10-23-2007

N-Defendant Litigation and Settlement: The Impact of Joint and Several Liability

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N-DEFENDANT LITIGATION AND SETTLEMENT: THE IMPACT OF JOINT AND SEVERAL LIABILITY[*]

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21 April 2008

Abstract

This article generalizes the analysis of settlement under joint and several liability from lawsuits involving one plaintiff and two defendants to those involving $n \geq 2$ defendants. We demonstrate that, depending upon the correlation of outcomes among the defendants, but regardless of the distribution of liability shares, joint and several liability may encourage plaintiffs to select some defendants for litigation while settling with those remaining. In particular, under joint and several liability, when each defendant is grouped with others sharing correlated outcomes, and the plaintiff's probability of success against any defendant is sufficiently high, we show that, in all Nash equilibrium, the plaintiff litigates against the defendant from each group with the smallest share and settles with all other defendants. There are a continuum of equilibria but they are all payoff-equivalent up to permutation. Plaintiff's payoff is identical in all equilibria; only the payoffs of the members with the smallest share within a group are permuted. We also show that, for sufficiently high plaintiff's probability of success, this result holds even when the defendants are allowed to form coalitions, and derive the set of offers in the behaviorally unique coalition proof Nash equilibrium; as before, these offers induce litigation against the defendant with the smallest share in each group and settlement with all other defendants.

1 Introduction

Joint and several liability has remained a focus of controversy in a number of areas of law and has received extensive attention in the economic analysis of law. For example, advocates of tort reform continue to call[1] for the restriction or abolishment of joint and

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[1]These efforts have met with some success. According to the American Tort Reform Association (ATRA), thirty-seven states have enacted some form of legislative restriction on joint and several liability. http://www.atra.org/issues/index.php?issue=7345.

several liability.[2]   Joint and several liability also remains an important issue in the ongoing debate over Federal antitrust laws.[3] The issue of the imposition of joint and several liability under the Federal Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA)[4] provides a further example, arising not only in attempts to reform the application of the rule, but also in the recurring debate over the renewal of CERCLA.

Understanding the effects of joint and several liability and, in particular, its effect on settlement, clearly has important policy implications for both these debates and those in the other areas of law where joint and several liability is applied. Kornhauser and Revesz (1994) introduced the analysis of settlement under joint and several liability.[5] That article solved a settlement game between one plaintiff and two defendants. Lawsuits involving multiple defendants under tort, antitrust, environmental laws, as well as other types of law employing joint and several liability, however, regularly involve more than two defendants. Yet, with the exception of Chang and Sigman (2000), the subsequent joint and several liability literature has also restricted attention to the case of two defendants. Also, the subsequent joint and several liability literature does not address the effects of coalition or side payments among

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[2]For example, ATRA identifies joint and several liability reform as part of its "Legislative Agenda." http://www.atra.org/issues/. According to its website, "ATRA was co-founded in 1986 by the American Medical Association and the American Council of Engineering Companies...working to bring greater fairness, predictability and efficiency to America's civil justice system." ATRA states that it has been "[s]ince 1986, the only national organization exclusively dedicated to reforming the civil justice system." http://www.atra.org/about.

[3]On April 2, 2007, the Antitrust Modernization Commission submitted its final Report and Recommendations to Congress and the President on Antitrust Modernization Committee which sets forth, *inter alia*, proposed changes to joint and several liability in Federal law based antitrust actions. See, http://www.amc.gov/ and the Antitrust Modernization Commission Report And Recommendations (April 2007), which appears at http://www.amc.gov/report_recommendation/amc_final_report.pdf. The Antitrust Modernization Commission was created pursuant to the Antitrust Modernization Commission Act of 2002 (Pub. L. No. 107-273, §§ 11051-60, 116 Stat. 1856) and consists of 12 members, 4 of whom were appointed by the President, 4 of whom were appointed by the leadership of the Senate, and 4 of whom were appointed by the leadership of the House of Representatives. Id. § 11054(a). The Commission is charged by statute to: (1) examine whether the need exists to modernize the antitrust laws and to identify and study related issues; (2) solicit views of all parties concerned with the operation of the antitrust laws; (3) evaluate the advisability of proposals and current arrangements with respect to any issues so identified; and (4) prepare and submit to Congress and the President a report." http://www.amc.gov/about_commission.htm.

[4]42 U.S.C. §§ 9601-75 (2000).

[5]Both the *ex ante* and *ex post* incentive effects of joint and several liability have been examined in the literature on economic analysis of law. In this essay we restrict attention to the effects of joint and several liability on the choice between settlement and litigation.

The earliest economic analyses of settlement under joint and several liability appear in Easterbrook, Landes and Posner (1980) and Polinsky and Shavell (1981). These papers addressed the problem of settlement and claim reduction in antitrust litigation. Both papers implicitly assumed that plantiff's prospects of success against all defendants are perfectly positively correlated.

Kornhauser and Revesz (1994) provide a general model of settlement under joint and several liability that recognizes the importance of the degree of correlation in plantiff's prospects of success against each of two defendants.

Subsequent treatments of settlement under joint and several liability generally modify the model developed in Kornhauser and Revesz (1994). See for example Klerman (1996) and Dopuch, Ingberman and King (1997).

Settlement generally has received much more extensive treatment. See Daughety (2000) and Spier (2006) for surveys.

defendants.

In this article we examine the effects of joint and several liability on the choice between settlement and litigation for any number, n, of defendants. Our analysis significantly extends the model introduced by Chang and Sigman.[6] In our analysis, we show that the current literature has misunderstood the implications of the two-defendant model for the n-defendant context. We demonstrate that depending on the level of correlation of the defendants' outcomes, but regardless of the distribution of liability shares, joint and several liability will encourage plaintiffs whose chances of succeeding at trial are sufficiently high to litigate against the defendants with the smallest shares of liability, while settling with those remaining. In particular, under joint and several liability, when correlated defendants are grouped together, we show that a plaintiff always prefers litigating against the defendant with the smallest share in each group while settling with the other defendants to litigating with all defendants and this profile and the associated set of offers form the pure strategy Nash equilibrium of the game. Under appropriate conditions of correlation among plaintiff's prospects of success against defendants, a plaintiff prefers this selective litigation to settling with all defendants and this profile and the offers that induce it are the pure-strategy Nash equilibrium. These Nash equilibria are payoff-equivalent up to permutation (defined below).

We further demonstrate that this result holds even when the defendants are permitted to form coalitions. In other words, for appropriate p, full selective litigation characterizes pure strategy Nash equilibria of the game, when defendant coalitions are allowed. These equilibria are again payoff-equivalent up to permutation.

We also analyze the effect of allowing side payments among defendants. When side payments are allowed, the maximum the plaintiff can collect is the return for the profile in which all defendants litigate and the profits in which either all defendants litigate or that in which all settle for the amount they world pay if they all had litigated comprise the Nash equilibria of the game for all p.

We also directly extend the conclusions of Kornhauser and Revesz (1994) and show that when the n defendants' outcomes are independent, the plaintiff will litigate against all defendants. If the defendants' outcomes are perfectly correlated, the plaintiff will settle with all defendants when liability shares are uniform and, when the plaintiff's probability of success is sufficiently high, will litigate with the smallest share defendant.

The discussion proceeds as follows. In the next section, we introduce our model through a discussion of the two most relevant prior articles: Kornhauser and Revesz (1994) and Chang and Sigman (2000). Section 3 summarizes our analysis of both the groupwise correlation and uniform correlation models. Section 4 concludes with a brief discussion of the policy implications of our analysis. Proofs of the theorems, lemmata, and propositions appear in

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[6]Our results apply to two classes of correlation matrices. The first, introduced by Chang and Sigman, assumes that the n defendants can be divided into m disjoint subsets. The plaintiff's prospects of success against each defendant in a given subset are perfectly, positively correlated while the plaintiff's prospects of success against defendants in different subsets are independent. We call this first class of matrices *grouped* or *groupwise* correlation matricies. In the second class of correlation matrices considered, the plaintiff's prospects of success against any two defendants is identical. We call this class of matrices the *uniform correlation* model. A complete extension to n defendants would solve the game for every possible n x n matrix of correlations among plaintiff's prospects of success against each defendant.

the Appendix.

2   The Model

We model settlement under a rule of joint and several liability with contribution among non-settling defendants and with a *pro tanto* set-off rule. Under a *pro tanto* set-off rule, when a plaintiff with a claim for V settles with one or more defendants for an amount S, her claim against the remaining defendants is reduced by the settlement amount. That is, her claim against remaining defendants is $V - S$. We also assume that each defendant j is responsible for a share r_j of the plaintiff's claim. Without loss of generality, we assume that the plaintiff's claim has value $V = 1$. For convenience, we assume that all claims are joined in a single litigation.

We consider a sequential game Γ with $n + 1$ players. The plaintiff is player 0 and players 1 to n are the defendants. In stage 1, the plaintiff makes a take-it-or-leave-it offer ω_j to each defendant j with $\omega_j \ \epsilon \ [0, \infty)$; her strategy space Ω thus consists of all non-negative vectors (ω_j). At stage 2, each defendant decides, non-cooperatively, whether to settle, s, or to litigate, $\neg s$. Phrased differently, at stage 2, the defendants play a non-cooperative subgame in which each has the (behavioral) strategy set $\{s, \neg s\}$.[7] Let $\sigma = (\zeta_1, ..., \zeta_n)$ be the vector of behavioral responses of defendants and S the set of possible behavioral profiles for the defendants. At stage 3, the plaintiff decides whether to litigate against any non-settling defendants. The defendants each face litigation costs of λ_d if they do not settle, while the plaintiff faces litigation costs of λ_π for each defendant with whom the plaintiff litigates. We generally restrict attention to situations in which the plaintiff's costs of litigation are small; we thus generally assume that the plaintiff litigates against all non-settling defendants.[8]

We must now specify the structure of uncertainty created by litigation. We assume that the plaintiff has a probability p of prevailing against a single defendant at trial. In general, the plaintiff's prospects of success in a litigation against multiple defendants will depend on the degree of correlation among her prospects of success against different defendants. Kornhauser and Revesz (1994) noted that the degree of correlation would depend on the degree of commonality of the facts that the plaintiff must prove to prevail against different defendants. This degree of commonality might range from negative one (where proving fact F insures the responsibility of defendant j while exonerating defendant k and proving not-F exonerates j but implicates k) to positive 1 (where proving F implies liability for both j and k or exonerates both). If F bears on the responsibility of j but not on k, then the plaintiff's prospects of success against j and k exhibit some independence.

Thus, a general model of settlement under joint and several liability would identify the equilibrium strategies of the plaintiff and defendants for any correlation matrix $M = \left[\rho_{ij} \right]$, in other words, any positive semi-definite matrix with $\rho_{ii} = 1$. For negatively correlated defendants, the model must be adjusted to accommodate the property that negative correlation

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[7]In the full game, each defendant's strategy space is much more complex. A strategy for defendant i is a function f from Ω to $\{s, \neg s\}$.

[8]We focus on the effects of joint and several liability where litigation costs are small because increasing the size of the litigation costs merely masks the effect of the liability rule itself by increasing the attractiveness of settlement for all parties.

in the case of lawsuits is not "perfect" – if the plaintiff fails to make its proofs, for example in a judgment as a matter of law under Rule 50 of the Federal Rules of Civil Procedure, the plaintiff will lose against all defendants, even though those defendants are negatively correlated in the sense that if the plaintiff were to prove her case against one defendant, the other is exonerated and vice versa.[9]

For the case $n = 2$ studied by Kornhauser and Revesz, this analysis is straightforward because the correlation among the defendants is uniform and class of correlation matrices comprises the 2×2 family with the single parameter $\delta_{ij} \geq 0$ off the diagonal. The dimensionality of the problem, however, grows with the square of n; the problem has dimensionality $\frac{n(n-1)}{2}$. Moreover, construction of valid correlation matrices can itself be problematic for arbitrary numbers of defendants.[10] Finally, as noted *supra*, the model requires additional structuring specifically to accommodate negative correlation. To make the problem tractable, we restrict attention to two classes of matrices. In the uniform correlation case we assume that $\delta_{ij} = \delta$ for all $i \neq j$. In the groupwise correlation case, we assume, following Chang and Sigman (2000), that the set of n defendants can be partitioned into m subgroups, such that within groups, the plaintiff's prospects of success are perfectly correlated while, across groups, her prospects are independent. Thus, for these groups $\delta_{ij} = 1$ when i and j belong to the same group while $\delta_{ij} = 0$ when i and j belong to different groups.

Our analysis significantly extends the analysis of the groupwise correlation model introduced in Chang and Sigman by removing two major restrictions they impose on their model. First, Chang and Sigman restrict their analysis to the case where, for all i, $r_i = \frac{1}{n}$. In essence, they analyze a rule of joint and several liability that divides responsibility equally among the responsible parties. Second, they compare plaintiff's payoffs for only two strategy profiles of defendants: that in which all defendants settle is compared to that in which all defendants litigate.[11] Our present model, however, compares the plaintiff's payoffs for *all* pure strategy profiles. Comparing only the two profiles, Chang and Sigman argue that a plaintiff prefers the "all settle" profile to the "all litigate" profile. Though this claim is true, we show that there exists a p^* sufficiently large such that for $p \geqslant p^*$, the plaintiff prefers to make offers that induce the member from each group with the smallest share to litigate and all other members to settle, to the "all settle" equilibrium. We then show that as m increases to n, p^* goes to 0. Finally, we show that, for the case $r_i = \frac{1}{n}$ studied by Chang and Sigman, even when $p < p^*$, any offer vector that induces the "all settle" equilibrium among defendants might also induce "all litigate" equilibrium among defendants. As the "all litigate" equilibrium *pareto* dominates the "all settle" equilibrium for the defendants, a rational plaintiff might prefer the offer vector that induces one member from each group to litigate while inducing settlement from all other defendants. In fact, as we demonstrate below, when the defendants' liability shares are uniform as in Chang and Sigman, and if

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[9]An example of such a situation is a case where a plaintiff has been harmed by a machine because that machine had safety devices removed by either the operator/owner of the machine or by the manufacturer. If the plaintiff shows the owner removed the guards and is liable, the manufacturer is exonerated and vice versa. The plaintiff may, however, fail to adduce sufficient evidence at trial to prove either parties liability, in which case, both owner and manufacturer would prevail.

[10]See, *e.g.* Budden, *et al.* (2006) at 53-54: "We contend that even statisticians and mathematicians find the problem of developing feasible correlation matrices to be challenging."

[11]In addition, Chang and Sigman do not analyze the case in which $m = n$.

there is a group containing only one defendant, the profile in which the defendant from the single defendant group litigates while the other defendants settle will be preferred to the profile in which all defendants settle.

Chang and Sigman do not consider the possibility of coalitions among the defendants coordinating their behavior in respond to the plaintiff's offers and neither Chang and Sigman nor Kornhauser and Revesz consider the effects of side payments, analyses we conduct in this paper. The main result cited above proves to be robust to these concerns..

3   Groupwise and Uniform Correlation Summary

In this section, we state our two major theorems and then outline the structure of the proofs which appears in the Appendix. We then provide intuitions underlying the theorems in the context of our extension of the uniform correlation model and provide an example illustrating the conclusions.

To begin, we introduce some notation. Let $(\neg S_k, S_{n-k})$ be the profile of defendant strategies in which one defendant from k distinct groups litigates while all other defendants settle. Let (S) be the profile of defendant strategies in which all defendants settle; and let $(\neg S)$ be the profile of defendant strategies in which all defendants litigate.[12]

We shall say that an equilibrium is *payoff-equivalent up to permutation* when in all equilibria, plaintiff's payoff is identical, the payoff of every defendant whose share is not the smallest in his group is identical, and the payoffs of those defendants in each group with smallest share are a permutation of the their payoffs in other equilibria.

3.1   The Main Theorem and the Structure of its Proof

We now state our main theorem:

Theorem 1 *(a) For $1 < m \leq n$, there exists a p^* such that there is at least one Nash equilibrium in which the plaintiff makes offers that induce the behavioral strategy profile $(\neg S_m, S_{n-m})$ for all $p > p^*$. (b) For all such $p > p^*$, the plaintiff litigates against the defendant with the smallest liability share in each of the m groups, and settles with the remaining $n - m$ defendants. All Nash equilibria are payoff-equivalent up to permutation.*

The Nash equilibrium is unique up to permutation as there may be more than one defendant with the smallest share of liability in a given group; the plaintiff will litigate against only one of them but litigating against either is equally good for the plaintiff. Moreover, though, for those defendants with whom plaintiff settles, there is a unique offer, many offers will induce the defendant with the smallest share to litigate. So the plaintiff's vector of offers at stage one of the game is not unique in the strict sense; from an economic perspective, however, all offers that induce litigation are equivalent from the plaintiff's perspective.

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[12]In this article, we denote the strategy profile in which all defendants settle by (S); that in which all defendants litigate by $(\neg S)$; and the profile in which the plaintiff litigates against one defendant from each of the m correlated groups and settles with the remaining $n - m$ defendants as $(\neg S_m, S_{n-m})$. The utility to the plaintiff of a strategy profile σ is denoted $u_\pi(\sigma)$.

Our proof of part (a) proceeds by showing that the payoff $u_\pi\left(\neg S_m, S_{n-m}\right)$ to the plaintiff exceeds her payoff from some key profiles of defendants' behavioral strategies. In particular, we first show that $u_\pi\left(\neg S_m, S_{n-m}\right) > u_\pi(\neg S)$. We then show that $u_\pi\left(\neg S_m, S_{n-m}\right)$ exceeds her payoff from any other vector of offers that induces any profile of defendants' behavioral strategies in which more than one defendant from some group litigates. We then consider profiles in which $l < m$ defendants litigate and show that plaintiff's payoff from offers that induce the profile $\left(\neg S_m, S_{n-m}\right)$ is highest when p is sufficiently high.

The proof of part (b) relies on three observations. First, from the proof of part (a), we know that for appropriate p, the plaintiff will prefer to litigate against one defendant from each of the m groups while settling with the remaining defendants. Thus, we only need compare the returns for the profiles in which the plaintiff litigates against one defendant from each group and find the profile with the maximal plaintiff's return. Second, we observe that the plaintiff's return for such profiles is strictly increasing in the total value of all settlements, O_{n-m}, and as we know from the proof outline for Theorem 1, adding or interchanging the defendants who litigate has no effect on the plaintiff's return from litigation. This is clear from inspection of the plaintiff's total payoff function: $u_\pi\left(\neg S_m, S_{n-m}\right) = P(1 - O_{n-m}) + O_{n-m} = P + (1 - P)O_{n-m}$ (where P is the probability weighting of outcomes.) Finally, we note that each of the constraint functions on the settling defendants from each group is increasing in the size of that defendant's liability share. This is, in fact, merely a restatement of the common sense notion that, the larger the defendant's liability, the more that defendant will be willing to pay in settlement. Combining these observations, we demonstrate that by substituting the smallest defendant for the litigating defendant in each group, we do not affect the return from litigation, though, by settling the defendants with the larger liability shares in each group, the plaintiff's return increases. Observe that this identification of which defendant against whom to litigate is unique up to liability shares. In other words, if more than one defendant in a group has the same smallest share, the plaintiff is indifferent to interchanges of those smallest share defendants among the settling or litigating defendants.

Our proof of the main theorem establishes as a corollary, that as m increases p^* approaches 0 and $p^* = 0$ when $m = n$. For $n = m = 2$ we thus have the result first proven in Kornhauser and Revesz (1994) that, when plaintiff's prospects of success against each of two defendants are independent, she will litigate against both.

Notice that there are many Nash equilibria when $p > p^*$ because an equilibrium consists of a set of offers which induce exactly m defendants to litigate. Offers inducing defendant j to litigate, however, are not unique. Any offer that is sufficiently large will suffice. Consequently there will be continuum of Nash equilibria all of which are payoff-equivalent for the plaintiff. There may be additional multiplicity if there is more than one defendant in some group that has the smallest share. In this case, the plaintiff litigates against only one and the expected payoff of the litigating defendant of minimum size will differ from the payoff of any settling defendants of minimum size. The sum of payoffs of all minimum sized defendants in a given group does not vary across equilibria; indeed the payoffs of this group are simply permuted in different equilibria. When we ignore the names of the parties, the equilibria are thus, from an economic perspective, essentially unique.

With the exercise of care, some intuitions relating to these theorems can be extracted from the 2-defendant case analyzed by Kornhauser and Revesz. First, some caveats. As noted *supra*, Kornhauser and Revesz analyze a uniform correlation model where each defendant

has the same level of correlation as every other defendant. Our analysis for the uniform correlation cases $n = m$ (independent outcomes) and $m = 1$ (perfect correlation) show that the conclusions of Kornhauser and Revesz 2-defendant model extend directly to the n-defendant uniform correlation model. In our n-defendant case, when $m \neq n$ (groupwise correlation), however, the determinative factor is the probability of the success of the plaintiff at trial. In essence, when the plaintiff becomes sufficiently likely to succeed at trial, she will choose to litigate against someone, and, because there is no advantage to litigating with any more than one defendant per group, she will litigate with one defendant from each group and settle with the remaining defendants. By contrast, in the Kornhauser and Revesz independent outcomes analysis and our n-defendant uniform correlation extension, there is no such determinative factor and the plaintiff will choose to litigate against all defendants regardless of probability of success, relative liability share sizes, or any other factor.

In the case of perfect correlation in Kornhauser and Revesz and our n-defendant uniform correlation extension, the determinative factor is share size. When the share size is uniform in Kornhauser and Revesz, the plaintiff will settle with all defendants regardless of any other factor. In the n-defendant $n \neq m$ model, however, uniformity of share size either within each group or globally, does not change the conclusion about the existence of p^*. As noted above, in these features, the Kornhauser and Revesz model is directly comparable to the n-defendant models in which the correlation is uniform, either $n = m$ (independent outcomes) or $m = 1$ (perfect correlation). When $n = m$ in the n-defendant model, the plaintiff prefers to litigate against all defendants regardless of any other factor, exactly as in Kornhauser and Revesz. When $m = 1$ in the n-defendant model, the conclusion is identical with that for perfect correlation in Kornhauser and Revesz – in both, the determinative factor is again share size. With perfect correlation, when the share size is uniform, the plaintiff prefers to settle with all defendants regardless of any other factor for either our n-defendant or the Kornhauser and Revesz models.

With these caveats, for groupwise correlation, the extension to n defendants of the 2-defendant case of Kornhauser and Revesz provides an intuition for our main theorem. If we view each of the perfectly correlated groups as an entity, then we have m entities with independent outcomes, and both the 2-defendant case analyzed by Kornhauser and Revesz and our n-defendant uniform correlation extension tell us that the plaintiff should prefer to litigate against each uncorrelated entity. Inside each group/entity, the defendants are perfectly correlated, and both the 2-defendant case analyzed by Kornhauser and Revesz and our n-defendant uniform correlation extension tell us that the plaintiff should prefer to litigate with one defendant and settle with all other group members when p is sufficiently high. The plaintiff's return from litigation grows in the number of independent defendants against whom she litigates, but does not grow in the number of correlated defendants litigated against, while the plaintiff's return from settlement grows with every additional defendant who settles, up to the point where one defendant from each group litigates.

Uniformity of liability shares does not prevent this profile from being preferred – in the uniform correlation case, it is the differences in the share values that provide the surplus that causes the plaintiff to litigate against one defendant and settle with the remaining defendants, so that when the differences vanish so does the surplus. In the groupwise correlation case, because there are always uncorrelated defendants, the uniformity of shares does not have this effect.

Another way of viewing this intuition is to consider the case in which plaintiff's prospects of success against each defendant are independent of her prospects of success against the others. Kornhauser and Revesz showed that, even for zero litigation costs, the plaintiff litigates against both defendants. When her prospects of success are independent, the value to plaintiff of litigating against both defendants is $P = 1 - (1-p)^2 = p(2-p) > p$ for $p < 1$. The set-off rule, however, prevents the plaintiff from recovering in settlement with one or both defendants this surplus of $p(1-p)$. The same situation holds in the n-defendant case except that it is against each perfectly correlated group of defendants that her prospects of success are independent of her prospects of success against defendants in the other groups. As m increases, the probability that the plaintiff will prevail against *at least* one defendant and thereby recover her full loss of 1 increases. Again, the set-off rule prevents her from recovering the surplus in settlement. On the other hand, the plaintiff gains no additional surplus by litigating with more than one defendant from each of the m groups. Moreover, settling with a perfectly correlated defendant for an amount ω simply substitutes a certain payoff of ω for an uncertain return of $P\omega$.

The plaintiff's preferences across defendants' strategy profiles for $p < p^*$ is surprisingly complicated. For p near 0, the plaintiff's preferences for profiles depends on two main characteristics. First, the relative sizes of the defendants' liability shares have a critical impact on the plaintiff's preferences for p near 0, an effect that can overpower the effects of the second characteristic, the correlation structure. Here the term "correlation structure" relates to the number of defendants in the groups, particularly the number of defendants in the smallest group (by number of defendants) in relation to the total number of defendants. For example, as we demonstrate below, for p near 0 and for a number of different distributions of the liability shares, when the smallest group has a single member, the plaintiff will prefer to litigate against that one defendant group while settling with the other defendants to settling with all defendants. In fact, under Chang and Sigman's assumption of uniform liability shares, we show that, if there is a group consisting of a single member, for all p sufficiently near 0, the plaintiff will litigate against the lone defendant rather than settling with all defendants.

3.2   A Theorem on Coaltion-Proof Nash Equilibria

Our bargaining model requires each defendant to act non-cooperatively when deciding whether to settle or litigate. Nash equilibrium implies that no unilateral deviation from pattern of settlement and litigation induced by the selective lititgation offers of plaintiffs is possible, but it does not exclude the possibility that some coalition of defendants might collectively deviate from the equilibrium. That is, there might be some coalition C such that each defendant j in C is better off under the deviation than in the equilibrium in Theorem 1. Bernheim, Peleg, and Whinston (1987) introduced the concept of coalition-proof Nash equilibrium which restricts the set of Nash equilibria to those that are immune from deviations by coalitions. Here we use that refinement in the subgame among defendants at stage 2. We thus have a concept of defendant coalition-proof Nash equilibrium ("dCNPE").

In this subsection we show that, even when the defendants are allowed to form coalitions, for appropriately large p, the plaintiff prefers selective litigation and we show how the plaintiff can construct a set of offers that immunize the induced profile from coalitions while

maximizing plaintiff's return, thus forming the unique pure strategy Nash equilibrium for p near 1. The intuition for the construction is stepwise exclusion of each of the possible coalitions matching each coalition excluded to a particular defendant whose settlement the plaintiff would prefer to induce. The optimality of the payoff from this induced profile follows for the same reasons as those for the no coalition case.

When side payments are allowed, the plaintiff's return changes dramatically, with the defendants always able to limit their liability to their share for the profile in which all defendants litigate. If any defendant is paying more than that share, the defendant can pay up to the difference to those defendants who are paying less than their share under full litigation and the defendants will all litigate. This is a version of the Coase Theorem in which the efficient outcome, each defendant paying as if all defendants litigate will always be attained by the defendants bargaining to it. Allowing the plaintiff to make side payments has no effect, however, because any side payment itself can be seen merely as a component of the overall offer the plaintiff makes at stage 1 of the regular game, with the defendants facing the identical constraints as without plaintiff side payments.

We demonstrated in the main Theorem, that for sufficiently large p, the plaintiff will make offers to induce one defendant from each group to litigate and the remaining defendants to settle. As we demonstrate below, that for sufficiently large p, the introduction of the possibility of coalitions of defendants does not affect the profile the plaintiff would chose to induce.

The intuition is that we can create a series of offers for the defendants with whom the plaintiff chooses to settle which immunizes the profile against defendant coalitions of any size up to the total number of settling defendants. Suppose that as in full selective litigation, $(\neg S_m, S_{n-m})$, there are $n - m$ of such defendants. For the defendants who the plaintiff wishes to induce to litigate, all that matters is that the offers are large, say near 1. These defendants will never pay less by accepting the offer, so they will always litigate. Thus the plaintiff's only concern is avoiding coalitions of additional settling defendants among those with whom the plaintiff wishes to settle. We start by extracting the maximum settlement amount from a first defendant that would prevent that defendant from being in a coalition of $n - m$ litigating defendants. We do not need to be concerned about any offset here, because if the $n - m$ defendants litigate, then all defendants are litigating. Next, because we now need only worry about coalitions up to size $n - m - 1$ for the second defendant, we extract the maximum amount in settlement that prevents that second defendant from litigating in a coalition of $n - m - 1$ defendants, except that now we must reduce the amount that would be at stake by the settlement amount we have already extracted from the first defendant. We continue this process, matching each of the $n - m$ defendants the plaintiff wants to induce to settle with an immunization against a corresponding size coalition. Because there are a finite number of rearrangements of the $n - m$ defendants, we know one will have as good or better return than all others. We choose the corresponding vector of offers as the plaintiff's optimal set of offers.

Because any dCPNE proof profile from a set of offers would seem to require a similar form of inducement, and thus each profile is similarly constrained, it seems natural that the optimality of full selective litigation that we saw when there were no adjustment for coalitions still holds for sufficiently large p. This we demonstrate formally using similar reasoning as in the non-coalition-adjusted case. In the appendix we prove the following theorem:

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Theorem 2 *We can construct a vector of offers such that: 1) the offers induce full selective litigation; 2) the offers induce a defendant CPNE ("dCPNE"); 3) the offers provide the best return for the plaintiff for full selective litigation; 4) the offers provide a better plaintiff return than for any greater number of defendants litigating; and 5) provide a better return for plaintiff than for any fewer defendants litigating.*

3.3 An Example

Suppose that a plaintiff sues three defendants. For convenience, let us number these defendants as 1, 2 and 3. Defendants 1 and 2 will succeed or fail *together* if they litigate against the plaintiff, *i.e.*, their litigation outcomes are perfectly correlated. Defendant 3's outcome in litigation is, by contrast, uncorrelated with that of the other defendants.[13]

We designate the probability of the plaintiff succeeding in litigation against any one defendant as p. Here we assume $p = 0.55$. Thus, the probability for each defendant of prevailing in litigation is $1 - p = 0.45$. The distribution of liability is uniform across the defendants. Each defendant, therefore, has a liability share of $\frac{1}{3}$. For clarity, we assume away litigation costs in this example. The total amount at stake is set to 1.

We can now calculate the plaintiff's return, u_π, for various strategy profiles. Here, we examine: $u_\pi(S)$, the plaintiff's return for settling with all defendants; $u_\pi(\neg S)$, the return for litigating with all defendants; and $u_\pi(\neg S_2, S_1)$, the return for litigating with one member of each group of correlated defendants and settling with the remaining defendant. Notice that because the liability shares in this example are uniform, it makes no difference which defendant the plaintiff selects from the defendant 1 and 2 group for litigation. Therefore, without loss of generality, we assume that the plaintiff selects defendants 1 and 3 to litigate against and settles with defendant 2 in $(\neg S_2, S_1)$.[14]

For each defendant in (S), the maximum that that defendant will pay and still settle in equilibrium is the amount of her expected payout if she were the only litigating defendant. This is simply a statement of the equilibrium criteria on the defendants. This maximum amount is p multiplied by the amount at stake less the amounts that the other defendants have paid in settlement. Thus, for the defendant i to settle, the plaintiff's offer, ω_i, must be less than or equal to this amount.[15]

For defendant 1, this can be expressed as follows: $\omega_1 \leq 0.55\{1 - (\omega_2 + \omega_3)\}$. Summing these settlement offer amounts over all three defendants yields the total return to the plaintiff

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[13]The correlation matrix for this game is the following:

1 1 0
1 1 0
0 0 1

A scenario with such a correlation structure might arise, for example, in a products liability action where defendants 1 and 2 are co-owners of a company that manufactures the implicated product, while defendant 3 is a company that "repaired" the allegedly defective product. Clearly, defendants 1 and 2's litigation outcomes would be highly correlated, though not correlated with that of defendant 3.

[14]In the example, because of the uniformity of liability share size, any configuration of $(\neg S_2, S_1)$ is essentially identical, and therefore, configuration in the sense of which particular defendant litigates out of the 2-defendant group, is unimportant.

[15]Here, we assume a defendant settles if she is indifferent between settling and litigating. Without numerical substitutions, $\omega_1 \leq p\{1 - (\omega_2 + \omega_3)\}$.

for settling: $\omega_1+\omega_2+\omega_3 \le 0.55\{3-2(\omega_1+\omega_2+\omega_3)\}$. Rearranging,[16] $\omega_1+\omega_2+\omega_3 \le \frac{3(0.55)}{1+2(0.55)} = \sim 0.79$. Thus, if the plaintiff maximizes her return from settlement offers to the defendants, when all defendants settle, her total return, $u_\pi(S)$, is ~ 0.79.

If instead, the plaintiff litigates against all three defendants, *i.e.*, $(\neg S)$, her return is 1 unless *all* defendants win. Because the outcomes for defendants 1 and 2 are perfectly correlated, they essentially act as one for the purpose of calculating the plaintiff's return when all defendants litigate. Thus,[17] $u_\pi(\neg S) = 1 - (1 - 0.55)^2 = \sim 0.80$. Here, under the example's conditions, the plaintiff clearly prefers litigating against all defendants to settling with all defendants.

If, however, the plaintiff litigates against defendants 1 and 3 and settles with defendant 2, *i.e.*, $(\neg S_2, S_1)$, the payout in equilibrium by settling defendant 2 will be the amount defendant 2 would pay if defendant 2 lost when all three defendants were litigating. Again, this is simply the equilibrium condition on defendant 2. Because defendants 1 and 2 are perfectly correlated, defendant 2 would pay $\frac{1}{2}$ if defendants 1 and 2 lost (Pr = 0.55) while defendant 3 won (Pr = 0.45), and would pay $\frac{1}{3}$ if all three defendants lost (Pr = 0.55^2). These outcomes represent the totality of possible outcomes in which defendant 2 would pay anything. Thus,[18] $\omega_2 = \frac{1}{2}(0.55)(0.45) + \frac{1}{3}(0.55^2) = \sim 0.22$.

If defendant 1 loses (Pr = 0.55) and defendant 2 wins (Pr = 0.45) in $(\neg S_2, S_1)$, the payout by defendant 1 will be the amount at stake, less the amount paid by defendant 2 in settlement. If, however, both defendants 1 and 3 lose (Pr = 0.55^2) in $(\neg S_2, S_1)$, defendant 1 will pay her weighted share of the amount at stake, 1, less the amount paid by defendant 2 in settlement. Because the defendants each have the same share of liability, defendant 1's weighted share when two defendants lose is $\frac{1}{2}$.[19] Together, these outcomes are represented by the following:[20] $(0.55)(0.45)(1-0.22) + \frac{1}{2}(0.55^2)(1-0.22) = \sim 0.31$. Defendant 3's expected payout for $(\neg S_2, S_1)$ is identical. The plaintiff's expected return is the sum of these payouts and settlement amounts[21]: $u_\pi(\neg S_2, S_1) = 0.22 + 2(0.55)(0.45)(1-0.22) + (0.55^2)(1-0.22) = \sim 0.84$.

Thus, the plaintiff prefers the profile $(\neg S_2, S_1)$ to both $(\neg S)$ and (S). In fact, as will be shown later, $(\neg S_m, S_{n-m})$ dominates $(\neg S)$.

Note that the amount of return to the plaintiff for $(\neg S_2, S_1)$ when the effects of defendant 2 settling is factored away is the contribution from defendant 1, $(0.55)(0.45) + (0.55^2)\frac{r_1}{r_1+r_3}$ combined with that from defendant 3, $(0.55)(0.45) + (0.55^2)\frac{r_3}{r_1+r_3}$, which is $2(0.55)(0.45) + (0.55^2) = \sim 0.80$. This is exactly the same value as $u_\pi(\neg S)$, so that the amounts related to defendant 2 settling represent a *surplus* of $u_\pi(\neg S_2, S_1)$ over $u_\pi(\neg S)$. This fact will be utilized later to demonstrate that $(\neg S_m, S_{n-m})$ dominates $(\neg S)$. Notice also that ignoring the effect of defendant 2 settling, this amount does not depend on the distribution of the liability shares – when summed over all losing defendants here, the liability share weighting equals 1 regardless of the values that are selected for the r_i.

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[16] $\omega_1 + \omega_2 + \omega_3 \le \frac{np}{1+(n-1)p}$.

[17] $u_\pi(\neg S) = 1 - (1 - p)^2$.

[18] $\omega_2 = \frac{r_2}{r_1+r_2}p(1 - p) + \frac{r_2}{r_1+r_2+r_3}p^2$.

[19] Defendant 1's share divided by the sum of defendant 1 and defendant 2's shares = $\frac{1/3}{1/3+1/3} = \frac{1}{2}$.

[20] $p(1 - p)(1 - \omega_2) + \frac{1}{2}p^2(1 - \omega_2)$.

[21] $u_\pi(\neg S_2, S_1) = \omega_2 + 2p(1 - p)(1 - \omega_2) + p^2(1 - \omega_2)$.

Observe also, that although defendants 1 and 2 are essentially identical, sharing the same probability of success, identical shares of liability and being perfectly correlated, defendant 1 expects to pay more than defendant 2. Defendant 2 is allowed to settle for ~ 0.22, while defendant 1 is forced to litigate and face an expected cost of ~ 0.31. Notice, moreover, that this was true even with the assumptions that litigation costs are zero and that all defendants are infinitely solvent.

4 Behavior below p^*: One Litigating Defendant And All Defendants Settling

The prior section characterizes the equilibria when p is sufficiently large. When $p < p^*$, however, many different types of equilibria are possible. From lemma 5, we know that in these equilibria $0 \leq k < m$ defendants litigate. Many different equilibria are possible; which equilibrium arises depends not only on p, but also on the distribution of shares r_i among defendants, and the specific correlation structure. The behavior of the plaintiff's preferences below p^* exhibits some of the complexity that arises out of even a correlation structure restricted as we have in our model. In this section, we examine the relationship of the profiles in which one defendant litigates and the remaining defendants settle, $(\neg S_1, S_{n-1})$, and the profile in which all defendants settle, (S). The litigating defendant must be one of two types: a member of a group including other defendants; or the only member of one of the m groups, what we shall refer to as a "singleton." Whether such a singleton group exists is an important property in determining when there exists a p at or range of p in which the plaintiff prefers to settle with all defendants. The discussion in this section demonstrates this property and also shows effects of the relative sizes of the defendants' liability shares r_i to plaintiff's preferences over strategy profiles. We show, for example, that when the shares are uniform or "close" to uniform and there is a singleton, the plaintiff will never settle with all defendants. In the following propositions, proven in the appendix, we establish a threshold for the return from the settling defendants necessary for the plaintiff to prefer to litigate with one defendant over settling with all defendants. To begin, however, we extend the example of the prior section.

4.1 An Example Continued

Some of these effects can be seen in an extension of the three defendant, two group example above. Here, we assume a p sufficiently small to exhibit the effects, $p = 0.001$ while keeping the distribution of liability uniform across the defendants.[22] Thus, the probability for each defendant of prevailing in litigation is $1 - p = 0.999$ and each defendant has a liability share of $\frac{1}{3}$. As above, we assume away litigation costs.

To calculate the plaintiff's payoff for full selective litigation, $(\neg S_2, S_1)$, we first need to calculate the amount of settlement the plaintiff could obtain from defendant 2. The

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[22]Litigating only against the singleton defendant remains the Nash equilibrium for much larger p, on the order of .3. Even in this simple example, however, p^* is the solution to a seventh order polynomial which is not obviously solvable.

payout in equilibrium by settling defendant 2 will be the amount defendant 2 would pay if defendant 2 lost when all three defendants were litigating. Again, this is simply the equilibrium condition on defendant 2. Because defendants 1 and 2 are perfectly correlated, defendant 2 would pay $\frac{1}{2}$ if defendants 1 and 2 lost ($\Pr = 0.001$) while defendant 3 won ($\Pr = 0.999$), and would pay $\frac{1}{3}$ if all three defendants lost ($\Pr = 0.001^2$). These outcomes represent the totality of possible outcomes in which defendant 2 would pay anything. Thus, $\omega_2 = \frac{1}{2}(0.001)(0.999) + \frac{1}{3}(0.001^2) = \sim 0.0005$. Then $u_\pi(\neg S_2, S_1) = 0.0005 + 2(0.001)(0.999)(1 - 0.0005) + (0.001^2)(1 - 0.0005) = \sim 0.00250$.

For each defendant in (S), the maximum that that defendant will pay and still settle in equilibrium is the amount of her expected payout if she were the only litigating defendant. This is simply a statement of the equilibrium criteria on the defendants. This maximum amount is p multiplied by the amount at stake less the amounts that the other defendants have paid in settlement. Thus, for defendant i to settle, the plaintiff's offer, ω_i, must be less than or equal to this amount.

For defendant 1, this can be expressed as follows: $\omega_1 \leq 0.001[1 - (\omega_2 + \omega_3)]$. Summing these settlement offer amounts over all three defendants yields the total return to the plaintiff for settling: $\omega_1 + \omega_2 + \omega_3 \leq 0.001[3 - 2(\omega_1 + \omega_2 + \omega_3)]$. Rearranging, $\omega_1 + \omega_2 + \omega_3 \leq \frac{3(0.001)}{1+2(0.001)} = \sim 0.002994$. Thus, if the plaintiff maximizes her return from settlement offers to the defendants, when all defendants settle, her total return, $u_\pi(S)$, is $\sim 0.0.002994$.

However, for $(s, s, \neg s)$, defendants 1 and 2 will each be willing to pay the same amount in settlement which is the amount, for example, defendant 1 would pay if defendant 1 were to litigate when defendant 3 also litigates. That amount is:

$\omega_1 \leq [0.001^2 \frac{r_1}{r_1 + r_3} + 0.001(1 - 0.001)][1 - \omega_2] = [0.001^2(\frac{1}{2}) + 0.001(1 - 0.001)][1 - \omega_2]$

$= \sim 0.000999$. Thus, $u_\pi(s, s, \neg s) = 0.001997 + (0.001)(1 - 0.001997) = \sim 0.002995 > \sim 0.00250 = u_\pi(\neg S_2, S_1)$ and $u_\pi(s, s, \neg s) = \sim 0.002995 > \sim 0.002994 = u_\pi(S)$. At $p = 0.001$ therefore, the plaintiff prefers to litigate against defendant 3 and settle with the other defendants, to settling with all defendants and to full selective litigation.

4.2 Characterizing the Equilibria

Proposition 1 *The plaintiff prefers the profile in which one defendant litigates, $(\neg S_1, S_{n-1})$, to settling with all defendants, (S), when the return from the settling defendants in the profile $(\neg S_1, S_{1-m})$, $\sum_{i=1}^{n-1} \omega_i$, is greater than $\frac{(n-1)p}{1+(n-1)p}$. When $\sum_{i=1}^{n-1} \omega_i$ is less than $\frac{(n-1)p}{1+(n-1)p}$, the plaintiff prefers (S) to $(\neg S_1, S_{1-m})$.*

Using this "threshold" return from settlement, we now examine the special case in which the profile $(\neg S_1, S_{n-1})$ includes a correlated group that has only one member. Another property that we will use in the arguments that follow is that that if there is such a "singleton," then there is a single defendant litigating profile in which the equilibrium constraints for the settlement offers to the $n - 1$ settling defendants all have the form: $\omega_j = (p - p^2 + \frac{1}{2}p^2)[1 - \sum_{i=1, i\neq j}^{n-1} \omega_i]$. Here, we assume without loss of generality that defendant n is the single litigating defendant.

Proposition 2 *When the defendants' shares of liability are uniform, and there exists a single member correlated group, the plaintiff never prefers the profile (S) to $(\neg S_1, S_{n-1})$.*

We now take advantage of the fact that the restriction of uniformity can be loosened somewhat, but the similar conclusions to Lemma 2 above still follow. Suppose, for example, that for all $i < n$, $\frac{r_i}{r_i + r_n} = b$. We then have the following proposition.

Proposition 3 *Where there is a single member correlated group (without loss of generality, we designate the member of that single defendant group as defendant n) and shares are uniform among the $n-1$ non-litigating defendants. Then for the plaintiff to prefer to settle with all defendants, it must be that $p > \frac{b}{1-b}$, where $b = \frac{r_i}{r_i + r_n}$.*

The proof of proposition 3 allows us to extend the characterization of equilibria to a broader class of distributions of shares. We have

Proposition 4 *Where there is a single member correlated group (without loss of generality, we designate the member of that single defendant group as defendant n), and for all $i < n$, $r_i < r_n$, then the plaintiff will prefer to litigate against defendant n over settling with all defendants, regardless of p.*

Note also the converse must be true with the maximum r_i which yields the following proposition.

Proposition 5 *Where there is a single member correlated group (without loss of generality, we designate the member of that single defendant group as defendant n), and when the maximum r_i of the settling defendants is less than r_n, when $p > \frac{b_{max}}{1-b_{max}}$, where $b_{max} = $ maximum $b_i = \frac{r_i}{r_i + r_n}$, the plaintiff will prefer to settle with all defendants to litigating against a single defendant.*

5 Conclusion

The effects on settlement of joint and several liability are directly tied to the level of correlation of the defendants. Thorough policy evaluations of the value of joint and several should therefore be informed by an understanding of the correlation structure and level of actions arising under the applicable law. Some causes of action may in fact, imply specific correlation structures among the defendants. For example, in civil antitrust conspiracy cases such as price fixing, the correlation of trial outcomes among the defendants is likely nearly perfect. Other types of actions, however, for example actions under CERCLA, may have a variety of expected correlation structures, including independence or grouped defendant correlation.

Moreover, because of the importance of underlying correlation structure and level to settlement under joint and several liability, it not sensible to characterize joint and several liability as either encouraging or discouraging settlement. In particular, given the complexity of the behavior of the plaintiff's preferences for p near $p = 0$ and because of the potential preferences for profiles in which certain defendants are selected for settlement while others are selected for litigation, characterizations such as "encouraging" or "discouraging" settlement fail to capture the nature of strategy profile and may mislead.

6   APPENDIX A

To prove part (a) of our Theorem, we first prove a series of lemmata.

Lemma 1 *Given n defendants partitioned into m groups of perfectly correlated defendants such that: (i) plaintiff's prospects of success against each member of a group is perfectly correlated with her prospects of success against every other member of the group; and (ii) plaintiff's prospects of success against each member of a group is uncorrelated with her prospects of success against any defendant in another group, the plaintiff will prefer litigating against a single member of each group while settling with the remaining defendants to litigating with all defendants.*

Proof: We first calculate the probability that, in a litigation against k defendants, one from each of k distinct groups, plaintiff will prevail against *at least one* defendant. Recall that p is the probability that plaintiff will prevail in a litigation against one defendant (and $1 - p$ the probability she will lose). Thus, the probability she will lose a litigation against k defendants is $(1 - p)^k$ and the probability that she prevails against at least one defendant is $1 - (1 - p)^k$. Let $P = P_m = 1 - (1 - p)^m$ be the probability that plaintiff prevails against at least one defendant when she litigates against one defendant from each group. Notice that litigating against more than these m defendants does not increase the probability that plaintiff will prevail because plaintiff's prospects of success against the new defendant will be perfectly correlated with her prospects of success against one of the m original defendants. We conclude, therefore, that plaintiff's payoff $u_\pi(\neg S)$ from litigating against all defendants is simply P.

Let I_{n-m} be the set of indices of defendants with whom plaintiff settles in the profile $(\neg S_m, S_{n-m})$ and $W_{n-m} = \sum_{I_{n-m}} \omega_k$ her total return from settlement. Then plaintiff's expected payoff $u_\pi(\neg S_m, S_{n-m}) = P + (1 - P)W_{n-m}$ as, with probability P, she prevails against at least one defendant and recovers her entire claim of 1 while, with probability $1 - P$, she loses to all m defendants and recovers only the settlement amount W_{n-m}. Now, clearly $u_\pi(\neg S_m, S_{n-m}) - u_\pi(\neg S) = (1 - P)W_{n-m} > 0$. ∎

This result is strong in the following sense. As noted above, litigating with one defendant from each group and settling with the other defendants always offers the plaintiff a higher utility than litigating with all defendants. The plaintiff can always make a set of offers such that $(\neg S_m, S_{n-m})$ is the unique[23] Nash equilibrium for that vector of offers. Moreover, this vector of offers still creates a surplus over the plaintiff's return for $(\neg S)$.

Lemma 2 *There is always a set of offers such that there is a positive surplus for which the strategy profile $(\neg S_m, S_{n-m})$ is a Nash equilibrium and $(\neg S)$ will not be a Nash equilibrium.*

Proof: First, note that an "offer" is the amount a defendant would have to pay to settle. Thus, it is clear that for sufficiently low offers to the settling defendants in the strategy profile $(\neg S_m, S_{n-m})$, that profile will be in equilibrium while $(\neg S)$ will not be in equilibrium.

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[23]This is uniqueness up to litigating against a single defendant from each group. The essence of the analysis that follows is that profiles including more than one litigating defendant from each group can be excluded as simultaneous Nash equilibria.

Note also that the offers to defendants who litigate in $(\neg S_m, S_{n-m})$ do not have any effect on the equilibrium offers for those who settle as long as the offers to those who litigate in $(\neg S_m, S_{n-m})$ are large enough to induce those defendants to litigate.

Consider the Nash equilibrium condition for each settling defendant i. He will settle as long as the settlement amount is less than his expected loss from litigation conditional on all other defendants playing their specified strategies in $(\neg S_m, S_{n-m})$. We refer to this payout from litigation as d_i.

Then:

$$d_i = p[1 - \sum_{I_{n-m} \ \& \ j \neq i} \omega_j].$$

Clearly, $d_i > 0$. Note that for all offers smaller than d_i, defendant i would choose to settle in $(\neg S_m, S_{n-m})$.

Then, for each settling defendant in $(\neg S_m, S_{n-m})$, choose $\omega_i = d_i$.[24] This selection guarantees that the offer is low enough that the defendant will choose to settle in $(\neg S_m, S_{n-m})$, while excluding the defendant from litigating in $(\neg S)$.[25]

Now consider each litigating defendant i in $(\neg S_m, S_{n-m})$. He will litigate as long as the plaintiff's settlement offer ω_i exceeds the defendant's expected loss from litigation, conditional on all other defendants adhering to the specified strategy profile. We name this payout for litigation f_i. As with d_i, $f_i > 0$. Note that for offers smaller than f_i, defendant i in $(\neg S)$ will settle in equilibrium and that therefore for offers smaller than f_i, $(\neg S)$ will not be in equilibrium. For each litigating defendant in $(\neg S_m, S_{n-m})$, we can choose an ω large enough to make that defendant litigate. This is possible because the offer can be made arbitrarily large while still guaranteeing that the defendant will litigate. Then, with this selection, $(\neg S_m, S_{n-m})$ will be a Nash equilibrium while $(\neg S)$ will not. Moreover, as the ω_i for the settling defendants are positive, the surplus will be positive and therefore, $u_\pi(\neg S_m, S_{n-m}) > u_\pi(\neg S)$.∎

Lemma 3 *The plaintiff can select a vector of offers that results in $(\neg S_m, S_{n-m})$ being a unique Nash equilibrium, and such that $u_\pi(\neg S_m, S_{n-m}) > u_\pi(\neg S)$.*

Proof: By making the offers sufficiently large, the plaintiff can always make offers so that the defendants who litigate in $(\neg S_m, S_{n-m})$ must litigate in any other profile in equilibrium. In other words, offers can be selected to exclude from being in equilibrium any profile without the property that at least the defendants litigating in $(\neg S_m, S_{n-m})$ are also litigating in this profile. Thus, the plaintiff can exclude any profile that does not include at least m litigating defendants. This, of course, applies to the profile in which all defendants settle. Also, this exclusion can clearly be made with no effect on the plaintiff's return for $(\neg S_m, S_{n-m})$.[26] For any profile including at least the m litigating defendants of $(\neg S_m, S_{n-m})$ (excluding

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[24]This relies on our assumption that if a defendant is indifferent between settling and litigating, the defendant will litigate. In the absence of this assumption, we can choose an $\varepsilon > 0$ and set $\omega_i = d_i - \varepsilon > 0$ – we can always choose an $\varepsilon > 0$ small enough that $\omega_i > 0$ – and achieve the same effect.

[25]It is important to note that this selection of ω_i does not maximize the plaintiff's return for $(\neg S_m, S_{n-m})$ in equilibrium.

[26]That is because, as noted above, the return for litigating remains the same regardless of the number of defendants who litigate.

$(\neg S_m, S_{n-m})$ itself), let i' be the index of one of the additional defendants litigating in that profile. Then we use the same procedure to choose an offer that guarantees that defendant i' settles as used in the proof of Proposition 2 relating to excluding $(\neg S)$ as a simultaneous Nash equilibrium, with the modification that the Min is taken over $d_{i'}$, $f_{i'}$ and $g_{i'}$, where $g_{i'}$ is the bound on $\omega_{i'}$ for litigation by defendant i' in this new profile.■

Lemma 4 *The plaintiff's payoff from the profile in which one defendant litigates from each of the m groups while the remaining defendants settle, is at least as great her payoff from any other profile in which at least one defendant from each group litigates. Plaintiff's payoff is greater if more than m defendants litigate.*

Proof: Because the outcomes for defendants in the same group are perfectly correlated, litigating defendants in a group lose as a group, paying a *pro rata* share of the amount at stake. Thus, an increase in the number of litigating defendants does not increase the plaintiff's return from litigation directly. The amount at stake, however, may change with an increase in the number of litigating defendants because the number of defendants from whom the plaintiff can extract settlements decreases. We can therefore focus our attention on settlement amounts. A strategy profile that decreases or increases the plaintiff's return from settlement relative to litigating against one defendant from each group will likewise decrease or increase respectively the plaintiff's overall return.

One way to view the effect of an increase in the number of litigating defendants is through an examination of the feasible set formed by the equilibrium constraints on the settlement amounts. We know from basic theorems of linear programming that an optimal return to the plaintiff must arise at an extreme point of this feasible set. Essentially, each "vertex" of the $(n - m)$-dimensional polytope that forms the hull of the feasible set can be thought of (with the "proper subset" restriction set forth below) as a vector of settlement values.[27]

In fact, as we will show, each feasible set for profiles including additional litigating defendants beyond one litigating defendant per group, forms a *proper subset* of the feasible set for the profile in which one defendant from each group litigates. Maximizing the plaintiff's return subject to equilibrium constraints of the $(n - m)$-dimensional polytope will therefore provide a return that the plaintiff's return for profiles including additional litigating defendants never exceeds.

To show this formally, we look first at the linear programming, simplex algorithm tableau with the sum of the settlement amounts as the objective function, subject to the $n - m$ equilibrium constraints on the settlement amounts. The system then has the form: Maximize:[28]

$$\sum_{\kappa \epsilon K} \omega_\kappa.$$

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[27]Because the constraint functions change in a fundamental fashion when all defendants litigate (the amount at stake is 1), the conclusions in this section are not applicable to the profile in which all defendants litigate.

[28]Here we designted the m litigating defendants - one from each correlated group - as defendants 1 to m. Note that there are also non-negativity conditions on the ω_i.

Where $K = $ (set of all settling defendants), subject to:

$$\omega_i \leq (1 - \sum_{\kappa \epsilon K'} \omega_\kappa) P_i.$$

Where $K' = $ (set of settling defendants other than i) and P_i has the form:

$$
\begin{aligned}
P_i = [\quad & (1-p)^{m-1} p \left(\frac{r_i}{r_i + r_{\lambda_1}} + ... + \frac{r_i}{r_i + r_{\lambda_m}} \right) \\
+ \quad & (1-p)^{m-2} p^2 \left(\frac{r_i}{r_i + r_{\lambda_1} + r_{\lambda_2}} + ... + \frac{r_i}{r_i + r_{\lambda_{m-1}} + r_{\lambda_m}} \right) \\
+ \quad & (1-p)^{m-3} p^3 \left(\frac{r_i}{r_i + r_{\lambda_1} + r_{\lambda_2} + r_{\lambda_3}} + ... + \frac{r_i}{r_i + r_{\lambda_{m-2}} + r_{\lambda_{m-1}} + r_{\lambda_m}} \right) \\
+ \quad & (1-p)^{m-4} p^4 \left(\frac{r_i}{r_i + r_{\lambda_1} + r_{\lambda_2} + r_{\lambda_3} + r_{\lambda_4}} + ... + \frac{r_i}{r_i + r_{\lambda_{m-3}} + r_{\lambda_{m-2}} + r_{\lambda_{m-1}} + r_{\lambda_m}} \right) \\
+ \quad & ... \\
+ \quad & p^m \left(\frac{r_i}{r_i + r_{\lambda_1} + r_{\lambda_2} + ... + r_{\lambda_m}} \right)].
\end{aligned}
$$

where r_{λ_i} is the liability share of litigating defendant i and r_κ is the liability share of settling defendant κ. Notice that, should settling defendant κ litigate and lose, the other litigating member of the group to which she belongs would also lose the litigation. This tableau has the form:

ω_1	ω_2	...	ω_{n-m}	sv_1	sv_2	...	sv_{n-m}		
1	$-P_1$	...	$-P_1$	1	0	...	0		$-P_1$
$-P_2$	1	...	$-P_2$	0	1	...	0		$-P_2$
$-P_3$	$-P_3$	...	$-P_3$	0	0	...	0		$-P_3$
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
$-P_{n-m}$	$-P_{n-m}$	...	1	0	0	...	1		$-P_{n-m}$
-1	-1	...	-1	0	0	...	0		0

where the sv_i are slack variables and where we assume, without loss of generality that the first $n - m$ indexes correspond to an index in an ordered set of the defendants who settle.

The maximum return occurs when using elementary row operations, there are non negative entries in the last row of the tableau. Because the matrix of the equilibrium constraints:

$$\begin{array}{ccccc}
1 & -P_1 & -P_1 & \ldots & -P_1 \\
-P_2 & 1 & -P_2 & \ldots & -P_2 \\
-P_3 & -P_3 & 1 & \ldots & -P_3 \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
-P_{n-m} & -P_{n-m} & -P_{n-m} & \ldots & 1
\end{array}$$

is linearly independent (inspection, *i.e.* Gaussian elimination to upper diagonal form, shows this matrix clearly has, for example, a non-zero determinant), the only way to get non-negative entries in the last row of the tableau is for each preceding row to have "participated" in a row operation with the last row, and therefore, for each ω_i to have a value greater than zero.

Now we show that any profile including additional litigating defendants induces a set of constraints that form a feasible set that is a subset of the feasible set for one defendant from each group litigating. We can see this by examining the zeros of the equilibrium constraint half-planes for corresponding settling defendants in the one from each group and the additional litigant profiles.[29]

The constraint conditions for settlers in the one litigant from each group appear above. For the additional litigant system, the constraint system is:

$$\omega_i' \leq (1 - \sum_{\kappa \epsilon K} \omega_\kappa') P'.$$

Because of the intragroup perfect correlation, the denominators of the fractions in the equation for P' have additional rs with an r_j for each additional defendant who litigates.[30] Thus $P' < P$.

To find one set of zeros, we set all ω_k and ω_k' equal to 0. We then have $\omega_i = P$, $\omega_i' = P'$ and therefore, at that zero, $\omega_i < \omega_i'$. Now choosing one ω_k and one ω_k' and making the remaining be zero we have: $0 = [1 - \omega_j]P$ or $\omega_j = 1$ and $0 = [1 - \omega_j']P$ or $\omega_j' = 1$.

Thus, the feasible set for the profile with additional litigant is a subset of the feasible set where the constraints are from the profile in which one defendant from each group litigates and therefore, litigating against one defendant from each group provides a better payoff for the plaintiff than any profile including additional litigating defendants.

To complete the proof we need first to show that the same result holds when more than one additional defendant chooses to litigate. The inclusion of the feasible set of this profile within the feasible set generated by exactly one defendant litigating from each group follows immediately from the fact that additional litigants changes only the size of the denominator. Second, notice that we can use this property for any strategy profile with additional litigants by introducing the additional litigating defendants in any sequence that includes all of them.∎

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[29] In what follows, we will indicate all variables associated with the additional litigant profiles by appending a $'$ to each such variable.

[30] Note that the number of terms in the expansion of P' is the same as for P.

Lemma 5 *There exists $p^* < 1$ such that for all $p > p^*$, $u_\pi(\neg S_m, S_{n-m}) > u_\pi(\neg S_k, S_{n-k})$ for all k, $0 < k < m$.*

Proof: First, we examine the case comparing profiles that differ by one litigating defendant. That is, if $(\neg S_j, S_{n-j})$ is a profile in which one defendant from each of j groups litigates and $(\neg S_k, S_{n-k})$ is a profile in which one defendant from each of $k = j - 1$ groups litigates such that $(\neg S_k, S_{n-k})$ is identical to $(\neg S_j, S_{n-j})$ except that one additional defendant settles, then there exists a $p^*(j,k) < 1$ such that for $p > p^*$, $u_\pi(\neg S_j, S_{n-j}) > u_\pi(\neg S_k, S_{n-k})$.

Observe that for each profile that results in $(\neg S_j, S_{n-j})$, there exist j possible profiles $(\neg S_k, S_{n-k})$, such that $(\neg S_k, S_{n-k})$ is a profile in which one defendant from each of $k = j - 1$ groups litigates such that $(\neg S_k, S_{n-k})$ is identical to $(\neg S_j, S_{n-j})$ except that one additional defendant settles.

Fix one of the j profiles $(\neg S_k, S_{n-k})$. We introduce the notation $S(n-i) = \sum_{j \epsilon I} \omega_j$, where I is the set of settling defendants for the profile in which i defendants litigate.

Note that $S(n-k) > S(n-j) > 0$. Let $D = [1 - (1-p)^j][1 - S(n-j)] + S(n-j) - \{[1 - (1-p)^k][1 - S(n-k)] + S(n-k)\}$ for $k < j$. D is plaintiff's net benefit from litigating with one additional litigant. Thus, when $D > 0$, $u_\pi(\neg S_j, S_{n-j}) > u_\pi(\neg S_k, S_{n-k})$.

Simplified, the requirement for $D > 0$ is that $D = p + (1-p)S(n-j) - S(n-k) > 0$.

Observe that $S(n-k) < \frac{(n-k)p}{1+(n-k)p}$. Then for $p > \frac{(n-k)p}{1+(n-k)p}$, $D > 0$.

In other words for $1 - (n-k) + (n-k)p > 0$, $D > 0$.

Thus, for all $p > \frac{n-k-1}{n-k}$, $D > 0$ (note that for all n, k, the right hand side of the inequality is < 1).

Let $p^*(j,k) = \frac{n-k-1}{n-k}$. Then, for all $p > p^*(j,k) = \frac{n-k-1}{n-k}$, $D > 0$, and therefore, for all $p > p^*(j,k)$, $u_\pi(\neg S_j, S_{n-j}) > u_\pi(\neg S_k, S_{n-k})$.

Note that $p > p^*(j,k)$ does not depend on j, except that $j > k$. The partial derivative $\frac{\partial f}{\partial k}$ of $f = \frac{n-k-1}{n-k}$ is $\frac{-1}{(n-k)^2}$ which is less than zero for all n, k and therefore, $p^*(j,k)$ is decreasing in k. The maximum value k can attain is $m - 1$ and thus, for any $p > p^*(m, m-1) = \frac{n-m-2}{n-m-1}$, $u_\pi(\neg S_j, S_{n-j}) > u_\pi(\neg S_k, S_{n-k})$ for any $j > k$, $j \le m$, $k \ge 1$. Observe also, that $\frac{n-m-2}{n-m-1} > 0$. ∎

For our purposes here, there are only four types of profiles: full selective litigation, $(\neg S_m, S_{n-m})$; partial selective litigation, $(\neg S_k, S_{n-k})$ where $m > k > 0$; full settlement, (S); and full litigation, $(\neg S)$. We have shown that full selective litigation dominates full litigation. Thus, to guarantee that the plaintiff will prefer full selective litigation, $(\neg S_m, S_{n-m})$, to any other profile, we need p such that p is greater than the maximum of: 1) the $p^*(m, 0)$ which will be shown below as the p above which the plaintiff prefers full selective litigation to settling with all defendants or $u_\pi(\neg S_m, S_{n-m}) > u_\pi(S)$; and 2) $p^*(m, m-1)$. Call this maximum, p^*. Because each of $p^*(m, 0)$ and $p^*(m, m-1)$ is less than 1, in other words $p^*(m, 0)$ and $p^*(m, m-1)$ exist, p^*, their maximum is less than 1, and will likewise exist. Then for all $p > p^*$, the plaintiff will prefer full selective litigation to any other strategy profile.

We now examine the relationship between selective litigation and settling with all defendants. Unless the probability of winning a litigation, p, is low, the plaintiff will prefer litigating against one defendant from each group and settling with the rest to "total" settlement or additional litigation.

The plaintiff will receive full compensation from litigation when any of the m litigating defendants loses. The plaintiff does not get a return from litigation only when all defendants

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win. The plaintiff's return from litigating with one defendant from each of the m groups is:[31]

$$\sum_{i=1}^{n-m} \omega_i + [1 - (1-p)^m](1 - \sum_{i=1}^{n-m} \omega_i) - m\lambda_\pi$$

$$= 1 - (1-p)^m[1 - \sum_{i=1}^{n-m} \omega_i] - m\lambda_\pi.$$

To calculate the return to the plaintiff for settling with all n defendants, we use a procedure similar to that in the 3-defendant numerical example. Thus, we begin by identifying the equilibrium condition on defendant i for settling when all other defendants settle. When the profile S is in equilibrium, each defendant i will settle for:[32]

$$\omega_i \leq p(1 - \sum_{j \neq i} \omega_j) + \lambda_d.$$

Summing these amounts over the n defendants:

$$\sum_{j=1}^{n} \omega_j \leq np - (n-1)p \sum_{j=1}^{n} \omega_j + n\lambda_d.$$

Rearranging the above yields the plaintiff's expected return for settling with all the defendants:

$$\sum_{j=1}^{n} \omega_j \leq \frac{np + n\lambda_d}{1 + (n-1)p}.$$

Taking the difference of these returns:

$$u_\pi(\neg S_m, S_{n-m}) - u_\pi(S) = 1 - (1-p)^m[1 - \sum_{i=1}^{n-m} \omega_i]$$

$$-\frac{np}{1 + (n-1)p} - m\lambda_\pi - n\lambda_d.$$

From this difference[33] we construct a function χ whose sign allows us to analyze the behavior of the difference $u_\pi(\neg S_m, S_{n-m}) - u_\pi(S)$.

Because the common denominator of the difference is positive, we are only concerned with the sign of the numerator. If it is positive, the plaintiff prefers litigating with one defendant

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[31] We have assumed, without loss of generality, that the litigating defendants are the last m defendants by subscript.

[32] Here, as in the numerical example, we impose a convention that when a defendant is indifferent between settling and litigating, the defendant will chose to settle.

[33] It is clear from inspection and unsurprising that an increase in litigation costs encourages settlement. Thus, for clarity, we now let λ_π and λ_d go to zero.

from each group, and if it is negative, the plaintiff prefers settling with all defendants. The expression for the numerator is: [34]

$$1 - (1-p)^{m-1}[1 - \sum_{i=1}^{n-m} \omega_i][1 + (n-1)p].$$

We will now also ignore the $1 - \sum_{i=1}^{n-m} \omega_i$ term, thus strengthening the result in favor of $u_\pi(\neg S_m, S_{n-m})$ by guaranteeing that when the remaining difference is positive, the plaintiff clearly prefers $(\neg S_m, S_{n-m})$ to settling with all defendants.[35] Another important benefit of ignoring this term is that the results that follow in this article are *independent of the distribution* of the liability shares r_i.

We call the remaining function χ. When $\chi = 1 - (1-p)^{m-1}[1 + (n-1)p]$ is positive, the plaintiff certainly prefers $(\neg S_m, S_{n-m})$ to settling with all defendants. Notice that χ is a polynomial in p of degree m.[36]

As will be shown, when $1 < m < n$, χ is 0 at $p = 0$, becomes negative immediately to the right of $p = 0$, reaches a local minimum, then increases, becomes positive and assumes the value $\chi = 1$ at $p = 1$. As m increases, the region of $p \epsilon [0, 1]$ where $\chi < 0$ "shrinks" and then disappears when $m = n$. Thus, for $m = n$, $\chi \geq 0$ for all $p \epsilon [0, 1]$.

We now formally examine the behavior of χ on $p \epsilon [0, 1]$. First notice that at the endpoints of the interval χ assumes the values, $\chi = 0$ at $p = 0$ and $\chi(1) = 1$ at $p = 1$.

To find where χ is increasing or decreasing and to find the local minima and maxima of χ on the relevant interval, we calculate $\frac{\partial \chi}{\partial p}$.

$$\begin{aligned}
\frac{\partial \chi}{\partial p} &= -(1-p)^{m-1}(n-1) + [1 - (n-1)p](m-1)(1-p)^{m-2} \\
&= (1-p)^{m-2}[m - n + pm(n-1)].
\end{aligned}$$

Thus, for $\frac{\partial \chi}{\partial p} = 0$, either $p = 1$ or $p = \frac{n-m}{m(n-1)}$, and thus the local minima or maxima of χ are at these points.

We can now prove part (a) of the main theorem:

Theorem 1 (a) For $1 < m < n$, there exists a $p^* \epsilon (0, 1)$ such that $u_\pi(\neg S_m, S_{n-m}) > u_\pi(S)$ for all $p \epsilon (p^*, 1]$.

Proof: Observe that when $p = 0$, $\frac{\partial \chi}{\partial p} = m - n$. Therefore, at $p = 0$, $\frac{\partial \chi}{\partial p} < 0$. Thus, χ is decreasing at $p = 0$ when its value is $\chi = 0$, and therefore χ must become negative just to the right of $p = 0$.

For $m < n$, on $p \epsilon [0, \infty)$, $\frac{\partial \chi}{\partial p} = 0$ means $p > 0$. We also need to know, however, whether this local minimum or maximum occurs for $p < 1$, so that it would be contained in the relevant interval. Thus, we look at $\frac{n-m}{m(n-1)}$ to observe the conditions under which the ratio is

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[34] The litigation costs in the numerator are $(m\lambda_\pi + n\lambda_d)[1 + (n-1)p]$.

[35] Ignoring the $1 - \sum_{i=1}^{n-m} \omega_i$ term means that even if the difference in negative, the plaintiff may still prefer $(\neg S_m, S_{n-m})$.

[36] An exception occurs when $m = 1$, in which case, $\chi = (1-n)p$, a linear function in p.

less than or equal 1, and observe:

$$n - m \leq m(n-1)$$
$$n \geq 0.$$

Thus, $0 < \frac{n-m}{m(n-1)} < 1$. Therefore, there is a single local minimum/maximum, at $p = \frac{n-m}{m(n-1)}$ on $p \in (0,1)$. At this p, χ is negative.

Recall that $\frac{\partial \chi}{\partial p} = (1-p)^{m-2}[m - n + p(nm - m)]$. For $p^* > \frac{n-m}{m(n-1)}$, $\frac{\partial \chi}{\partial p} = (1-p^*)^{m-2}[m - n + p^*(nm - m)]$.

Clearly, $(1-p^*)^{m-2} > 0$ for $p^* \in [0,1)$. The second term of the partial derivative is positive for such p^*. Thus, $\frac{\partial \chi}{\partial p} > 0$ for $p^* > \frac{n-m}{m(n-1)}$ and $p^* < 1$. Therefore, for $p^* > \frac{n-m}{m(n-1)}$, χ is increasing up to $p = 1$ where $\chi = 1$ and therefore χ must become positive on $p \in (\frac{n-m}{m[n-1]}, 1)$.

Thus, the values of p for which χ is negative are restricted to an interval immediately to the right of $p = 0$.■

We now prove part (b) of Theorem 1.

Theorem 1 (b) For $1 < m \leq n$, there exists a p^* such that there is a unique Nash equilibrium in which plaintiff makes offers that induce the behavioral strategy profile $(\neg S_m, S_{n-m})$ whenever $p > p^*$. For sufficiently small p, the plaintiff will prefer settling with all defendants over full selective litigation.

Proof: For the first part, see the discussion above.

The proof that for sufficiently small p, the plaintiff will prefer settling with all defendants over full selective litigation is as follows. We examine first, the difference between $u_\pi(S) = \frac{np}{1+(n-1)p}$ and $u_\pi(\neg S_m, S_{n-m})$, in other words: $\frac{np}{1+(n-1)p} - \{1 - (1-p)^{m-1}[1 - \sum_{i=1}^{n-m} \omega_i][1 + (n-1)p]\}$. Noting that $1 - (1-p)^{m-1}[1 - \sum_{i=1}^{n-m} \omega_i][1 + (n-1)p] > 1 - (1-p)^{m-1}[1 - \sum_{i=1}^{n-m} \omega_i]$, and that $\sum_{i=1}^{n-m} \omega_i < S(n-m)$, where $S(n-m)$ is the sum of settlement amounts for a litigation in which there are $n-m$ defendants all of whom settle, *i.e.* $S(n-m) = \frac{(n-m)p}{1+(n-m-1)p}$, (this follows from the fact that $S(n-m)$ has larger coefficients in each of its equilibrium constraints than any possible set of settlement constraints for $n-m$ defendants settling while other defendants litigate), we have $\frac{np}{1+(n-1)p} - \{1 - (1-p)^{m-1}[1 - \sum_{i=1}^{n-m} \omega_i][1 + (n-1)p]\} > \frac{np}{1+(n-1)p} - \{1 - (1-p)^{m-1}[1 - \sum_{i=1}^{n-m} \omega_i]\}$ and $H(p) = \frac{np}{1+(n-1)p} - \{1 - (1-p)^{m-1}[1 - \sum_{i=1}^{n-m} \omega_i]\} > D(p) = \frac{np}{1+(n-1)p} - \{1 - (1-p)^m[1 - \frac{(n-m)p}{1+(n-m-1)p}]\}$.

Thus, whenever $H(p) > 0$, the plaintiff clearly prefers (S) to full selective litigation, $(\neg S_m, S_{n-m})$.

But $H(0) > D(0) = 0$, so for p in the neighborhood of 0, the plaintiff prefers (S) over $(\neg S_m, S_{n-m})$, full selective litigation.■

From the proof of lemma 5 we have the following corollary:

Corollary 1 *As m increases, p^* goes to 0 and $p^* = 0$ at $m = n$.*

6.0.1 Behavior below p^*: One Litigating Defendant And All Defendants Settling

Proposition 1 : The plaintiff prefers the profile in which one defendant litigates, $(\neg S_1, S_{n-1})$, to settling with all defendants, (S), when the return from the settling defendants in the

profile $(\neg S_1, S_{1-m})$, $\sum_{i=1}^{n-1} \omega_i$, is greater than $\frac{(n-1)p}{1+(n-1)p}$. When $\sum_{i=1}^{n-1} \omega_i$ is less than $\frac{(n-1)p}{1+(n-1)p}$, the plaintiff prefers (S) to $(\neg S_1, S_{1-m})$.

Proof: For the single defendant litigating profile, $(\neg S_1, S_{n-1})$, to be preferred over (S), the following needs be true: $p[1 - \sum_{i=1}^{n-1} \omega_i] + \sum_{i=1}^{n-1} \omega_i > u_\pi(S)$. Thus, to satisfy this requirement, $(1-p) \sum_{i=1}^{n-1} \omega_i + p > \frac{np}{1+(n-1)p}$, or simplified: $\sum_{i=1}^{n-1} \omega_i > \frac{(n-1)p}{1+(n-1)p}$.

Because the inequality: $p[1 - \sum_{i=1}^{n-1} \omega_i] + \sum_{i=1}^{n-1} \omega_i > u_\pi(S)$ is "exact" in the sense that when $p[1 - \sum_{i=1}^{n-1} \omega_i] + \sum_{i=1}^{n-1} \omega_i \leq u_\pi(S)$, the plaintiff prefers (S), the converse follows for the same reasons.∎

Using this "threshold" return from settlement, we now examine the special case in which the profile $(\neg S_1, S_{n-1})$ includes a correlated group that has only one member. Another property that we will use in the arguments that follow is that that if there is such a "singleton," then there is a single defendant litigating profile in which the equilibrium constraints for the settlement offers to the $n-1$ settling defendants all have the form: $\omega_j = (p - p^2 + \frac{1}{2}p^2)[1 - \sum_{i=1, i \neq j}^{n-1} \omega_i]$. Here, we assume without loss of generality that defendant n is the single litigating defendant.

Proposition 2 : When the defendants' shares of liability are uniform, and there exists a single member correlated group, the plaintiff never prefers the profile (S) to $(\neg S_1, S_{n-1})$.

Proof: The uniform distribution of the r_i implies $\frac{r_i}{r_i + r_n} = \frac{1}{2}$ and from above, $\omega_j = (p - p^2 + \frac{1}{2}p^2)[1 - \sum_{i=1, i \neq j}^{n-1} \omega_i]$. This implies that, $\sum_{j=1}^{n-1} \omega_j = \frac{(n-1)[p-p^2+\frac{1}{2}p^2]}{1+(n-2)[p-p^2+\frac{1}{2}p^2]}$. Thus, from Lemma 1, for the plaintiff to prefer to settle with all defendants, it must be that: $\sum_{i=1}^{n-1} \omega_i = \frac{(n-1)[p-p^2+\frac{1}{2}p^2]}{1+(n-2)[p-p^2+\frac{1}{2}p^2]} \leq \frac{(n-1)p}{1+(n-1)p}$.

Simplifying, the requirement becomes: $p[1-p] \leq 0$, which is a cannot occur. Thus, $u_\pi(\neg S_1, S_{n-1}) > u_\pi(S)$ for all p, in other words, the plaintiff never prefers (S) if there is a single member group and liability shares are uniform.∎

We now take advantage of the fact that the restriction of uniformity can be loosened somewhat, but the similar conclusions to Lemma 2 above still follow. Suppose, for example, that for all $i < n$, $\frac{r_i}{r_i + r_n} = b$. We then have the following proposition.

Proposition 3: Where there is a single member correlated group (without loss of generality, we designate the member of that single defendant group as defendant n) and shares are uniform among the $n-1$ non-litigating defendants. Then for the plaintiff to prefer to settle with all defendants, it must be that $p > \frac{b}{1-b}$, where $b = \frac{r_i}{r_i + r_n}$.

Proof: For the plaintiff to prefer to settle with all defendants, we must have $\sum_{i=1}^{n-1} \omega_i = \frac{(n-1)[p-p^2+bp^2]}{1+(n-2)[p-p^2+bp^2]} \leq \frac{(n-1)p}{1+(n-1)p}$.

Simplifying, this restriction becomes that $p > \frac{b}{1-b}$.

Notice that for $r_i = r_j$ for $i < n$, $j < n$, if $\frac{b}{1-b} > 1$, there is no feasible p such that the plaintiff will prefer (S) over $(\neg S_1, S_{n-1})$. Observe also that $\frac{b}{1-b} > 1$ means $b < \frac{1}{2}$, or $\frac{r_i}{r_i + r_n} < \frac{1}{2}$. In other words, for $r_i < r_n$, the plaintiff prefers to litigate against one defendant.∎

Finally, observe that $\frac{r_i}{r_i + r_n} < \frac{1}{2}$ also holds for any arbitrary distribution of r_i, as long as for all $i < n$, $r_i < r_n$.

Taking advantage of this property, we have the following proposition.

Proposition 4: Where there is a single member correlated group (without loss of generality, we designate the member of that single defendant group as defendant n), and for all $i < n$,

$r_i < r_n$, then the plaintiff will prefer to litigate against defendant n over settling with all defendants, regardless of p.

Proof: We can also derive the following restrictions where the r_i have an arbitrary distribution. Let b_{min} be the minimum $b = \frac{r_i}{r_i+r_n}$. Then $\sum_{i=1}^{n-1} \omega_i > \sum min[\omega_i]$ where $min[\omega_i]$ are the constraints on the offers with b_{min} substituted for b. Then if u exceeds the threshold for $u_\pi(\neg S_1, S_{n-1})$ to be greater than $u_\pi(S)$, clearly $u_\pi(\neg S_1, S_{n-1}) > u_\pi(S)$.

As with the above, when $p[b + pb - p] > 0$, $u_\pi(\neg S_1, S_{n-1})$ is preferred and thus when $p < \frac{b_{min}}{1-b_{min}}$, $(\neg S_1, S_{n-1})$ is clearly preferred. Thus, when $\frac{b_{min}}{1-b_{min}} \geq 1$, the plaintiff prefers $(\neg S_1, S_{n-1})$ to (S) for any p. Observe that, $\frac{b_{min}}{1-b_{min}} \geq 1$ means $b_{min} \geq \frac{1}{2}$ or that the minimum r_i be greater than or equal to r_n.∎

Note also the converse must be true with the maximum r_i which yields the following proposition.

Proposition 5: Where there is a single member correlated group (without loss of generality, we designate the member of that single defendant group as defendant n), and when the maximum r_i of the settling defendants is less than r_n, when $p > \frac{b_{max}}{1-b_{max}}$, where $b_{max} = $ maximum $b_i = \frac{r_i}{r_i+r_n}$, the plaintiff will prefer to settle with all defendants to litigating against a single defendant.

Proof: This follows by the same reasoning as the preceding proposition.∎

6.1 The Effect Of Allowing Coalitions Of Defendants

We now prove that the same payoff-equivalent up to permutation equilibria are coalition proof.

Theorem 2: We can construct a vector of offers such that: 1) the offers induce full selective litigation; 2) the offers induce a defendant CPNE ("dCPNE"); 3) the offers provide the best return for the plaintiff for full selective litigation; 4) the offers provide a better plaintiff return than for any greater number of defendants litigating; and 5) provide a better return for plaintiff than for any fewer defendants litigating. The equilibria are payoff-equivalent up to permutation.

Proof of 1) and 2): The procedure for construction of the $n-m$ elements corresponding to the $n-m$ defendants who are to settle in full selective litigation follows.

Take a defendant whom the plaintiff would like to induce to settle and label this defendant, defendant $n-m$. Offer defendant $n-m$, w_{n-m}, the amount of expected payment of defendant $n-m$ when all defendants litigate. This offer will prevent $(\neg S)$ from being a coalition. Thus, $w_{n-m} = 1 - (1-p)^m P_{n-m}$ where P_{n-m} is the probability and share weighting of the form set forth above in Lemma 4.

Using w_{n-m} as the offset, take a second defendant, defendant $n-m-1$, and offer w_{n-m-1}, the amount of expected payment for 1 defendant settling (with the w_{n-m} offset) – this will prevent $n-m-1$ litigating defendants from being a coalition. Thus, $w_{n-m-1} = (1 - w_{n-m})P_{n-m-1}$, where P_{n-m-1} has the form (where there are k_{n-m-1} other defendants

the plaintiff would like to induce to settle in the correlation group of defendant $n-m-1$):

$$
\begin{aligned}
P_{n-m-1} = [\quad & (1-p)^{m-1}p\left(\frac{r_{n-m-1}}{R_{n-m-1}}\right) \\
+ \quad & (1-p)^{m-2}p^2(\frac{r_{n-m-1}}{R_{n-m-1}+R_{n-m-2}} + ... + \frac{r_{n-m-1}}{R_{n-m-1}+R_1}) \\
+ \quad & (1-p)^{m-3}p^3(\frac{r_{n-m-1}}{R_{n-m-1}+R_{n-m-2}+R_{n-m-3}} + ... + \frac{r_{n-m-1}}{R_{n-m-1}+R_2+R_1}) \\
+ \quad & ... \\
+ \quad & p^{m-1}(\frac{r_{n-m-1}}{R_{n-m-1}+R_{n-m-2}+...+R_1})].
\end{aligned}
$$

where $R_i = \sum_{j\epsilon\gamma_i} r_j$ with $\gamma_i = \{$indices of the defendants in defendant i's correlation group$\}$.

We continue this procedure for all $n - m$ defendants the plaintiff chooses to induce to settle. The payoff is clearly greater than the dCPNE of offers of 1 for all defendants the plaintiff chooses to induce to litigate and zero for the $n - m$ defendants the plaintiff chooses to induce, and is greater than an offer of 1 for litigating defendants and the amount expected for $(\neg S)$ for the $n-m$ settlers. Moreover, thee offers satisfy the defendants' Nash equilibrium conditions of the $n - m$ defendants the plaintiff chooses to induce to settle (and, of course, the m litigating defendants equilibrium conditions also).

Notice, however, that the order in which this is done matters to the plaintiff's ultimate return because of the differing constraints when the settlements are intra- or extra-group. Additional litigants outside a group only affect the offset, while intragroup share and offset are affected, share sometimes dramatically. Thus picking groups according to size might matter as the amount of surplus (ignoring offsets due to settlement) that can be extracted from a group gets closer and closer to $\frac{1}{m}[1 - (1-p)^m]$ as the group size gets larger (if the group size is 2, the best return would be $(\frac{1}{2})(\frac{1}{m})$, ignoring offsets due to settlement).

Note that there are a finite number of ways, $(n - m)!$, to sequence the defendants. We select the sequence providing the best total plaintiff payoff.

Proof of 3) *and* 4): This is an optimal return for the plaintiff for the following reasons. Using the same numbering scheme for the defendants as in the construction, if the plaintiff were to try to extract any more from any defendant, say defendant i, then defendants 1 to i will pay less by litigating than by settling and will form a coalition of size i doing just that. This is because if defendant i litigates, the loss of the offset of defendant i's settlement amount to the remaining defendants – and this includes sharing differences, because the constraints setting the offers reflected them – will cause the offers for those defendants to be larger than what they would pay by litigating. Likewise, if the plaintiff were to attempt to extract amounts over the procedure offers from any collection of defendants, the highest index of that group being i', then all defendants up to defendant i' would litigate in a coalition for the identical reasons. Notice, that such a construction of offers does not depend on uniformity of shares.

The intuition that the return is better than for a profile with any greater number of litigating defendants is obvious from the procedure – all that happens with a greater number of litigating defendants is that there are fewer offers. Formally, Lemma 4 shows that the

plaintiffs' return is decreasing in the number of additional litigating defendants because each feasible set polytope for increasing litigating defendants is contained in that of full selective litigation. The same holds here.

Proof of 5): Here we construct an analog to Lemma 5 showing that the plaintiff's best dCPNE for full selective is better than any dCPNE with greater numbers of settling defendants.

Recall that j is the number of litigating defendants, $j \geq m$. Let $k = j - 1$ (Increasing the number of settlers over $n - j$ to $(n - j) + 1$).

Let $D = p + (1 - p)S(n - j) - S(n - k)$.

Because of the need to "coalition proof" the profiles, both of the following hold:

$S(n-k) = S(n-j)+S'$ and $S' = P[1-S(n-j)]$ where $P =$ share weighted probabilities.

Thus, $D = p + (1 - p)S(n - j) - S(n - j) - S'$, so that $D = p - pS(n - j) - S'$, inserting from above, $D = p[1 - S(n - j)] - P[1 - S(n - j)]$.

Notice that $P < p$ (by inspection, can factor a p out of P and leave at most a binomial.)

Thus $D > 0$ for all p. Then use transitivity down the chain from $j = m$ to $j = 1$. We need not worry about whether an optimal chain for full selective is different than an optimal chain for more defendants settling. The same optimal chain can be used for more defendants settling – up to full selective litigation, the plaintiff's payoff cannot be greater for this chain than the optimal chain for full selective litigation.∎

6.2 The Effect Of Side Payments

Our analysis focusses on the effects of side payments among defendants. Side payments from the plaintiff to defendants are merely offers – the individual defendants criteria for accepting a side payment from the plaintiff is the no different than that defendants criteria for accepting or rejecting a regular offer, except that such side payments might allow a defendant an offset when that defendant litigates. But it makes no sense in our present model for the plaintiff to pay a defendant to litigate – the plaintiff can always get d defendant to litigate at no extra cost by refusing to settle with that defendant (*i.e.*, by making the settlement offer very high.)[37] Thus, the defendants behave no differently with a provision for the plaintiff to defendant side payments than they would without one.

Side payments among defendants, however, have a profound effect. When such payments are permitted, no defendant would ever pay more than their share of the payout for $(\neg S)$,

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[37]In actual litigation, the plaintiff may have such an incentive, but this incentive most often is to alter p, or to increase a defendants uncertainty of the outcome. This may be through changing witnesses potential biases in testimony, or as on aid to discovery. For example, in an action that requires proof of conspiracy between codefendants, side payments and an agrecment with one defendant could aid the plaintiff in achieving success at trial against to remains defendants, in essence, increasing p. Such plaintiff agreements include "Mary Carter" agreements, which the Federal Judicial Center notes: "Many varieties of such agreements have developed, including loan-receipt agreements and agreements to dismiss during the case or not to execute a judgment if the defendant does not take an aggressive posture against the plaintiff's claims. These agreements have been criticized as unfair to nonsettling defendants, because they align the interests of the "settling" defendant, who remains in the litigation, with those of the plaintiff (usually covertly), eliminating their normal adversarial relationship." *Manual for Complex Litigation Fourth*, Federal Judicial Center (2004), p. 177. Our present model does not, however, allow for such adjustments to p.

the profile in which everyone litigates. The reasons are analogous to those in the Coase Theorem. To see this, imagine that all defendants but one get offers of what they would pay under $(\neg S)$. The remaining defendant gets an offer that is some amount, for convenience we shall call it $n\varepsilon$, in excess what that defendant would pay under $(\neg S)$. That defendant could give each of the others ε to litigate and all defendants, including the defendant making the payments would do better by ε. The situation is the same for any collection of defendants who are paying more than they would under $(\neg S)$. If some defendants are paying less than what they would under $(\neg S)$, in order to do better than $(\neg S)$, the plaintiff would have to make up those deficits on other defendants. But those defendants would simply transfer the excess that they paid to the underpaying defendants and all would litigate. The plaintiff could offer each defendant 1 for settlement – all would litigate and pay the plaintiff $u(\neg S)$ in total, or the plaintiff could offer each defendant what that defendant would have paid under $(\neg S)$. With the latter, plaintiff would again collect $u(\neg S)$, but all defendants would choose to settle being by indifferent to settling.

6.3 The Cases $m = n$ and $m = 1$: Extension of Kornhauser And Revesz

When $m = n$ the following proposition shows that *for any p*, the plaintiff prefers "selective" litigation to settling with all defendants.

Proposition 6 *When the number of groups, m, is equal to the number of defendants, n, the payoff to the plaintiff for the profile in which one defendant litigates from each of the m groups while settling with the remaining defendants, is greater than the payoff for settling with all defendants, for all $p \epsilon [0,1]$.*

Proof: Recall the function $\chi = 1 - (1-p)^{m-1}[1 + (n-1)p]$. Earlier we saw that when χ is positive, the plaintiff certainly prefers $(\neg S_m, S_{n-m})$ to settling with all defendants. Notice that for $m = n$, $\frac{\partial \chi}{\partial p} = 0$ when $p = 0, 1$. Thus, for $m = n$, on $p \epsilon [0,1]$, χ reaches its local minimum value, $\chi = 0$ at $p = 0$. Therefore, because of this fact and because χ has no local minima on $p \epsilon (0,1)$, χ must be non-negative on the entire relevant interval. This also follows from the fact that when $m = n$, $\frac{\partial \chi}{\partial p} > 0$ for all $p \epsilon (0,1)$, which means χ is increasing on the entire interval from a value of $\chi = 0$ at $p = 0$. Under either analysis, $\chi(p)$ is non-negative for all $p \epsilon [0,1]$ so that $u_\pi(\neg S_m, S_{n-m}) > u_\pi(S)$.∎

Thus, when $m = n$, *i.e.*, for the case when all defendant are uncorrelated and each group only contains one defendant, the plaintiff prefers to litigate against one defendant from each group to settling with all defendants. Because in the case $m = n$, there is only one defendant per group, this means that the plaintiff prefers to litigate against *all* defendants to settling with all defendants.

The $m = n$ grouped correlation formulation is isomorphic to an extension of the Kornhauser and Revesz 2-defendant uniform correlation model with uncorrelated defendant outcomes to n defendants.

In the 2-defendant model of Kornhauser and Revesz, when the defendants' litigation outcomes are uncorrelated, the plaintiff prefers to litigate against all defendants over any

other strategy profile. Thus, the analysis has demonstrated that the conclusion of Kornhauser and Revesz for two uncorrelated defendants extends to n uncorrelated defendants.

The case $m = 1$ is isomorphic to an extension of the Kornhauser Revesz model to n defendants where the defendants' litigation outcomes are all perfectly correlated. Recall that when $m = 1$, $\chi = (1 - n)p$, a linear function in p. Thus, at $p = 0$, $\chi = 0$ and at $p = 1$, $\chi = 1 - n < 0$. Because χ is linear, $\chi \leq 0$ on the entire relevant interval. This strongly suggests[38] that for $m = 1$, the plaintiff prefers settling with all defendants to "selective" litigation and therefore also to litigating with all defendants.[39] The defendant also prefers settlement with all defendants to litigation with all defendants, $u_\pi(\neg S) = p < \frac{np}{1+(n-1)p} = u_\pi(S)$, because $n + (n - 1)p > 1$ for $n \geq 2$ and $0 \leq p \leq 1$. The conclusion that the plaintiff prefers (S) to $(\neg S)$ is what Kornhauser and Revesz showed for two perfectly correlated defendants.

Finally, the following comparative statics results demonstrate that in equilibrium, as m increases to n, the interval in which the plaintiff prefers settling with all defendants to litigating with one defendant from each group "shrinks" as m approaches n and then vanishes when $m = n$. In other words, as the number of groups gets larger, the plaintiff increasingly prefers litigating with one defendant from each group to settling with all defendants.

Proposition 7 *The region R with the property that for $p \, \epsilon \, R$ the payoff to the plaintiff for the profile in which one defendant litigates from each of the m groups while remaining defendants settle is less than that for all defendants settling, i.e. the interval for p such that $u_\pi(\neg S_m, S_{n-m}) < u_\pi(S)$ decreases in length as m increases and vanishes when $m = n$.*

Proof: Recall that it was demonstrated above that when $m = n$, $u_\pi(\neg S_m, S_{n-m}) > u_\pi(S)$ for all $p \, \epsilon \, [0, 1]$.

For m not equal to n, first fix p and m in the difference function, $\chi = 1 - (1 - p)^{m-1}[1 + (n - 1)p]$, and observe that χ is linear decreasing in n. However, this is with m fixed.

Now, suppose we fix n and $m < n$ and find the $p^* \, \epsilon \, (0, 1)$ such that $\chi = 0$. There is at most one such p^*, and that if there is no such p^*, χ would be positive on $(0, 1)$. From the discussions above characterizing χ, it is clear there is only one such p^* and that for $p > p^*$, $\chi > 0$.

As m is allowed to increase, one can see that an increase in m results in an increase in χ and χ therefore becomes positive for p^*. In fact, an increase in m causes an increase in χ for any fixed $p' \, \epsilon \, (0, 1)$.[40]

Thus, as m increases toward n, the right hand boundary of the region where $\chi < 0$ moves toward the origin and the interval of $p \, \epsilon \, (0, 1)$ on which χ is negative becomes smaller for an increase in m with n fixed.

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[38] The caveat here is that χ has been set up to "favor" (S) by ignoring the 1 - sum of settlements term. Thus, $\chi < 0$ is not a guarantee that $u_\pi(S)$ is preferred.

[39] This is because, as proven above, selective litigation dominates all defendants litigating. In particular, for $m = 1$, $u_\pi(\neg S) = p < p + (1 - p) \sum \omega_i = u_\pi(\neg S_1, S_{n-1})$. As a side note, observe that the highest payoff for $(\neg S_1, S_{n-1})$ occurs when the litigating defendant i' has the smallest r_i. This is because the equilibrium constraint on ω_j for a settling defendant j includes a factor of the form $\frac{r_j}{r'_i + r_j}$ which achieves a maximal value when r'_i is as small as possible.

[40] At the endpoints, $p = 0$ and $p = 1$, however, χ remains unchanged.

Another way to observe the effect of increasing m is to note how much larger m has to be so that for some fixed probability p^*, $\chi < 0$. First, fix some $p^* = \frac{1}{a}$ where $a > 1$. Then, for $\chi < 0$, m and n must satisfy: $1 - (\frac{1}{a})^{m-1}[1 + (n-1)\frac{1}{a}] > 1$. Rearranging this becomes: $n > a^m - a - 1$. Because of the assumption that $a > 1$, it is clear that n must grow *exponentially* to keep χ negative for increasing m.∎

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