:

Histograms and PDFs of the horizontal and vertical forces are shown in Fig. (8). The probabilistic characteristics of these forces are presented in Table (3). From a statistical viewpoint, these results are in accord with the central limit theorem (Ang and Tang, 1975). The majority of the horizontal and vertical force values are found to range between $0.5 \text{ and } 6 \text{ kN}$ and between $0.2 \text{ and } 3 \text{ kN}$, respectively. The shape parameters of the horizontal and vertical forces are $\xi = 0.449$, $\xi = 0.447$, respectively. This means that the dispersions of these forces are very important and should be taken into consideration in the reliability analysis. Furthermore, the horizontal and vertical force values were positive for each set of tillage system parameters. In fact, the vertical force value depends on the rake angle. The positive vertical forces can be attributed to the rake angle of $45°$ considered in this study. Zhang and Kushwaha (1995) and Godwin (2007) reported that the vertical force becomes negative when the rake angle is larger than $60°$.

The correlation coefficient between the horizontal and vertical forces is found to be $\rho(P_H, P_V) = 0.93$. This means that the relationship between the two forces is positive and quasi-linear, as illustrated in Fig. (9). In reality, the horizontal force P_H and vertical force P_V are calculated by combining the total force with the force of adhesion (McKyes, 1985). The effect of the total force on the horizontal and vertical forces is greater than the effect of the adhesion force such that the value of correlation coefficient is close to one. The correlation between P_H and P_V don't reflect a causal relation between these forces but it is due to the fact that these forces were calculated according to Equations (3) and (4).

4.4 Failure probability evaluation:

The results obtained by the MCS technique and the FORM method for the failure probability are almost identical, for the assumption of non-correlation, $P_{f-MCS} = 1.07 \times 10^{-3}, P_{f-FORM} = 1 \times 10^{-3}$, and correlation, $P_{f-MCS} = 1.5 \times 10^{-3}, P_{f-FORM} = 1.47 \times 10^{-3}$ existing between the tillage forces, Table (4). This is due to the quasi-linearity of the limit state function at the design point (Zhao and Ono, 1999). In addition, the correlation between the horizontal and vertical forces has no significant effect on the failure probability. Therefore, it is concluded that the FORM method can be used to estimate the failure probability, without taking into account the correlation between the horizontal and vertical forces, with sufficient accuracy. This makes the estimation of failure probability simpler and less time-consuming (Haldar and Mahadevan; 2000b), as compared to MCS calculations. This is because the calculation of the failure probability using the MCS technique requires one million iterations, while the SQP algorithm needs only a few iterations (between 6 and 10 iterations) to find the design point and calculate the failure probability.

5. Conclusions

This work aimed at proposing a probabilistic approach for modeling the variability of tillage forces by taking into account the variability of soil engineering properties, tool design parameters and operational conditions. This approach was implemented for modeling the variability of tillage forces for the shank of a chisel plough. The results allow us to draw the following conclusions:

1- The soil engineering properties do not have the same probability distributions and only the internal friction angle has a normal distribution.

2- The effect of the variability of soil cohesion on both the vertical and horizontal forces is the largest as compared with the effects of the variability of the other tillage system parameters. In addition, only the variability of the surcharge pressure has no significant effect on either the horizontal or vertical forces.

3- Both the horizontal and vertical forces have lognormal distributions with $\mu = 0.872, \xi = 0.449$ and $\mu = 0.004, \xi = 0.447$ for the horizontal and vertical forces, respectively. The relationship between the horizontal and vertical forces is positive and quasi-linear with $\rho(P_H, P_V) = 0.93$.

4- The MCS technique and the FORM method provide nearly identical results for the failure probability, although the FORM method led to simpler and faster calculations, when assuming non-correlation between the tillage forces ($P_f = 1.47 \times 10^{-3}$). Correlations between the vertical and horizontal forces only slightly changed the reliability level.

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Appendix I: Samples of soil engineering properties

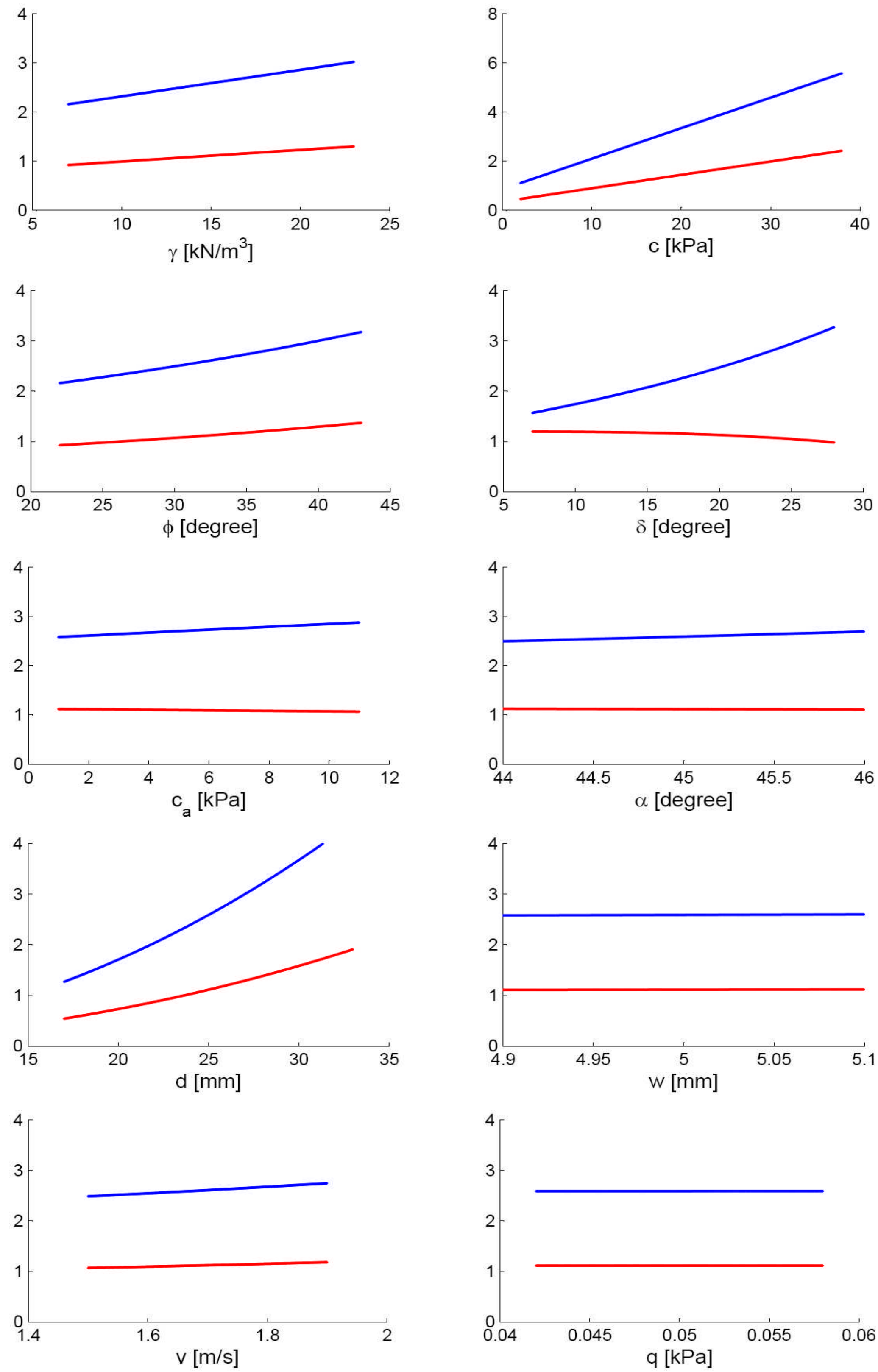
The following table presents the samples of soil engineering properties (soil specific weight γ, soil cohesion c, angle of internal friction ϕ, angle of soil-tool friction δ, soil-tool adhesion c_a), collected from the literature, used in this work.

N°	$\gamma\ (kN/m^3)$	$c\ (kPa)$	$\phi\ (°)$	$\delta\ (°)$	$c_a\ (kPa)$
1	14.70	4.60	37.5	22.0	0.00

2	10.80	0.00	34.0	22.0	0.00
3	14.61	8.90	23.3	18.8	0.00
4	15.01	2.26	35.0	23.0	0.00
5	15.70	3.63	35.0	23.0	0.00
6	14.70	4.60	35.0	23.0	0.00
7	15.30	10.5	30.8	24.0	0.00
8	19.00	31.7	42.0	24.0	0.00
9	16.38	6.00	32.0	24.0	0.00
10	14.02	23.0	22.0	22.0	8.00
11	18.05	20.4	34.0	25.0	0.00
12	16.98	15.5	31.8	23.0	0.00
13	15.79	15.3	30.3	22.0	0.00
14	16.98	15.5	31.8	23.0	0.00
15	14.34	7.19	34.5	23.5	3.29
16	11.50	33.5	37.3	27.3	9.40
17	11.00	35.3	29.8	25.2	8.10
18	14.50	6.30	36.0	23.3	2.20
19	13.20	11.9	33.1	22.1	2.70
20	14.70	2.00	30.0	15.2	7.66
21	14.12	6.00	35.0	20.0	0.00
22	16.38	6.00	32.0	24.0	0.00
23	13.73	9.00	35.0	29.0	0.00
24	14.02	23.0	22.0	22.0	8.00
25	13.23	9.23	29.0	22.0	0.00
26	14.71	12.1	30.2	22.3	0.18
27	14.91	13.3	29.6	23.6	0.21
28	15.30	24.5	36.5	24.7	0.29
29	15.01	22.6	34.5	23.1	0.35
30	14.62	20.5	32.2	24.0	0.31
31	13.05	6.70	39.3	23.8	0.60
32	14.22	11.7	36.8	24.0	8.30
33	12.50	5.00	35.0	24.5	3.25
34	12.80	10.2	38.0	22.0	5.27
35	13.50	11.0	32.5	24.8	3.22
36	12.50	5.00	35.0	24.5	3.21
37	12.75	8.60	32.6	22.4	0.00
38	12.75	7.00	31.4	13.1	0.00
39	12.75	9.30	29.2	14.4	0.00
40	14.72	22.7	29.3	16.0	0.00

41	14.72	17.0	30.6	18.9	0.00
42	14.72	16.0	30.9	15.6	0.00
43	14.72	11.7	30.8	25.0	0.00
44	14.72	8.10	31.4	19.8	0.00
45	14.72	9.20	30.8	18.3	0.00
46	16.19	19.5	37.6	11.9	0.00
47	16.19	30.7	26.6	13.3	0.00
48	16.19	20.3	30.8	24.0	0.00
49	16.19	18.6	27.4	24.1	0.00
50	16.19	13.2	28.4	21.6	0.00
51	16.19	13.9	29.1	20.9	0.00
52	17.66	16.7	33.5	23.0	0.00
53	17.66	22.0	29.2	15.9	0.00
54	17.66	12.8	29.8	17.2	0.00
55	17.66	11.6	30.9	18.8	0.00
56	19.62	29.9	28.8	19.9	0.00
57	19.62	21.3	27.1	14.8	0.00

Appendix II: Tillage forces-tillage system parameters relationships



— Horizontal force [kN] — Vertical force [kN]

Tillage forces-tillage system parameters relationships

Figures

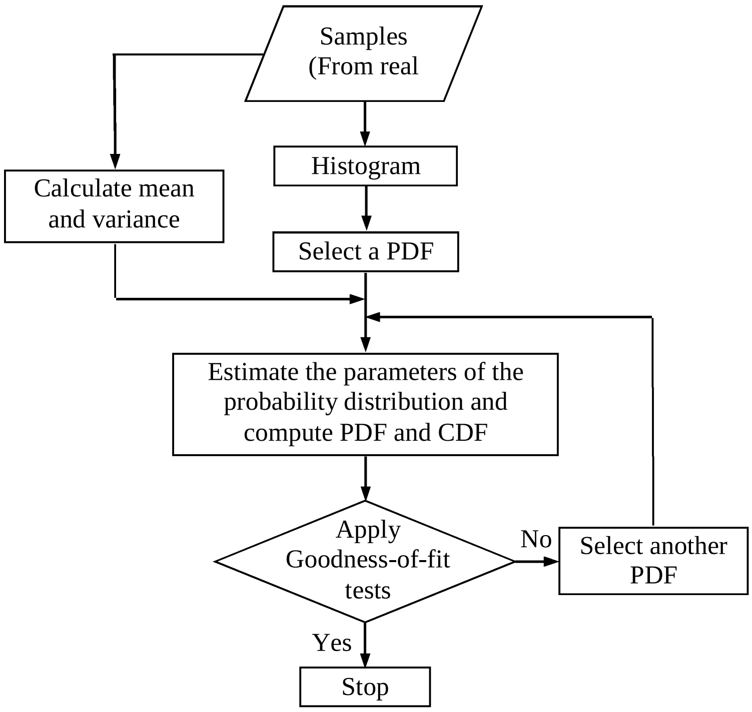


Fig. 1 – Proposed method for modeling the variability of soil engineering properties

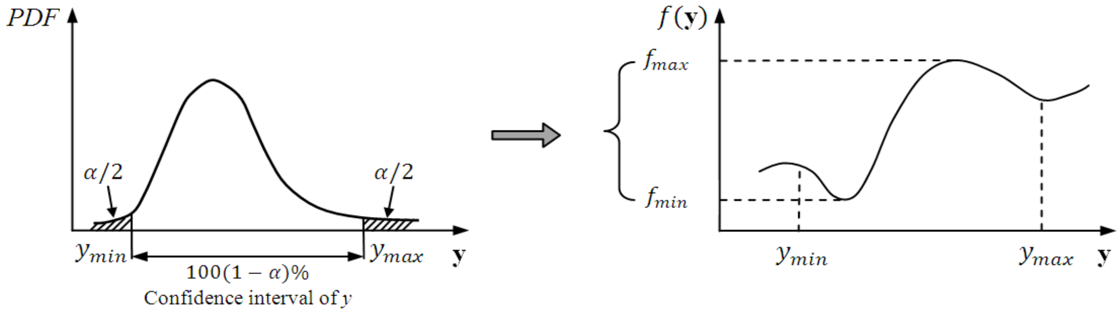


Fig. 2 - The concept of estimating the effects of input random variable dispersion (**y**, input random variable; $f(\mathbf{y})$, output random variable; y_{max}, y_{min}, confidence interval bounds; f_{max}, f_{min}, maximum and minimum values of output variable in confidence interval; $100(1-\alpha)\%$, confidence interval of input variable)

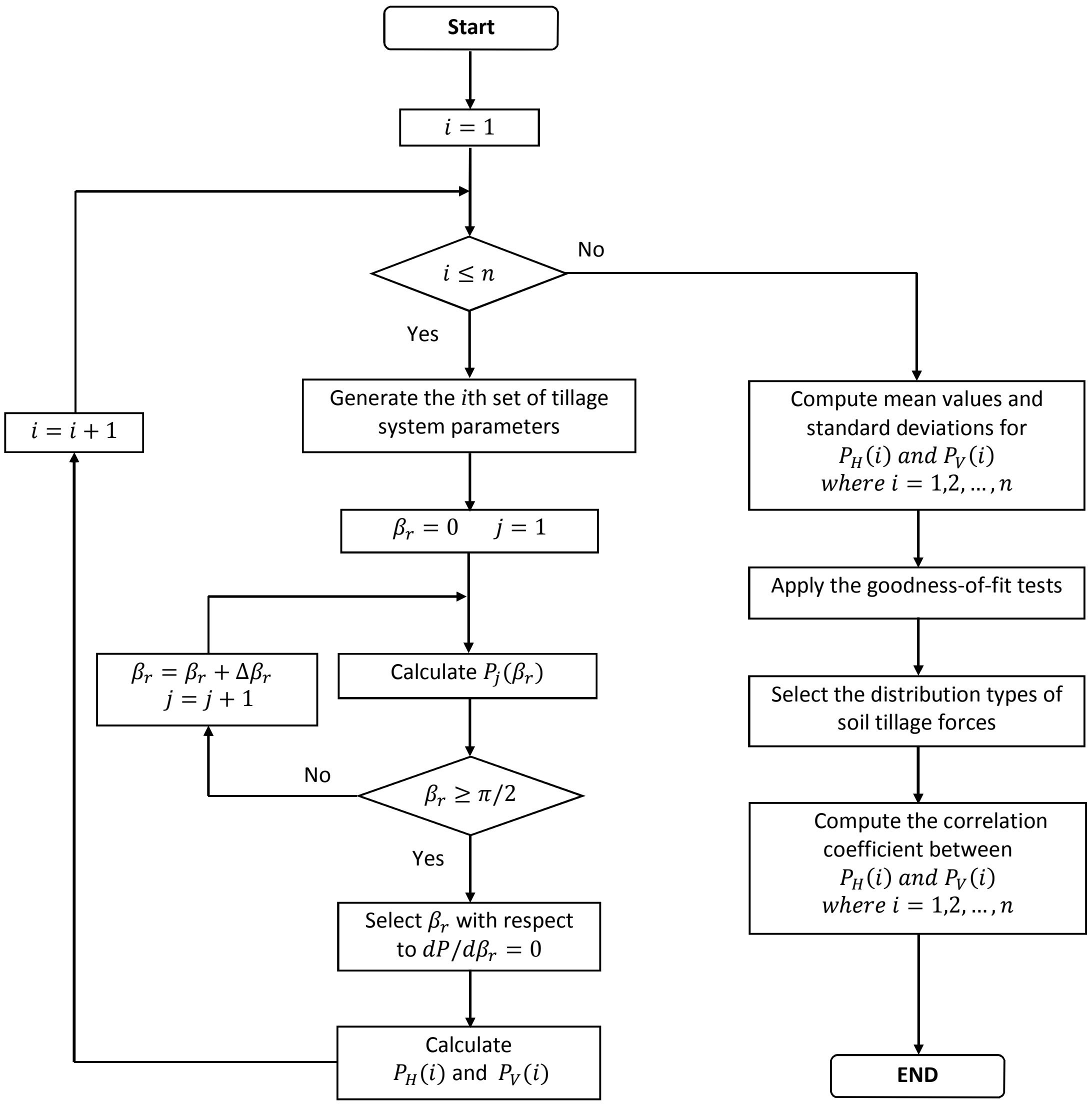


Fig. 3 – Flow chart of the steps involved in estimating the variability of tillage forces based on the variability of tillage system parameters

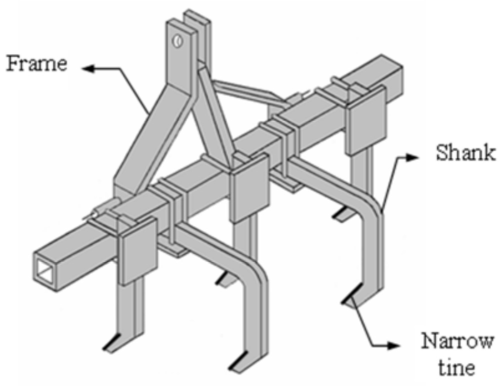


Fig. 4 - Illustration of a five-shank chisel plough (tine width ; rake angle ; tillage depth ; tool speed)

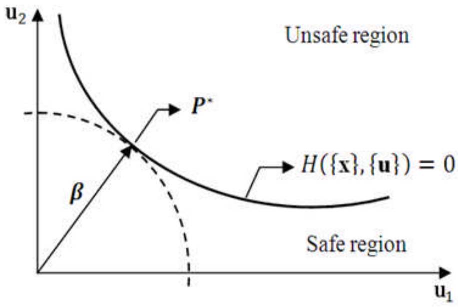


Fig. 5 - The concept of the reliability index in normalized space ($\boldsymbol{P}^*$, design point; β, reliability index; $H(\{\mathbf{x}\},\{\mathbf{u}\}) = 0$, limit state function in normalized space)

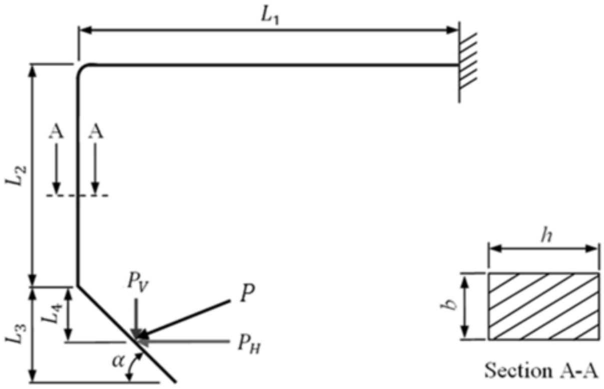


Fig. 6 - A schematic drawing of the chisel plough shank with acting forces (

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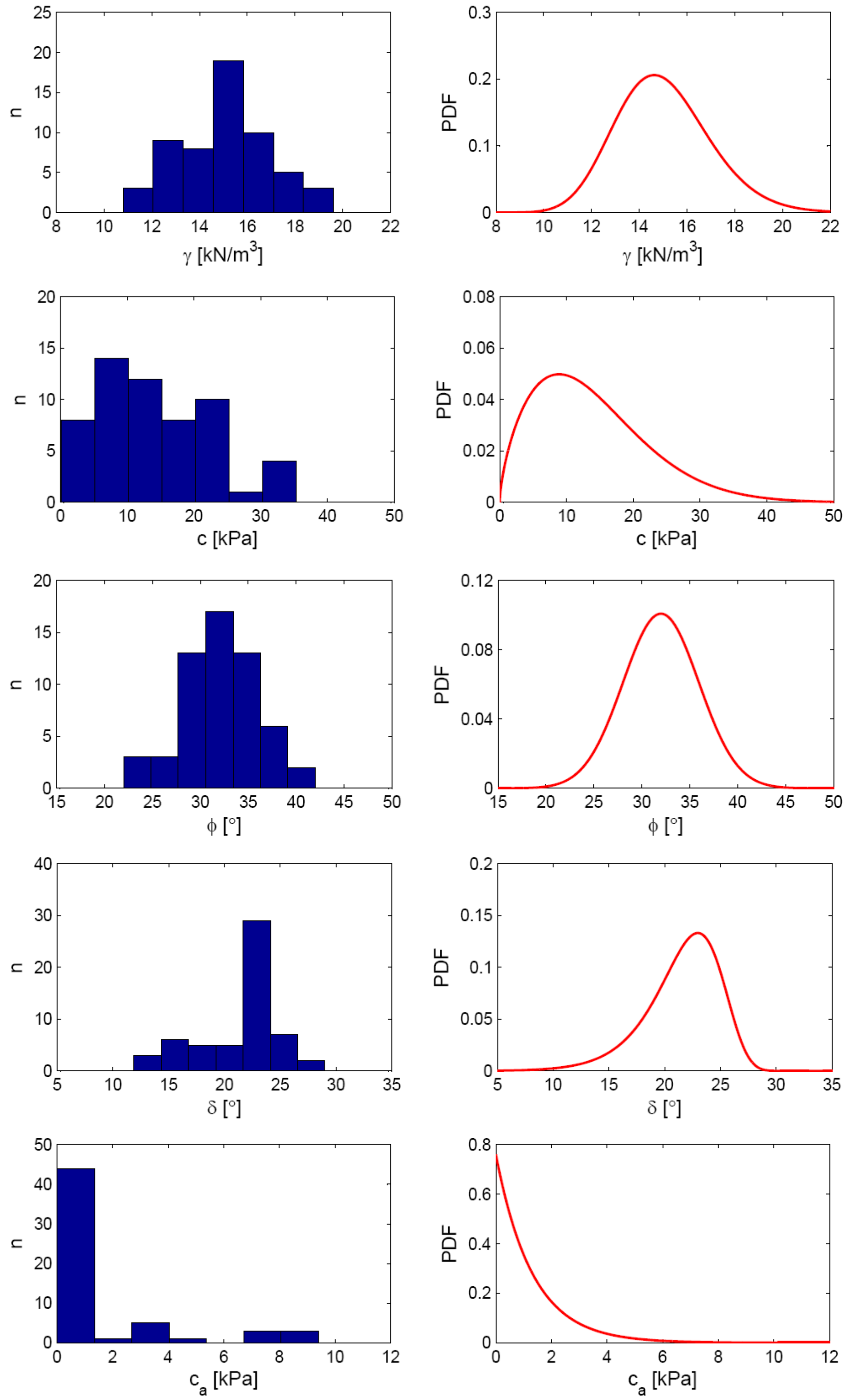

Fig. 7 - Histograms and probability density functions for soil engineering properties

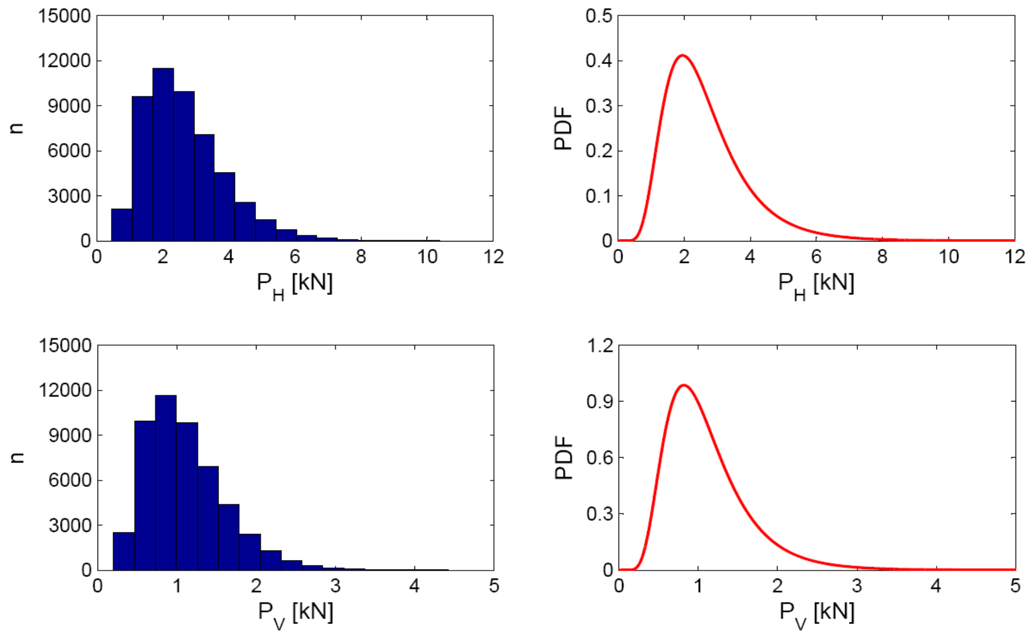

Fig. 8 - Histograms and probability density functions for horizontal and vertical forces

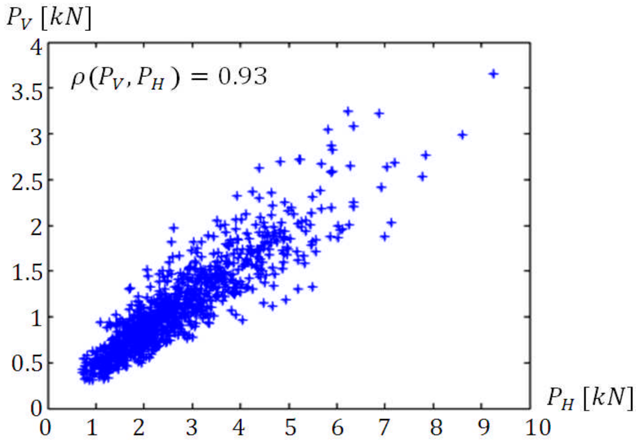

Fig. 9 - Correlation of tillage forces

Table 1 Probabilistic characteristic of soil engineering properties

Soil engineering properties	Type of distribution	Distribution parameters
Soil specific weight, $\mathrm{kN.m^{-3}}$	Lognormal	$\xi = 0.13, \mu = 2.7$
Soil cohesion, kPa	Weibull (2P)	$k = 15.51, \lambda = 1.66$
Internal friction angle, deg	Normal	$m = 32, \sigma = 3.96$
Soil-tool friction angle, deg	Weibull (3P)	$\varepsilon = -64.08, \tau = 87.14, \omega = 31.52$
Soil-tool adhesion, kPa	Exponential	$\eta = 0.76$

ξ and μ are the shape and scale parameters of a lognormal distribution; ε, τ and ω are, respectively, the location, scale and shape parameters of a Weibull distribution; m, σ are, respectively, the location and scale parameters of a normal distribution; η is the scale parameter of a exponential distribution.

Table 2 Results of sensitivity analyses using differential sensitivity method and the proposed sensitivity method

Soil tillage parameters	Differential sensitivity method		Proposed sensitivity method	
	$\frac{\partial P_H}{\partial y_i} \times 10^{-6}$	$\frac{\partial P_V}{\partial y_i} \times 10^{-6}$	ΔP_H, kN	ΔP_V, kN
Soil specific weight, $\mathrm{kN.m^{-3}}$	13.224	5.7921	0.414	0.181
Soil cohesion, kPa	24.114	10.563	4.020	1.763
Soil-tool adhesion, kPa	0.0517	-0.0082	0.145	-0.023
Internal friction angle, deg	47.852	20.984	0.741	0.325
Soil-tool friction angle, deg	40.789	-6.4631	1.164	-0.171
Rake angle, deg	200.76	-19.923	0.199	-0.019
Tillage depth, m	1.2217	0.5334	0.963	0.418
Tool width, m	1.5080	0.5965	0.011	0.004
Surcharge pressure, kPa	0.0005	0.0002	0.002	0.001
Forward speed, $\mathrm{m.s^{-1}}$	1.7708	0.7766	0.208	0.091

Table 3 Probabilistic characteristics of tillage forces

Force type	Distribution type	Distribution parameters
P_H, kN	Lognormal	$\mu = 0.872, \xi = 0.449$
P_V , kN	Lognormal	$\mu = 0.004, \xi = 0.447$

Table 4 Calculating failure probabilities using Monte Carlo simulation (MCS) and the first-order reliability method (FORM)

		Failure probability, P_f
Uncorrelated variables	MCS	1.07×10^{-3}
	FORM	1.00×10^{-3}
Correlated variables	MCS	1.50×10^{-3}
	FORM	1.47×10^{-3}