PRESERVICE TEACHERS' CONTENT KNOWLEDGE OF FUNCTION CONCEPT WITHIN A CONTEXTUAL ENVIRONMENT

A Dissertation

by

IRVING ANTHONY BROWN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

Major Subject: Curriculum and Instruction

Preservice Teachers' Content Knowledge of Function

Concept within a Contextual Environment

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Approved by:

Co-Chairs of Committee,	Gerald O. Kulm
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ABSTRACT

Preservice Teachers' Content Knowledge of Function Concept
within a Contextual Environment. (August 2011)

Irving Anthony Brown, B.S., Lamar University; M.S., Tarleton State University

Co-Chairs of Advisory Committee: Dr. Gerald O. Kulm
Dr. Dennie L. Smith

The overarching goal of this dissertation research was to develop and measure the psychometric properties of an instrument to assess preservice teachers' content knowledge of the function concept embedded in contextual problems. This goal was accomplished through two research projects described in two studies. The first study reports on the collective case study that was used to pilot test the instrument and the second study details the rationale used in item selection and the psychometric properties of the new instrument. Unlike existing research studies that examine a broad range of function related topics using various forms of symbolic, tabular, and graphical representations as the basis for questions and problems, this study focused solely on function problems immersed in various real world contexts. Since this is not a common approach to measuring content knowledge of the function concept, the existing instruments in published studies were not found to be suitable for this specialized purpose. The psychometric measurements of the instrument did not suggest that the instrument was valid or reliable so more research will be required to validate the instrument. However, based on the preliminary results from testing, several potential

suggestions can be made to teacher educations programs. Inferences drawn from the mathematical problem-solving cognition will aid in the development and validation of future instruments to assess the content knowledge of the mathematical function concept of preservice teachers as they complete contextualized problem-solving tasks.

DEDICATION

"For by him all things were created…"
Colossians 1:16

This work is dedicated to the elements of my being; God and my family. In all of my works I give the honor to God who is the head of my life and his son, my lord and savior, Jesus Christ. I am nothing without him in my life and my work would be meaningless if I did not work to serve him. I, as with every good thing I do, am created by him and for him.

I am truly blessed to have a wonderful, supportive and loving family that has been my anchor my entire life. Without their guidance and encouragement I would have not had the fortitude to pursue my dreams. Though my grandmother, Pricilla Nunnally, my aunt Dorothy Nunnally and my cousin Cofer L. McIntosh are no longer with us on earth, I carry them with me in my heart and their spirits will be with us forever.

I would be but half a man without my wife; Valerie is the center of me and my partner in every good thing I do. If anyone would ever look upon my accomplishments with favor, they should know that I am only the lesser half of the story. This was, is, and always will be an IrVal idea. These first 29 years have only marked the beginning of the wonders that God has in store for us.

"Forever Lovers,
Forever Friends.
A lifetime's a Short Time,
When Love Never Ends" (Davis, 1976)

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I would like to thank my committee co-chairs, Dr. Dennie Smith and Dr. Gerald Kulm, and my committee members, Dr. Jane Schielack, and Dr. Myeongsun Yoon, for their guidance and support throughout the course of this research. The support from Drs. Kulm and Smith extend far beyond the reach of this study for they were instrumental in bringing me into the doctoral program and providing a strong start to my research career.

Thanks also go to my friends and colleagues and the department faculty and staff for making my time in the Department of Teaching, Learning, and Culture at Texas A&M University a wonderful experience. I also want to extend my gratitude to the faculty members in Texas and North Carolina, who served on my Subject Matter Expert panel, and to all the Texas and North Carolina professors and their preservice teachers who were willing to participate in the study.

I extend my heartfelt thanks to my parents, Vernon and Julia Brown, my sister, Melissa Brown and our extended family of Browns, Nunnallys and Picketts for their encouragement and support over these many years of my educational and professional journey. I include in my family my mentor of the last two decades, Dr. Richard L. Price or “Doc” as he is known on the campus of Lamar University. Dr. Price has been my steadfast supporter and a beacon of light on my academic path.

I owe a special debt of gratitude to my wonderful daughter Chasity. In addition to her love and support, she has been my data transcriber and proof-reader for many parts of this study and her help has been invaluable to my work. Last in this list, but first in my heart, I wish to thank my beautiful, gift-from-God wife for her love, patience and understanding. Valerie, I thank you for your sacrifice; few people know what the spouse of a Ph.D. student must endure and you are a shining exemplar of a supportive wife.

NOMENCLATURE

ACP	Alternative Certification Program - an alternate route to state teacher licensure from the traditional university based undergraduate teacher education programs.
ETS	Educational Testing Service
Middle School	The state of Texas had traditionally held certification for middle school mathematics teachers to include teachers in fourth through eighth grade classrooms. In 2008, that scope was narrowed to only include sixth through eighth grade teachers. It should also be noted that the term “middle” school is sometimes used synonymously used with “junior high” school and can only encompass the seventh and eighth grades. For the purposes of this study, the term middle school will mean grades four through eight.
Praxis™	The series of examinations were created by ETS to provide several states a means of assessment of preservice teachers’ general academic knowledge, specific subject knowledge and pedagogical knowledge for the various teacher certification programs across the U.S.
SACS	The Commission on Colleges of the Southern Association of Colleges and Schools is the regional body for the accreditation of degree-granting higher education institutions in the Southern states. These states include: Alabama, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Texas, and Virginia.
SBEC	State Board for Educator Certification
TEA	Texas Education Agency
TExES™	Texas Examinations of Educator Standards™ () - is the product of a collaborative effort between the State of Texas’ State Board for Educator Certification (SBEC), the Texas Education Agency (TEA) and the Educational Testing Service (ETS) to create a series of examinations to be used by state agencies as a partial requirement of teacher certification in the state of Texas.

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CHAPTER I

INTRODUCTION

The genesis of this research project occurred after I conducted several mathematics content knowledge review sessions for two groups of preservice teachers enrolled in alternative certification programs (ACP). In working with preservice middle school mathematics and secondary school mathematics/science teachers, I found that many of them struggled to understand the requirements and context of the problems more than the specific mathematical operations required for a solution. After I explained the problem context and requirements, most of the preservice teachers were able to perform the correct mathematical operations to find a solution. These experiences led me to form a basic hypothesis: middle and secondary school teachers do not have sufficient content knowledge of the function concept in contextual environments to perform at the high expectations of today's STEM curriculum.

Generally speaking, teachers lack depth in mathematics content knowledge and the concept of function (Even, 1993; Kulm, 2008; Leinhardt, Zaslavsky, & Stein, 1990; Sherin, 2002; Stein, Baxter, & Leinhardt, 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979). Learning complex concepts, such as the concept of function, is

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This dissertation follows the style of the *Journal for Research in Mathematics Education*.

much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995). Mathematics education researchers need a specialized instrument to ascertain the level of teachers' content knowledge of the function concept in a contextualized environment but such an instrument was not found in the literature. Having such an instrument would allow researchers to determine what steps were needed to improve teachers' content knowledge.

This study details the development of such an instrument through two empirical research projects. The first project examines preservice teachers' contextualized function problem-solving cognition by analyzing qualitative data collected in task-based interviews and serves as the pilot test of the instrument. The second project describes the item selection, development, and the psychometric properties of the instrument. The two projects are highly connected through an iterative process of design and testing.

In 1975, the National Advisory Committee on Mathematics Education (NACOME) paved the way for the public school reform efforts that restructured the teaching of mathematics in general and algebraic concepts in particular (O'Callaghan, 1998). Algebra, misunderstood by some to be merely a study of variables and symbolic manipulation (Driscoll, 1999), plays a central role in students' mathematical development. Algebra can be thought of as the mathematical "bridge" across which secondary students must pass to reach advanced mathematical concepts in high school (Dooren, Verschaffel, & Onghena, 2002) as well as post secondary studies in the science, technology, engineering, and mathematics (STEM) subject areas.

Robert Moses, the noted leader of the Algebra Project spoke of math literacy, and more specifically algebra, as the new focal point in civil rights:

> Today, I want to argue, the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy. I believe that the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of registered Black voters in Mississippi was in 1961 (Moses & Cobb, 2001, p. 5).

Even though our nation's schools have made math literacy a priority in education, a persistent gap in algebraic achievement between students from minority groups and White students exists (Ladson-Billings, 1997; Moses & Cobb, 2001; Richardson, 2009).

Of the algebraic topics covered in middle and secondary schools, researchers recognize the concept of function as the single most important tool in algebra regarding a student's ability to apply mathematical concepts in sciences, engineering and other related contexts (Hollar & Norwood, 1999; Knuth, 2000; Lloyd & Wilson, 1998; O'Callaghan, 1998; Zbiek, 1998). Underdeveloped knowledge of the function concept hinders the mathematical development of students. The National Council of Teachers of Mathematics (NCTM) expects all students, beginning in grades six, to be able to "model and solve contextualized problems using various representations such as graphs, tables, and equations" (National Council of Teachers of Mathematics, 2000b), which requires students to possess a working knowledge of functions in contextualized environments.

With the focus of mathematics education research on students' achievement, it is important to note that teacher knowledge is the most important factor influencing student learning (Dooren et al., 2002; Lappan & Ferrini-Mundy, 1993; National Council of Teachers of Mathematics, 1991). Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (N. Webb, 1979) and earlier research found that teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). The lack of full understanding of the function concept impacts the preparation of teachers by bringing a marginalized view of functions to their classrooms (Leinhardt et al., 1990; Lloyd & Wilson, 1998; Stein et al., 1990).

Teachers' knowledge is not a static core of ideas but rather an ever changing pool of ideas that are supplemented, refined, and at times diminished during the instructional life of the teacher (Sherin, 2002). It is important that the initial pool of preservice teachers mathematical knowledge is as *deep and clear* as possible (Hill, Rowan, & Ball, 2005; Kulm, 1982; Li & Kulm, 2008). A rich source of knowledge for the teacher reduces the number of modifications to existing mathematical misconceptions and misrepresentations teachers must correct as they gain experience in the classroom. This pool of knowledge forms long before the preservice teacher enters a college classroom, but it is the college experience that is expected to deepen and widen the pool.

Some research suggests that the best way to learn the concept of function is over an extended period (years in some cases) (Yerushalmy, 2000). Through these years of mathematics instruction, are the preservice teachers actually absorbing the knowledge in such a way that it will provide an initial foundation that can be used in the classroom? Finding teachers that have acquired and maintained a strong concept of function can be particularly challenging in an environment where it has been reported that 65% of the middle school teachers neither have a mathematics degree nor have certification in mathematics (Li & Kulm, 2008).

Traditional algebraic instruction stresses the memorization of algebraic facts and symbolic manipulation at the expense of problem-solving skills and conceptualization (Hollar & Norwood, 1999; Karsenty, 2002; O'Callaghan, 1998). Research has uncovered the following disturbing corollary to this shortsighted view of algebraic instruction: students have been found to internally characterize word problems as artificially contrived classroom based scenarios that have little or no relation to problems of the real-world (Greer, Verschaffel, & De Corte, 2002; Verschaffel, De Corte, & Borghart, 1997). This suggests students feel free to suspend common sense approaches and the benefit of their personal real-world experiences and attack the word problem with simple mathematical facts and algorithms they have previously learned.

In their *Connections* standard, NCTM prescribes "Instructional programs from prekindergarten through grade 12 should enable all students to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (National Council of Teachers of Mathematics, 2000a, p. 63). NCTM takes the position that mathematical lessons learned by students in connected curricula are richer and are better retained than a lesson of isolated mathematical thoughts. They also contend that through contextual lessons, students gain a better feel for both the utility of mathematics in other subject areas as well as a deeper understanding of the related content area of which the mathematics lesson is connected. Since research reflects more than one definition of contextual lessons (Roth, 1996), it is sensible to examine definitions and descriptions of contextual teaching and learning.

Williams (2007) describes contextual teaching as “a methodology of teaching that connects academic concepts to real-world conditions and encourages students to see how what they learn relates to their lives” (p. 572). Contextual teaching is also described as an integration of social constructivism, situated cognition, multiple intelligences and brain based learning theories (Lynch & Harnish, 2003; Williams, 2007). Roth (1996) cautions that contextualized problems are not just word problems with added verbiage, but rather “an expression is contextualized as part of meaningful practices rather than through an increase in (sign-based) situation descriptions” (p. 489). Roth’s minority opinion of contextual teaching follows a perspective held by John Dewey (Menand, 2001) that it is necessary for students to actively (kinesthetically) engage in the learning process. Current mathematics education research does not show a strong link between Dewey’s kinesthetic learning perspective and contextual teaching.

A significant aspect of the notion of contextual learning and problem-solving lies beyond the definition or description and entails how students are impacted by it. On the surface, research suggests the importance of “finding meaning by connecting academic work with daily lives” (Lynch & Harnish, 2003, p. 6) , and the idea that contextual lessons add meaning to both the mathematical topic as well as justify increased interest in academics (Lynch & Harnish; Wiseley, 2009). However, a deeper rationale for this study lies in the way contextual lessons help students convert seemingly correct mathematical answers into solutions based on "real-world" thinking. The following two problems best illustrate this point (Verschaffel et al., 1997, p. 340):

> Steve has bought 4 ropes of 2.5 metres each. How many ropes of 0.5 metre can he cut out of these 4 ropes? Steve has bought 4 planks of 2.5 metres each. How many planks of 1 metre can he saw out of these 4 planks?

Students can compute a correct response to the first problem without considering real-world consequences; 4 ropes * 2.5 meters/rope * 1 rope section/0.5 meters = 20 rope sections. But the same straightforward application of mathematics applied to the second problem leaves the student with a seemingly correct mathematical solution of 10 planks, which of course is impossible based on the context of the problem.

Research has shown empirically that students have a strong tendency not to use their common sense in solving word problems but would rather rely on absent-minded repetition of the drills practiced in school mathematics (Greer et al., 2002; Verschaffel et al., 1997). The following diagram (Figure 1) of the problem-solving process shows the dependence on interpreting the mathematical solution in light of contextual constraints to arrive at a "real" solution.

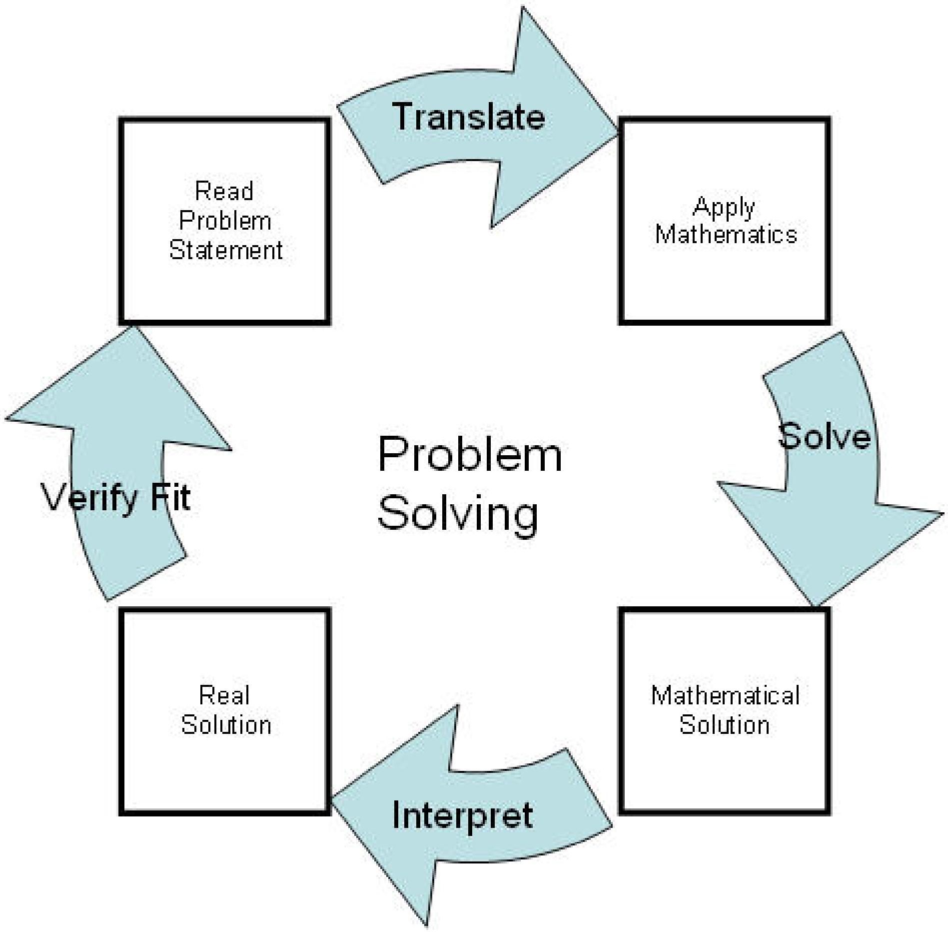

Figure 1 *Kulm's Problem-solving Model* (Kulm, 1982)

Since mathematics teachers have the greatest influence in students' mathematical development (Dooren et al., 2002), it is imperative that teachers possess sufficient depth of mathematical content knowledge and experience in contextual problem-solving to effectively lead student learning. Researchers believe one of the contributing factors to the disconnect between real-world insights in students' problem-solving abilities is "the way in which these problems are considered and used in current instructional practice and culture, and more specifically the lack of systematic attention to the modeling perspective by the teacher"(Verschaffel et al., 1997, p. 340). Likely reasons teachers avoid modeling highly contextualized problems in their classrooms are their inexperience and lack of confidence in solving this type of problem.

A teacher's ability to solve problems in a contextual (using real-world insights) environment needs to be assessed to give teacher educators insight into possible modifications to preservice programs and to provide school administrators a clearer perspective into the professional development needs of mathematics teachers. This need, and the fact that a contextual function instrument was not found in the literature, drives the rationale for creating and validating such an instrument.

Statement of the Problem

Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (N. Webb, 1979). More specifically, teachers lack depth in their conception of the mathematical function (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979). Learning complex concepts, such as the concept of function, is much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995) but research suggests that the primary use of contextual, real-world instruction is relegated to developmental mathematics courses in post secondary education (Grubb & Kraskouskas, 1992; Stone, Alfeld, Pearson, & Lewis, 2006; Wiseley, 2009) even though state teacher certification examinations are rich in contextualized problems (Educational Testing Service, 2009a, 2009b; Texas Education Agency, 2009).

If we, as mathematics educators, expect to improve preservice teachers' content knowledge of the function concept in a contextualized environment, then the first step will be to assess their current knowledge. Only after we have assessed their current level of knowledge can we plan modifications to preservice teacher education and perhaps teacher professional development programs. In order to make a sound assessment, we must have a reliable and valid instrument at our disposal. There are many extant survey instruments to measure a wide range of mathematical attitudes, affective behaviors, and content knowledge dimensions, including general function concepts, but an instrument that specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published.

Statement of Purpose

The overarching goal of this dissertation research was to develop and validate an instrument to assess preservice teachers' content knowledge of the function concept embedded in contextual problems. This goal was accomplished through two research projects described in two central chapters. Chapter II reports on the collective case study that was used to pilot test the instrument, and Chapter III details the rationale used in item selection and the psychometric properties of the new instrument. Unlike existing research studies that examine a broad range of function-related topics using various forms of non-contextual symbolic, tabular, and graphical representations as the basis for questions and problems, this study focuses solely on function assessment problems immersed in various real-world contexts.

Since this is not a common approach to measuring content knowledge of the function concept, the existing instruments in published studies were not found to be suitable for this specialized purpose. An instrument dedicated to measurement of content knowledge of functions using problems that are immersed in real-world contexts was developed and tested for validity and reliability measures.

Research Questions

The overarching question that defined and guided both research projects described in this study was, "how can preservice mathematics teachers' content knowledge of the function concept be assessed?" The following questions form the basis of this investigation:

1. How do preservice teachers demonstrate their knowledge of conceptual and procedural problem-solving skills related to contextualized mathematical function problems?
   a. How do preservice teachers decode the imbedded function concept from a contextualized problem?
   b. Which procedural approaches do preservice teachers use in problem-solving?
   c. How do preservice teachers demonstrate their conceptual knowledge of functions in problem-solving?

2. What are the key items in assessing contextual function concepts that should be included in an instrument to assess preservice mathematics teachers' knowledge?
3. What are the psychometric properties of an instrument developed to assess preservice middle and secondary mathematics teachers' knowledge of the mathematical concept of function within a contextual environment?

CHAPTER II
OBSERVING PRESERVICE TEACHERS' CONTEXTUALIZED FUNCTION PROBLEM-SOLVING THROUGH TASK-BASED INTERVIEWS

Introduction

In 1975, the National Advisory Committee on Mathematics Education (NACOME) paved the way for the public school reform efforts that restructured the teaching of mathematics in general and algebraic concepts in particular (O'Callaghan, 1998). Algebra, misunderstood by some to be merely a study of variables and symbolic manipulation (Driscoll, 1999), plays a central role in students' mathematical development. Algebra can be thought of as the mathematical "bridge" across which secondary students must pass to reach advanced mathematical concepts in high school (Dooren et al., 2002) as well as post secondary studies in the science, technology, engineering, and mathematics (STEM) subject areas.

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Problem

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Learning complex concepts, such as the concept of function, is much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995) but research suggests that the primary use of contextual, real-world instruction is relegated to developmental mathematics courses in post secondary education (Grubb & Kraskouskas, 1992; Stone et al., 2006; Wiseley, 2009) even though state teacher certification examinations are rich in contextualized problems (Educational Testing Service, 2009a, 2009b; Texas Education Agency, 2009).

There are many extant instruments to measure a wide range of mathematical content topics and attitudes, but an instrument that specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published.

The purpose of this study was to describe the mathematical problem-solving processes related to contextualized mathematical function problems for six preservice middle and secondary school mathematics teachers at a large Southwestern, research intensive university. The results of the study will contribute to the validation of a new instrument to assess the content knowledge of the mathematical function concept of preservice teachers as they complete contextualized problem-solving tasks. Conclusions about the participants' mathematical problem-solving processes were derived from the verbal responses to interview questions, the written responses to the assessment instrument, and the interpretation of the researcher's observation of the preservice teachers' responses during task-based interviews. Inferences drawn from the mathematical problem-solving cognition will aid in the development and validation of an instrument to assess preservice mathematics teachers' knowledge of how to connect their knowledge mathematical concept of function to a contextual setting.

Research Questions

The research questions were driven by a desire to gain a richer understanding of the internal cognitions and procedures used by preservice teachers as they attempt to understand and apply the concept of function in a richly contextualized environment. The central question that guided this study was: How do preservice teachers demonstrate their knowledge of conceptual and procedural problem-solving skills related to contextualized mathematical function problems?

The following sub questions served to sharpen the focus of the study:

1. How do preservice teachers decode the imbedded function concept from a contextualized problem?

2. Which procedural approaches do preservice teachers use in problem-solving?

3. How do preservice teachers demonstrate their conceptual knowledge of functions in problem-solving?

Theoretical Framework

This study proposes to discover aspects of the participants' knowledge related to the task of solving contextualized mathematical function problems. Since it is not possible to directly access a person's thoughts, Piaget's clinical examination forms the theoretical basis for this study (Piaget, 1965). As Goldin (1997, 2000) builds on Piaget's base and focuses the work on mathematics education research through task-based interviews, he cautions that the outcomes from clinical interviews cannot be considered to be thoughts or cognitions. Goldin's perspective is guided by, "First, it is crucial to maintain carefully the scientific distinction between that which is observed and inferences that are drawn from observations" (1997, p. 52). Therefore a foundational theoretical postulation for this research is based on the notion that it is not possible to observe a mathematical construct within a student's mind, but rather the observations allow us to infer something about the student's internal mathematical constructs.

In addition to allowing us to infer qualities of the participant's internal mathematical cognitions, the theoretical framework should also describe how the characteristics of the mathematical tasks under study interact with those internal cognitions of the participant (Goldin, 1997). A task's language, mathematical content and structure, mathematical appropriateness, and the interview context are examples of the characteristics of the task. Kulm and others describe these as "task variables" in problem-solving research (Kulm, 1979; N. Webb, 1979). In this research project, the task's mathematical content and structure and the mathematical appropriateness of the task are principally based on the TExES™ and Praxis™ teacher certification examinations, upon which the primary research instrument is based (I. A. Brown, 2011).

The task's language and the interview context are key characteristics under study in this research project. This research assumes the theoretical position that the language of the contextualized function problems of the primary instrument will require the participant to work at a much higher level of cognition than when solving straightforward, "given these conditions, find the value of x" type of problems. By moving beyond the simple recall of definitions and basic algorithms, the participant will have the opportunity to demonstrate their deeper understanding of the central concepts of mathematical functions and their ability to use this knowledge to aid in their problem-solving approaches. The interview context is based on the design principle that holds the participant's free expression of mathematical ideas is of the utmost importance in the interview. Answers are to be accepted as "good" by the interviewer (researcher) whether the answers are correct or not. By using only non-directive follow-up questions and heuristic suggestions, the participant's responses are expected to yield rich data that will allow the researcher to infer qualities of the participant's internal mathematical constructs.

The theoretical foundation for the task-based interviews, as described by Goldin (1997) is based on three fundamental assumptions. First, it is assumed that the student's internal mathematical cognitions or "competencies and structures of such competencies" (p.55) are able to be inferred from external behaviors. Specifically, these internal structures are able to be inferred from the data gathered from task-based interviews when the task and the interview elicit certain cognitive and behavioral responses from the interviewee. The second assumption "is the idea that competencies are encoded in several different kinds of internal representations and that these interact with one another and with observable, external representations during problem-solving" (Goldin, p. 55). The third assumption is that a student's representational actions are based on internal and/or external configurations which "symbolize" other representational configurations.

Method

Participants

The participants in this study were preservice middle school mathematics teachers enrolled in a preservice teachers' mathematics education problem-solving course at a large Southwestern, research intensive university. Purposeful sampling was used to choose six students from the class based on their willingness to participate in this pilot study. To effect a *maximum variation* type of sampling (Creswell, 2007, p. 127), two of the highest scoring students and two of the lowest scoring students were chosen from the group that volunteered to participate along with two students that scored near the mean of the class scores.

Five of the six preservice teachers were preparing to teach middle school mathematics (education majors) while the sixth participant, Teacher4, was a mathematics major preparing to teach secondary school mathematics. One of the participants, Teacher6, was previously a mathematics major before becoming an education major. All six of the participants were classified as university juniors; four of which were White females; there was one Black female and one White male.

Instrument

There were two instruments, both created by the researcher, employed in this study. The primary instrument, the “Contextual Function Instrument”, was an initial draft of an instrument which consisted of fifteen multiple choice questions designed to assess middle and secondary school preservice mathematics teachers’ knowledge of the mathematical function concept in a contextually rich environment. This primary instrument was the focus of the pilot study and is the instrument that will ultimately be validated in a separate research study.

Prior to creating a new instrument, research literature was searched in an attempt to find an instrument that would satisfy the requirements of being highly contextualized as well as focusing on pre-calculus level function concepts. Included in the review of research literature were the following databases available on the Texas A&M University Library website: Academic Search Complete (EBSCO), ERIC, ScienceDirect, and the Web of Science (Texas A&M University, 2010). Additionally, function instruments were sought using GoogleScholar, and the test collection at Educational Testing Service (ETS).

The questions on the primary instrument were created by the researcher and are based on common competencies described in the TExES™ and Praxis™ teacher certification examinations for middle and secondary school preservice mathematics teachers (Educational Testing Service, 2009c; Texas Education Agency, 2009). Texas middle school mathematics teachers are required to demonstrate successful performance on the “115 Mathematics 4 – 8” examination and Texas secondary school mathematics teachers are required to demonstrate successful performance on the “135 Mathematics 8 – 12” examination (Texas Education Agency, 2008). North Carolina middle school mathematics teachers are required to demonstrate successful performance on the “Middle School Mathematics (0069)” examination and secondary school mathematics teachers are required to demonstrate successful performance on the “Mathematics: Content Knowledge (0061)” examination (Educational Testing Service, 2009a, 2009b, 2009c).

The second instrument was a task-based interview protocol script of five primary questions asked of the participants during the interview. In the instances when the five primary questions did not consume the entire 45 minute interview time slot, a sixth auxiliary question was used. The development of the interview protocol, which is shown in Appendix A, is based on the work of Goldin’s (1997) task based interview protocol creation guidelines:

> The script is written so that for each main question, exploration proceeds in four stages: (a) posing the question ("free" problem-solving) with sufficient time for the child to respond and only nondirective follow-up questions (e.g., "Can you tell me more about that?”); (b) heuristic suggestions if the response is not spontaneous (e.g., "Can you show me by using some of these materials?"); (c) guided use of heuristic suggestions, again to the extent that the requested description or behavior

> does not occur spontaneously (e.g., "Do you see a pattern in the cards?"); and (d) exploratory (metacognitive) questions (e.g., "Do you think you could explain how you thought about the problem?") (p. 45).

The goal of the instrument's design was to guide the interviewer in eliciting complete, coherent mathematical thoughts from the participant for each of the questions.

Data Collection

Qualitative data for this collective case study were collected face to face by the researcher (interviewer) in a conference room on campus, using the aforementioned protocol in 45 minute, semi structured, individual interviews. The interviews were audio recorded and those recordings were transcribed for data analysis. The researcher also made notes during the interview to supplement the transcription and the participants' written responses to the tasks presented.

Results

Data Analysis

The data consisted of the audio recorded interviews, the written problem-solving solutions from the participants, as well as the interviewer's notes. The audio tapes were transcribed and read several times by the researcher to help form initial open coding. Direct interpolation was also used to analyze data within each individual case. An in-depth depiction of the cases using narrative analysis was created to give a clearer image of the individual participants.

Each of the participants' responses (transcribed from audio recordings) was subdivided into "logical segments" of information that allowed the researcher to infer something of the participants thinking relating to the assigned task. The interview transcripts yielded a total of 209 logical segments for all six participants. Using a constant comparison approach (Boeije, 2002; Glaser, 1965; Maxwell, 2010), 18 researcher-defined codes were developed and used to interpret these 209 logical segments. For example, when asked about the method used to arrive at a particular answer, Teacher3's response included the statement, "I'm trying to remember, like, the equation for surface area and I'm thinking that since we really don't need to know how much it costs that would eliminate answer A and answer C." That logical segment was assigned the code, "eliminate answer choices." To validate the primary researcher's coding process four other researchers reviewed, supplemented, and modified the coding scheme.

The three research sub-questions were used as themes and categorical aggregation was employed to classify data across the six cases. The following tables show the codes (Table 1), the frequency of their occurrence (Table 2) and their rank within the specific groups (Table 3).

Table 1. *Frequency of Preservice Teachers' Strategies to Decode the Imbedded Function Concept from Contextualized Problems.*

Rank	Decoding Approach	Frequency
1	Decode the context and eliminate the extraneous text without getting overwhelmed by the amount of text.	61.4%
2	Use function notation and proper mathematical language to re-state and understand the problem context.	18.2%
3	Convert problem into their own words to better understand what the problem is asking.	11.4%
4	Relate the embedded function in familiar "x" and "y" terms even when the Cartesian coordinate system has nothing to do with the problem at hand.	4.5%
5	Assume the given mathematics problem is actually a "physics" problem and therefore that it is outside of their knowledge base.	4.5%

Table 2. *Frequencies of Procedural Approaches Used by Preservice Teachers Use in Contextual Problem-Solving.*

Rank	Procedural Approach	Frequency
1	Eliminate answer choices to reduce the probability of making a wrong answer selection.	35.4%
2	Use to mathematical definitions (from memory) to serve as foundations in their problem-solving strategies.	25.7%
3	Use graphing calculators.	10.6%
4	Use dimensional analysis to resolve disparities between the units given in the problems statement and the solution choices.	9.7%
5	Used conceptual understanding of mathematics and functions to guide their solution formulation.	8.8%
6	Decide on a potential formula for a solution, then "plugged in" one of the given numerical values and checked to see if the solution matches an answer choice.	4.4%
7	Apply proportional reasoning as an analytic method of solution.	2.7%
8	Use knowledge from prior problems and filtering of given information to find solution.	2.7%

Table 3. *Frequencies of Preservice Teachers Demonstration of Conceptual Knowledge of Functions in Contextual Problem-Solving.*

Rank	Demonstrated Approach	Frequency
1	Demonstrate the ability to apply real-world knowledge as constraints within their mathematical problem-solving.	38.5%
2	Use the definitions of a mathematical function as well as function notation.	30.8%
3	Use multiple representations to conceptualize the problem and its solution.	23.1%
4	Struggle with *unknown* constants in the problem even when the value of the problem is not necessary to find a solution.	3.8%
5	*Filter* decoded functional concepts to find the most appropriate use.	3.8%

The logical segments that included the five codes in Table 1 accounted for about 21% of the total segments across the six cases (44 out of 209). An analysis of the original questions asked, the participants' statements that contained the logical segments and the correct answers to the questions revealed that the participants were 100% successful when they converted the problem into their own words (rank #3) and when they related the problems to the Cartesian coordinate system (rank #4). The participants were found to be totally unsuccessful (0%) when the assumed the problem was actually a "physics" problem (rank #5). They were successful in about 63% of the cases when they tried to use function notation and proper mathematical language (rank #2), but only successful at a rate of about 26% when they attempted to decode the context of the problem and eliminate extraneous text (rank #1).

The logical segments related to the eight codes in Table 2 accounted for about 54% of the total segments across the six cases (113 out of 209). An analysis of the original questions asked, the participants' statements that contained the logical segments and the correct answers to the questions revealed that the participants were 93% successful when they eliminated answer choices (rank #1). The participants were found to be successful in 83% of the cases when they used graphing calculators (rank #3) and 80% successful when they used their conceptual understanding of mathematics and functions to guide their solution formulation (rank #5). They were successful in about 67% of the cases both when they applied proportional reasoning (rank #7) and when they used knowledge from prior problems and filtering of given information to find the solution (rank #8), but only successful at a rate of about 40% when after deciding on a potential formula for a solution, they "plugged in" one of the given numerical values and checked to see if the solution matched an answer choice (rank #6). Finally, the participants were found to be successful about 64% of the time when they used dimensional analysis to resolve disparities between the units given in the problem statement and the solution choices (rank #4).

The logical segments related to the five codes in Table 3 accounted for about 25% of the total segments across the six cases (52 out of 209). An analysis of the original questions asked, the participants' statements that contained the logical segments and the correct answers to the questions revealed that the participants were 100% successful when filtering decoded functional concepts to find the most appropriate (rank #5) and 95% successful when they demonstrated their ability to apply real-world knowledge as constraints (rank #1). The participants were found to be totally unsuccessful (0%) when they struggled with "unknown" constants in the problem (rank #4). They were successful in about 83% of the cases when they used multiple representations to conceptualize the problem and its solution (rank #3), and successful at a rate of about 75% when they attempted to use their definitions of a mathematical function as well as function notation (rank #2).

Even though the teachers' correct answers were not the primary focus of this study, it is still worth noting that the individual scores ranged from a low of 10% correct to a high of 80% correct. Overall, the six participants answered a total of 26 task related questions correctly out of the 59 task questions posed (44%). Due to the time constraint of the interview and the fact that some participants answered much more quickly than others, each of them were not asked the same number of questions. Participants were asked as few as eight questions and as many as 12 questions with the average being about 9.8 task questions.

It is interesting to note that none of the preservice teachers interviewed had a strong definition of a mathematical function in memory. When asked, "What is a mathematical function", the responses varied significantly. One participant responded:

> A mathematical function is a lot of things hmm, in my opinion, it's where (pause) I don't know how to explain, it's something I'm so used to doing, when I have to explain what it actually is, it's been so long since I've been told a definition that I just go through it. I wouldn't know how to explain it; it's something that comes with experience.

Another participant offered a specific definition that characterized a one-to-one correspondence between variables, and another said a mathematical function is something that shows a relationship between two or three things. Yet, even with a weak demonstrated function definition, all of the participants were successful about three-quarters of the time when they attempted to use their definitions of a mathematical function as well as functional notation. Rizzuti (1991), using 12^{th} grade mathematics students in her study, also found that her participants' weak function definitions did not hinder their function-related problem-solving ability.

Interpretation of Individual Participants' Responses

Teacher1

My initial impression of Teacher1 was that she was uncomfortable with the mathematics involved. Of the six participants, she displayed the least positive body language and she made the most negative statements about the questions on the instrument and her ability to answer the questions. My experience with students of mathematics suggests to me that her affective display was more likely related to her lack of confidence rather than lack of knowledge or skill. She answered two of the nine questions correctly, but more importantly she had a very diverse (and at times conflicting) range of responses to the questions.

She struggled with the definition of a function as well as the concept of direct and inverse proportionality, but during a discussion of a different problem she was able to demonstrate her knowledge of the function concept by implicitly expanding the functional relationship between heat and evaporation loss.

> …but a lot of things in the real-world isn't just linear, it has a lot of variability, variables throughout the day. I mean it could be sun shining one day so you are going to lose… with heat you are going to lose more

> water with evaporation whereas some days it's not sun shining so it's not going be; whereas [solution] D [she refers to a linear graph of evaporation losses versus pool radius] makes it seems like it will be sunshine every single day and in life it's really not. Some days are cloudy some days are not, so that it's not going to be perfect, staggering and that's why I say [solution] B [she refers to a quadratic graph of evaporation losses versus pool radius] because it has that curve so it kind of leaves more leeway to the days.

This problem explicitly states, "…that water evaporation is directly proportional to the surface area of the pool…", yet she was able to extend this idea based on prior knowledge that the evaporation rate of water is a function of heat. Unfortunately, this added knowledge may have played a role in the confounding of the answer. She chose the correct answer, but the line of thinking she used to arrive at that conclusion was flawed.

In her discussion of problem solutions, she demonstrated the ability to apply proper contextual meaning to the mathematics with statements like, "I would eliminate C because I know the radius wouldn't be negative; [there is] really not such a thing as a negative radius". But she was not consistent in this demonstrated ability, as shown by her notion in the passage above that the graph of a linear function suggests something "perfect" whereas the smooth curve of a quadratic function implies a more natural and "variable" response.

On several occasions Teacher1 made statements like, "… there's a lot of information in the problem and I'm trying to separate it in my mind so that I don't get anything confused", which served as examples of her demonstrated difficulties in decoding the imbedded function concept from contextualized problems. When she was able to fully grasp the concept within the contextual problem, she was able to demonstrate adequate mathematical procedural problem-solving skills. But these procedural skills were still overshadowed by statements like, "A mathematical function is a lot of things hmm, in my opinion, it's where (pause). I don't know how to explain, it's something I'm so used to doing, when I have to explain what it actually is (pause), it's been so long since I've been told a definition that I just go through it. I wouldn't know how to explain it; it's something that comes with experience." Teacher1 demonstrated knowledge of problem-solving procedures, like immediately eliminating as many answer choices as possible, but her ability to clearly demonstrate her conceptual knowledge of functions was not apparent.

Teacher2 and Teacher5

Teacher2 and Teacher5 are grouped together because they were quite similar in more than just demographics (White, female, preservice middle school teachers), they showed similar problem-solving approaches and responses. They had similar ratios of correct response to interview questions (2/10 and 3/12, respectively) and they both answered the same questions correctly and used similar reasoning in arriving at answer choices. In their solution to a problem that defined a function such that "the water evaporation is directly proportional to the surface area of the pool", both focused on a self-defined concept of change to explain their conceptual knowledge of functions. Teacher2 correctly chose answer choice B and explained, “I’m thinking that because it’s directly proportional, that means that the proportion is not changing or wouldn’t be changing and so I feel like it should be a straight line and not a curve or maybe I don’t know.” (It should be noted that independent of her self defined concept of change in graphs and its relationship to the term *directly proportional*, her answer choice contradicts her statement; choices B and C show the graphs of quadratic functions and choices A and D show the graphs of linear functions.) Teacher5 also correctly chose answer choice B and explained, “I would go with B just for the fact that that function is changing and I think it would change.” When asked why she would eliminate choice A (the graph of the linear function) she continued to explain, “Because of the changing like that one is constant…” Teacher5, like Teacher2, also used good real-world knowledge to constrain her choice to those with positive values on the independent axis knowing that the pool radius could not take on negative values.

Teacher2 and Teacher5 both struggled to decode the imbedded function concept from the contextualized problems. “Difficult, … like when they throw in the extra stuff that’s not needed” and “I think that the length of the problems and this has been the case with most of them, it’s just so overwhelming when you’re trying to think about everything at the same time” were characteristic responses by both teachers. Both teachers employed the “eliminate answer choices” problem-solving strategy as their primary procedural approach to problem-solving without giving sufficient cause for the elimination.

When they demonstrated their conceptual knowledge of functions in problem-solving, Teacher2 and Teacher5 both showed strengths in their ability to use real-world knowledge to properly constrain their problem solutions. However, they both struggled with foundational function concepts; they were unable to remember proper function definitions or were unable to apply some that they did recall. An exception to this difficulty was noted in their approach to find the zeros of a quadratic function. In this case, both teachers clearly demonstrated their understanding of the need to set the given function equal to zero and solve for the independent variable.

Teacher3

Teacher3 offered quite a contrast in style and demonstrated knowledge from Teacher1, Teacher2 and Teacher5. Teacher3 approached the problems with a calm, confident manner that allowed her to work through the most difficult problems without showing signs of frustration. She correctly answered 7 of the 10 questions, and one of the three incorrect responses was due to a simple calculation error rather than a conceptual shortcoming. Most interesting was the fact that she was the first interviewee that actually “spoke aloud her thoughts” as she solved the problems with very little prompting from the interviewer. Though it was not explicitly part of the task-based interview, Teacher3 displayed strong pedagogical skills in her explanation of her thoughts and problem-solving techniques.

She was able to consistently decode the imbedded function concept by carefully translating the text of the stated problem into her own words. She was then able to use these translated ideas to explicitly state what she believed the problem was asking and to link to definitions and concepts that she was able to recall. Additionally, Teacher3 was able to determine which parts of the text were clearly extraneous and eliminate them from future consideration as she continued her problem solution.

The primary procedural approach to problem-solving used by Teacher3 was to apply her knowledge of mathematical definitions and function concepts to properly set the problems up for an analytic solution. When posed with a problem that involved inverse proportional function, Teacher3 explained her thought in the following way, “I kind of saw that inversely proportional to the quantity so I knew that the proportion, the quantity wouldn’t be on top, it would be on bottom and I knew I would divide because when you find the inverse it’s one over something.” On several occasions she also relied on the graphing calculator as part of her problem-solving approach, but she also made it clear that, “if I can, I’ll solve it without using the calculator to better understand why it's doing what it's doing.” While she also used the technique of the elimination of answers, she was more likely to actually work through the problem and compare her answer with the answer choices available.

Teacher3’s primary means of demonstrating her conceptual knowledge of functions involved the way she interpreted and set-up her problem solution strategy. For example she stated, “We know that they’re looking for how much evaporation loss is lost per day with the gallons of water and I know that the water evaporation is directly proportional to the surface area.” She continued by describing the mathematical operations that will be necessary to complete the calculations and properly used dimensional analysis to convert a given quantity into a quantity more suitable for problem solution. Teacher3 also showed strength in her ability to use real-world knowledge to properly constrain her problem solutions.

Teacher4

Teacher4 offered the greatest contrast between his demonstrated knowledge as evidenced by the number of correct answers to the posed questions (4/8) and the quality of his responses during the interview. Like Teacher3, Teacher4 was quite impressive as he “spoke aloud his thoughts” as he solved the problems. He did this with minimal prompting from the interviewer. Also like Teacher3, he displayed strong pedagogical skills in his explanation of his thoughts and problem-solving techniques and he was very confident and relaxed during the interview. As a preservice secondary school mathematics teacher, Teacher4’s additional mathematics training was evident in his discussions. In three of the four incorrect answers to questions posed, Teacher4 knew exactly how to set up and solve the problem. It seems that he rushed the solutions (in two cases, he did the problems in his head) and therefore gave incorrect answers to concepts and procedures that he clearly understood. There was only one problem for which Teacher4 did not understand how to arrive at a correct solution.

Like Teacher3, he was able to consistently decode the imbedded function concept by carefully translating the text of the stated problem into his own words. He was then able to use these translated ideas to explicitly state what he believed the problem was asking and to link to the wealth of mathematical definitions and concepts that he easily recalled. Furthermore, Teacher4 was also able to determine which parts of the text were clearly extraneous and eliminate them from future consideration as he continued his problem solution.

Teacher4 also mirrored Teacher3's proclivity for using an analytical approach to problem-solving, but unlike Teacher3 he demonstrated a mild aversion to the use of technology in his solutions. He openly confessed, "Then my lack of technology skills, umm, made it difficult to figure out the exact point and I'm very slow at using a calculator." His secondary procedural approach to problem-solving was the proper use of elimination of answers as he worked through the problems.

Teacher4 demonstrated his conceptual knowledge of functions in statements like, "It says it's a modulating activity therefore I was thinking that the graph will have a series of ups and downs…" and "and then the fact that the formula has a square we knew that the graph should increase exponentially not linearly." Like Teacher2, Teacher3 and Teacher5, he also showed as a strong point his ability to use real-world knowledge to properly constrain his problem solutions. Teacher4 was particularly adept at using dimensional analysis to verify his problem solution strategy.

Teacher6

Teacher6 was a preservice middle school mathematics teacher, but unlike the others, she was a mathematics major before switching to education. It seemed that this additional training in mathematics allowed her to perform at a higher level of competence than all of the other preservice teachers that were interviewed in this study. Teacher6 also approached the problems with a calm, confident manner that allowed her to work through the most difficult problems without showing signs of frustration. She correctly answered eight of the ten primary questions, and since she showed signs of having a deeper understanding of the concepts, she was asked several conceptual "follow-up" questions which she also answered correctly. Teacher6 also "spoke aloud her thoughts" as she solved the problems with very little prompting from the interviewer and like Teacher3 and Teacher4, she displayed strong pedagogical skills in her explanation of her thoughts and problem-solving techniques.

Unlike Teacher3 and Teacher4, she was unable to initially decode the imbedded function concept as she translated the text of the stated problem into her own words. She freely confessed, "If I would have been paying more attention and read the problem more carefully, it might have taken a lot less time." It seemed that she merely needed time to become acclimated to the dense text in these contextualized problems, because after reading the first few problems, Teacher6 was also able to determine which parts of the text were extraneous and eliminate them from future consideration.

Teacher6 also mirrored Teacher3's proclivity for using an analytical approach to problem-solving, but unlike Teacher4 she demonstrated a higher level of affinity for the use of technology (the graphing calculator) in her solutions than all of the other teachers interviewed. After describing how to solve the problem conceptually she added, "Then, since it gave you the equation, you can pretty much just plug it into the calculator and find out where it's equal to zero." Her secondary procedural approach to problem-solving was the proper use of elimination of answers as she worked through the problems.

Teacher6 demonstrated her conceptual knowledge of functions in statements like, "Ok so it gives you a function of the activity of the thing which is t^2+3 times t^2- 5t + 4 and it says you only can remove the capsule when the activity is at zero, so we need to find the roots of that function" and "the dependent variable would be the evaporation loss per gallon per day, and [the independent variable is] the radius of the circle of the pool." Like most of the other teachers, she also showed her ability to use real-world knowledge to properly constrain her problem solutions. Teacher6 also paid attention the units of the problem and was adept at using dimensional analysis to verify her problem solution strategy.

Discussion

Decoding Strategies

One of the questions under consideration in this study was to better understand how preservice teachers decode the embedded function concept from a contextualized problem. The most challenging aspect of this question is, "How much context makes a good contextual problem and how much does it take to simply muddy the waters?" It was not a goal of this study to answer that question directly, but that question remains a part of the conversation as we consider the results. A priori assumptions concerning this research question included the preservice teachers being either confounded by the language and sheer volume of text in the problems and/or being unable to recall or apply enough function concepts and definitions to be effective problem solvers.

The results showed that the most frequently used approach to decode the embedded function concept was to "eliminate the extraneous text" but the participants who used that approach were only successful about a quarter of the time. On the other hand, participants were always successful when they “converted problem into their own words for better understanding” or “related the embedded function in familiar ‘x’ and ‘y’ terms even when the Cartesian coordinate system has nothing to do with the problem at hand.” However, these more successful approaches were rarely used. These results show that the methods used by the preservice teachers most often were not the methods that they were most successful with and the most successful methods were among the least used by the preservice teachers. Though the studies are quite different, these results are contrary to finding by Nathan and Koedinger where beginning algebra students employed the methods “that gave the highest likelihood of success (around 70%) and led to the greatest number of correct solutions when they were applied” (2000, p. 176). A similar study using preservice teachers as the participants has not been discovered.

Procedural Approaches

The preservice teachers showed a wider range of procedural approaches than they did in either decoding or demonstrating function concept knowledge. Their procedural approaches accounted for about half of the total observed strategies. The most frequently used procedure, “eliminating answer choices”, was also the method that produced success on almost every item. Eliminating answer choices, along with using dimensional analysis, “top-down” analytic solution methods and multiple representations were the strategies hypothesized the preservice teachers would use, but it was not anticipated that eliminating answer choices would be used to that extent. Nor was it anticipated that the eliminating answer choices procedure would be used in such a casual manner as using the statement “it doesn’t seem right” as justification.

Eliminating answer choices is a popular test-taking strategy that is taught throughout most of a student’s K – 12 academic experiences, but it is best applied after careful consideration of the correct answer to the problem (Hong, Sas, & Sas, 2006; Texas Education Agency, 2009). Students should not eliminate an answer choice simply because “it doesn’t seem right”, but rather they should have a logical basis for answer elimination. “Students should NOT eliminate answer choices that they are not sure about, only those that they can logically show are wrong using either information in the question or facts that they know” (Noel, 2010).

Prior to data collection, the researcher hypothesized a strong demonstration of their knowledge of conceptual and procedural problem-solving skills related to contextualized mathematical function problems would have included more “top-down” analytic solutions (which would include the use of correct mathematical definitions in their problem-solving strategies), more dimensional analysis used in problem-solving as well as more discussions in multiple representations. Only the top two teachers displayed these types of problem-solving skills. These skills add to the richness of problem-solving methods as mathematics students are tasked with complex STEM problems (Connally et al., 2004).

A promising result in this study was observed in the preservice teachers’ ability to apply real-world knowledge as constraints within their mathematical problem-solving. This strategy was their most frequently demonstrated approach as well as a very successful approach for them. The ability to properly apply constraints to a novel mathematical problem to create a useful real-world solution is a key attribute in project-based learning environments, which are central to STEM curricula (Prince & Felder, 2006). Another promising result was found in their ability to correctly use multiple representations to find or discuss problem solutions. Unfortunately, it was only used about a quarter of the time.

Implications and Conclusions

Since today's middle and high school mathematics teachers provide the academic foundation for future STEM college graduates, it is important for mathematics educators to be able to assess just how well prepared the mathematics teachers are in integrating contextual function problem-solving into the 6 – 12 curriculum. In the setting of a university mathematics course, a freshman or sophomore student can recite function definitions, examples and counter-examples of mathematical functions. The path to academic achievement can seem quite linear; the instructor lectures on the function concept, the students reinforce the lecture with homework assignments and within a relatively short time frame the students demonstrate their acquisition of this new knowledge by passing an exam. But what happens a year or two later when that same preservice teacher is asked to demonstrate their knowledge of the function concept? As these preservice teachers make ready to transition to in-service mathematics teachers, how much conceptual knowledge of functions do they maintain? How do they apply this knowledge in problem-solving? How well can preservice teachers adapt classroom function concepts in contextual applications that tend to be buried in extraneous details and text?

The ability to retain problem-solving skills is not a matter of memory retention related to specific problems or problem types, but rather a profound knowledge of the foundational concepts that form the backbone of the mathematical problems (L. Ma, 1999). Researchers need a way to measure teachers' knowledge to develop better preservice programs and summative assessments prior to professional licensure. If a reliable and valid instrument is employed to establish preservice teachers' content knowledge of mathematical function, then that instrument can be used with induction year in-service middle and secondary school mathematics teachers to establish the quality and quantity of problems solving skills retained in the transition from student to teacher.

The ability of a teacher to adapt content knowledge into pedagogical content knowledge and to employ it in the classroom in such a way as to increase student learning of mathematics is a central theme in mathematics education research (Darling-Hammond & Youngs, 2002; Hill, 2007; Hill & Ball, 2004; Hill et al., 2005; Kulm, 2008; Lappan & Ferrini-Mundy, 1993). Results from this study have provided a view into how preservice teacher decode function concepts embedded in context similar to traditional problems found in other STEM courses. The use of this instrument, and its future generations, can provide mathematics education researchers quantifiable evidence of the interaction between preservice teachers' knowledge and the methods they use to convert that knowledge into useful problem-solving skills.

There are many extant instruments to measure a wide range of mathematical content topics and attitudes, but an instrument which specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published. In the effort to validate a new content knowledge of mathematical function instrument, a pilot test was required to gain further insight into the mathematical thinking of preservice teachers while they complete contextualized mathematical function problem-solving tasks. The purpose of this case study was to describe the mathematical problem-solving cognition related to contextualized mathematical function problems for preservice middle and secondary school mathematics teachers. Inferences drawn from the mathematical problem-solving cognition will aid in the development and validation of an instrument to assess preservice mathematics teachers' knowledge of the mathematical concept of function.

Results, in general, will help establish the mathematical content and style of the next generation of the contextual function instrument. Results from the decoding strategies employed by the preservice teachers in this study will impact the complexity of the context in future problems and the finding from the procedural approaches used will likely push future instrument development towards a free-response instrument rather than a multiple-choice instrument.

CHAPTER III

THE DEVELOPMENT AND PSYCHOMETRIC MEASUREMENT OF AN INSTRUMENT TO ASSESS PRESERVICE MATHEMATICS TEACHERS' CONTENT KNOWLEDGE IN CONTEXTUAL FUNCTION PROBLEM-SOLVING

Background

Of the algebraic topics covered in middle and secondary schools, researchers recognize the concept of function as the single most important tool in algebra regarding a student's ability to apply mathematical concepts in sciences, engineering and other related contexts (Hollar & Norwood, 1999; Knuth, 2000; Lloyd & Wilson, 1998; O'Callaghan, 1998; Zbiek, 1998). The National Council of Teachers of Mathematics (NCTM) expects all students, beginning in grade six, to be able to "model and solve contextualized problems using various representations such as graphs, tables, and equations" (National Council of Teachers of Mathematics, 2000a), which require students to possess a working knowledge of functions in contextualized environments.

With the focus of mathematics education research on students' achievement, it is important to note that teacher knowledge is the most important factor influencing student learning (Dooren et al., 2002; Lappan & Ferrini-Mundy, 1993; National Council of Teachers of Mathematics, 1991; The Education Alliance, 2006). Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (Kulm, 2008) and earlier research found that teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990).

Finding highly qualified teachers can be particularly challenging in an environment where it has been reported that 68.5% of the middle school teachers in the US have neither a major nor have certification in mathematic (Li & Kulm, 2008). Do middle and secondary school mathematics teachers have sufficient content knowledge of the function concept in contextual environments to perform at the high expectations of today's STEM curriculum?

Since mathematics teachers have the greatest influence in students' mathematical development (Dooren et al., 2002), it is imperative that teachers possess the depth of mathematical content knowledge and the experience of contextual problem-solving to effectively lead students. Researchers believe one of the contributing factors to the disconnect between real-world insights in students' problem-solving abilities is "the way in which these problems are considered and used in current instructional practice and culture, and more specifically the lack of systematic attention to the modeling perspective by the teacher"(Verschaffel et al., 1997, p. 340).

Teachers' Knowledge of the Concept of Function

A question that might serve to focus our attention could be phrased simply as, "What do mathematics teachers need to know to be effective in teaching functions in middle and secondary mathematics classrooms?" Generally speaking, middle and secondary school mathematics teachers are responsible for diverse range of subjects ranging from pre-algebra through AP® calculus and the concept of function is a common thread woven through all of these courses (Educational Testing Service, 2009c; W. Ma & Freedson, 2002; National Council of Teachers of Mathematics, 2000a; Texas

Education Agency, 2008, 2009). To be successful in any of these courses, a mathematics teacher must have a firm grasp of both the pedagogical knowledge necessary for the classroom instruction as well as mathematical content knowledge of the material they are teaching (Kulm, 2008; Mohr, 2008). Shulman (1986) cautions against viewing these notions as being mutually exclusive or even independent thoughts, but rather he introduces the notion of "pedagogical content knowledge" to provide a paradigm bridge between these complementary ideas. But there are still unanswered questions concerning teacher knowledge.

Shulman regards the following as "the central questions for disciplined inquiry into teacher education":

> What are the domains and categories of content knowledge in the minds of teachers? How, for example, are content knowledge and general pedagogical knowledge related? In which forms are the domains and categories of knowledge represented in the minds of teachers? What are promising ways of enhancing acquisition and develop? (p.9)

These questions are used to form the basis of his theoretical framework of teacher knowledge, which he divides into three categories; subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Of these three, this study will focus on subject matter content knowledge.

If content knowledge is viewed as an independent construct that only served the individual teacher, it could be discounted as irrelevant to the goals of classroom instruction. Research has shown that teacher's knowledge is not an independent construct within only the teacher, but rather that a teacher's mathematical content knowledge is directly related to pedagogical content knowledge and therefore has a direct effect on student learning (N. Webb, 1979). Just knowing that a teacher has specific content knowledge is not enough, we need to know that they have the appropriate knowledge for the content domain being taught and we need to know that they have a sufficient level of that knowledge (Sherin, 2002). It was the aim of this study to develop and validate an instrument to measure middle and secondary school preservice mathematics teachers' content knowledge of the function concept in a contextual environment.

Test Development

Testing, which has been in existence for about 3000 years (Allen & Yen, 2002), is used to measure a characteristic, trait or quality in a person or object. A test can also be defined as "an evaluative device or procedure in which a sample of an examinee's behavior in a specified domain is obtained and subsequently evaluated and scored using a standardized process" (American Educational Research Association, American Psychological Association, National Council on Measurement in Education, & Joint Committee on Standards for Educational and Psychological Testing (U.S.), 1999, p. 3). Carmines and Zeller (1979) describe measurement as a "process" of quantifying empirical data with abstract concepts. The creation and subsequent validation of a "test" or assessment instrument is firmly grounded in measurement theory. "Measurement theory is a branch of applied statistics that attempts to describe, categorize and evaluate the quality of measurements, improve the usefulness, accuracy and the meaningfulness of measurements, and propose methods for developing new and better measurement instruments" (Allen & Yen, 2002, p. 2). In the quest improve the usefulness, accuracy and meaningfulness of quantitative instruments, two common metrics are often used: reliability and validity.

When developing new instrumentation for testing, great care should be taken to show that the newly created instrument is both reliable and valid. Without verification of these two key qualities, the proposed instrumentation cannot be trusted to provide a useful link between observable phenomenon and abstract ideas (Carmines & Zeller, 1979). Generally speaking, reliability is a measure of how consistent an instrument is in measuring the same quality repeatedly. "Fundamentally, reliability concerns the extent to which an experiment, test, or any measure procedure yields the same results on repeated trials" (Carmines & Zeller, 1979, p. 11).

Common methods of assessing reliability in social science research are internal-consistency estimates of reliability and include the split halves method which can yield the Spearman-Brown coefficient for tests where the halves can be classified as parallel, or Cronbach's alpha (α) when the test halves are considered essentially τ-equivalent (Allen & Yen, 2002; Carmines & Zeller, 1979). The Kuder-Richardson #20 (K-R 20) coefficient, like the Spearman-Brown coefficient and Cronbach's alpha, is also a measure of an instrument's internal consistency reliability. These forms of internal consistency reliability testing procedure display a contrast to procedures that aim to measure reliability over a period of time, such as the test-retest reliability procedure (Huck, 2008).

Reliability is a measure of an instrument's consistency whereas validity quantifies how well an instrument is able to measure its intended dimension. This could commonly be phrased, "how well a test measures what it is supposed to measure" (Orcher, 2005). Sireci (2007, p. 478) summarizes key aspects of validity in the following:

- Validity is not a property of a test. Rather, it refers to the use of a test for a particular purpose.
- To evaluate the utility and appropriateness of a test for a particular purpose requires multiple sources of evidence.
- If the use of a test is to be defensible for a particular purpose, sufficient evidence must be put forward to defend the use of the test for that purpose.
- Evaluating test validity is not a static, one-time event; it is a continuous process.

The three major types of validity used in research in the social science are content validity, criterion-related validity, and construct validity (Allen & Yen, 2002; Carmines & Zeller, 1979; Orcher, 2005). This study focused on content and construct validity for the new instrument.

Problem

Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (Kulm, 2008). More specifically, teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979). Learning complex concepts, such as the concept of function, is much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995) but research suggests that the primary use of contextual, real-world instruction is relegated to developmental mathematics courses in post secondary education (Grubb & Kraskouskas, 1992; Stone et al., 2006; Wiseley, 2009) even though state teacher certification examinations are rich in contextualized problems (Educational Testing Service, 2009a, 2009b; Texas Education Agency, 2009). There are many extant instruments to measure a wide range of mathematical content topics and attitudes, but an instrument which specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published (I. A. Brown, 2010).

The purpose of this study was to create an instrument to assess mathematics teachers' content knowledge of the function concept in a contextual environment and measure the psychometric properties of the instrument for the purpose of measuring middle and secondary school preservice mathematics teachers' content knowledge of the function concept in a contextual environment. The results from this study will aid mathematics education researchers in developing future assessments of a teacher's ability to solve problems in a contextual environment. This may offer teacher educators insight into possible modifications to preservice programs and provide school administrators a clearer perspective into the professional development needs of mathematics teachers.

Research Questions

The research questions were driven by a desire to gain a deeper understanding of the complex nature of instrument development and the validation of an instrument for a particular mathematical assessment. The questions reflect the iterative *design-test-redesign* method used in this study. The initial set of problems used in the instrument was developed based on research findings and items used in the TExES™ and Praxis™ examinations (Brown, 2010). The questions were pilot tested with a sub-sample of middle school preservice mathematics teachers which prompted a redesign of the problem set. This redesigned problem set was then evaluated by the subject matter experts which again led to further redesign of the instrument. The goal of this design-test-redesign method was to determine which items should be included in the instrument and how those individual items chosen performed together as an instrument. The questions that formed the basis of this study were:

1. What are the key items in assessing contextual function concepts that should be included in an instrument to assess preservice mathematics teachers' knowledge?
2. What are the psychometric properties of an instrument developed to assess preservice middle and secondary mathematics teachers' knowledge of the mathematical concept of function within a contextual environment?

These two questions served as foundations to guide the research project reported in this paper.

Theoretical Framework

"Validity refers to the degree to which evidence and theory support the interpretations of test scores entailed by proposed uses of tests. Validity is, therefore, the most fundamental consideration in testing and evaluating tests" (American Educational Research Association et al., 1999, p. 9). Validity is not a characteristic of a test but rather it is a metric related to how the test is used nor is it a singular measurement but rather validity is established by an amalgamation of validity evidence (American Educational Research Association et al., 1999; Lissitz, 2009; Sireci, 2007). The *Standards for educational and psychological testing* "outline various sources of evidence that might be used in evaluating a proposed interpretation of test scores for particular purposes" (American Educational Research Association et al., 1999, p. 11) which are evidence based on test content, response processes, internal structure, relations to other variables, and consequences to testing. This research attempts to establish validity for the proposed use of the instrument through the processes of item review and factor analysis.

Item Selection and Review

"Evidence of content validity is needed whenever performance on a sample of items (the test) is used to make inferences about the broader domain of which the test items are a sample" (F. G. Brown, 1983, p. 132). This is particularly true of cognitive achievement tests like the instrument developed in this study. In defining content validity, the *Standards* define test content in following manner, "Test content refers to the themes, wording, and format of the items, tasks, or procedures regarding administration and scoring" (American Educational Research Association et al., 1999, p.

11). The goal of establishing content validity for this study is to show that the instrument represents a subset of the skills or competencies of the broader domain, namely preservice teachers' knowledge of the function concept in a contextual environment. The primary method for establishing content validity is by conducting a review of test items by an expert panel (Carmines & Zeller, 1979; Huck, 2008).

Standard 1.6 and Standard 1.7 serve to guide the item selection and review processes:

> Standard 1.6: When the validation rests in part on the appropriateness of test content, the procedures followed in specifying and generating test content should be described and justified in reference to the construct the test is intended to represent.
>
> Standard 1.7: When a validation rests in part on the opinions or decisions of expert judges, observers, or raters, procedures for selecting such experts and for eliciting judgments or ratings should be fully described. The qualifications, and experience, of the judges should be presented. The description of procedures should include any training and instructions provided, should indicate whether participants reached their decisions independently, and should report the level of agreement reached. If participants interacted with one another or exchanged information, the procedures through which they may have influenced one another should be set forth. (American Educational Research Association et al., 1999)

A pathway to provide evidence of validity can be developed by providing a logical structure that demonstrates the relationships between the test items and the construct under examination and providing the method for properly identifying and qualifying the experts who will ultimately judge the validity claims.

Measurement Issues

It is important to recognize that even if the sample chosen is truly representative of the population under study, there can still be errors associated with the collected data. One of the most obvious sources for error is due to survey non-response (Ott & Longnecker, 2001). To reduce the potential effects due to non-responsive participants, the researcher worked closely with most of the faculty at the universities under study to improve faculty *buy-in* on the research. The faculty members have also been offered the opportunity to join the researcher in future potential journal publications that use the collected data from their respective programs.

One area of bias that the instrument has very little protection against is respondent bias (Abbasi, 2000). Since the survey is anonymous, and consists of non-personal type questions, the natural instinct to protect ones self should not be as strong as with other types of surveys. Errors due to processing errors can be guarded against by using structure checks, duplication checks, and omission checks and by having the checks verified by an independent, third party (graduate student) to the research (Abbasi, 2000; Ott & Longnecker, 2001).

Due to the highly contextual nature of the instrument, there is also the potential for leading questions as well as unclear or poorly worded questions. The initial item review, discussed in a following section, should allow most of these type errors to be identified and corrected or eliminated. Care has been taken in the design of the items to make them both clear and neutral in language, but the language itself is another area of concern. It is assumed that preservice teachers are able to read at or above the level of the language used in the creation of the instrument, but as mentioned in the next section, the expert panel will provide feedback concerning any potential language level concerns they may perceive.

Factor Analysis

Factor analysis is a statistical method of data analysis that is used to help determine construct validity. “A prime use of factor analysis has been in the development of both the operational constructs for an area and the operational representatives for the theoretical constructs” (Gorsuch, 1983, p. 350). A construct, for the purposes of this study, can be thought of as a theoretical concept that logically binds test items. In developing an instrument to measure mathematical cognitive skills or knowledge, it is reasonable to question whether all of the test items are based on one particular mathematical concept or are several concepts represented in the items. Each of these constructs that bind groups of items, of a particular mathematical concept for example, will be a factor in the factor analysis of the instrument.

Since the problems were developed from three specific function content areas in the TExES™ and Praxis™ examinations, it is assumed that these three areas will also constitute the three factors in the instrument. It is also true that the test items were developed based on three of the four *Levels of Demands of Mathematical Tasks* as set forth by Smith and Stein (Smith & Stein, 1998). We hypothesize that a factor analysis will show that the items selected for use in the instrument are related based on the three mathematical function concepts under examination, namely linear functions, quadratic functions, and exponential functions. We further hypothesize that a factor analysis, based on different factors, will show that the items are correlated based on three of the levels of demand of mathematical tasks, namely Lower-Level Demands (procedures without connections), Higher-Level Demands (procedures with connections), and Higher-Level Demands (doing mathematics). Therefore, confirmatory factor analysis (CFA) will be used to verify the a priori hypotheses concerning the model structure, factors, and factor loading of the items in the instrument.

When qualifying model structures, several indices are used to guide the model refinement process. Brown (2006) describes the overall goodness of by saying:

> Goodness-of-fit indices provide a global descriptive summary of the ability of the model to reproduce the input covariance matrix, but the other two aspects of fit evaluation (localized strain, parameter estimates) provide more specific information about the acceptability and utility of the solution (p. 133).

In addition to the goodness-of-fit index, the Root Mean Square Error of Approximation (RMSEA), the Weighted Root Mean Square of Residual (WRMR), and the Comparative Fit Index / Tucker-Lewis Index (CFI/TLI) should be used to determine the model's psychometric properties (Albright & Park, 2009; T. A. Brown, 2006).

Method

Development of the Instrument/Item Creation

The initial draft of the instrument consisted of fifteen multiple choice questions to assess middle and secondary school preservice mathematics teachers' knowledge of the mathematical function concept in a contextually rich environment. A sample question follows:

> Pricilla's newly formed company, Aggie Pools, Inc., is in the business of installing round, in ground swimming pools. She offer pools from the economy sized (20 feet in diameter) to the "Hummer" of backyard pools (80 feet in diameter). A common concern of her customers is the amount of water lost each day due to evaporation. To maintain a constant level in the swimming pool, each gallon of water lost to evaporation must be made up with fresh water from the city water department. The local fresh water supply rate is \$4.80 for the first 2000 gallons of water and \$1.96 for each additional thousand gallons. Armed with the knowledge that water evaporation is directly proportional to the surface area of the pool and that evaporation loss rates in her service area average about 1/120 gallons per hour per ft^2 of pool surface area, she decides to create a formula that relates evaporation loss per day as a function of the radius of the pool.
>
> Of the four graphs below, which of the graphs best represents the real-world use of the above function?

The question above was followed by four graphical representations of functions. One of the four graphs will best represent the function described by the problem statement while the remaining three graphs will be incorrect or less desirable solutions to the problem.

The question above and fifteen similar questions were created by the authors and are based on common competencies described in the TExES™ and Praxis™ teacher certification examinations for middle and secondary school preservice mathematics teachers (Educational Testing Service, 2009c; Texas Education Agency, 2009). Texas middle school mathematics teachers are required to demonstrate successful performance on the "115 Mathematics 4 – 8" examination and Texas secondary school mathematics teachers are required to demonstrate successful performance on the "135 Mathematics 8 – 12" examination(Texas Education Agency, 2008). North Carolina middle school mathematics teachers are required to demonstrate successful performance on the "Middle School Mathematics (0069)" examination and North Carolina secondary school mathematics teachers are required to demonstrate successful performance on the "Mathematics: Content Knowledge (0061)" examination(Educational Testing Service, 2009a, 2009b, 2009c).

For the middle school function content, common requirements between the TExES™ (domains II & V) and the Praxis™ (content categories I & III) examinations included models, representations, and transformations, linear and non-linear functions, properties of functions and their graphs, mathematical models of real-world situations, analysis and evaluation models, use of multiple representations of a concept, and use mathematical models in other academic disciplines. The secondary school function content maintains all of the middle school function content with the additional requirement to be able to use the properties of polynomial, exponential, and trigonometric functions to analyze, graph, model, and solve problems based on TExES™ (domain II) and the Praxis™ (content categories IV-Trigonometry & V-Functions). The initial content in the instrument was based on four specific domain areas; linear, quadratic, trigonometric, and exponential functions.

A fundamental goal in item selection was to ensure alignment between the standards, in this case the TExES™ and Praxis™ examinations, and the instrument. The items in this version of the instrument were created to be consistent between (1) a common category of content between the standards, (2) Bloom's Taxonomy for the cognitive domain (Krathwohl, 2002), and (3) cognitive demand as defined by Webb's (2002, p. 4) "depth-of-knowledge". Webb (2007, p. 2) defines the following four levels of depth of knowledge:

> Level 1 (recall and reproduction) is the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple science process of procedure. A student answering a Level 1 item either knows the answer or does not.
>
> Level 2 (skills and concepts) includes the engagement of some mental processing beyond recalling or reproducing a response. The content knowledge or process involved is more complex than in Level 1. Keywords that generally distinguish a Level 2 item include 'classify,'

> 'organize,' 'estimate,' 'make observations,' 'collect and display data,' and 'compare data.'
>
> Level 3 (strategic thinking) requires reasoning, planning, using evidence, and higher level of thinking than the previous two levels. The complexity results because the multistep task requires more demanding reasoning.
>
> Level 4 (extended thinking) Tasks at this level have high cognitive demands and are very complex. Students are required to make several connections, to related ideas within the content area or among content areas—and have to select or devise one approach among many solution alternatives. This level requires complex reasoning, experimental design and planning, and probably will require an extended period of time, either for the science investigation required by an objective, or for carrying out the multiple steps of an assessment item.

A careful review of TExES™ and Praxis™ examinations reveals there are no "Level 4" depth-of-knowledge competencies required, therefore there are no "Level 4" items in this instrument.

Table 4, adapted from Hess' work on "cognitive complexity" (Hess, 2006, p. 6), shows the association between Bloom's Taxonomy and Webb's Depth of Knowledge (DoK) levels. The areas highlighted in bold print emphasize the intersection of the cognitive complexity items and the common competencies between the TExES™ and Praxis™ mathematics examinations relating to linear, quadratic, and exponential function problems that are of interest in this study.