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Are the 1D and 2D Constitutive Equations of Piezoelectric Materials Right?

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Piezoelectric materials, having direct and converse piezoelectric effects, provide useful sensing and actuation functions and lead to a variety of macro/micro sensors and actuators devices. The sensors and actuators utilize either the longitudinal piezoelectric coefficient, e.g., thin film bulk acoustic wave resonators (FBAR), or transversal piezoelectric coefficient, e.g., unimorph and bimorph structures such as cantilevers, fixed-fixed beams, and membranes. Evaluation of such macro/micro piezoelectric device performance is basic and essential work and the right use of piezoelectric coefficients is the first step for design, analysis, and development. The following *e* or *d* form constitutive equations for *z*-polarized piezoelectric materials in 2D plate analyses are often used in literature and text books, which simply come from the 3D constitutive equations of piezoelectric materials without changes in stiffness, compliance, and piezoelectric coefficients:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} - \begin{bmatrix} e_{31} & e_{31} & 0 \end{bmatrix}^{\mathrm{T}} E_3 \tag{1}$$

$$D_3 = \begin{bmatrix} e_{31} & e_{31} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} + \varepsilon_3^{\varepsilon} E_3 \tag{2}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & 0 \\ s_{12} & s_{22} & 0 \\ 0 & 0 & s_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} + \begin{bmatrix} d_{31} & d_{31} & 0 \end{bmatrix}^{\mathrm{T}} E_3 \tag{3}$$

$$D_3 = \begin{bmatrix} d_{31} & d_{31} & 0 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} + \varepsilon_3^{\sigma} E_3 \tag{4}$$

Where σ_i is the component of stress vector, ε_i is the component of strain vector, c_{ij} is the component of stiffness matrix of piezoelectric materials measured at constant electrical fields, s_{ij} is the component of compliance matrix of piezoelectric materials measured at constant electrical fields, E_3 is the electric field applied in the *z*-direction, D_3 is the electric displacement generated in the *z*-direction, e_{ij} is the piezoelectric stress coefficient at constant mechanical strain, d_{ij} is the piezoelectric strain coefficient at constant mechanical stresses, ε_3^{σ} is the dielectric coefficient in the *z*-direction measured at constant mechanical stresses, $\varepsilon_3^{\varepsilon}$ is the dielectric coefficient measured at constant mechanical strains. Eqs. (1) and (2) represent that the piezoelectric coefficient, e_{31} or d_{31}, is responsible for 2D plate deformations only and there is no effect of e_{33} or d_{33} on 2D plate deformations. Surprisingly, we found that Eqs. (3) and (4) perfectly work for 2D plate analyses but Eqs (1) and (2) don’t.

The main motivation for this paper is to correct the mistake in literature and text books by derivations of right *e* and *d* forms of constitutive equations of *z*-polarized piezoelectric materials in 1D and 2D analyses in the cases that (1) 2D plate stress state, (2) 1D long thin beam, and (3) for 1D thick beam for macro/micro sensor and actuator designs and analyses, which contain reduced stiffness and enhanced piezoelectric coefficients. Three modified coefficients are proposed by introduction of the new concepts: reduced and enhanced effects in the constitutive equations, enabling to give understanding of how piezoelectric coefficients affect designs of sensors, actuators and generators (energy harvesting devices). The work is based on 3D constitutive equations and so applicable to the range of piezoelectric materials from bulk down to thick/thin film.