Measurement of Branching Fractions and Search for $CP$-Violating Charge Asymmetries in Charmless Two-Body $B$ Decays into Pions and Kaons

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We present measurements of the branching fractions and a search for CP-violating charge asymmetries in charmless hadronic final states of kaons and pions. The results are based on a data sample of approximately 23 million $BB$ pairs collected by the BABAR detector at the PEP-II asymmetric $B$ Factory at SLAC. We find the following branching fractions: $B(B^0 \to \pi^+\pi^-) = (4.1 \pm 1.0 \pm 0.7) \times 10^{-6}$, $B(B^0 \to K^+\pi^-) = (16.7 \pm 1.6 \pm 1.3) \times 10^{-6}$, $B(B^+ \to K^+\pi^0) = (10.8^{+2.3+1.0}_{-1.9-1.0}) \times 10^{-6}$, $B(B^+ \to K^0\pi^+) = (18.2^{+3.3+2.0}_{-3.6-2.0}) \times 10^{-6}$, $B(B^0 \to K^0\pi^0) = (8.2^{+3.7+1.2}_{-2.7-1.2}) \times 10^{-6}$. We also report the 90% confidence level upper limits $B(B^0 \to K^+K^-) < 2.5 \times 10^{-6}$, $B(B^+ \to \pi^+\pi^0) < 9.6 \times 10^{-6}$, and $B(B^+ \to \bar{K}^0K^+) < 2.4 \times 10^{-6}$.

In addition, charge asymmetries have been measured and found to be consistent with zero, where the statistical precision is in the range of $\pm 0.10$ to $\pm 0.18$, depending on the decay mode.

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The study of $B$ meson decays into charmless hadronic final states plays an important role in the understanding of CP violation. In the Standard Model, all CP-violating phenomena are a consequence of a single complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. Recently, the Belle and BABAR collaborations published results on measurements of CP-violating asymmetries in $B$ decays into final states containing charmonium, leading to constraints on the angles of the CKM Unitarity Triangle. Measurements of the rates and charge asymmetries for $B$ decays into the charmless final states $\pi\pi$ and $K\pi$ can be used to constrain the angles $\alpha$ and $\gamma$ of the Unitarity Triangle.

In this Letter we present new measurements of the branching fractions for $B$ meson decays to the charmless hadronic final states $\pi^+\pi^-$, $K^+\pi^-$, $K^+\pi^0$, $K^0\pi^+$ and $K^0\pi^0$. In addition, we search for charge asymmetries in the modes $B^0 \to K^+\pi^-$, $B^+ \to K^+\pi^0$ and $B^+ \to K^0\pi^+$. Previous measurements of these decays were reported by the CLEO Collaboration.

The data sample used in these analyses was collected with the BABAR detector at the PEP-II $e^+e^-$ collider at the Stanford Linear Accelerator Center. It corresponds to an integrated luminosity of 20.6 fb$^{-1}$ taken on the $\Upsilon(4S)$ resonance (“on-resonance”) and 2.61 fb$^{-1}$ taken at a center-of-mass (CM) energy 40 MeV below the $\Upsilon(4S)$ resonance (“off-resonance”), which are used for continuum background studies. The on-resonance sample corresponds to $(22.57 \pm 0.36) \times 10^6$ $BB$ pairs. The collider is operated with asymmetric beam energies, producing a boost ($\beta\gamma = 0.56$) of the $\Upsilon(4S)$ along the collision axis. The boost increases the momentum range of two-body $B$ decay products from a narrow distribution centered near 2.6 GeV/c to a broad distribution extending from 1.7 to 4.3 GeV/c.

The BABAR detector is a spectrometer of charged and neutral particles and is described in detail in Ref. 3. Charged particle (track) momenta are measured in a tracking system consisting of a 5-layer, double-sided, silicon vertex detector and a 40-layer drift chamber (DCH) filled with a gas mixture of helium (80%) and isobutane (20%), both operating within a 1.5 T superconducting solenoidal magnet. Photons are detected in an electromagnetic calorimeter (EMC) consisting of 6580 CsI(Tl) crystals. Charged hadron identification is based on the Cherenkov angle $\theta_c$ measured by a unique, internally reflecting Cherenkov ring imaging detector (DIRC).

Hadronic events are selected based on track multiplicity and event topology. Backgrounds from non-hadronic events are reduced by requiring the ratio of Fox-Wolfram moments $H_2/H_0$ to be less than 0.95 and the sphericity of the event to be greater than 0.01.

All tracks (except $K^0_S$ decay products) are required to have a polar angle within the tracking fiducial region $0.41 < \theta < 2.54$ rad and a Cherenkov measurement from the DIRC. The latter is satisfied by 91% of the tracks in the fiducial region. We require a minimum number of Cherenkov photons associated with each $\theta_c$ measurement in order to improve the resolution. The efficiency of this requirement is 97% per track. Tracks with a $\theta_c$ within $3\sigma$ of the expected value for a proton are rejected. Electrons are rejected based on specific ionization $(dE/dx)$ in the DCH system, shower shape in the EMC, and the ratio of shower energy to track momentum.

Candidate $K^0_S$ mesons are reconstructed from pairs of oppositely charged tracks that form a well-measured vertex and have an invariant mass within $3.5\sigma$ of the nominal $K^0_S$ mass. The measured proper decay time of the $K^0_S$ candidate is required to exceed 5 times its error.

Candidate $\pi^0$ mesons are formed from pairs of photons with an invariant mass within $3\sigma$ of the nominal $\pi^0$ mass. Photons are defined as showers in the EMC that have the expected lateral shape, are not matched to a track, and have a minimum energy of 30 MeV. The $\pi^0$ candidates are then kinematically fitted with their mass constrained to the nominal $\pi^0$ mass.
B meson candidates are reconstructed in four topologies: $h^+h'^-$, $h^+\pi^0$, $K^{0}\bar{h}$, and $K^{0}\pi^0$, where the symbols $h$ and $h'$ refer to $\pi$ or $K$. The kinematic constraints provided by the $\Upsilon(4S)$ initial state and relatively precise knowledge of the beam energies are exploited to efficiently identify $B$ candidates. We define a beam-energy substituted mass $m_{ES}$ as $\sqrt{E_b^2 - \vec{p}_B^2}$, where $E_b = (s/2 + \vec{p}_i \cdot \vec{p}_j)/E_i$, $\sqrt{s}$ and $E_i$ are the total energies of the $e^+e^-$ system in the CM and lab frames, respectively, and $\vec{p}_i$ and $\vec{p}_B$ are the momentum vectors in the lab frame of the $e^+e^-$ system and the $B$ candidate, respectively. To improve the resolution in modes containing $\pi^0$ mesons, the $B$ candidate is kinematically fitted with the energy constrained to the CM beam energy. For all modes, the $m_{ES}$ resolution is dominated by the beam energy spread and is approximately 2.5 MeV/c$^2$. Candidates are selected in the range $5.2 < m_{ES} < 5.3$ GeV/c$^2$.

We define an additional kinematic parameter $\Delta E$ as the difference between the energy of the $B$ candidate and half the energy of the $e^+e^-$ system, computed in the CM system, where the pion mass is assumed for all charged decay products of the $B$. The $\Delta E$ distribution is peaked near zero for modes with no charged kaons and shifted on average $-45$ MeV ($-91$ MeV) for modes with one (two) kaons, where the exact separation depends on the laboratory kaon momentum. The resolution on $\Delta E$ is mode dependent. For final states that contain no $\pi^0$ mesons the resolution is about $26$ MeV. For modes with $\pi^0$ mesons the resolution is about $42$ MeV and is asymmetric due to underestimation of the $\pi^0$ energy in the EMC. Candidates are accepted in the following $\Delta E$ ranges (given in GeV): $[-0.15, 0.15]$ ($h^+h'^-$), $[-0.2, 0.15]$ ($h^+\pi^0$), $[-0.115, 0.075]$ ($K^{0}\bar{h}$) and $[-0.2, 0.2]$ ($K^{0}\pi^0$).

Detailed Monte Carlo simulation, off-resonance data, and events in on-resonance $m_{ES}$ and $\Delta E$ sideband regions are used to study backgrounds. The contribution due to other $B$-meson decays, both from $b \rightarrow c$ and charmless decays, is found to be negligible. The largest source of background is from random combinations of tracks and neutral particles produced in the $e^+e^- \rightarrow q\bar{q}$ continuum (where $q = u, d, s$ or $c$). In the CM frame this background typically exhibits a two-jet structure that can produce two high momentum, nearly back-to-back particles, in contrast to the spherically symmetric nature of the low momentum $\Upsilon(4S) \rightarrow B\bar{B}$ events.

We exploit this topology difference by making use of two event-shape quantities. The first variable is the angle $\theta_\parallel$ [11] between the sphericity axes of the $B$ candidate and of the remaining tracks and photons in the event. The distribution of $|\cos \theta_\parallel|$ in the CM frame is strongly peaked near 1 for continuum events and is approximately uniform for $B\bar{B}$ events. We require $|\cos \theta_\parallel| < 0.9$, which rejects 66% of the background that remains at this stage of the analysis.

The second quantity is a Fisher discriminant $F$ constructed from the scalar sum of the CM momenta of all tracks and photons (excluding the $B$ candidate decay products) flowing into nine concentric cones centered on the thrust axis of the $B$ candidate. Each cone subtends an angle of 10° and is folded to combine the forward and backward intervals. Monte Carlo samples are used to obtain the values of the coefficients, which are chosen to maximize the statistical separation between signal and background events. The distributions of $F$ for Monte Carlo simulated $B^0 \rightarrow h^+h'^-$ decays and background events in the $m_{ES}$ sideband region $5.20 < m_{ES} < 5.27$ GeV/c$^2$ are displayed in Fig. 1(a).

![FIG. 1: (a) The distributions of the Fisher discriminant for Monte Carlo simulated $B^0 \rightarrow h^+h'^-$ decays (histogram) and background events (points) in the $m_{ES}$ sideband region $5.20 < m_{ES} < 5.27$ GeV/c$^2$; (b) The $K^0\pi^0$ separation, in units of standard deviations, as a function of momentum, derived from the Cherenkov angle measurements of kaon and pion tracks in a $D^{*+} \rightarrow D^0\pi^+$ control sample, as described in the text.](image_url)
fluctuations in the total number of observed events $N$. For the $K^\pm \pi^\mp$, $\pi^\pm \pi^0$, $K^\pm \pi^0$, $K^0 \pi^\pm$, and $K^0 \pi^0$ terms, the yields are rewritten in terms of the sum $n_f + n_\bar{f}$ and the asymmetry $A = (n_f - n_\bar{f})/(n_f + n_\bar{f})$, where $n_f (n_\bar{f})$ is the fitted number of events in the mode $B \to f (\bar{B} \to \bar{f})$. The numbers of events, $N$, entering the maximum likelihood fit for each topology are 16032 ($h^+h^-$), 16452 ($h^+\pi^0$), 3623 ($K^0h^+$), and 1503 ($K^0\pi^0$).

The parameters for background $m_{ES}$ and $\Delta E$ PDFs are determined from events in on-resonance $\Delta E$ sideband regions. The signal $m_{ES}$ and $\Delta E$ PDF parameters are determined from fully reconstructed $B^+ \to D^0\pi^+$ and $B^+ \to D^0\rho^+(\rho^+ \to \pi^+\pi^0)$ decays. Events in on-resonance $m_{ES}$ sideband regions and Monte Carlo simulated signal decays are used to parameterize the Fisher discriminant PDFs for background and signal, respectively (see Fig. 3(a)). Alternative parameterizations obtained from off-resonance data and Monte Carlo simulation are used as cross-checks and for determination of systematic uncertainties. The $\theta_c$ PDFs are derived from kaon and pion tracks in the momentum range of interest from approximately 42000 $D^{*+} \to D^0\pi^+$ ($D^0 \to K^-\pi^+$) decays. This control sample is used to parameterize the $\theta_c$ resolution $\sigma_{\theta_c}$ as a function of track polar angle. The resulting $K^-\pi^+$ separation, defined as $|\theta^K - \theta_c|/\sigma_{\theta_c}$, where $\theta^K$ ($\theta_c$) is the expected Cherenkov angle for a kaon (pion), is shown as a function of momentum in Fig. 3(b).

The results of the fit are summarized in Table 1, where the statistical error for each mode corresponds to a 68% confidence interval and is given by the change in signal yield $n_i$ that corresponds to a $-2\ln L$ increase of one unit. Signal significance is defined as the square root of the change in $-2\ln L$ with the corresponding signal yield fixed to zero. For the three modes that have statistical significance less than 4$\sigma$ we report Bayesian 90% confidence level upper limits. In addition, for the purpose of combining with measurements from other experiments, we report the branching fractions corresponding to the fitted signal yields: $B(B^+ \to \pi^+\pi^0) = (5.1^{+2.0}_{-1.8} \pm 0.8) \times 10^{-6}$, $B(B^0 \to K^+K^-) = (0.85^{+0.81}_{-0.66} \pm 0.37) \times 10^{-6}$ and $B(B^+ \to K^0\pi^+) = (-1.3^{+3.4}_{-2.8} \pm 0.7) \times 10^{-6}$. The upper limit on the signal yield for mode $i$ is given by the value of $n_i^0$ for which $\int_0^{n_i^0} \mathcal{L}_{\text{max}}^i \, dn_i / \int_0^{\infty} \mathcal{L}_{\text{max}}^i \, dn_i = 0.90$, where $\mathcal{L}_{\text{max}}$ is the likelihood as a function of $n_i$, maximized with respect to the remaining fit parameters. Branching fraction upper limits are calculated by increasing the signal yield upper limit and reducing the efficiency by their respective systematic errors.

Table 1: Summary of results for detection efficiencies ($\varepsilon$), fitted signal yields ($N_S$), statistical significances ($S$), measured branching fractions ($B$), and charge asymmetries. The efficiencies include the branching fractions for $K^0 \to K^0 \to \pi^+\pi^-$ and $\pi^0 \to \gamma\gamma$. Equal branching fractions for $T(4S) \to B^+\overline{B}$ and $B^+\overline{B}$ are assumed. The 90% confidence level (C.L.) intervals for the charge asymmetries include the systematic uncertainties, which have been added in quadrature with the statistical errors.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\varepsilon$ (%)</th>
<th>$N_S$</th>
<th>$S$ ($\sigma$)</th>
<th>$B(10^{-6})$</th>
<th>$A$</th>
<th>$A$ 90% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>45</td>
<td>$41 \pm 10 \pm 7$</td>
<td>4.7</td>
<td>$4.1 \pm 1.0 \pm 0.7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^+\pi^-$</td>
<td>45</td>
<td>$169 \pm 17 \pm 13$</td>
<td>15.8</td>
<td>$16.7 \pm 1.6 \pm 1.3$</td>
<td>$-0.19 \pm 0.10 \pm 0.03$</td>
<td>$[-0.35, -0.03]$</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>43</td>
<td>$8.2^{+7.8}_{-6.4} \pm 3.5$</td>
<td>1.3</td>
<td>$&lt; 2.5$ (90% C.L.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^+\pi^0$</td>
<td>32</td>
<td>$37 \pm 14 \pm 6$</td>
<td>3.4</td>
<td>$&lt; 9.6$ (90% C.L.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^+\pi^0$</td>
<td>31</td>
<td>$75 \pm 14 \pm 7$</td>
<td>8.0</td>
<td>$10.8^{+2.7}_{-1.5} \pm 1.0$</td>
<td>$0.00 \pm 0.18 \pm 0.04$</td>
<td>$[-0.30, +0.30]$</td>
</tr>
<tr>
<td>$K^0\pi^+$</td>
<td>14</td>
<td>$59^{+11}_{-10} \pm 6$</td>
<td>9.8</td>
<td>$18.2^{+3.0}_{-3.0} \pm 2.0$</td>
<td>$-0.21 \pm 0.18 \pm 0.03$</td>
<td>$[-0.51, +0.09]$</td>
</tr>
<tr>
<td>$\overline{K}^0\pi^+$</td>
<td>14</td>
<td>$-4.1^{+4.5}_{-3.8} \pm 2.3$</td>
<td>$-$</td>
<td>$&lt; 2.4$ (90% C.L.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^0\pi^0$</td>
<td>10</td>
<td>$17.9^{+6.8}_{-5.8} \pm 1.9$</td>
<td>4.5</td>
<td>$8.2^{+3.1}_{-2.7} \pm 1.2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG. 2: The $m_{ES}$ and $\Delta E$ distributions for the various modes, using likelihood ratio requirements described in the text. The solid curves represent the fit predictions for both signal and background; the dashed curve represents the given signal mode only and the dotted curve represents other modes of the same topology.

1σ of their measured uncertainties or by substituting alternative PDFs from independent control samples. The systematic errors in the signal yields due to PDF uncertainties depend on decay mode as shown in Table 1.

The $D^{*+}$ control sample of kaon and pion tracks is used to estimate systematic uncertainties in the asymmetries arising from possible charge biases in the $\theta_c$ quality requirements, as well as from differences in $\theta_c$ reconstruction for different charge species. From these studies we conservatively assign a systematic uncertainty of ±0.01 on $A$ for all the modes. Charge biases in the detector and track reconstruction chain are studied in high statistics samples of charged tracks in multihadron events. These studies show differences in reconstruction efficiencies for positively and negatively charged tracks of less than 0.005. We assign an overall systematic uncertainty of ±0.01 on $A$ for possible charge-correlated biases in track reconstruction and particle identification. All measured background asymmetries are consistent with zero with statistical uncertainties less than 0.03. The fitted signal yields and asymmetries for off-resonance data and on-resonance $\Delta E$ sidebands are also consistent with zero.

The overall systematic errors on the branching fractions and charge asymmetry measurements are computed by adding in quadrature the PDF systematic uncertainties and the systematic uncertainties on the efficiencies or due to possible charge biases, respectively.

In summary, we have measured branching fractions for the rare charmless decays $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow K^+\pi^-$, $B^+ \rightarrow K^+\pi^0$, $B^+ \rightarrow K^0\pi^+$, and $B^0 \rightarrow K^0\pi^0$, and set upper limits on $B^0 \rightarrow K^+K^-$, $B^+ \rightarrow \pi^+\pi^0$, and $B^+ \rightarrow K^0K^+$. We find no evidence for direct CP violation in the observed decays and set 90% C.L. intervals. These measurements are in good agreement with previous results [6, 7].

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[5] Charge conjugate states are assumed throughout, except where explicitly noted.


