

*Decision Engineering Report Series*

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**HANDLING INTEGRATED QUANTITATIVE  
AND QUALITATIVE SEARCH SPACE IN  
ENGINEERING DESIGN OPTIMISATION  
PROBLEMS**

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August 2003

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ISBN 1-861940-98-X

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## **Abstract**

Since information in engineering design problems can be both quantitative ( $Q^T$ ) and qualitative ( $Q^L$ ) in nature, combining both types of information can result in more realistic solutions for real world optimisation problems. However, most of the approaches reported in the literature are incapable of conducting optimisation searches in such a mixed environment. Therefore this report proposes a mathematically proven methodology for handling integrated  $Q^T$  and  $Q^L$  search space in real world optimisation problems. The report begins by presenting the definition of these optimisation problems, an analysis of the challenges that they pose for existing optimisation strategies and related research. The report then presents the proposed solution strategy and the mathematical proof. Furthermore, a case study on a rod rolling problem is presented to validate the effectiveness of the proposed methodology. The report concludes with a brief outline of limitations and future research activities.

**Keywords:** *Design optimisation, Evolutionary computing, Qualitative and quantitative information, Search space, Rolling system*

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## 1. Introduction

Information in real world engineering design problems can be both quantitative ( $Q^T$ ) and qualitative ( $Q^L$ ) in nature [Oduguwa et al. 2003].  $Q^T$  models are very popular in real world design optimisation problems. Even though such models have been very useful in providing detailed information about the design problem, they can be ineffective in situations where the mathematical formulation of a design problem is not available or is partially defined. In such cases  $Q^L$  information can provide a valuable access to the design problem by taking advantage of human approximate reasoning to improve the complex design problem representation. Integrated  $Q^T$  and  $Q^L$  search space can therefore be defined as the combination of both types of information within a framework that enables an optimisation algorithm to facilitate a search towards a desirable goal. This tends to improve the efficient use of information and can result in more realistic solutions.

Such mixed forms of information within real world design optimisation problems can either complement, substitute or contradict each other. This report focuses on the forms that contradict each other. Here the mixed type of information are conflicting in nature. There are various approaches reported in the literature for dealing with such mixed information engineering design problems. When used with design search scenarios, most of these approaches do not explore the trade-off relationships that exist between  $Q^T$  and  $Q^L$  search space. This can bias the search toward sub-optimal regions and can result in unrealistic solutions.

This report presents a solution approach using a multi-objective optimisation algorithm to explore the trade-off relationships that exist in the conflicting nature of the  $Q^T$  and  $Q^L$  information inherent in design engineering problems. This should provide a practical alternative to compare and choose compromised optimal realistic solutions. The report begins by reviewing related research for dealing with  $Q^T$  and  $Q^L$  information in engineering design optimisation problems. This is followed by a mathematical justification of the functional relationship between the mixed information and the solution strategy adopted in this report. A real world case study is then presented to illustrate the concept and the report concludes with an outline of challenges and future research directions.

## 2. Challenges Of $Q^T$ And $Q^L$ In The Integrated Search Paradigm

There are several challenges that can inhibit the wider applications of current optimisation strategies for real world design problems with contradicting  $Q^L$  and  $Q^T$  information. Some of these are outlined as follows:

- It is difficult to develop solution strategies that combine both types of information within an optimisation framework since most optimisation techniques deals with  $Q^T$  models only.

- Solving real world problems could present scalability issues. The computational cost required to generate  $Q^L$  models when simulating the problem is exponential with increasing number of variables.
- Developing  $Q^L$  and  $Q^T$  search procedures for objectives greater than two can be complex. Higher number of  $Q^L$  objectives has the tendency to increase the fragmentation in the search space. This is largely due to the discreteness in the  $Q^L$  search space.
- It is difficult to ensure the appropriate correlation of the granularity of the  $Q^L$  models with the measurement scale of the  $Q^T$  models. Inappropriate correlation could deceive the genetic search to a local optimum.
- Developing  $Q^L$  models that represents a broad range of physical phenomena from different perspectives at a level which allows useful and verifiable inference to be drawn can be computationally expensive and non-trivial.
- Developing procedures to help extract explanatory rules from genetic search would be quite difficult, since the evolutionary procedure does not keep the history of each generation.

### 3. Related Research

It is observed that although some attempt has been made to separately handle the  $Q^L$  and  $Q^T$  knowledge within a design optimisation framework, there is not much reported work on handling the two types of knowledge simultaneously within an integrated design optimisation framework. This section briefly reviews approaches for handling  $Q^L$  and  $Q^T$  knowledge in engineering design.

Interval analysis is a deterministic technique that can be used to incorporate the  $Q^L$  knowledge into design problems. The technique is used to compute imprecise variables [Moore 1979], where each number of design variables is replaced with a range of values (interval) and the outputs values of a system response are also indicated as range of possible values. There are several applications of interval analysis to reason about imprecision in engineering designs [Wu and Chang 2003]. Since this technique output values at the boundaries of the intervals [Antonsson and Wood 1989], it is not suitable for design problems that requires information within the intervals.

Standard sensitivity analysis can also be used to reason qualitatively about engineering design problems. They can be used to evaluate the rate of change (sensitivities) of a performance parameter with respect to a design variable. These sensitivities are achieved by evaluating the partial derivatives or Lagrange multipliers of the system equations [Reklatis *et al.* 1983]. Standard sensitivity analysis method provides information on a single operating point only for every evaluation and assumes no interaction between input variables which can be too crude a simplification of the underlying model [Mirjam *et al.* 2003].

Probabilistic analysis approaches can also be used to represent imprecision in design problems [Siddall 1983]. However the calculus of probability does not permit the relationships between inputs and outputs to be found [Jensen and Sepulveda 2000]. For example, it is not possible to determine the design variables from a system performance that shows a low likelihood by using probability calculations alone. Probabilistic methods are being developed in which the statistical distributions of input values of model are incorporated into the modelling process. The underlying principle of these approaches is that the input parameters of the model are defined by a statistical distribution and not by a single value. Such a distribution can take any mathematical form uniquely defined by a mean and standard deviation. Examples include Normal or Gaussian distribution. Input values of the design parameters are then sampled randomly for the appropriate distribution and used in the experimental model. Examples of such formulations are the Taguchi methods and the experimental design techniques. These methods are powerful design tools to determine experimental points in a noisy space, but when used alone can be time consuming for exploring the design space.

There are several approaches developed based on the mathematics of fuzzy sets to incorporate  $Q^L$  knowledge into design. Most of the applications of fuzzy sets within the field of decision making consist of fuzzification of classical theories, where the fuzzy theories attempt to deal with the imprecision and vagueness in human reasoning of design variable preferences, constraints and goals. Fuzzy set theory have been used as a mathematical formulation to represent  $Q^L$  knowledge in decision making [Bellman and Zadeh 1970]. Two of the earlier work dealing with optimization of fuzzy systems was by Tanaka *et al.* (1974), and Zimmerman (1974). Since then several variations of fuzzy based approaches have been reported in the literature. Approaches based on fuzzy mathematical programming include fuzzy goal programming, flexible programming, fuzzy multi-objective optimisation, possibilistic programming with fuzzy preference operators and fuzzy linear programming. Antonsson and Wood (1989) also developed a fuzzy based approach referred to as the Method of Imprecision for engineering design problems where designers are given preference over a range of design values. Most of these fuzzy based approaches fundamentally fuzzifies the elements (constraints, goals or design variables) of an underlying mathematical formulation and do not combine the  $Q^L$  evaluation within the optimisation search. There are a number of other fuzzy based approaches reported in the literature where  $Q^L$  knowledge has been used in conjunction with  $Q^T$  models. Fuzzy Genetic Algorithms (FGA) manages problems in an imprecise environment. It combines fuzzy concepts with genetic algorithms. Approaches using fuzzy fitness evaluation function for the GA chromosomes has been reported in the literature [Koskimaki and Goos 1996; Dahal *et al.* 1999]. In fuzzy optimisation Hsu *et al.* (2001) adopted fuzzy optimization algorithm for determining the optimal gap openings of the programming points in the blow moulding process and in fuzzy controlled simulation optimisation. Medaglia *et al.* (2002) proposed an approach that incorporated  $Q^L$  knowledge into the optimisation strategy. Roy (1997) developed a design optimisation framework where both types of criteria or knowledge are handled separately. The report identified multiple 'good' design solutions based on the principle  $Q^T$  criteria and later evaluated them individually based on the other  $Q^L$  criteria. Most of the approaches reported above simply do not provide the means to deal with both  $Q^T$  and  $Q^L$  information simultaneously within an optimisation framework. Recently,

Oduguwa *et al.* (2003.) extended the work of Roy (1997) by developing an integrated  $Q^T$  and  $Q^L$  evaluation optimisation approach which combines  $Q^L$  evaluation from designers with  $Q^T$  formulation of the design problem within an optimisation framework. The elaborate approach adopts the principle of multi-objective optimisation to explore the functional relationship between the  $Q^T$  and  $Q^L$  knowledge. Fuzzy logic was used to incorporate  $Q^L$  knowledge where both the membership function values and the defuzzified domain values were used as fitness function values in the optimisation algorithm to guide the search.

In previous work, the authors did not justify the functional relationship between the  $Q^T$  and  $Q^L$  information. The presence of such a functional relationship was treated as an empirical observation from previous work. This however presents a weakness for the proposed solution strategy. Therefore, this report presents a mathematical justification of the functional relationship between the mixed information. This enhances the rigour of the proposed solution strategy. Furthermore, a case study on rod rolling problem is presented to validate the effectiveness of the proposed methodology.

#### 4. Multi-objective Optimisation

Most real world problems are characterised by several non-commensurable, conflicting objectives. Multi-objective optimisation seeks to minimise the  $n$  components  $f(x) = (f_1(x), \dots, f_n(x))$ , of a possibly non-linear vector function  $f$  of a decision variable  $x$  in the search space. Each of these objectives has a different optimal solution. There is no unique, (Utopian) solution to a multi-objective problem but a set of non-dominated solutions referred to as Pareto-optimal set. A solution to this class of problem is Pareto-optimal if from a point in the design space, the value of any other solution cannot be improved without deteriorating at least one of the others. The objective for a complex multi-objective optimisation problem is to find different solutions close and well distributed on the true Pareto-optimal front. The conditions for a solution to become dominated with respect to another solution are described as follows.

For a problem having more than one objective function (say,  $f_j$ , where  $j = 1, \dots, M$  and  $M > 1$ ), a solution  $x(1)$  is said to dominate solution  $x(2)$  if the following conditions are satisfied:

- a) The solution  $f_j(x(1))$  is no worse than  $f_j(x(2))$  for all  $j = 1, 2, \dots, M$  objectives.
- b) The solution  $x(1)$  is strictly better than  $x(2)$  in at least one objective.

Several genetic algorithm based multi-objective optimisation techniques have been reported in the literature. The main thrust of such algorithms is to produce a spread of multiple optimal solutions rather than a single optimal solution. VEGA [Schaffer 1985] was one of the first, since then possibilities of improving computational efficiency and solution accuracy have been explored by other researchers [Fonseca and Fleming 1993; Zitzler and Thiele 1998; Knowles and D.W. 1999; Veldhuizen 1999; Deb *et al.* 2000; Tiwari 2001]. The genetic algorithm based multi-objective optimisation technique NSGAI [Deb *et al.* 2000] is adopted in this study.

## 5. Handling Integrated $Q^T$ and $Q^L$ Search Space

$Q^T$  search space in the sense of engineering design problems is such that for every design point identified in the parameter space there is corresponding objective function value. Therefore it is widely accepted that a functional relationship exists between the design parameters in the parameter space and the objective function values. This functional relationship was nicely defined by Bottazzini (1986). This is stated as follows:

*Let  $A$  and  $B$  be two sets, which may or may not be distinct. A relation between a variable element  $x$  of  $A$  and a variable element  $y$  of  $B$  is called a functional relation in  $y$  if, for all  $x$  in  $A$ , there exists a unique  $y$  in  $B$  which is in the given relation with  $x$ .*

By the same analogy for solutions lying on the Pareto front, for every  $Q^T$  solution to a given design problem, there exists a corresponding  $Q^L$  evaluation expressing the designer's opinion about the problem. This  $Q^L$  evaluation varies in a unique fashion with the  $Q^T$  solution.

This suggests that under the conditions determined by the theorem, in the next section, there exists a functional relationship between both  $Q^T$  and  $Q^L$  Pareto optimal solution of design problems. The section that follows presents a mathematical justification for this functional relationship and describes a solution technique that exploits this for addressing integrated  $Q^T$  and  $Q^L$  search problems. For example consider an engineering design problem such as the cantilever beam shown in Figure 1 with two decision parameters, i.e length ( $L$ ) and diameter ( $D$ ) to withstand a load  $P$ . Two possible objectives are considered, such as the minimisation of the deflection ( $f_1$ ) expressed by a  $Q^T$  model and the design desirability of the beam section in terms of the weight ( $f_2$ ) expressed by the  $Q^L$  evaluation of human reasoning.

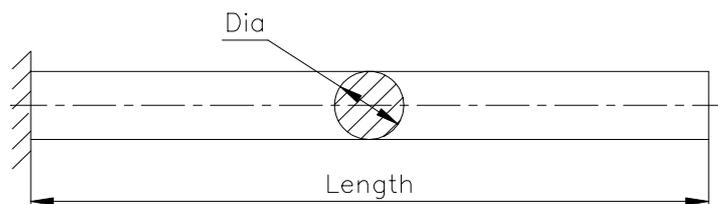


Figure 1: A layout of a cantilever beam

The deflection model can be obtained from the equation given by:

$$\delta = \frac{64PL}{3E\pi D^4}$$

The  $Q^L$  evaluation expresses the designers preference based on the  $Q^T$  values taken by the sectional parameters of the rod diameter and the length. Table 1 shows a selection of solutions used in this example. Figure 2 shows a plot of the  $Q^T$  and  $Q^L$  search space. For every points lying of the Pareto fronts there is a unique functional relationship between the  $Q^T$  solutions and the  $Q^L$  evaluations as shown by solutions 1-6. These solutions also demonstrate the conflicting nature of this type of relationship. Here the value of any one of the solution cannot be improved without deteriorating at least one of the others in the parlance of multi-objective optimisation.

Table 1: Solutions for cantilever beam design

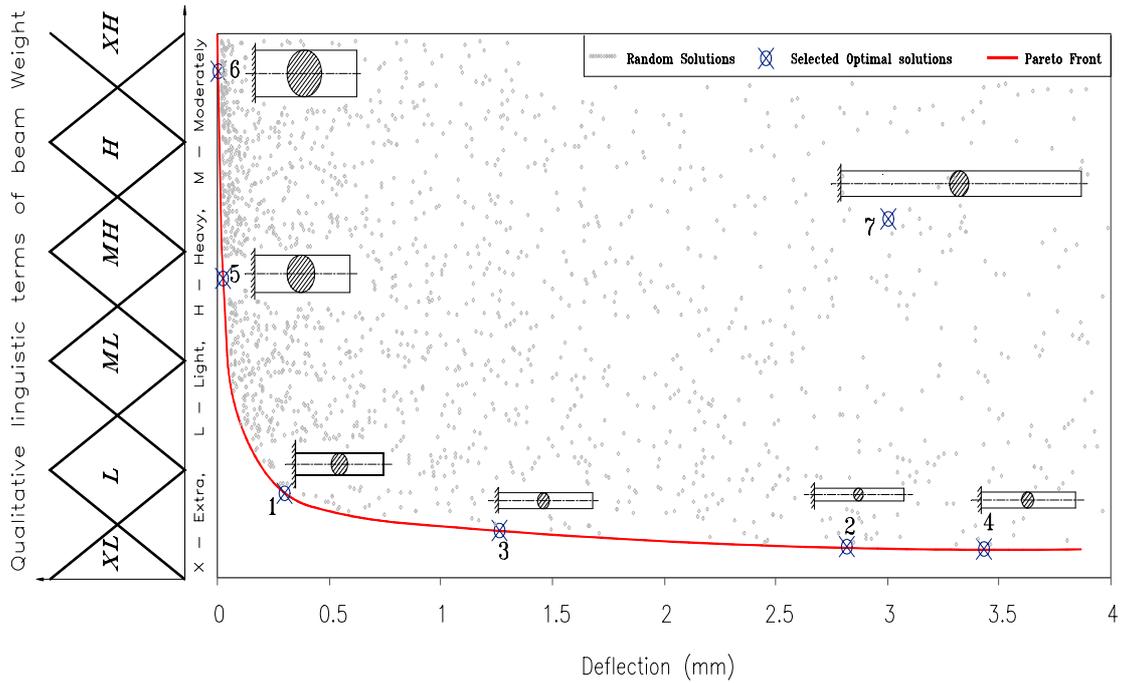
Solution	D (mm)	L (mm)	Deflection (mm)	Design Preference
1	18.5	100	0.28	L
2	10.7	102	2.65	XL
3	13.4	107	1.25	XL/L
4	10.6	110	3.46	XL
5	31.4	108	0.042	MH
6	39.2	115	0.02	XH/H
7	21.6	273	3.01	MH/H

This suggests that there is a functional relationship between the  $Q^T$  and  $Q^L$  search space. This section that follows presents a mathematical formalism for the functional relationship between the  $Q^T$  and  $Q^L$  information and describes a solution technique for addressing such problem.

## 5.1. Mathematical Justification Functional Relationship

### 5.1.1. Theorem

There exist a functional relationship between both  $Q^T$  and  $Q^L$  Pareto optimal solution of design problems.


 Figure 2:  $Q^T$  and  $Q^L$  objectives space

### 5.1.2. Definitions

The following definitions are used in conjunction with the mathematical justification.

#### Definition 1:

$Q^L$  evaluation is a proposition of the form “if  $A$  then  $B$ ” semantically expressing the designers opinion with respect to inputs of parameter values into the objective function values of a given  $Q^T$  model.

This is represented as  $\tilde{A} = \{\tilde{f}(x) | x \in X\}$  where the tilde represents the fuzziness in the  $Q^L$  evaluations, and modelled as stated in definition 2.

#### Definition 2:

If  $X$  is a collection of objects denoted generically by  $x$  then a fuzzy set  $\tilde{A}$  in  $X$  is a set of ordered pairs:  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$  where  $\mu_{\tilde{A}}(x)$  is the membership function of  $x$  in  $\tilde{A}$  which maps  $X$  to the membership space  $M$ .

#### Definition 3:

The  $Q^T$  model and the  $Q^L$  model (obtained from definition 2) represent two independent objective spaces explaining different behavioural aspects of an overall design problem.

**Definition 4:**

Two propositions  $\tilde{A}$  and  $\tilde{B}$  are equivalent if and only if the membership function values induced by  $\tilde{A}$  and  $\tilde{B}$  are equal such that:  $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x), \forall x \in X$

**Definition 5:**

$Q^T$  objective function value is a function of the form  $y = f(x), x \in X$ , where  $y$  is the objective function value and  $x$  is a location in the search space.

The theorem is therefore stated mathematically as follows:

Let  $\{\tilde{A} \in \mathfrak{R}^n \times I \mid I \in [1,0]\}$ ,  $B \subseteq \mathfrak{R}$  such that:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$$B = \{f(y) \mid y \in Y\}$$

Then  $K = \{[\mu_{\tilde{A}}(x), f(y)] \mid (x, y) \in X \times Y\}$  is a functional relationship on the  $Q^L$  evaluation  $A$  and the associated objective function value of the  $Q^T$  model.

**5.1.3. Identification Of Validity Conditions**

This section identifies the conditions under which the proposed theorem is valid. For a functional relationship to exist between the variables  $x$  and  $y$ , such that  $y = f(x)$ , where  $x \in X$ , the following four conditions must be satisfied.

1.  $x_1 = x_2 \Rightarrow y_1 = y_2$
2.  $x_1 \neq x_2 \Rightarrow (y_1 = y_2) \vee (y_1 \neq y_2)$
3.  $y_1 = y_2 \Rightarrow (x_1 = x_2) \vee (x_1 \neq x_2)$
4.  $y_1 \neq y_2 \Rightarrow (x_1 \neq x_2)$

The conditions above are standard for functional relationships for  $Q^T$  based models. However, the following four conditions are specified as propositions P1 – P4, for functional relationship between both  $Q^T$  and  $Q^L$  to exist.

P1:  $f(x_k) = f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1})$

P2:  $f(x_k) \neq f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1}) \vee \tilde{f}(x_k) \neq \tilde{f}(x_{k+1})$

P3:  $\tilde{f}(x_k) = \tilde{f}(x_{k+1}) \Rightarrow f(x_k) = f(x_{k+1}) \vee f(x_k) \neq f(x_{k+1})$

P4:  $\tilde{f}(x_k) \neq \tilde{f}(x_{k+1}) \Rightarrow f(x_k) \neq f(x_{k+1})$

Proposition 1 (P1):  $f(x_k) = f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1})$

This proposition states that for a given set of identical objective function values ( $f(x_k)$ ,  $f(x_{k+1})$ ), the associated  $Q^L$  evaluations are equal. The equality expression on the right hand side is treated in accordance with definition 4. There are clearly two cases to be considered in this proposition.

Case I:  $x_k = x_{k+1}$

This is the case when the two designs are the same. This implies that both their  $Q^T$  and  $Q^L$  evaluations are also equal. Therefore, P1 is unconditionally true for cases where the two designs under consideration are the same.

Case II:  $x_k \neq x_{k+1}$

This is the case when the two designs are different. If the  $Q^T$  evaluation of two different designs are equal, then one of the following conditions is true:

- The corresponding  $Q^L$  evaluation of the two designs are equal.
- The corresponding  $Q^L$  evaluations of the two designs are different.

However as stated in this proposition, the proposed theorem is valid only if the equality of the  $Q^T$  evaluations of the two different designs implies the equality of the corresponding  $Q^L$  evaluations. This condition is mathematically stated in Lemma 1. Therefore Lemma 1 provides a necessary condition for the proposed theorem to be valid.

**Lemma 1:**  $\{f(x_k) = f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1})\} | (x_k \neq x_{k+1})$

This lemma provides a condition that must be satisfied for the proposed theorem to be valid.

Proposition 2 (P2):  $f(x_k) \neq f(x_{k+1}) \Rightarrow \tilde{f}(x_k) = \tilde{f}(x_{k+1}) \vee \tilde{f}(x_k) \neq \tilde{f}(x_{k+1})$

There are two cases to be considered in this proposition.

Case I:  $x_k = x_{k+1}$

This is the case when the two designs are the same. This case does not exist for P2 since the  $Q^T$  evaluation of the two designs is different. This cannot be the case if the designs are same.

Case II:  $x_k \neq x_{k+1}$

This is the case when the two designs are different. If the  $Q^T$  evaluations of two different designs are different, then one of the following conditions is true:

- The corresponding  $Q^L$  evaluations of the two designs are equal.

- The corresponding  $Q^L$  evaluations of the two designs are different.

The above conditions match with the conditions in the proposition. Therefore, this proposition is true in all cases and hence does not impose any conditions on the validity of the proposed theorem.

Table 2: Alternative beam design solutions P2

Solution	D (mm)	L (mm)	$\delta (Q^T)$ (mm)	Design Preference ( $Q^L$ )
2	10.7	102	2.65	XL
4	10.6	110	3.46	XL

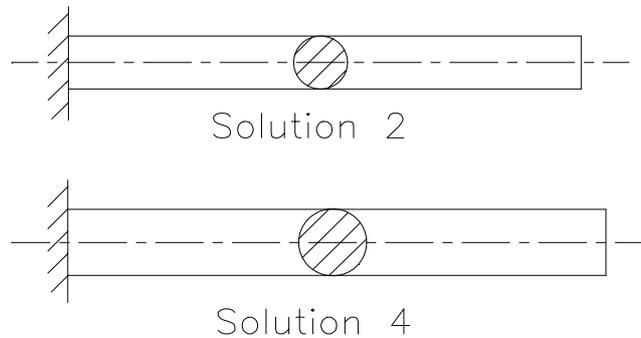


Figure 3: Beam Section for P2

For example from the solutions in Table 2, design solutions 2 and 4 supports case II of P2. Two different designs 2(10.7, 102) and 4(10.6, 110) with deflection values of 2.65 and 3.46 respectively correspond to equal  $Q^L$  evaluations.

Proposition 3 (P3):  $\tilde{f}(x_k) = \tilde{f}(x_{k+1}) \Rightarrow f(x_k) = f(x_{k+1}) \vee f(x_k) \neq f(x_{k+1})$

There are two cases to be considered in this proposition.

Case I:  $x_k = x_{k+1}$

This is the case when the two designs are the same. In this case both  $Q^T$  and  $Q^L$  evaluations are also equal. Therefore, P3 is unconditionally true for cases where the two designs under consideration are the same.

Case II:  $x_k \neq x_{k+1}$

If the  $Q^L$  evaluations of two different designs are equal, then one of the following conditions is true.

- The corresponding  $Q^T$  evaluations of the two designs are equal.

- The corresponding  $Q^T$  evaluations of the two designs are different.

Therefore this proposition is true in all cases and hence does not impose any conditions to the validity of the proposed theorem.

The example given to show support P2, also supports P3 in a similar manner.

Proposition 4 (P4):  $\tilde{f}(x_k) \neq \tilde{f}(x_{k+1}) \Rightarrow f(x_k) = f(x_{k+1}) \vee f(x_k) \neq f(x_{k+1})$

There are two cases to be considered in this proposition.

Case I:  $x_k = x_{k+1}$

This is the case when the two designs are the same. This case does not exist for P4 since the  $Q^L$  evaluations of these two designs is different. This cannot be the case if the designs are the same.

Case II:  $x_k \neq x_{k+1}$

If the  $Q^L$  evaluations of two different designs are different, then one of the following conditions is true.

- The corresponding  $Q^T$  evaluations of the two designs are equal.
- The corresponding  $Q^T$  evaluations of the two designs are different.

However, as stated in this proposition, the proposed theorem is valid only if the inequality of the  $Q^L$  evaluations of the two different designs implies the inequality of the corresponding  $Q^T$  evaluations. This condition is mathematically stated in Lemma 2. Therefore Lemma 2 provides a necessary condition for the proposed theorem to be valid.

**Lemma 2:**  $\{\tilde{f}(x_k) \neq \tilde{f}(x_{k+1}) \Rightarrow f(x_k) \neq f(x_{k+1})\} | (x_k \neq x_{k+1})$

This lemma provides a condition that must be satisfied for the proposed theorem to be valid.

Table 3: Alternative beam design solutions P4

Solution	D (mm)	L (mm)	$\delta(Q^T)$ (mm)	Design Preference ( $Q^L$ )
1	18.5	100	0.28	L
6	39.2	115	0.02	XH/H

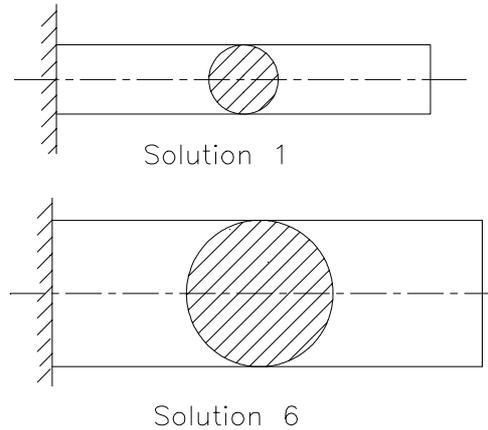


Figure 4: Beam Section for P4

For example from the solutions in Table 3, design solutions 1 and 6 supports P4. Two different designs 1(8.5, 100) and 6(39.2, 115) with deflection values of 0.28 and 0.02 respectively correspond to different  $Q^L$  evaluations of  $L$  and  $XH/H$ .

## 5.2. Solution Strategy For Integrated $Q^T$ And $Q^L$ Search Space Problems

The fundamental principle for combining the  $Q^T$  and  $Q^L$  information is based on transforming the  $Q^L$  information into cardinal information with the subsequent use of multi-criteria method. Evolutionary multi-objective optimisation solution approach is proposed as a solution strategy for the integrated  $Q^T$  and  $Q^L$  search space problem. The rationale for adopting this strategy is based on the following considerations:

- The cardinality of the objectives is greater than one, where the problem comprises of two search spaces obtained from the  $Q^T$  and  $Q^L$  information.
- This problem is such that the nature of this relationship exhibits a conflict. From the example given in Section 5, the two conflicting objectives are: maximise the design preference and to minimise the beam deflection. From the first objective, it is desirable to obtain small cross section beams. Maximum design preference would tend to result to beams that will not be sufficiently rigid and the beam deflection will be large. On the other hand, large cross section is expected for minimum beam deflection.
- A structured method is required to explore the conflicting behaviour of the two objectives. This can be used to obtain a spectrum of solution that provides the best compromise for the two objectives. This can be useful for designers to compare and choose optimal solutions.

In spite of these conditions there are exceptions where the cardinality of this mixed search space can be reduced to one. In such case the problem becomes a single objective problem and the multi-objective solution approach is no longer suitable. The following proposition is provided to identify the condition for which the solution strategy applies.

- The  $Q^T$  and  $Q^L$  objectives derived in relation to definitions 1, 2 and 3 represent objective cardinality greater than one.
- The proposed theorem is valid only if the equality of the  $Q^T$  evaluations of two different designs implies the equality of the corresponding  $Q^L$  evaluations (Lemma 1).
- The proposed theorem is valid only if the inequality of the  $Q^L$  evaluation of the two different designs implies the inequality of the corresponding  $Q^T$  evaluations (Lemma 2).

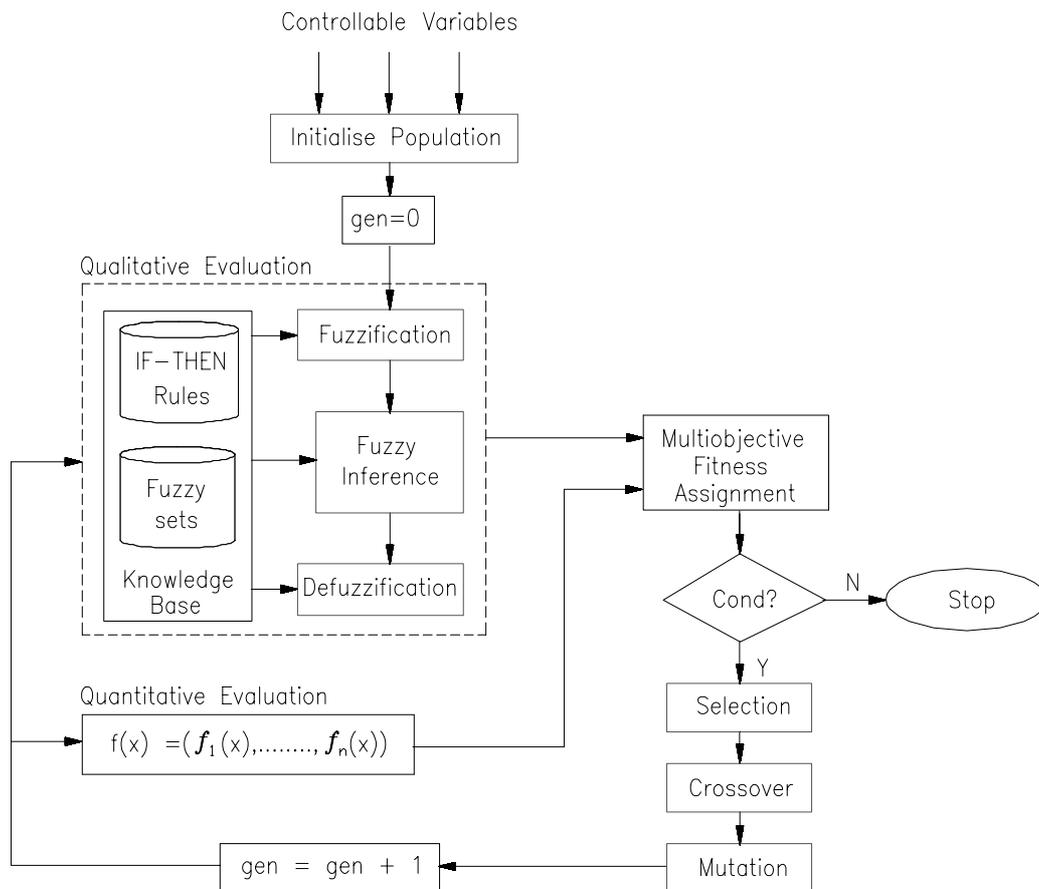


Figure 5: Optimisation framework for integrated  $Q^T$  and  $Q^L$  search space problems

### 5.3. Solution Approach

The optimisation algorithm as shown in Figure 5 is based on the genetic algorithm (GA) integrated with a fuzzy reasoning module. The basic GA are adaptive methods used to solve search and optimisation problems, based on genetic processes of biological organisms [Goldberg 1989]. They simulate the genetic state of a population of individuals through operators such as natural selection, mutation, and crossover. GA search by using a population of points and as a result of the parallel search GAs are

effective in finding the global optimum. These features are used to assign values to each decision variables. These values indicate the vector of each variable in the design space.

The fuzzy reasoning module consists of fuzzification, fuzzy inference and defuzzification routines. Values of the decision variables from individual members of the population are fuzzified, and fuzzy IF-THEN rules are applied within the fuzzy inference mechanism. The evaluation of a proposition produces a single fuzzy set associated with each model solution variable. For example, in evaluating the following propositions, *If a is Y then S is P*, *If b is X then S is Q*, *If c is Z then S is R*, the consequent fuzzy set P, Q R is correlated to produce a fuzzy set representing the solution variable S. An appropriate method of defuzzification is used to find a scalar value and the corresponding membership grade that best represents the information contained in the consequent fuzzy set D. The scalar value represents the approximate  $Q^L$  evaluation of the design problem and it is used to assign a fitness value to the  $Q^L$  aspect of the individual member of the population. The  $Q^L$  fitness evaluation also takes into account the membership grade to ensure that membership grade below a selected threshold is penalised using the penalty function method.

Final fitness solution of each member of the population is based on a ranking mechanism that considers the fitness values from the  $Q^T$  and  $Q^L$  models. In order to select the fittest member of the population, each individual is ranked based on the Pareto dominance criteria stated in section 2. Individual members of the population are assigned a fitness value based on the  $Q^T$  and  $Q^L$  models. The multi-objective ranking mechanism then performs a non-domination ranking procedure on each member where it is assigned a ranking value based on its location in the objective space.

## 6. A Real World Case Study: Rod Rolling Problem

The proposed approach is illustrated using a rod rolling design problem. In complex hot rolling of rods, FE analysis is often used to study the effect of roll design and complex thermo-mechanical interactions during high temperature rolling on key properties such as shape, microstructure, rolling loads and torque [Mori 1990; Kim *et al.* 1991; Shin *et al.* 1994]. Owing to their discrete nature, FE models have been used to develop a detailed understanding of the rolling process at meso-scale level. Although FE techniques allow an entire rolling sequence to be studied, it is still time consuming (mostly in 3D) [Kang *et al.* 1996; Yoshida and Kihara 1996; Lenard *et al.* 1997] despite improvement in both hardware and software to use them as embedded into a general optimizer for optimizing roll design sequences. There are very few cases reported in the literature where soft computing techniques have been used to solve design optimisation of the rolling problem. Several authors have applied fuzzy reasoning, for roll pass design [Shivpuri and Kini 1998; Pataro and Helman 1999; Jung and Im 2000] and GAs in other related metal forming problems [Hwang and Chung 1997; Myllykoski *et al.* 1998; Chung and Hwang 2002; Chakraborti and Kumar 2003]. Most of these reported applications do not combine  $Q^L$  evaluation within a design optimisation framework and therefore are not able to explore solutions with functional relationship between the  $Q^T$  and  $Q^L$  knowledge. The rod design problem is a two objective optimisation problem

(maximising the shape of the rod profile and minimising the deformation load). It is used to illustrate an optimisation problem based not only on  $Q^T$  information but also on the engineer's  $Q^L$  knowledge for solving complex engineering design problems.

The shape condition is a roundness measure of the rod profile often measured using classical numerical models. Since the rod profiles tend to emerge as non-smooth most of the shape conditions evaluated using classical models do not tend to correlate with the designer's representation. Here, a  $Q^L$  model is proposed to capture the designer's representation of the shape condition.

In this study, the shape and the load required for rod deformation are modelled using fuzzy reasoning and meta-modelling technique respectively. The simultaneous optimisation of both responses is treated as a multi-objective problem. The problem is considered multi-objective in nature due to the conflicting relationship between the two objectives. In practice, for a given stock size a perfect shape condition requires large roll pockets. This implies a high contact of the stock with the roll, which results in high loads.

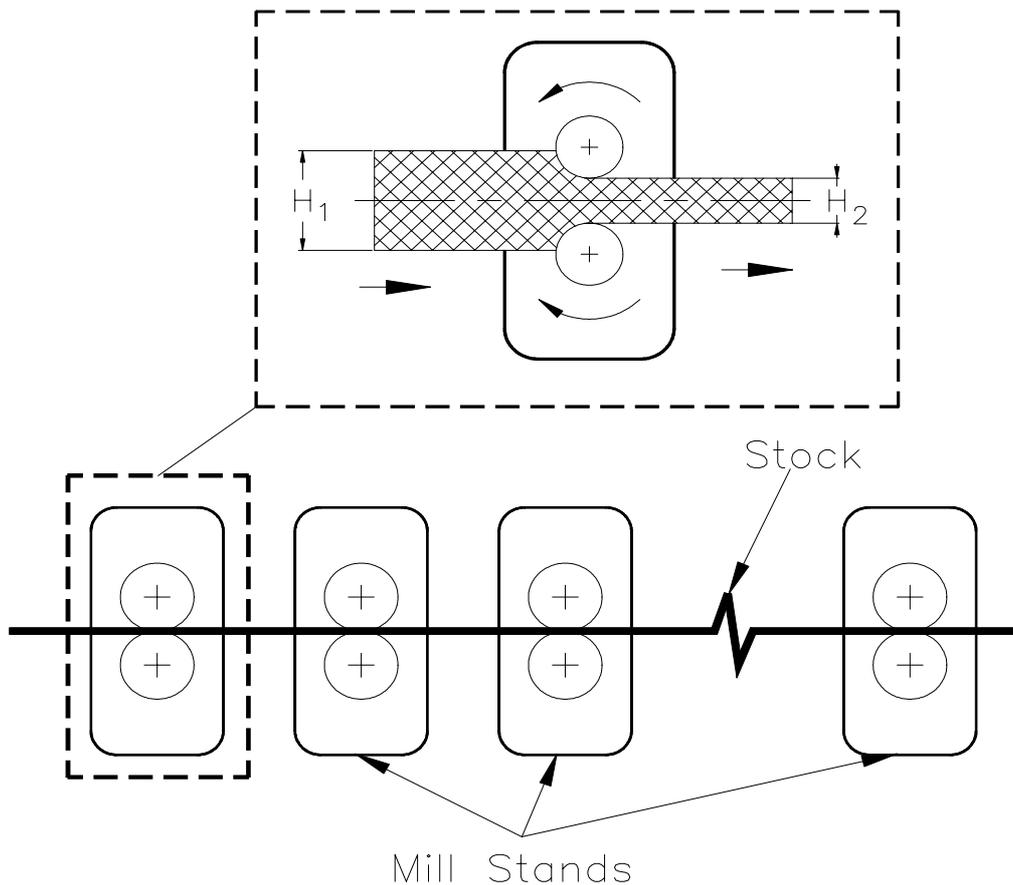


Figure 6: Rolling Process

## 6.1. Rod Rolling Process

A schematic layout of a rod rolling process is shown in Figure 6. The process is a continuous manufacturing process whereby a square billet (dimension ranging from 100 mm to 150 mm) referred to as the stock is deformed into a rod size ranging from 5 mm to 12 mm. The rolling operation is a high speed, high production process in which a pair of rolls rotates at the same peripheral speed in opposite directions. The stock is continuously deformed by passing it through a series of high rolling mill stands.

## 6.2. Experimental Procedure And Model Development

A single roll pass was modelled using the ABAQUS Explicit FE simulation software. The case study described in this report deals with the shape and load optimisation of a single oval to round wire rod pass. The geometrical parameters relevant to the present study that affects these objectives were solicited from the domain expert and categorised as: (a) initial thickness  $h_1$ , (b) initial width  $w_1$ , (c) work roll radius  $R$ , (d) pass radius  $Pr$ , (e) roll gap  $Rg$  and (f) temperature  $T$ .

The genetic search for optimal solution requires a model definition that quantifies the ‘goodness’ of each solution according to the formulation of the optimisation problem. Here specific model details of the objectives are shown in the sections that follow. Details of the  $Q^T$  and  $Q^L$  model development process for shape and load are detailed elsewhere [Oduguwa and Roy 2003] and therefore are omitted in this report.

### 6.2.1. Quantitative Modelling

Advanced computational simulation is becoming a key component of engineering research and product development. However despite improvement in both hardware and software the function evaluation tends to be computationally expensive. This problem increases significantly especially for evolutionary algorithm based optimisation approach where large number of population samples is required.

In order to address these problems, approximate metamodels are developed using Response Surface Methods (RSM). The metamodel is a typical example of functional approximation defined as a model of an underlying simulation model [Kleijnen 1975; Friedman 1996]. The RSM is used in this study since it is one of the most popular method of constructing approximate models in the design optimisation literature [Montgomery and Peck 1992]. RSM can be used to create smooth approximations of the response data. The most widely used approximate models are the linear and quadratic polynomials generated using ordinary least square regression on the set of analysis data. In its simplistic sense, RSM involves (a) choosing an experimental design for generating data, (b) choosing a model to represent the data, and then (c) fitting the model to the observed data.

Due to the high computational cost of the FE runs, a low cost small composite design (SCD) fractional sampling method was adopted for the experimental design. The general SCD is a special resolution III fraction of a  $2k$  augmented with axial points and runs in the centre of the design. These designs are capable of estimating second order

model and are mostly suitable for problems with low computational cost [Montgomery 1997]. The SCD matrix adopted in this study is a two-level, 6 factor fractional design augmented with axial points ( $\alpha$ ) with a value of 1.719 and two centre points. This design matrix was used to generate the input values for the FE simulations.

From the observation of the FE results, the following design parameters specified in Section 6.2 were used to develop  $Q^T$  models (load) used in this study. For each run, values of the measured output for the responses were recorded.  $Q^T$  models of the responses were generated by fitting a second order model (main effects, interaction effects and quadratic effects). The fit with the lowest sum of squares error (highest  $R^2$ ) was selected, this resulted in the following experimental models (initial stock area/roll area (SAR), form factor (FF) and the roll radius/material height ratio (RRMR)) as predicted using ANOVA.

$$\begin{aligned} \text{Load} = & -2023520.422 + 50112.96 h_1 - 853.369 h_1^2 + 35728.755 w_1 + 434.706 w_1^2 - \\ & 39003.709 Pr + 604.149 Pr^2 + 19369.967 Rg - 1271.041 Rg^2 + 578.474 Rr - 2.206 Rr^2 \\ & + 2799.198 T - 1.039 T^2 + 1135 h_1 w_1 - 177.396 h_1 Pr - 305.971 w_1 Pr - 2075.8333 w_1 Rg \\ & + 57.274 w_1 Rr - 77.103 w_1 T + 417.083 Pr Rg + 14.183 Pr T - 0.413 Rr T \end{aligned} \quad (1)$$

$$\begin{aligned} \text{SAR} = & -1.976 + 0.1106 h_1 - 0.00157 h_1^2 + 0.184 w_1 - 0.0012 w_1^2 - 0.104 Pr + 0.0025 \\ & Pr^2 + 0.0046 Rr - 2.708E-6 Rr^2 + 11.24E-6 T + 0.0026 h_1 w_1 - 5.728E-4 h_1 Pr - \\ & 1.0455E-4 h_1 Rr - 0.002 w_1 Pr - 0.0036 w_1 Rg - 1.207E-4 w_1 T \end{aligned} \quad (2)$$

$$\begin{aligned} \text{RRMR} = & 6.155 - 0.375 h_1 + 0.0056 h_1^2 + 0.061 Rr + 5.877E-5 h_1 Rg - 9.319E-4 h_1 Rr - \\ & 1.267E-5 Rg Rr \end{aligned} \quad (3)$$

$$\begin{aligned} \text{FF} = & 11.109 - 0.190 h_1 + 0.0022 h_1^2 - 0.525 w_1 + 0.0077 w_1^2 + 0.0022 Pr^2 + 0.176 Rg \\ & + 0.00561 Rg^2 - 0.0035 Rr + 5.191E-6 Rr^2 - 0.0061 T + 9.765E-7 T^2 - 8.722E-4 h_1 w_1 + \\ & 0.0011 h_1 Pr + 7.015E-5 h_1 Rr - 0.0061 w_1 Rg - 4.1E-5 w_1 Rg + 2.532E-4 w_1 T - 0.0011 \\ & Pr Rg - 1.695E-4 Rg Rr + 4.319E-5 Rg T \end{aligned} \quad (4)$$

### 6.2.2. Qualitative Modelling

$Q^L$  evaluation of the design solutions is performed by the experts to determine the suitability of each design. In this study, fuzzy modelling technique is used to represent the  $Q^L$  knowledge used by the experts to reason about the design solutions. Fuzzy models that were developed by formulating the response variables of Equations 2, 3 and 4 as the antecedent part of the rules, and modelling the expert's reasoning of the FE outputs as the consequent part of the rules. Nine rules were developed. Details of the rule development process are as follows.

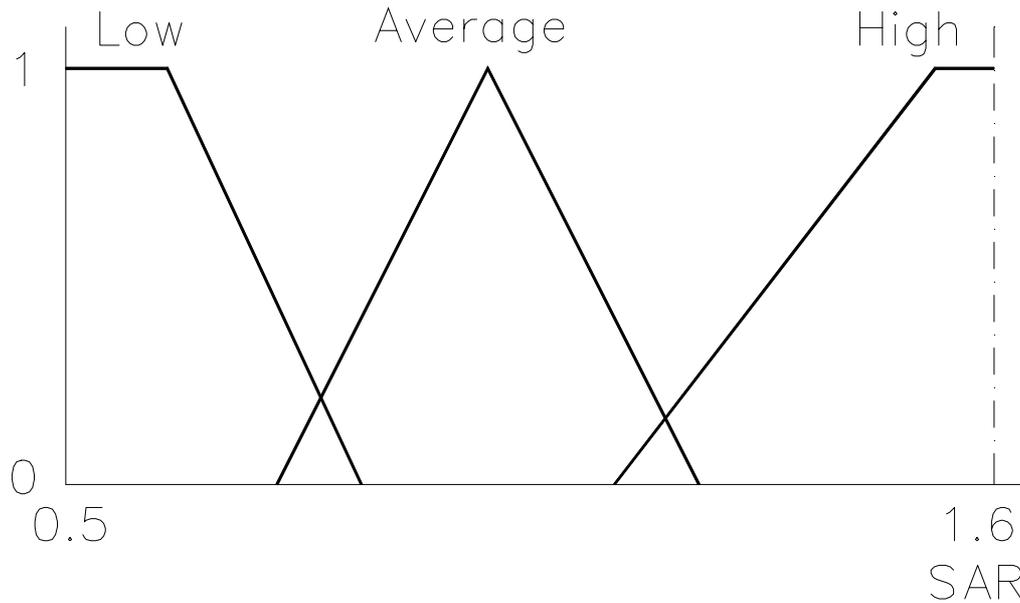


Figure 7: Membership Functions for Fuzzy Set SAR

The antecedent parts of the fuzzy rules were developed by fuzzifying the response variables of Equations 2, 3 and 4. For each of the response variables, the fuzzy sets shown in Figure 7, Figure 8 and Figure 9 were created with triangular membership functions and their corresponding linguistic labels *low*, *average* and *high*. These fuzzy sets correspond to the expert's interpretation of the variable's behaviour with respect to the rod roundness phenomenon. Membership functions were developed using the fuzzy sets to facilitate the rule development process. The membership function for each fuzzy variable shows the degree of membership of each value in the variable's fuzzy sets for the range of interest. For example Figure 7 shows the membership function for stock area (SAR). At a value of 0.77, the degree of membership is 0.375 in the fuzzy set 'Low', a degree of membership 0.1 in the fuzzy set 'Average', and a degree of membership 0 in the fuzzy set 'High'. The membership functions for the h/w ratio and form factor were also developed as shown in Figure 8 and Figure 9 respectively.

The consequent part of the fuzzy rule was developed to represent the expert's  $Q^L$  evaluation of the roundness of the rod profile. This was achieved by initially classifying the FE output of the rod profiles into five main categories as shown in Figure 10. These five categories were then formulated into the following five fuzzy sets (as shown in Figure 11) with bell shaped membership functions. The corresponding linguistic labels are: *Elliptical (E)*, *Fairly Elliptical (FE)*, *Flat Round (FLTR)*, *Fairly Round (FR)*, and *Round (R)*. The membership function of each fuzzy set and the degree of overlap were developed using the engineer's interpretation of the FE outputs. These represent the way experts' reason about the roundness of the rod profile.

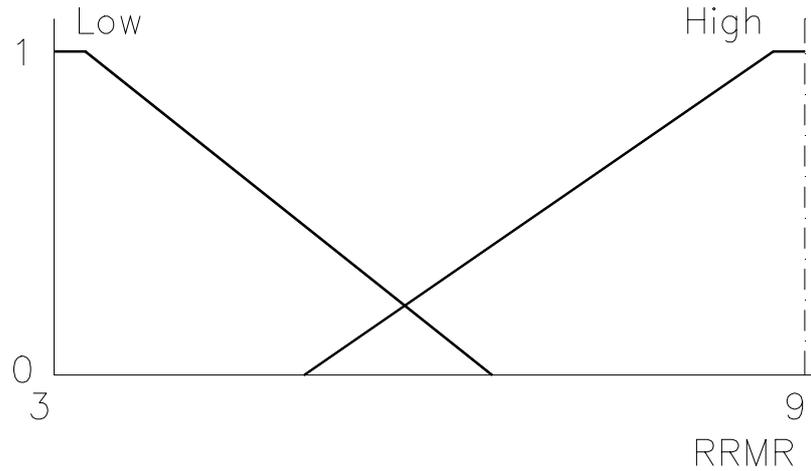


Figure 8: Membership Functions for Fuzzy Set RRMR

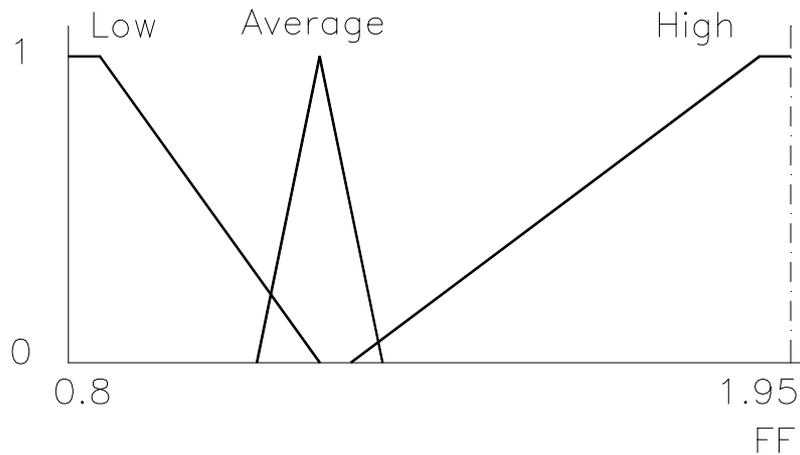


Figure 9: Membership Functions for Fuzzy Set Form Factor

A rule base that specifies the  $Q^L$  relationship between the output parameter (shape condition) and the input parameters: initial stock area/roll area (SAR), form factor (FF) and the roll radius/material height ratio (RRMR) was formulated as shown in Table 4. These rules were developed by interactive interview with the domain experts. For example, rule 1 shows that if the area ratio is *average*, the RRMR is *low*, and the form factor is *low* then rod profile is predicted as '*round*'.

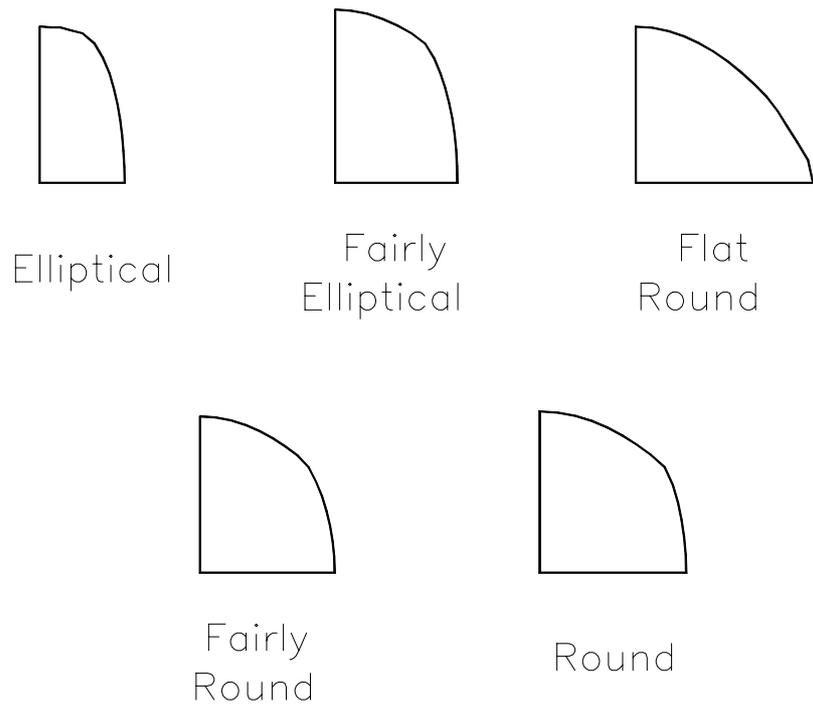


Figure 10: Classification of FE Rod Shape Profiles

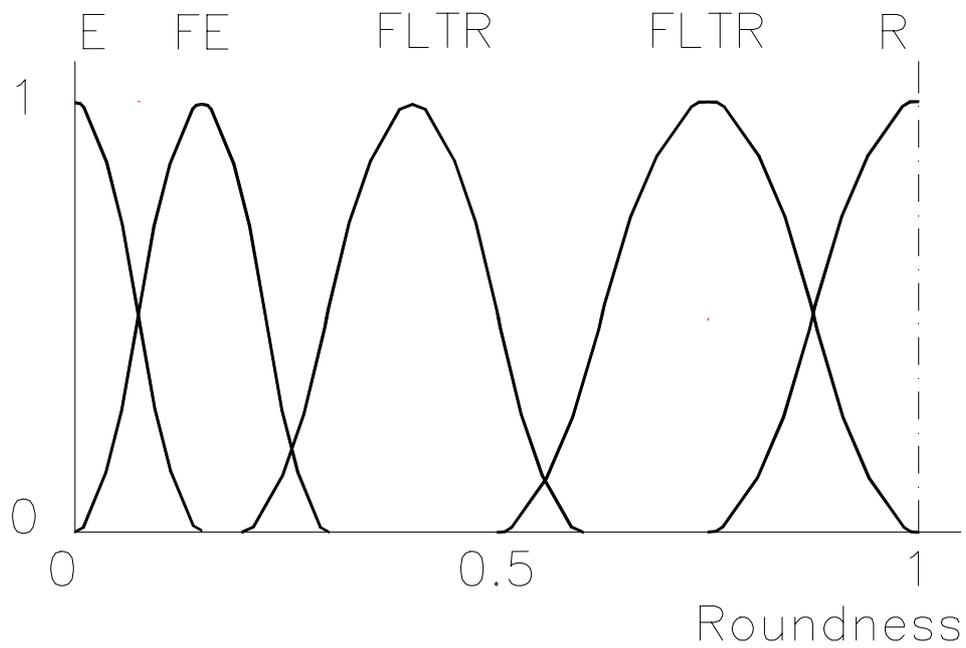


Figure 11: Membership Functions for Roundness

Table 4: Fuzzy Rule Base

Rule no	SAR	RRMR	FF	Roundness
1	Ave	L	L	R
2	H	H	L	FLTR
3	H	L	L	FLTR
4	Ave	H	H	FE
5	Ave	H	Ave	FR
6	L	L	H	E
7	Ave	L	H	E
8	L	H	H	E
9	Ave	H	L	R

The compensatory weighted mean operator was used to aggregate the fuzzy sets in the antecedent part of the rule. This ensures that the cumulative effect of the other rules influences the determination of the strain distribution. These fuzzy sets were then converted into a scalar value using the centroid method of defuzzification in the final step of the fuzzy inference cycle.

The fuzzy sets, input, output fuzzy variables and fuzzy rule base all constitute the  $Q^L$  model that is used within the optimisation module to evaluate the  $Q^L$  aspect (shape condition) of the design problem. These fuzzy sets are then converted into a scalar value by a chosen method of defuzzification in the final step of the fuzzy inference cycle. A centroid method of defuzzification is used in this study. The defuzzified scalar value best represents the fuzzy solution sets.

### 6.3. Definition Of The Optimisation Problem

The rod design problem is a two objective optimisation problem. The aim of this module is to solve a two objective rod design optimisation problem using simplified method of dealing with the membership function. The design problem consists of two cardinal objectives: to maximise the shape of the rod profile using  $Q^L$  models of the rod profile and minimise the deformation load using the  $Q^T$  model. The multi-objective optimisation problem is formulated as shown below:

$$\text{Minimise Load} \quad f_1(\mathbf{x}) = P \quad (5)$$

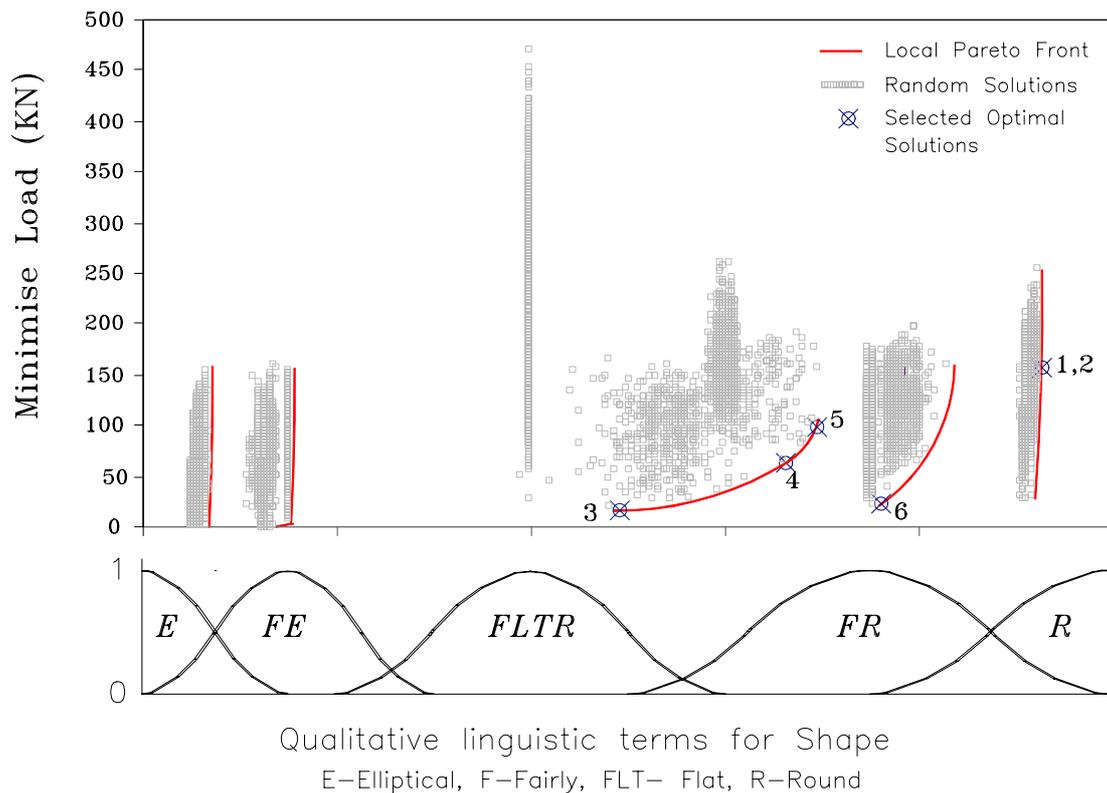
$$\text{Maximise Shape} \quad f_2(\mathbf{x}) = \tilde{A}(\mathbf{x}) \quad (6)$$

$$\text{Subject to} \quad P > 0 \quad (7)$$

$$\mu_{\tilde{A}}(\mathbf{x}) > 0.5 \quad (8)$$

where fuzzy terms are denoted by the tilde,  $\mu_{\tilde{A}}(x)$  is the membership grade of the shape condition.  $P$  is the  $Q^T$  models given by equation 1 in Section 0. Equation 7 is a constraint that ensures the deformation load is not negative while Equation 8 controls the influence of the membership function values on the search space. These constraints were dealt with using the penalty function method.

NSGAI [Deb *et al.* 2000] was adopted in the study since it is one of the most popular multi-objective GA. NSGAI was used to rank each member of the population in terms of the fitness from the  $Q^T$  model and the  $Q^L$  model. Fitness from the  $Q^L$  model consists of defuzzified scalar values and the associated membership grade from the fuzzy inference mechanism. This describes the shape condition of the rod profile and the deformation load for the rod design. Solutions having membership grades below a chosen threshold (0.5 in this study) are considered infeasible for the rod problem and are replaced by feasible solutions obtained by conducting a local search. Similarly, fitness from the  $Q^T$  models expresses the positive load deformation.



### Maximise Roundness

Figure 12:  $Q^T$  and  $Q^L$  Search Space of Design Problem

## 7. Test Results And Discussion

The proposed approach was used to optimise two objectives: the maximisation of the fuzzy output values (defuzzified domain value and the associated membership grade) and the minimisation of the deformation load. The  $Q^T$  and  $Q^L$  models outlined in Section 6.2 were used for this purpose. The proposed algorithm was implemented for the test problem using C++ code on a Pentium 4 PC. The performances of the NSGAI for different values of crossover probability (CP) and mutation probability (MP) were first investigated. Ten independent GA runs were performed in each case using a different random initial population. A population size of 100 was used with a total of 100,000 iterations. In most of the cases examined, seven out of ten runs obtained similar results.

A random search was conducted on the problem to explore the nature of the search space. Figure 12 shows the search space of the multi-objective  $Q^T$  and  $Q^L$  search space in this study. Due to the discrete nature of the search space, five local Pareto fronts were identified. Here the functional relationship of the  $Q^T$  and  $Q^L$  objectives is validated with respect to the theorem given in Section 5.

Table 5: Rod design solutions for P1

No	(h)	(w)	Pass Rad	Roll Gap	Roll Rad	Temp	Shape	Load (kN)
1	35.0	20.1	27.0	4.1	145.0	779.7	FR/R	162
2	32.9	19.6	26.1	4.7	215.5	796.9	FR/R	162

From Table 5, two different design solutions 1 and 2 support P1. For two identical  $Q^T$  evaluations the corresponding  $Q^L$  evaluations are also equal (*FR/R*).

Table 6: Rod design solutions for P2 and P3

No	(h)	(w)	Pass Rad	Roll Gap	Roll Rad	Temp	Shape	Load (kN)
3	28.2	18.4	24.3	2.6	162.2	1071.9	FLTR	14.0
4	31.1	16.0	17.4	4.5	198.3	958.4	FR	138.7
5	28.9	20.6	18.5	5.3	272.2	1061.6	FR	53.2

Table 6 shows that the selected solutions 3, 4 and 5 (also as shown Figure 12) supports P2 the functional relationship between  $Q^T$  and  $Q^L$  information. According to P2, two different designs 3 and 4 with different  $Q^T$  evaluations 14 and 138.7 respectively corresponds to different  $Q^L$  evaluations *FLTR* and *FR* respectively. Solutions 4 and 5 also satisfies the corresponding identical  $Q^L$  evaluation *FR* given two different designs where the  $Q^T$  evaluations are unequal (138.7, 53.2). This example also supports P3.

Table 7: Rod design solutions for P4

No	(h)	(w)	Pass Rad	Roll Gap	Roll Rad	Temp	Shape	Load (kN)
1	38.2	14.9	20.7	1.0	205.2	827.8	FR/R	162.3
6	30.9	15.5	21.5	2.8	275.6	920.6	FR	120.1

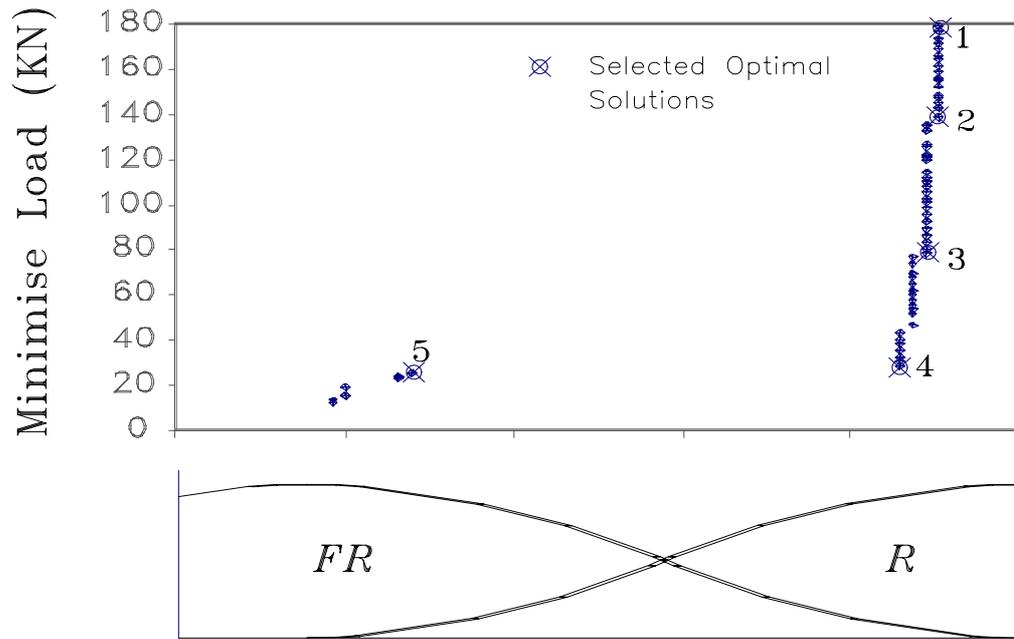
Table 7 shows that the selected solutions 1 and 6 (also as shown Figure 12) supports P4 for the functional relationship between  $Q^T$  and  $Q^L$  information. According to P4, two different designs 3 and 4 with different  $Q^T$  evaluations 162.3 and 120.1 respectively corresponds to different  $Q^L$  evaluations *FR/R* and *FR* respectively.

This illustration confirms the functional relationship between the  $Q^T$  and  $Q^L$  objectives, and it also indicates the conflicting nature of this relationship. This conflicting behaviour and the two-objective cardinality therefore confirm the appropriateness of the multi-objective solution approach. In the sections that follows, the NSGAII results of the multi-objective problem and some of the challenges poised by optimising within integrated  $Q^T$  and  $Q^L$  search space are discussed.

### 7.1. Experimental Results

The trade-off solutions between roundness and load located in the optimal region by the NSGA II optimisation algorithm is shown in Figure 13. Despite the complexity of the problem, NSGAII was able to find solutions in the optimal region of the design space. Non-dominated solutions were obtained from the experimental runs. The Pareto optimal solution plot shows the spread of the optimal solutions in the two dimensions. Table 8 also shows a selection of optimal solutions and their variable values from the experimental runs. It demonstrates the diversity of the vectors of the decision variables in the parameter space. Since solutions on these fronts are all equally good, further higher level criteria could be applied to select a suitable solution for the problem.

The selected solutions provides insight into the solution space both in the parameter and the objective space which is useful for designers to select suitable solutions for the given problem. For example, solutions number 3 and 5 has a roundness and load value of (round , 78.22 kN) and (flat round, 26.12 kN) respectively. Since both solutions are equally good, designers might prefer to tradeoff solution in the parameter space by selecting solution 3 based on lower temperature (728.1°C). Similarly, designers might prefer to tradeoff solution in the objective space by selecting solution 5 based on lower deformation load (26.12 kN).



Qualitative linguistic terms for Shape  
 F—Fairly, R—Round

### Maximise Roundness

Figure 13: NSGA II Pareto Solution Plot

Table 8: Selected Solutions

No	(h)	(w)	Pass Rad	Roll Gap	Roll Rad	Temp	Shape	Load (kN)
1	29.1	22.2	30.1	4.3	254.2	769.3	R	179.55
2	28.1	21.2	30.4	4.3	254.2	731.5	R	138.5
3	28.1	22.2	30.4	3.6	114.4	728.1	R	78.2
4	29.5	19.2	30.9	0.9	119.8	1070	R	28.6
5	27.8	19.1	26.8	1.1	162.6	1062	FR	26.1

## 7.2. Limitations

There are four main issues that reflect the limitations of the proposed approach to design optimisation problems with  $Q^L$  evaluation:

- Visibility of the  $Q^L$  parameter space to the search algorithm is lost due to the transformation of the  $Q^L$  information into cardinal information. As a result, it becomes difficult to control the equivalent correlation between granularity of the  $Q^L$  models with the measurement scale of the  $Q^T$  models.

- The approach is mostly suitable for real world problems with lower number of objectives, as higher number of  $Q^L$  objectives has the tendency to increase the fragmentation in the search space. This is largely due to the discreteness in the  $Q^L$  search space.
- The approach mainly deals with  $Q^T$  and  $Q^L$  information that are conflicting in nature. It is not suitable for mixed form of information that are complementary in nature. This limitation is due to the fundamental solution strategy adopted in the approach.
- Since this approach inherently supports multiple problem representation from different perspective, it is important to ensure that the original problem structure is preserved in the sub-system models. This can ensure that the optimum solution achievable by aggregating the sub-system models is closely identical to the system if it were to be wholly solved. However, it can be quite challenging to develop strategies that help preserve the global characteristics of the original problem.

### 7.3. Future Research

Future research activities are required to address the main limitations described in the previous section and the challenges outlined in Section 2. This section briefly describes main research directions.

- Studies are required to develop optimisation algorithms that can deal with various combinations of  $Q^T$  and  $Q^L$  information in a single framework. This is a desirable feature to improve the robustness of such techniques for real world problem since the nature problem space in most case can be unknown.
- Scalability of integrated  $Q^T$  and  $Q^L$  design optimisation strategies to higher dimensional problems is an important success criteria for wider applications. This is influenced by the feature of the problem (large number of parameters) and the nature of the resulting search space (fragmentation). This is due to the discontinuity present in real world problems and  $Q^L$  design space. Therefore it is hoped that studies in these areas presents an interesting line of research.
- Techniques are required for representing the native parameter space of the  $Q^L$  information within the optimisation framework. This could provide capabilities for tuning the correlation between the granularity of the  $Q^L$  models with the measurement scale of the  $Q^T$  models. Search algorithm that considers such features of the problem should give better performance.

## 8. Conclusions

Most real world engineering design problems can be  $Q^T$  and  $Q^L$  in nature. A review of the literature reveals that most optimisation algorithms are not capable of dealing with such mixed information simultaneously within a design optimisation framework. This

tends to restrict optimisation techniques to problems with only  $Q^T$  information and inhibits the wider application optimisation techniques to real world problems.

This report proposes a methodology to deal with the challenges posed by integrated  $Q^T$  and  $Q^L$  search space in real world optimisation problems. The mathematical proof of the solution strategy was also presented. A case study based on multi-objective rod rolling problem was presented to validate the effectiveness of the proposed methodology.

The results obtained show  $Q^T$  solutions and their functional relationships with the  $Q^L$  evaluations in the optimal region of the search space. This demonstrates that the proposed solution approach can be used to solve real world problems having integrated  $Q^T$  and  $Q^L$  information.

## Acknowledgements

The authors wish to acknowledge the support of the Engineering and Physical Sciences Research Council (EPSRC) and Corus R, D and T, Swinden Technology Centre, UK.

## References

- Antonsson, E. K. and K. L. Wood (1989) Computations with Imprecise Parameters in Engineering Design: Background and Theory. *Journal of Mechanisms, Transmissions and Automation in Design* 111: 616-625.
- Bellman, R. E. and L. A. Zadeh (1970) Decision Making in a Fuzzy Environment. *Management Science* 17(4): B141-B164.
- Bottazzini, U. (1986) *The Higher Calculus: A History of Real and Complex Analysis from Euler to Weierstrass*, Springer-Verlag.
- Chakraborti, N. and A. Kumar (2003) The Optimal Scheduling of a Reversing Strip Mill: Studies Using Multi-population Genetic Algorithms and Differential Evolution. *Journal of Material Processing Technology* Submitted for publication.
- Chung, J. S. and S. M. Hwang (2002) Process Optimal Design in Forging by Genetic Algorithm. *Journal of Manufacturing Science and Engineering* 124: 397-408.
- Dahal, K. P., C. J. Aldridge and J. R. McDonald (1999) Generator maintenance scheduling using genetic algorithm with a fuzzy evaluation function. *Fuzzy Sets and Systems* 102: 21-29.
- Deb, K., S. Agrawal, A. Pratap and T. Meyarivan (2000) A Fast Elitist Non-dominated Sorting Genetic Algorithm for Multi-objective Optimization: NSGA-II. *Parallel Problem Solving from Nature VI (PPSN-VI)*, 849-858.
- Fonseca, C. M. and P. J. Fleming (1993) Genetic Algorithms for multiobjective optimization: Formulation, discussion and generalization. *Proceedings of the Fifth International Conference*, Morgan Kaufmann, San Mateo, CA.: 416-423.
- Friedman, L. W. (1996) *The Simulation Metamodel*. Dordrecht, Netherlands, Kluwer.
- Goldberg D. E (1989) *Genetic Algorithm in Search, Optimization and Machine Learning*. Massachusetts, Addison Wesley.

- Hsu, Y., T. Liu, M. Hsu and F. Thibault (2001) A fuzzy optimization algorithm for blow moulding process. 6th International Conference on CSCW Design, Ontario, Canada.
- Hwang, S. M. and J. S. Chung (1997) Application of genetic algorithm to optimal design of the die shape in Extrusion. *Journal of Materials Processing Technology* 72: 69-77.
- Jensen, H. A. and A. E. Sepulveda (2000) "Use of approximation concepts in fuzzy design problems." *Advances in Engineering Software* 31: 263-273.
- Jung, J. Y. and Y. Im, T. (2000) Fuzzy algorithm for calculating roll speed variation based on roll separating force in hot rolling. *International Journal of Mechanical Sciences* 42(2): 249-272.
- Kang, B.-S., J.-H. Lee and J.-H. Lee (1996) Process design in multi-stage cold forging by the finite element method. *Journal of Material Processing Technology* 58: 174-183.
- Kim, N. S., S. Kobayashi and T. Altan (1991) Three-dimensional analysis and computer simulation of shape rolling by the finite and slab element method. *International Journal of Machine and Tool Manufacture*(31): 553-563.
- Kleijnen, J. P. C. (1975) A comment on Blanning's metamodel for sensitivity analysis: the regression metamodel in simulation. *Interfaces* 5(3): 21-23.
- Knowles, J. D. and C. D.W. (1999) The Pareto archived evolution strategy: A new baseline algorithm for multiobjective optimisation. *Proceedings of the 1999 Congress on Evolutionary Computation*, 98-105, New Jersey.
- Koskimaki, E. and J. Goos (1996) Fuzzy Fitness Function for Electric machine Design by Genetic Algorithm. *Proceedings of the Second Nordic Workshop on Genetic Algorithms and Their Applications (2NWGA)*, 237-244, Vaasa, Finland.
- Lenard, J. G., A. Said, A. R. Ragab and M. Abo Elkhier (1997) The Temperature, roll force and roll torque during hot bar rolling. *Journal of Material Processing Technology*: 147-153.
- Medaglia, A. L., S.-C. Fang and H. L. W. Nuttle (2002) Fuzzy controlled simulation optimization. *Fuzzy Sets and Systems* 127: 65-84.
- Mirjam, N., A. Vonk Noordegraaf and J. Kleijnen, P, C (2003) Sensitivity analysis by experimental design and metamodeling: Case study on simulation in national animal disease control. *European Journal of Operational Research* 146(3): 433-443.
- Montgomery, D. C. (1997) *Design and Analysis of Experiments*, John Wiley & Sons.
- Montgomery, D. C. and E. A. Peck (1992) *Introduction to Linear Regression Analysis*. New York, Wiley.
- Moore, R. E. (1979) *Methods and applications of interval analysis*, SIAM, Philadelphia.
- Mori, K. (1990) General purpose fem simulator for 3-d rolling. *Advanced Technology of Plasticity* 4: 1773-1778.
- Myllykoski P. Larkiola J. Korhonen A.S. and Cser L. (1998) The role of neural networks in the optimisation of rolling processes. *Journal of Material Processing Technology* 80-81: 16-23.
- Oduguwa, V. and R. Roy (2003 (submitted for publication)) An Integrated Design Optimisation approach for Quantitative and Qualitative Search Space. *Proceedings of ASME: 2003 ASME Design Engineering Technical Conference, Chicago, Illinois*, (Accepted for publication).
- Oduguwa, V., R. Roy and D. Farrugia (2003) Fuzzy Multi-Objective Optimisation Approach for Rod Shape Design in Long Product Rolling. *Fuzzy Sets and Systems* -

- IFSA 2003, 10th International Fuzzy Systems Association World Congress, 636-643, Istanbul, Turkey, Springer-Verlag.
- Oduguwa, V., A. Tiwari and R. Roy (2003) Genetic Algorithm in process optimisation. CDROM Proceedings of the 8th Online World Conference on Soft Computing in Industrial Applications (WSC-8), Finland (accepted for publication).
- Pataro, C., D. M, and H. Helman (1999) Direct determination of sequences of passes for the strip rolling process by means of fuzzy logic rules. Proceedings of the Second International Conference on Intelligent Processing and Manufacturing of Materials. IPMM '99, 549 -554.
- Reklatis, G. V., A. Ravindran and K. M. Ragsdell (1983) Engineering Optimisation Methods and Applications, John Wiley and sons.
- Roy, R. (1997) Adaptive Search and the Preliminary Design of Gas Turbine Blade Cooling System, PhD Thesis, 1997, University of Plymouth: Plymouth.
- Schaffer J.D. (1985) Multiple Objective Optimization with Vector Evaluated Genetic Algorithms. First International Conference on Genetic Algorithms, 93-100, Pittspergh.
- Shin, H. W., D. W. Kim and N. S. Kim (1994) A Study on the Rolling of I-Section Beams. International Journal of Machine and Tool Manufacture 34(147-160).
- Shivpuri, R. and S. Kini (1998) Application of Fuzzy Reasoning Techniques for Roll Pass Design Optimisation. Proceedings of the 39th Mechanical Working and Steel Processing Conference and Symposium on New Metal Forming Processes, 755-763.
- Siddall, J. N. (1983). Probabilistic Engineering Design. New York, Marcel Dekker.
- Tanaka, H., T. Okuda and K. Asai (1974) On fuzzy mathematical programming. Journal of Cybernetics 3: 37-46.
- Tiwari, A. (2001) Evolutionary computing techniques for handling variable interaction in engineering design optimisation, PhD Thesis, 2001, Cranfield University: Cranfield,UK.
- Veldhuizen, D. V. (1999) Multiobjective Evolutionary Algorithms: Classifications, Analysis, and New Innovations, PhD Thesis, 1999, Air Force Institute of Technology: OH: Dayton.
- Wu, C. C. and N. B. Chang (2003) Global strategy for optimizing textile dyeing manufacturing process via GA-based grey nonlinear integer programming. Computers & Chemical Engineering 27: 833-854.
- Yoshida, S. and S. Kihara (1996) Some experiences of shape-forming: experiments and numerical simulations. Engineering Computations 13(2/3/4): 98-110.
- Zimmermann, H.-J. (1974) Optimization in fuzzy environment. XXI International TIMS and 46th Conference, San Juan, Puerto Rico.
- Zitzler, E. and L. Thiele (1998) An evolutionary algorithm for multiobjective optimization: The strength Pareto approach. Zurich, Switzerland, Computer Engineering and Networks Laboratory.

