

Author's Accepted Manuscript

Warranty claim analysis considering human factors

Shaomin Wu

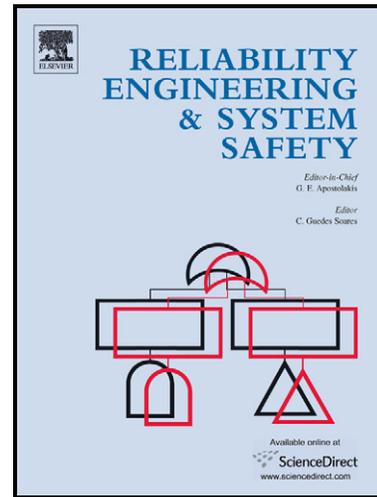
PII: S0951-8320(10)00183-3
DOI: doi:10.1016/j.ress.2010.07.010
Reference: RESS 4361

To appear in: *Reliability Engineering and System Safety*

Received date: 7 December 2009
Revised date: 26 July 2010
Accepted date: 29 July 2010

Cite this article as: Shaomin Wu, Warranty claim analysis considering human factors, *Reliability Engineering and System Safety*, doi:[10.1016/j.ress.2010.07.010](https://doi.org/10.1016/j.ress.2010.07.010)

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting galley proof before it is published in its final citable form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



www.elsevier.com/locate/ress

Warranty claim analysis considering human factors

Shaomin Wu¹

Cranfield University, School of Applied Sciences, Cranfield, Bedfordshire MK43 0AL, UK

Abstract

Warranty claims are not always due to product failures. They can also be caused by two types of human factors. On the one hand, consumers might claim warranty due to misuse and/or failures caused by other human factors. Such claims might account for more than 10% of all reported claims; on the other hand, consumers might not be bothered to claim warranty for failed items that are still under warranty, or claim warranty after they have experienced several intermittent failures. These two types of human factors can affect warranty claim costs. However, research in this area has received rather little attention.

In this paper, we propose three models to estimate the expected warranty cost when the two types of human factors are included. We consider two types of failures, intermittent and fatal failures, which might result in different claim patterns. Consumers might report claims after a fatal failure has occurred, and upon intermittent failures they might report claims after a number of failures have occurred. Numerical examples are given to validate the results derived.

Keywords: warranty claim, non-failed but reported (NFBR), failed but not reported (FBNR), human factor, intermittent failure, fatal failure.

Nomenclature

$\lambda_1(t)$	Intensity function of a non-homogeneous Poisson process of fatal failures
$\lambda_2(t)$	Intensity function of a non-homogeneous Poisson process of intermittent failures
$F_2(t)$	Cumulative distribution function for lifetime t due to intermittent failures
p_{2k}	Probability that the cause of an intermittent failure at the k th warranty claim is not successfully detected
$S_2(t)$	Probability of successfully identifying and then repairing the cause of intermittent failures at time t
$H_3(t)$	Cumulative distribution function due to a NFBR claim
$q_1(t)$	Probability of a claim being made at time t , given that a fatal failure has occurred
q_2	Probability that an intermittent failure results in a warranty claim
c_1	Cost on a claim due to a fatal failure
c_2	Cost on detecting the cause of intermittent failures
\tilde{c}_2	Cost on fixing the cause of an intermittent failure
c_{20}	Cost on detecting and fixing the cause of intermittent failures per unit time
c_{31}	Administration cost on per NFBR claim
c_{32}	Expected cost on fixing the cause of an NFBR claim
w	Length of warranty periods

¹ Email address: shaomin.wu@cranfield.ac.uk. Telephone: +44 1234 754 963.

1. Introduction

A warranty is a contractual obligation incurred by a manufacturer (vendor or seller) in connection with the sale of a product. In broad terms, the purpose of warranty is to establish liability in the event of a premature failure of an item or the inability of the item to perform its intended function [1]. Product warranty has become increasingly more important in consumer and commercial transactions, and is widely used to serve many different purposes [2]. The US Congress has enacted several acts (UCC, Magnusson Moss Act, Tread Act, etc.) over the last 100 years. The European Union (EU) passed legislation requiring a two-year warranty for all products sold in Europe [3].

Analysing warranty claims can provide manufacturers with useful information on their products, as warranty claim data are collected from the field that reflects the real operating conditions and usage intensity. Research on analysing warranty claims data has mainly been concentrated on dealing with incomplete warranty claims data (see [4-6], for example) and developing improved techniques to model warranty claims data (see [7-10], for example). After the field reliability of products has been estimated, warranty servicing cost analysis becomes another topic that needs to be focused. In this area, optimising warranty policies under different cost settings (see [11-13], for example), and selecting maintenance policies for given warranty policies are two main research focuses (see [14,15], for example). For more detailed information on warranty claims data analysis and warranty servicing cost analysis, the reader is referred to the review papers [3,16-20] and the three books [19,21,22].

However, our literature review shows that most of the existing research makes the following two assumptions:

- (i) failed products will be reported warranty, and
- (ii) claims reported are due to product failures.

The above two assumptions do not necessarily hold and are associated with consumers' behaviours towards warranty claims.

On Assumption (i), most of the research assumes that an absence of warranty claim is a 'no failure' situation [2,5,23-26]. Only two publications, from the same authors, consider the cases where all of reported products are not failed (see [27,28], for example). Patankar and Mitra [27,28] consider consumer behaviour in exercising warranties and describe it with warranty execution functions. They assume that all consumers may not exercise the warranty even if the product fails during the warranty period, or FBNR (failed be not reported).

For Assumption (ii), most authors assume that reported claims are due to product failures. Little research considers situations where reported products might be due to misuse, other human factors, or even non-failed products. In this paper, all of such claims are called non-failed but reported (NFBR) claims.

Consumers might also execute warranty claims after they have experienced a number of intermittent failures. An item might stop working due to a fatal failure or an intermittent failure. A product with a fatal failure will stop working until it is repaired. In practice, not all of failures are fatal; some are intermittent. An intermittent failure is the loss of some functions or performance characteristics of a product for a limited period of time until subsequent recovery of the function. In the case of intermittent failures, consumers may experience

a failure and restart the product (for example, computers) and it runs OK. When the product is taken to a service agent, the repairman might not experience this failure when the item is being detected. The claims due to intermittent failures can constitute a quite large proportion of the entire claim population, as the percentage of the “no-fault found” (NFF) event can be as high as 50% of all failures in electronic products while intermittent failure is one of the main causes of NFF [29]. An intermittent failure example can be as follows.

A global variable in an electronic product is read and rewritten over another global variable; a miscalculation can then arise and lead to product failure. The users can therefore mistakenly believe that the product has failed and claim warranty, the product is then sent to a service agent. However, when the global variables are reset, perhaps upon rebooting the computer, the product can return to normal function.

Although intermittent failures are a main cause of failures, little research has been found to model their warranty costs.

In this paper, we derive warranty costs for three situations: NFBR claims, FBNR phenomenon, and their combination, and we assume two types of failures: fatal and intermittent failures.

The novelty of this work lies in:

- to our knowledge, it is the first paper modelling warranty claim cost due to NFBR claims;
- it considers manufacturer’s ability to rectify intermittent failures and assume such ability is increasing over time; and
- warranty claims are usually considered for individual products. This paper assumes that a manufacturer’s ability to fix the cause of intermittent failures develops over a batch of products.

The paper is structured as follows. Section 2 discusses human factors in warranty claims. Section 3 develops three models considering both physical reliability and human factors for repairable products. Section 4 offers numerical examples to validate the models developed above. Section 5 concludes the findings.

2. Human factors in warranty claims

In this section, we discuss three factors that might cause warranty claims: non-failed but reported (NFBR), failed but not reported (FBNR), and claims arising from intermittent failures.

2.1 Non-failed but reported (NFBR) claims

There are situations when consumers might claim warranty, although failures are not due to product reliability but due to human factors. Some examples are as follows.

- Claims for failures due to misuse, damage, accident, neglect, or lack of care. For example, a consumer by accident poured water into a laptop computer, which makes the computer failed. The consumer then claims warranty. If a product is damaged by human factors, consumer can be responsible for part of fees, for example, returning and shipping fee.
- Fraudulent claims. Consumer might be driven by warranty or insurance claims, and replace multiple items to repair one fault or resort to fraudulent reporting of a problem which never occurred. For example, the AAA insurance stated “At least 10 percent of all reported claims are fraudulent in some way, according to industry reports” [29].

- Some other forms of such claims can arise due to various reasons such as complexity of products, lack of sufficient training on product usage or faulty operational manual, product users might not be able to operate the products correctly, all of which can cause the products unable to work properly. The users can therefore mistakenly believe that the products have failed, and then claim warranty.

Here, we refer non-failed but reported claims (NFBR) to those claims that are purely due to human factors. It differs from claims upon intermittent failures that can occur but cannot be verified, replicated at will, or attributed to a specific failure site, mode, and mechanism. It also differs from claims arising from fatal failures that are due to products themselves but not human factors.

Responses of the manufacturers to NFBR claims can be different: (1) some manufacturers might even cease the warranty contract with consumers with NFBR claims, and (2) some manufacturers might not cease the warranty contract, as it is not easy to judge if a NFBF claim is intentionally or unintentionally committed.

However, a common feature is that both can incur costs to the manufacturers, and therefore should be considered in estimating warranty claim cost.

2.2 Failed but not reported (FBNR) events

Failed items might not be reported warranty. This can happen due to various reasons, for example, technological advances cause some products (especially electronic products) to be updated frequently or become obsolete quickly. Although such items are protected by warranty contracts, their users might not bother to claim warranty for failed items, especially when failed items have been served for a quite long time and/or they are not expensive. We call such a phenomenon as Failed But Not Reported (FBNR). Patankar and Mitra [27,28] investigate consumer behaviour when they do not execute the full execution of warranty. They listed a number of influencing factors such as costs of executing the warranty, the type of rebate plan, etc.

There is a trend towards long-term warranties [20]. Rapid technological advances in many industries, especially the electronics manufacturing industry, make products obsolete quicker than before, which requires product manufacturers to provide long-term warranties to protect consumers' profits. Long term warranty and complex products will make FBNR events occur more often than short term warranty. This presents an incentive to estimate warranty claims for the FBNR claims.

2.3 Warranty claims arising from intermittent failures

As intermittent failures might involve more testing to find the causes of the failures, costs on intermittent failures are different from those on fatal failures. Consumers might also claim warranty after intermittent failures have occurred for several of times. The claim patterns upon intermittent failures are therefore different from those claims arising from fatal failures. Hence, it is vitally important to consider consumer behaviour in analysing warranty claim data. For more discussion on intermittent failures or NFF, the reader is referred to [30,31], in which the authors discuss the concept, causes and impact of the "trouble not identified" phenomenon in the electronics industry, but they did not develop mathematical models to estimate warranty claims cost.

From a manufacturer's perspective, the ability to identify troubles arising from intermittent failures develops when more and more intermittent failures are investigated. It is reasonable to assume that the

probability of successfully detecting the causes of intermittent failures is increasing over time or over the number of claims. Such improvement might not affect the number of warranty claims as products have already been sold to consumers. The cost on repairing the intermittent failures, however, can decrease.

2.4 Warranty costs due to human factors

Normally, warranty cost incurs due to

- labour on diagnosing and repairing the failure;
- parts/materials used to repair the failed product;
- shipping including shipping new parts for replacement and/or failed products for repair.

Costs incurred by the above three types, claims upon NFBR claims, FBNR events, and intermittent failures differ from those claim costs due to fatal failures.

From a manufacturer's perspective, costs due to NFBR might only include costs on reporting (for example, delivering the non-failed products to their manufacturers), and cost due to FBNR might be zero as no report is conducted for a failed product.

In developing warranty claims models considering human factors such as NFBR and FBNR, an important requirement is that we should be able to differentiate claims due to NFBR from those due to FBNR in the models. This is because we need to estimate the cost on claims due to NFBR and FBNR.

The impact of intermittent failures can be profound. Due to their characteristics, manufacturers may assume a cause(s) rather than spend the time and cost to determine a root-cause. This can result in increased maintenance costs, decreased equipment availability, increased consumer inconvenience, reduced consumer confidence, damaged company reputation, and in some cases potential safety hazards [29].

The probability distributions of NFBR claims and FBNR events also have their own characteristic features. The proportion of NFBR claims in the whole product population might decrease over time since the products have entered service, whereas the proportion of FBNR events might increase over time. The NFBR for repairable products might seldom occur after the products have failed and repaired once, because the users can be assumed to have learnt how to operate the products from this failure and shall not make more mistakes of reporting non-failed products. It is therefore reasonable to assume that a consumer makes at most one NFBR claim.

3. Model development

Suppose that the following general assumptions hold.

- (1) Two types of failures are considered: fatal and intermittent failures. Fatal failures require rectification to restore the products to operational state, and intermittent failures do not require rectification action to make it operational but need action to rectify the cause of such failures. The occurrences of fatal and intermittent failures are assumed to be statistically independent.
- (2) Three types of claims are considered: claims upon fatal failures, claims upon intermittent failures, and claims arising from NFBR events.

- (3) Time on repair is negligible. Repairs on fatal failures are minimal, that is, a product with a fatal failure is restored to the state where it was exactly before it failed. If an intermittent failure of a product cannot be verified, an identical product with the same age as the failed one will be used to replace it.
- (4) An individual consumer makes at most one NFBR claim. Upon NFBR claims, only administration cost is incurred to the manufacturer.
- (5) Only non-renewing warranty policy is considered.

In the rest of this section, we consider warranty costs for three situations: NFBR claims, FBNR phenomenon, and their combination. In all of the three situations, we assume two types of failures: fatal and intermittent failures.

3.1 Expected cost with fatal failure, intermittent failure, and NFBR

This section derives the expected warranty claim costs.

3.1.1 Expected cost on fatal failures

Based on Assumption (3) above, time on repair is negligible and repairs on fatal failures are minimal. Then the expected warranty cost with only fatal failures is given by

$$WC_1(w) = c_1 \int_0^w \lambda_1(t) dt \quad (1)$$

3.1.2 Expected cost on intermittent failures

We assume that

- every intermittent failure results in a warranty claim;
- the service agent can either detect the cause or not; and
- once the cause of intermittent failures has been detected and fixed, failures due to this cause will not occur again.

It should be noted that a manufacturer might receive warranty claims due to intermittent failures reported by different consumers, and then it tries to detect and fix the cause based on all of the claims. Hence, the manufacturer's ability to detect and further fix the cause develops over their experience learnt from treating all of claims. For this reason, it might not be correct to assume that the ability to detect and fix the cause of intermittent failures simply depends on claims from a single product/consumer. One should consider claims from all of the products sold. An alternative approach might be to assume that for an individual product, manufacturer's ability to detect and further fix the cause develops over time.

As such, we can consider the following two cases: the probability of successfully identifying and then repairing intermittent failures depends on (1) the number of claims due to intermittent failures; and (2) time.

In case that the probability of successfully identifying and then repairing intermittent failures is dependent on the number of claims, we can estimate the expected claim cost as follows.

Assume that n products are sold at the same date, and claims upon intermittent failures from all of the n products are reported according to a NHPP (nonhomogeneous Poisson process) or HPP (homogeneous Poisson process) with intensity function $\lambda_n(t)$. For example, if intermittent failures of an individual product occur

according to a HPP with intensity function is λ_0 , then $\lambda_n(t) = n\lambda$. Denote p_{2j} (where $p_{2j} > 0$ and $j \geq 1$) as the probability that the cause at the j th warranty claim is not detected. Then the probability of the first success in detecting the cause of the intermittent failures at the k th claim is $P_{2k} = (1 - p_{2k}) \prod_{j=1}^{k-1} p_{2j}$ with $k > 1$ (where $P_{21} = 1$ and $\sum_{k=1}^{\infty} P_{2k} = 1$). Note that k failures to occur in $[0, t)$ is given by the probability $[\Lambda(t)]^k / k!$, where $\Lambda_n(t) = \int_0^t \lambda_n(u) du$. Then the total expected warranty cost due to intermittent failures for an individual product is given by

$$WC_2(w) = \frac{1}{n} \sum_{k=1}^{\infty} \left([(k-1)c_2 + \tilde{c}_2] P_{2k} \frac{e^{-\Lambda_n(w)} [\Lambda_n(w)]^k}{k!} \right) + \frac{1}{n} \sum_{k=1}^{\infty} \sum_{m=k+1}^{\infty} \left([(m-1)c_2 + \tilde{c}_2] (1 - p_{2m}) \frac{e^{-\Lambda_n(w)} [\Lambda_n(w)]^k}{k!} \prod_{j=1}^{m-1} p_{2j} \right) \quad (2)$$

The first term in Eq. (2) is the expected warranty claim cost if the cause of intermittent failures is detected and fixed within warranty period; it implies that the manufacturer fails to detect the cause of the intermittent failures arising from the first $k-1$ claims but it is successful at the k th claim. The second term in Eq. (2) is the expected warranty claim cost if detecting the cause of intermittent failures has not been successful during warranty and it continues after warranty expires.

Remarks. In some cases, detecting the cause of intermittent failures might start from the first claim and from then such effort might continue until the cause is eventually detected and fixed or a new model of products is launched to replace the old ones. In this case, the probability of successfully detecting and then fixing the cause depends on time, instead of the number of intermittent failures. If we can set the time when the n products were sold to be 0, then the cumulative distribution function of time to the first failure (and then claim) is $F_2^{(n)}(t) = 1 - (1 - F_2(t))^n$. The probability that an intermittent failure occurs during the warranty period is given by $\int_0^w dF_2^{(n)}(t)$. If it occurs, the expected time length to fix or to a new generation is $\int_t^{T_n} u dS_2(u)$. Then the expected cost on warranty claims is given by

$$WC_{21}(w) = \frac{c_{20}}{n} \int_0^w \int_t^{T_n} u dS_2(u) dF_2^{(n)}(t) \quad (3)$$

where $S_2(t)$ is the cumulative distribution function of time to detect and remove the cause of intermittent failures from all of claims of n products, T_n is an estimated time when the manufacturer might give up trying to detect the cause (or the time when a new model of products is launched), and c_{20} is the cost on detecting and fixing the cause per unit time.

In what follows, we shall concentrate on $WC_2(w)$ in Eq. (2), $WC_{21}(w)$ in Eq. (3) will be analysed in our future work.

3.1.3 Expected cost on NFBR claims

We assume that time to a NFBR claim is a random variable Z with distribution function $H_3(t)$. We consider the following two scenarios.

Scenario 1-- A NFBR claim will not cause warranty to be ceased. Then the expected warranty cost is given by

$$WC_{31}(w) = c_{31}H_3(w) \quad (4)$$

where c_{31} is the administration cost per NFBR claim.

Scenario 2-- A NFBR claim will cause warranty to be ceased. Once the warranty ceases, there are no further costs to the manufacturer. Then the expected warranty cost given by

$$WC_{32}(w) = c_{32}H_3(w) \quad (5)$$

where c_{32} is the expected cost on fixing the cause of an NFBR claim

3.2 Model I --- combined effects from fatal, intermittent failures and NFBR claims

In this section, we assume that all failures are reported over the entire warranty period, and examine the combined effects from fatal failures, intermittent failures and NFBR, considering the two scenarios discussed in Section 3.1.3.

Scenario 1-- A NFBR claim will not cause warranty to be ceased. Then the expected warranty cost is given by

$$EC_{11}(w) = WC_1(w) + WC_2(w) + WC_{31}(w) \quad (6)$$

Scenario 2-- A NFBR claim will cause warranty to be ceased. If a NFBR claim occurs within warranty period, then the warranty ceases and no more claims on fatal failures or intermittent failures occur. The probability that this will occur is $H_3(z)$, where $z < w$, and the expected warranty claim cost is $WC_1(z) + WC_2(z)$ with $z < w$. If a NFBR claim occurs after warranty expires, then the expected warranty claim cost is $WC_1(w) + WC_2(w)$ with $z \geq w$.

On removing the conditioning we have the expected warranty cost

$$EC_{12}(w) = \int_0^w [WC_1(z) + WC_2(z) + c_{32}]dH_3(z) + [WC_1(w) + WC_2(w)](1 - H_3(w)) \quad (7)$$

3.3 Model II --- FBNR claims

Due to reasons such as technological advances, some products (especially electronic products) can become obsolete quickly. Although such products might sometimes be protected by a long-term warranty contract, their consumers might not claim warranty for failed products, especially when failed products have served for a quite long time and/or they are not expensive. In this section, we consider the situation when warranty claims are partially executed.

Upon fatal failures, the willingness of consumers to claim warranty might diminish with time. The probability of consumers being inclined to claim warranty for products due to a fatal failure is assumed to be $q_1(t)$, which is a decreasing function in time t .

Upon intermittent failures, the following three human factors need consideration.

- Consumers' willingness to claim warranty might diminish with time;

- Manufacturer's capability to identify the causes of intermittent failures is improving with time. Hence, cost on dealing with such claims can decrease.

Let q_2 denote the probability that an intermittent failure results in a warranty claim. Then we have a thinning process with intensity function (for warranty claims) given by $q_2\lambda_n(t)$. Hence, similar to the derivation of Eq. (2), we have the expected warranty cost given by

$$EC_2(w) = c_1 \int_0^w q_1(t)\lambda_1(t)dt + \frac{1}{n} \sum_{k=1}^{\infty} \left([(k-1)c_2 + \tilde{c}_2] P_{2k} \frac{e^{-q_2\Lambda_n(w)} [q_2\Lambda_n(w)]^k}{k!} \right) + \frac{1}{n} \sum_{k=1}^{\infty} \sum_{m=k+1}^{\infty} \left([(m-1)c_2 + \tilde{c}_2] (1-p_{2m}) \frac{e^{-q_2\Lambda(w)} [q_2\Lambda(w)]^k}{k!} \prod_{j=1}^{m-1} p_{2j} \right) \quad (8)$$

The first term in Eq (8) is the expected claim cost due to fatal failures. It considers both the probability of fatal failures and the probability of consumers being inclined to claim warranty for products due to these fatal failures. The meaning of the second and the third term are similar to those given for Eq. (2).

Patankar and Mitra [28] consider consumer behaviour in warranty execution and develop four warranty execution functions (WEFs), or called FBNR rates in this paper. Mathematically, these four WEFs can be categorised into the following two classes.

$$q_{11}(t, w_1, w, \varphi_1) = \begin{cases} 1 & 0 \leq t \leq w_1 \\ \frac{w - \varphi_1 w_1}{w - w_1} - \frac{(1 - \varphi_1)t}{w - w_1} & w_1 \leq t \leq w, \\ 0 & t > w \end{cases} \quad (9)$$

where $0 \leq \varphi_1 \leq 1$ and $0 \leq w_1 < w$, and

$$q_{12}(t, w_1, w, \varphi_2) = \begin{cases} 1 & 0 \leq t \leq w_1 \\ e^{-(t-w)/\varphi_2} & w_1 \leq t \leq w, \\ 0 & t > w \end{cases} \quad (10)$$

where $0 \leq w_1 < w$.

For fatal failures, the number of warranty claims, or WEF, is assumed to be

$$q_1(t) = e^{-\gamma_1 - \gamma_2 t}, \quad (11)$$

where $\gamma_1, \gamma_2 \geq 0$.

3.4 A hybrid model --- integrating both NFBR and FBNR cases

One can also combine both situations of NFBR claims and FBNR phenomenon and derive the expected cost as follows.

Scenario 1-- A NFBR claim will not cause warranty to be ceased. Then the expected warranty cost is given by

$$EC_{31} = EC_2(w) + WC_{31}(w) \quad (12)$$

Scenario 2-- A NFBR claim will cause warranty to be ceased. Then we have the following expected warranty cost.

$$EC_{32}(w) = \int_0^w [EC_2(z) + c_{32}]dH_3(z) + EC_2(w)(1 - H_3(w)) \quad (13)$$

3.5 Discussion

For the expected costs, we have the following special cases.

(1) If $p_{2j} = 1$, $q_1(t) = 1$, and $q_2 = 1$, where $j=1,2,\dots$, then the above expected costs can be obtained for the following situation, where

- all of failed products (including fatal and intermittent failures) are reported claims; and
- all of intermittent failures can be identified from the first instance.

(2) If $p_{2j} = 0$, $q_1(t) = 1$, and $q_2 = 0$, where $j=1,2,\dots$, then the above expected costs can be obtained for the following situations, where

- only fatal failure is considered; and
- all of failed products are reported claims.

(3) If $c_1, c_2 + \tilde{c}_2, c_{20}, c_{31}$, and c_{32} are set to 1, then the expected costs in Eqs. (1) -- (8) become the expected numbers of warranty claims for corresponding scenarios, respectively.

Apart from the human factors considered above, Rai and Singh [6,32] consider the fact that consumers experiencing non-critical failures might delay reporting of warranty claims till the coverage is about to expire, which can introduce a bias into the dataset.

4. Numerical data analysis

Assume that

$$\lambda_1(t) = \frac{\xi_2}{\xi_1} \left(\frac{t}{\xi_1}\right)^{\xi_2-1}, \quad (14)$$

$$\lambda_2(t) = \frac{t}{v_1}, \quad (15)$$

$$H_3(t) = 1 - e^{-(t/\alpha_1)^{\alpha_2}}, \quad (16)$$

$$p_{2j} = \eta^j \quad (17)$$

and set the values of the parameters in the above equations as in Table 1.

Table 1. A list of parameters

c_1	c_2	\tilde{c}_2	c_{31}	c_{32}	ξ_1	ξ_2	γ_1	γ_2	η	v_1	α_1	α_2	n	q_2	w
100	200	11	5	2	65	0.8	0.1	0.5	0.4	250	1000	1	50	0.8	36

If we change one of the parameters, we can investigate the relationship between the parameter and its impact on the expected cost. For simplicity, we investigate the following three situations:

- Situation 1 – Change parameter η in $p_{2j}(= \eta^j)$ in Eqs. (6)—(7), (12), and (13).
- Situation 2 – Change parameter q_2 in Eqs. (10)—(13), and
- Situation 3 – change parameters η_1 and η_2 in Eq. (11).

4.1 Expected costs against parameters η and n

If we change η from 0.1 to 1 with a step 0.1, and keep the other parameters fixed as shown in Table 1, the expected costs are shown in Table 2.

Table 2. The expected costs $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against η and n .

η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$EC_{11}(w)$ ($n=5$)	64.20	66.59	70.44	76.67	86.92	104.36	135.68	193.79	275.01	62.51
$EC_{12}(w)$ ($n=5$)	63.16	65.56	69.42	75.65	85.86	103.19	134.25	191.84	272.28	61.43
$EC_2(w)$ ($n=5$)	8.20	11.25	15.86	22.89	33.69	50.13	73.37	99.05	102.91	5.83
$EC_{31}(w)$ ($n=5$)	8.38	11.43	16.04	23.07	33.87	50.30	73.54	99.23	103.08	6.00
$EC_{32}(w)$ ($n=5$)	8.27	11.32	15.93	22.96	33.76	50.20	73.44	99.12	102.98	5.90
$EC_{11}(w)$ ($n=25$)	62.51	62.51	62.51	62.51	62.53	62.62	63.13	65.51	72.42	62.51
$EC_{12}(w)$ ($n=25$)	61.44	61.44	61.45	61.47	61.51	61.66	62.27	64.84	72.00	61.43
$EC_2(w)$ ($n=25$)	5.83	5.83	5.83	5.84	5.89	6.02	6.31	6.81	7.17	5.83
$EC_{31}(w)$ ($n=25$)	6.00	6.00	6.01	6.02	6.07	6.20	6.49	6.98	7.34	6.00
$EC_{32}(w)$ ($n=25$)	5.90	5.90	5.90	5.91	5.96	6.09	6.39	6.88	7.24	5.90

From Table 2, when the number n of products sold is smaller, the values of $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ change quickly over η . However, when $n > 30$, the values of $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ change very slowly. It also shows that the expected warranty claim costs $EC_{11}(w)$ and $EC_{12}(w)$ are much larger than $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$, which implies that the FBNR phenomenon takes effects.

Figure 1 shows the values of $EC_{32}(w)$ against η , for the cases when $n=5$ and $n=25$. It can be seen that $EC_{32}(w)$ increases much faster for the case $n=5$ than that for the case $n=25$. It can also be seen that $EC_{32}(w)$ reaches the smallest value when $\eta=1$. $\eta=1$ implies that the cause of intermittent failures can be detected immediately.

Figure 1: $EC_{32}(w)$ against η and n

4.2 Expected costs against parameter q_2 and n

If we change q_2 from 0.1 to 1 with a step 0.1, respectively, and keep the rest of the parameters unchanged as shown in Table 1, then we obtained the expected costs as shown in Table 3.

Figure 2 shows that the values of $EC_{32}(w)$ increase when q_2 changes from 0.1 to 0.6, then they decrease when q_2 becomes larger, say, when q_2 changes from 0.6 to 1. It can also be seen that the values of $EC_2(w)$ are much larger in the case of $n=5$ than those where the case of n becomes larger.

The findings about the relationship between parameter q_2 and $EC_2(w)$ is interesting. As one might expect, the larger values of q_2 imply more reports arising from intermittent failures and therefore can incur larger cost

to a manufacturer. From both Table 3 and Figure 2, we find that this is not always the case, due to the nonlinearity nature of the component $e^{-q_2 A_n(w)} [q_2 A_n(w)]^k$ in Eq. (8).

Table 3. The expected costs $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against q_2 and n .

q_2	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$EC_2(n=5)$	14.42	19.99	23.30	24.95	25.41	25.04	24.13	22.89	21.46	19.96
$EC_{31}(n=5)$	14.59	20.17	23.48	25.13	25.59	25.22	24.31	23.07	21.64	20.14
$EC_{32}(n=5)$	14.49	20.06	23.37	25.02	25.48	25.11	24.21	22.96	21.53	20.04
$EC_2(n=85)$	6.15	5.84	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83
$EC_{31}(n=85)$	6.33	6.02	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
$EC_{32}(n=85)$	6.22	5.91	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90
$EC_2(n=185)$	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83
$EC_{31}(n=185)$	6.01	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
$EC_{32}(n=185)$	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90

Figure 2: $EC_2(w)$, $EC_{31}(w)$ and $EC_{32}(w)$ against q_2 for $n=5$, $n=85$ and $n=185$, respectively.

4.3 Expected costs against parameters γ_1 , γ_2 and n

If we change γ_1 and γ_2 from 0.1 to 1.9 with a step 0.2, respectively, and keep the rest of the parameters fixed as shown in Table 1, all of the expected costs of $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ decrease, as shown in Table 4 and Table 5. We also notice that the gradient of the changes in $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ become very similar when n is larger. For example, in Figure 3 and Figure 4, the values are very close for the cases when $n=85$ and $n=185$.

Figure 5 shows how the expected cost $EC_{32}(w)$ changes over γ_1 and γ_2 . It shows that $EC_{32}(w)$ reaches the smallest value when both γ_1 and γ_2 are the smallest.

Table 4. The expected costs $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against γ_1 (when $\gamma_2=0.5$)

γ_2	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9
$EC_2(n=5)$	22.89	21.83	20.97	20.26	19.68	19.21	18.82	18.50	18.24	18.03
$EC_{31}(n=5)$	23.07	22.01	21.15	20.44	19.86	19.38	19.00	18.68	18.42	18.20
$EC_{32}(n=5)$	22.96	21.90	21.04	20.33	19.75	19.28	18.89	18.57	18.31	18.10
$EC_2(n=85)$	5.83	4.77	3.91	3.20	2.62	2.14	1.75	1.44	1.18	0.96
$EC_{31}(n=85)$	6.00	4.95	4.08	3.37	2.79	2.32	1.93	1.61	1.35	1.14
$EC_{32}(n=85)$	5.90	4.84	3.98	3.27	2.69	2.21	1.83	1.51	1.25	1.03
$EC_2(n=185)$	5.83	4.77	3.91	3.20	2.62	2.14	1.75	1.44	1.18	0.96
$EC_{31}(n=185)$	6.00	4.95	4.08	3.37	2.79	2.32	1.93	1.61	1.35	1.14
$EC_{32}(n=185)$	5.90	4.84	3.98	3.27	2.69	2.21	1.83	1.51	1.25	1.03

Table 5. The expected costs $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against γ_2 (when $\gamma_1=0.1$)

γ_2	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9
$EC_2(n=5)$	36.21	25.52	22.89	21.66	20.93	20.44	20.09	19.83	19.62	19.44
$EC_{31}(n=5)$	36.39	25.70	23.07	21.84	21.11	20.62	20.27	20.00	19.79	19.62
$EC_{32}(n=5)$	36.28	25.59	22.96	21.73	21.00	20.51	20.16	19.90	19.69	19.51
$EC_2(n=85)$	19.15	8.46	5.83	4.59	3.87	3.38	3.03	2.76	2.55	2.38
$EC_{31}(n=85)$	19.33	8.63	6.00	4.77	4.04	3.56	3.21	2.94	2.73	2.56
$EC_{32}(n=85)$	19.22	8.53	5.90	4.67	3.94	3.45	3.10	2.83	2.62	2.45
$EC_2(n=185)$	19.15	8.46	5.83	4.59	3.87	3.38	3.03	2.76	2.55	2.38
$EC_{31}(n=185)$	19.33	8.63	6.00	4.77	4.04	3.56	3.21	2.94	2.73	2.56
$EC_{32}(n=185)$	19.22	8.53	5.90	4.67	3.94	3.45	3.10	2.83	2.62	2.45

Figure 3: $EC_{32}(w)$ against η_1 when $n=5$, 85, and 185, respectively.

Figure 4: $EC_{32}(w)$ against η_2 when $n=5$, 85, and 185, respectively.

Figure 5: $EC_{32}(w)$ against η_1 and η_2 when $n=50$.

From Sections 5.1, 5.2 and 5.3, it can be seen that the number of products, n , is very important as the expected costs are sensitive to it.

5. Conclusions

Conventional research on warranty claims simply assumes that claims are only due to product failures and consumers will report claims upon product failure, which might not be true in reality. This paper models the expected warranty claim costs when consumer behaviour is taken into account for products protected by non-renewing warranty policy. The numerical examples in the paper show the relationships between parameters and the expected costs. The paper also shows that the expected claim costs are sensitive to the number of products sold.

With increasingly more accumulated warranty claim data, manufacturers should be able to develop more accurate warranty claim models to predict the expected cost and the expected number of claims. Such models should also include more relevant factors, such as failed but not reported phenomenon and non-failed but reported claims, which might impact warranty claims.

Our future research includes the following issues.

- The probability of not detecting the cause of intermittent failures and the probability of failed but not reported phenomenon were assumed to be dependent on the number of claims. Possible extensions are to assume them to be associated with both product age and the number of intermittent failures reported.

- In this paper, only one cause of intermittent failures was considered. More than one cause of intermittent failures should be studied.
- In the paper, when modelling the ability to detect the cause of intermittent failures, we assumed that products were sold at the same date. However, products shipped to retailers might not be sold at the same date. There might be delays between shipment dates and sales dates, known as sales delay, which should be considered in our future work.
- The paper only considered repairable products with minimal repair. Further work should also analyse warranty claims costs for non-repairable products or repairable products with different levels of maintenance quality (see [33] for maintenance models, for example).
- The paper only considered non-renewing warranty policy. Other warranty policy can also be considered.

Acknowledgment

The author would like to thank Professor DNP Murthy for the helpful discussion and his valuable comments. We are grateful to the reviewers for their helpful comments that lead a great improvement on the paper.

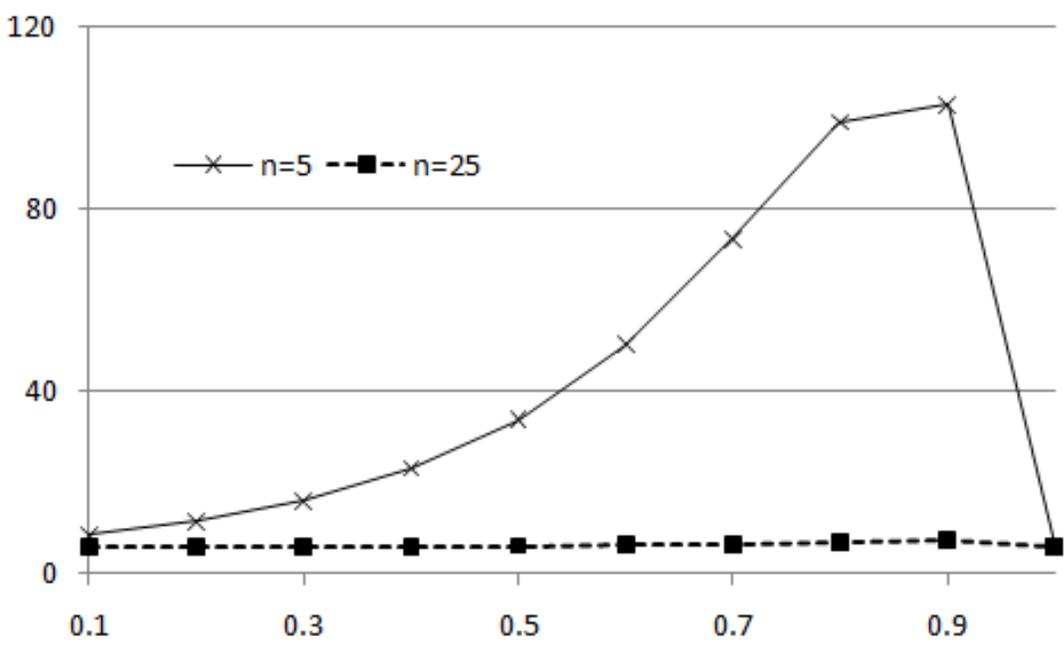
This research is supported by Engineering and Physical Sciences Research Council (EPSRC) of the United Kingdom (EPSRC Grant reference: EP/G039674/1).

References

- [1] Blischke WR, Murthy DNP. Product warranty management - I: A taxonomy for warranty policies. *European Journal of Operational Research* 1992;62(2):127-48.
- [2] Wu S, Li H. Warranty cost analysis for products with a dormant state. *European Journal of Operational Research* 2007;182(3):1285-93.
- [3] Murthy DNP, Djameludin I. New product warranty: A literature review. *Int J Prod Econ* 2002;79(3):231-60.
- [4] Oh YS, Bai DS. Field data analyses with additional after-warranty failure data. *Reliability Engineering and System Safety* 2001;72(1):1-8.
- [5] Rai B, Singh N. Hazard rate estimation from incomplete and unclean warranty data. *Reliability Engineering and System Safety* 2003;81(1):79-92.
- [6] Rai B, Singh N. Modeling and analysis of automobile warranty data in presence of bias due to customer-rush near warranty expiration limit. *Reliability Engineering and System Safety* 2004;86(1):83-94.
- [7] Attardi L, Guida M, Pulcini G. A mixed-Weibull regression model for the analysis of automotive warranty data. *Reliability Engineering and System Safety* 2005;87(2):265-73.
- [8] Kleyner A, Sandborn P. A warranty forecasting model based on piecewise statistical distributions and stochastic simulation. *Reliability Engineering and System Safety* 2005;88(3):207-14.
- [9] Jung M, Bai DS. Analysis of field data under two-dimensional warranty. *Reliability Engineering and System Safety* 2007;92(2):135-43.
- [10] Majeske KD. A non-homogeneous Poisson process predictive model for automobile warranty claims. *Reliability Engineering and System Safety* 2007;92(2):243-51.
- [11] Wu S, Clements-Croome D. Burn-in policies for products having dormant states. *Reliability Engineering and System Safety* 2007;92(3):278-85.
- [12] Wu C-, Chou C-, Huang C. Optimal burn-in time and warranty length under fully renewing combination free replacement and pro-rata warranty. *Reliability Engineering and System Safety* 2007;92(7):914-20.
- [13] Chattopadhyay G, Rahman A. Development of lifetime warranty policies and models for estimating costs. *Reliability Engineering and System Safety* 2008;93(4):522-9.
- [14] Jack N, Iskandar BP, Murthy DNP. A repair-replace strategy based on usage rate for items sold with a two-dimensional warranty. *Reliability Engineering and System Safety* 2009;94(2):611-7.
- [15] Jung KM, Park M, Park DH. System maintenance cost dependent on life cycle under renewing warranty policy. *Reliability Engineering and System Safety* 2010;95(7):816-21.
- [16] Thomas MU, Rao SS. Warranty economic decision models: A summary and some suggested directions for future research. *Operations Research* 1999;47(6):807-20.
- [17] Murthy DNP, Solem O, Roren T. Product warranty logistics: Issues and challenges. *European Journal of Operational Research* 2004;156(1):110-26.
- [18] Karim MR, Suzuki K. Analysis of warranty claim data: A literature review. *International Journal of Quality and Reliability Management* 2005;22(7):667-86.
- [19] Murthy DNP. Product warranty and reliability. *Annals of Operations Research* 2006;143(1):133-46.

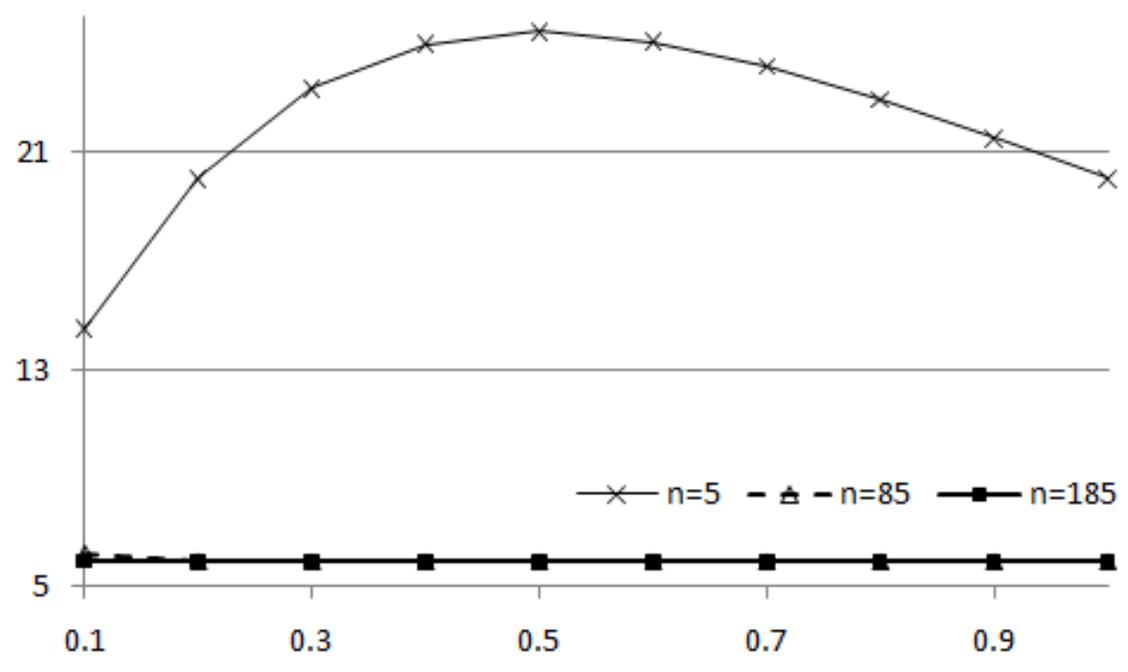
- [20] Rahman A, Chattopadhyay G. Review of long-term warranty policies. *Asia-Pacific Journal of Operational Research* 2006;23(4):453-72.
- [21] Thomas MU. *Reliability and warranties: methods for product development and quality improvement*. : Taylor & Francis Group; 2006.
- [22] Rai B, Singh N. *Reliability analysis and prediction with warranty data: Issues, Strategies, and Methods*. London: Taylor & Francis Group; 2009.
- [23] Lawless JF, Hu XJ, Cao J. Methods for the estimation of failure distributions and rates from automobile warranty data. *Lifetime Data Analysis* 1995;1(3):227-40.
- [24] Lu M-. Automotive reliability prediction based on early field failure warranty data. *Quality and Reliability Engineering International* 1998;14(2):103-8.
- [25] Kalbfleisch JD, Lawless JF. Some useful statistical methods for truncated data. *Journal of Quality Technology* 1992;24(3):145-52.
- [26] Wu S, Xie M. Warranty cost analysis for nonrepairable services products. *International Journal of Systems Science* 2008;39(3):279-88.
- [27] Patankar JG, Mitra A. Effects of warranty execution on warranty reserve costs. *Management Science* 1995;41:395-400.
- [28] Patankar JG, Mitra A. Warranty and consumer behavior: Warranty execution. *Product Warranty Handbook* 1996:421-38.
- [29] Qi H, Ganesan S, Pecht M. No-fault-found and intermittent failures in electronic products. *Microelectronics Reliability* 2008;48(5):663-74.
- [30] Thomas DA, Avers K, Pecht M. The "trouble not identified" phenomenon in automotive electronics. *Microelectronics Reliability* 2002;42(4-5):641-51.
- [31] Söderholm P. A system view of the No Fault Found (NFF) phenomenon. *Reliability Engineering and System Safety* 2007;92(1):1-14.
- [32] Rai B, Singh N. Customer-rush near warranty expiration limit, and nonparametric hazard rate estimation from known mileage accumulation rates. *IEEE Transactions on Reliability* 2006;55(3):480-9.
- [33] Wu S, Zuo MJ. Linear and nonlinear preventive maintenance models. *IEEE Transactions on Reliability* 2010;59(1):242-9.

Fig.1



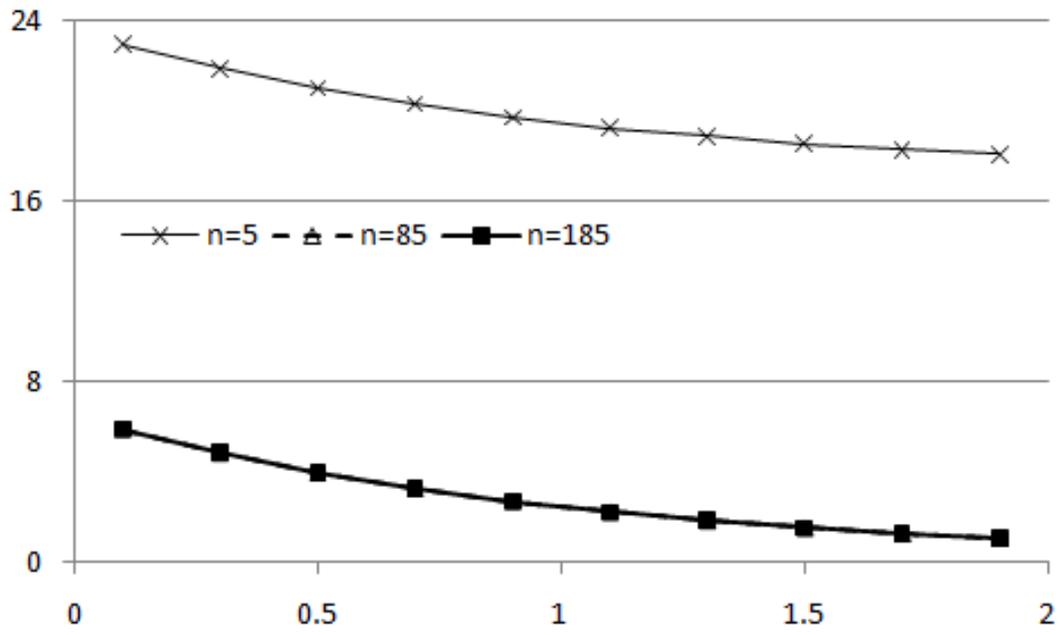
Accepted manuscript

Fig.2



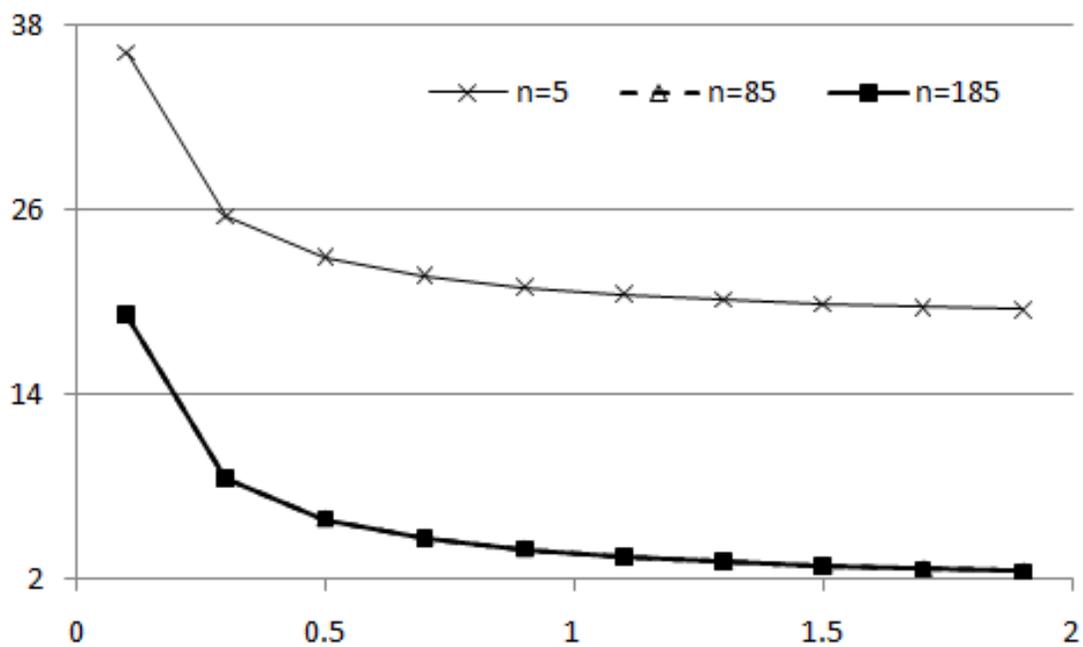
Accepted manu

Fig.3



Accepted manuscript

Fig.4



Accepted manuscript

Fig.5

