A New State of Turbulence: Elasto-Inertial Turbulence

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First simulations of EIT via early turbulence

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See Vincent Terrapon’s talk for more details
Investigative methods

Direct numerical simulation of periodic channel flow with a weak initial wall perturbation

$$Re = \frac{U_b H}{\nu}$$

$$N_x \times N_y \times N_z = 256 \times 151 \times 256$$

Initial wall perturbation designed to trigger turbulence at $Re=6000$ in water

$$v(x,y=\pm h,z,t) = \mathcal{H}(t) \left[ A \sin \left( \frac{8\pi}{L_x} x \right) \sin \left( \frac{8\pi}{L_z} z \right) + \epsilon(t) \right]$$

$$\mathcal{H}(t) = \begin{cases} 1 & \text{for } \frac{tU_b}{h} < 1 \\ 0 & \text{for } \frac{tU_b}{h} \geq 1 \end{cases}$$

$$\epsilon(t): \text{random noise}$$
Viscoelastic flow model

FENE-P model for Newtonian flow

polymer molecule

reduced model

q: end-to-end vector

continuum model

Polymer solution parameter

\[ C = C_{ij} = q \otimes q = q_i q_j \]

\[ W_i = \frac{\text{polymer solution relaxation time scale}}{\text{flow time scale}} \]

\[ L: \text{maximum polymer extension} \]

\[ \beta = 0.9 = \frac{\text{solvent viscosity}}{\text{zero-shear polymer solution viscosity}} \]
Viscoelastic flow model (FENE-P)

- Momentum transport equation
  \[ \partial_t u + \nabla u \otimes u = -\nabla p + \frac{\beta}{Re} \nabla^2 u + \frac{1 - \beta}{Re} \nabla \cdot T \]

- Polymer stress tensor
  \[ T = T_{ij} = \frac{1}{W_i} \left( \frac{C_{ij}}{1 - C_{kk}/L^2} - \delta_{ij} \right) \]

- Conformation stress tensor
  \[ \partial_t C + (u \cdot \nabla) C = C \cdot (\nabla u) + (\nabla u)^t \cdot u - T \]
  
  advection  polymer stretching-internal forces
Polymer effects

- Polymer solution viscosity decreases in shear flow for large concentration
- Polymer solution viscosity (dramatically) increases in extensional flows due to the increase in polymer extension
Realism of our DNS

- The hyperbolicity of the conformation tensor transport equation is respected as best as numerically possible (Dubief et al., 2005)

- Polymer parameters are such that the shear thinning effect is small but the extensional viscosity is large (increasing with $Wi$), as expected for low polymer concentrations

- The following movies are representative of the dominant dynamics of the respective flows over a very long simulation time

- Our simulations reproduce the evolution of the friction factor as a function of the Reynolds numbers observed in pipe flow experiments (Samanta, Dubief, Holzner, Shäfer, Morozov, Wagner, Hof, submitted)
Polymers create their own turbulence at subcritical Reynolds numbers.
Equations of Elasto-Inertial Turbulence

\[ \partial_t C + (u \cdot \nabla) C = C \cdot (\nabla u) + (\nabla u)^t \cdot C - T \]

\[ \nabla \cdot \left[ \partial_t u + \nabla u \otimes u = -\nabla p + \frac{\beta}{Re} \nabla^2 u + \frac{1 - \beta}{Re} \nabla \cdot T \right] \]

\[ \nabla^2 p = 2Q + \frac{1 - \beta}{Re} \nabla \cdot (\nabla \cdot T) \]
Flow is perfectly laminar in the absence of polymers

Polymer addition creates a self-sustained chaotic flow consisting of trains of cylindrical weakly rotational regions (positive $Q$) and weakly extensional regions (negative $Q$)

There is a hierarchy of cylindrical structures, the smallest one being of the order of the Kolmogorov scale

$$Re = \frac{U_b H}{\nu} = 1000, \quad Wi = \lambda_p \dot{\gamma} = 100$$

Isosurfaces of $Q$ the second invariant of velocity gradient tensor

$$Q = \frac{1}{2} (\Omega^2 - S^2) = \frac{1}{8} \left[ (\nabla u - \nabla u^t)^2 - (\nabla u + \nabla u^t)^2 \right]$$
$Re = \frac{U_b H}{\nu} = 1000, \ Wi = \lambda_p \dot{\gamma} = 100$

Contours of polymer extension and $Q$

$$\sqrt{\frac{\text{Trace}}{L^2}}$$

The polymer extension field is organized in sheets

Polymers cause the flow to evolve from pure shear flow to mix extensional-shear flow

The cylindrical $Q$ structures are attached to sheets of large polymer extension
\[ Re = \frac{U_b H}{\nu} = 6000, \quad Wi = \lambda_p \dot{\gamma} = 700 \]

Isosurfaces of \( Q \) the second invariant of velocity gradient tensor

\[ Q = \frac{1}{2} (\Omega^2 - S^2) = \frac{1}{8} \left[ (\nabla u - \nabla u^t)^2 - (\nabla u + \nabla u^t)^2 \right] \]

- The flow is at the maximum drag reduction state
- Vortices are highly intermittent
- Long periods of elasto-inertial turbulence
Equations of Elasto-Inertial Turbulence

\[
\partial_t C + (u \cdot \nabla) C = C \cdot (\nabla u) + (\nabla u)^t \cdot C - T
\]

\[
\nabla \cdot \left[ \partial_t u + \nabla u \otimes u = -\nabla p + \frac{\beta}{Re} \nabla^2 u + \frac{1 - \beta}{Re} \nabla \cdot T \right]
\]

\[
\nabla^2 p = 2Q + \frac{1 - \beta}{Re} \nabla \cdot (\nabla \cdot T)
\]

- Elasto-inertial turbulence results from the combination of the hyperbolic transport equation of \( C \) and the elliptic equation of \( p \)

- Pressure redistributes energy with trains of cylindrical structures to attenuate the anisotropy caused by sheets of extensional viscosity
Mechanism of Elasto-Inertial Turbulence

\[ \partial_t C + (u \cdot \nabla) C \]

Formation of sheets of \( C \)

\[ \nabla^2 p = 2Q + \frac{1 - \beta}{Re} \nabla \cdot (\nabla \cdot T) \]

Excitation of extensional sheet flow and elliptical pressure redistribution of energy

\[ C \cdot (\nabla u) + (\nabla u)^T \cdot C - T \]

Increase of extensional viscosity in sheets
Stick around

- Julio Soria will tell you everything you need to know about the flow topology of EIT
- Vincent Terrapon will show the long range interactions that trigger EIT in a bypass transition flow
- and all the other talks
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