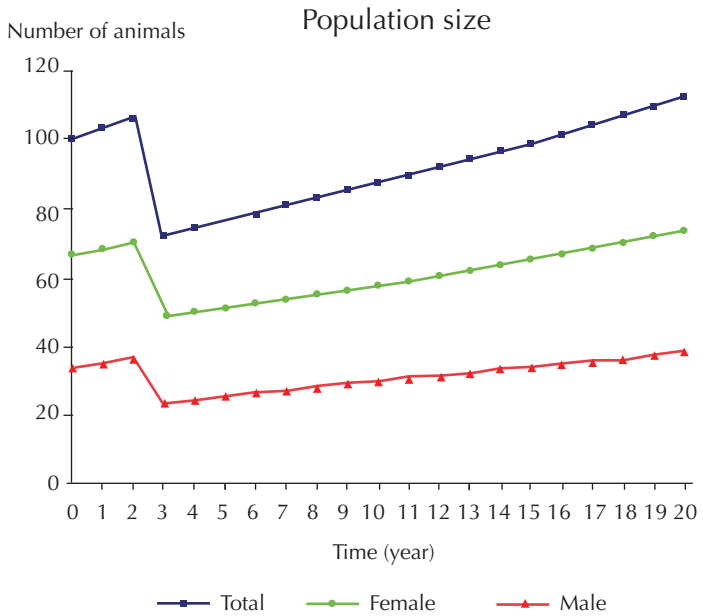
DynMod

A tool for demographic projections of tropical livestock populations under Microsoft Excel

User's Manual – Version 1



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Introduction

Livestock populations represent a major economic value for many developing countries. Tools for evaluating the impact of developing projects or shocks (like drought or outbreaks) on the livestock dynamics and production help the decision makers to better target strategies for improving economic situation of livestock smallholders. The present document is the user's manual of DynMod, which is a simple tool for demographic projection of tropical livestock populations. The tool was developed under Microsoft® Office Excel 2003 by CIRAD and ILRI.

DynMod is intended to researchers, developers, engineers and technicians of national services, or students dealing with demography of tropical livestock. Using simplified equations, DynMod is well adapted for implementing fast and crude ex ante or ex post diagnostics, in various applications as for example livestock population management, herd production estimation or exploration of scenarios in development projects.

DynMod represents animal populations that can be divided in juvenile, sub-adult and adult age groups and that have a reproduction distributed all over the year. The core of the model is the demographic simulation, i.e. the simulation of the dynamics of the number of living animals in the population and of the number of animals produced per year, based on a simplified approach. DynMod also provides results on live weights, meat production and secondary productions (milk, skin and hides, manure), as well financial outputs that can be used in more integrated financial calculations (e.g. benefit-costs ratios or internal return rates). Finally, DynMod provides estimates of the population feeding requirements in dry mater.

DynMod predicts what would happen under some hypothetical scenarios, not what will happen in the real world. In that sense, DynMod is more a projection model than a prediction model (Caswell, 2001). In a simulation, the user has to judge by itself of the relevancy of the scenario considered.

Two types of simulations can be carried out with DynMod:

- simulation of the dynamics and the production of a livestock population over a given period of time (the period can last from 1 to 20 years) in a possibly variable environment;

- simulation of the 1-year production of a population assuming the demographic steady state.

These two types of simulations are implemented with different spread-sheets: *dynmod_proj.xls* and *dynmod_steady.xls*, respectively. Both spread-sheets use the same demographic model and variables.

The handbook is composed of three chapters. Chapter 1 presents the general principles of DynMod. Chapter 2 describes its interface and chapter 3 some numerical examples of simulations. Lastly, an appendix details the concept of hazard rates and the equations used in the demographic model.

DynMod can be freely obtained from the author (matthieu.lesnoff@cirad.fr).

Chapter 1. General principles

1. Introduction

The model used in DynMod is deterministic. The user fixes the values of the model parameters (e.g., reproduction and mortality rates) on an annual basis. The time-interval used in computations is however the month for limiting the competing risks problem between demographic events (appendix 1). For simplification, no seasonal variation is represented although it is well-known that demographic rates of tropical livestock can have seasonality.

In projections over several years, the user can represent the environmental variability by varying "at hand" the values of the parameters of the considered years.

DynMod does not constraint the parameters – for instance, parameters are not population-density dependent or functions of resources (feed, water, etc.) or of economic variables (markets' offer/demand, animals' prices, etc.). Nevertheless, for preventing unrealistic growth, the user can define a maximum size for the livestock population, the surplus being automatically removed from the population.

Details on equations and parameters of DynMod are presented in appendix.

2. Structure and parameters of DynMod

In DynMod, the livestock population is divided by sex (female and male) and by age group (juveniles, sub-adults and adults) (Figure 1). Age group durations must be defined by the user. They can depend on biological or farming management traits, or on the way that the user needs to present the results (chapter 3). The dynamics of the population result from birth, mortality (i.e., deaths due to other causes than slaughtering, referred as natural deaths in this handbook), offtake (slaughtering, sales, departures in loans, etc.) and intake (purchases, arrivals in loans, etc.).

Only adult females are assumed to be reproductive. Births are determined by the reproduction rates (annual parturition and net prolificacy rates) and distributed between females and males following the parameter "proportion of females at birth". Note that DynMod can be run without gender. For instance, a female population can be simulated by dividing the parturition rate by two and setting the proportion of females at birth to 100%.

In each age group, animals can survive or be removed from the population by natural death or offtake. Natural deaths are simulated by applying mortality rates (by sex and age group) to animals living in the population. Offtake are simulated as follows:

- All the animals surviving until the end of the adult age group are automatically culled (this corresponds to the final culling) and are considered as offtake;

- Two additional ways can be used, jointly or separately. The first way is to define offtake rates (by sex and age group), which are applied to animals living in the population, while the second way is to define a maximum size (by sex and age group). At the end of each time-interval, any surplus (relative to this maximum size) is removed from the population and considered as offtake.

Being difficult to represent simply, intakes are frequently neglected in models on livestock dynamics. In DynMod, two ways can be used (jointly or separately). The first way is to include them into the offtake rates. In that case, intakes are not explicitly represented: offtake rates correspond to “net offtake rates”, which represent balances “offtake - intake”. The second way is to define a number (instead of a rate) of animals imported (by sex and age group). In that case, intakes are assumed to enter in the population at the middle of the considered year. For instance, the second way can be useful to represent major restocking of animals after a shock in the population.

Additional parameters can be specified in DynMod (by sex and age group): live weight and financial value of animals, milk/wool/manure productions and feed requirements.

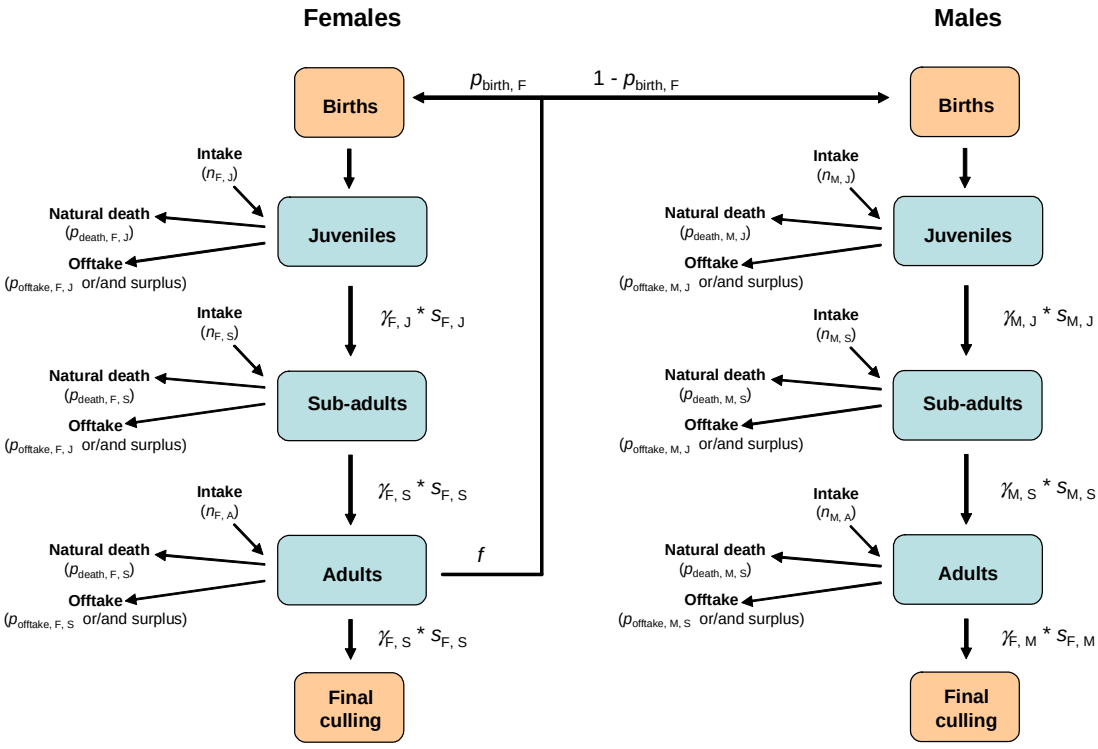

Figure 1 : *Structure of DynMod.*

Chapter 2. Interface under Excel

1. Dynmod_proj.xls

The Excel interface *dynmod_proj.xls* consists in the sheet *Projection model*. It is composed of three parts: "Parameters", "Results" and "Graphics".

An additional sheet *Tmp projection*, not described in the handbook, contains all the intermediary calculus (i.e. the Excel formulas; the interested user can refer to this sheet for details).

In *Projection model*, cells used for data entry are in white. Entered data appear in blue characters. Outputs are either in grey (for secondary variables, e.g. death and offtake probabilities calculated from the entered instantaneous hazard rates; appendix) or black (for projection results) characters. Both parts "Results" and "Graphics" are automatically updated when parameters are modified.

1.1. Parameters

The part "Parameters" is composed of four sections, corresponding to general characteristics of the population and of the projection (Figure 2) and to parameters related to demography (Figure 3), production (Figure 4) and feeding requirements (Figure 5).

1.1.1. General characteristics

- Durations of the age groups (month) (by sex). Based on these durations, DynMod calculates the exact ages (year) delimiting the age groups. For females, adults corresponds to the reproductive females;
- Initial number of animals in the population (initial population size) (by sex and age group). When a maximal population size is defined, the initial population size used for the projection is the one provided in the column "Initial c". Sex-by-age structures (global and intra-sex) are automatically calculated by DynMod;
- Duration of the projection (year).

1.1.2. Demography

<u>Reproduction</u>

- Parturition rate $h_{parturition}$ ($year^{-1}$; appendix 1) (average number of parturitions per year and reproductive female present all the year in the population);
- Net prolificacy rate (average number of born alive per parturition);
- Proportion of females at birth.

Mortality and offtake (by sex and age group)

- Mortality rate h_{death} (year^{-1}; appendix 1);
- Offtake rate $h_{offtake}$ (year^{-1}; appendix 1).

Mortality and offtake rates (as parturition rate) are instantaneous hazard rates (appendix 1) and must be expressed in year^{-1}, even if the age group duration is < or >1 year. As a help for the user, DynMod automatically calculates the probabilities p_{death} and $p_{offtake}$, corresponding to the entered h_{death} and $h_{offtake}$, as follows:

- over the age group duration if the duration <12 months;
- over 1 year if the duration >=12 months.

Consider the example in which h_{death} = 0.50 year^{-1} and $h_{offtake}$ = 0.40 year^{-1}. If for instance the duration of the age group is 6 months, probabilities displayed by DynMod correspond to a 6-month period (i.e., the period from the beginning to the end of the age group) (see appendix 1 for the following equations):

$$p_{death} = \frac{0.50}{0.50+0.40}\left[1-exp\left(-\left(6/12\right)\times\left(0.50+0.40\right)\right)\right] = 20\%,$$

$$p_{offtake} = \frac{0.40}{0.50+0.40}\left[1-exp\left(-\left(6/12\right)\times\left(0.50+0.40\right)\right)\right] = 16\%.$$

If the duration of the age group is 36 months, probabilities displayed by DynMod correspond to a 12-month period (i.e., the period from the beginning to the end of the year):

$$p_{death} = \frac{0.50}{0.50+0.40}\left[1-exp\left(-\left(0.50+0.40\right)\right)\right] = 33\%,$$

$$p_{offtake} = \frac{0.40}{0.50+0.40}\left[1-exp\left(-\left(0.50+0.40\right)\right)\right] = 26\%.$$

As noticed in Chapter 1, the instantaneous hazard rates entered in DynMod in year^{-1} are transformed to monthly rates ($h_{month} = h_{year}/12$), transformed to corresponding monthly probabilities and finally applied to the successive months of the considered year (see formulas in sheet "Projection Tmp")

Intake (by sex and age group)

- Number of animals eventually imported in the population (animals are assumed to be imported at mid-year).

1.1.3. Production

- Milk production per lactation;
- Live weight of the animal at the beginning of the age group (by sex and age group). DynMod automatically calculates the average live weights by age group, used for calculating the total live weight of the population;
- Carcass yield (%) used for calculating the meat production;
- Financial value per animal (by sex and age group);
  - For an offtake;
  - For an intake.
- Skin and hides production per animal and year (by sex and age group);
- Wool production per animal and year (by sex and age group);
- Manure production per animal and year (by sex and age group).

1.1.4. Feeding

- Daily requirements of dry matter per day and kg of live weight (by sex and age group).

1.2. Results

The part "Results" is composed of three sections corresponding to population size and structure (Figure 6), average stocks and productions (Figure 7, Figure 8, Figure 9) and feeding requirements (Figure 10).

1.2.1. Size and structure

For each time *t* of the projection (*t* = 0 is the beginning of the projection):

- Population size (number of animals);
- Population structure (global and intra-sex).

1.2.2. Production

For each year "*t* "of the projection (year "*t*" represents the period between time *t* and time *t* + 1; "*t*" = 1 is the first year of the projection):

- Ratio between the production in number of animals and the initial population size
  - Stock variation/Initial size. This represents the annual population growth rate;

  - Total annual production (= stock variation + offtake - intake)/Initial size;

- Production in number of animals, and productions in live weights, meat and financial equivalents
  - Average available living stock;
  - Stock variation;
  - Total production (= stock variation + offtake - intake);
  - Variation of the total production relatively to the previous year;

- Production of milk
  - Average production per female;
  - Total production;
  - Variation of the total production relatively to the previous year;

- Production of skin and hides (from offtake)
  - Total production;
  - Variation of the total production relative to the previous year;

- Production of wool
  - Total production;
  - Variation of the total production relative to the previous year;

- Production of manure
  - Total production;
  - Variation of the total production relatively to the previous year;

- Productivity indicators
  - Number of new sub-adults produced per reproductive female per year;
  - Number of new adults produced per reproductive female and year.

1.2.3. Feeding

For each year "*t* "of the projection:

- feeding requirements
  - Total feeding requirements;
  - Variation of the total production relatively to the previous year.

1.3. Graphics

The part "Graphics" is composed of two graphs (Figure 11) corresponding to the population size dynamics and the annual population growth rate.

Parameters — *PROJECTION*

Age group duration (month)			Exact age (year) from	to			Population size Initial	Max.	Initial c.	Population structure Global	Intra-sex
Female	Juvenile	12	0.0	1.0	F	J	9.00		9.00	9%	14%
	Sub-adult	36	1.0	4.0		S	19.70		19.70	20%	30%
	Adult	132	4.0	15.0		A	37.70		37.70	38%	57%
Male	Juvenile	12	0.0	1.0	M	J	8.80		8.80	9%	26%
	Sub-adult	36	1.0	4.0		S	15.20		15.20	15%	45%
	Adult	72	4.0	10.0		A	9.60		9.60	10%	29%
					Total	F	66.4		66.4	66%	100%
						M	33.6		33.6	34%	100%
						T	100.0		100.0	100%	

Number of years of projection: 20

Figure 2: *Part "Parameters" of dynmod_proj.xls – General characteristics.*

Demography			Year 1	2	3	4	5	6	7	8	9	10
Reproduction												
Parturition rate per year			0.60	0.60	0.40	0.60	0.60	0.60	0.60	0.60	0.60	0.60
Net prolificacy rate			1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Prob. of female at birth			0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Death and offtake												
	Female											
Hazard rates/year	Death	J	0.20	0.20	0.60	0.20	0.20	0.20	0.20	0.20	0.20	0.20
		S	0.07	0.07	0.14	0.07	0.07	0.07	0.07	0.07	0.07	0.07
		A	0.04	0.04	0.24	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	Offtake	J	0.06	0.06	0.08	0.06	0.06	0.06	0.06	0.06	0.06	0.06
		S	0.03	0.03	0.13	0.03	0.03	0.03	0.03	0.03	0.03	0.03
		A	0.03	0.03	0.30	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Probabilities	Death	J	18%	18%	44%	18%	18%	18%	18%	18%	18%	18%
		S	7%	7%	12%	7%	7%	7%	7%	7%	7%	7%
		A	4%	4%	19%	4%	4%	4%	4%	4%	4%	4%
	Offtake	J	5%	5%	6%	5%	5%	5%	5%	5%	5%	5%
		S	3%	3%	11%	3%	3%	3%	3%	3%	3%	3%
		A	3%	3%	23%	3%	3%	3%	3%	3%	3%	3%
	Male											
Hazard rates/year	Death	J	0.20	0.20	0.60	0.20	0.20	0.20	0.20	0.20	0.20	0.20
		S	0.07	0.07	0.14	0.07	0.07	0.07	0.07	0.07	0.07	0.07
		A	0.04	0.04	0.24	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	Offtake	J	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
		S	0.20	0.20	0.36	0.20	0.20	0.20	0.20	0.20	0.20	0.20
		A	0.21	0.21	0.71	0.21	0.21	0.21	0.21	0.21	0.21	0.21
Probabilities	Death	J	17%	17%	43%	17%	17%	17%	17%	17%	17%	17%
		S	6%	6%	11%	6%	6%	6%	6%	6%	6%	6%
		A	4%	4%	15%	4%	4%	4%	4%	4%	4%	4%
	Offtake	J	9%	9%	7%	9%	9%	9%	9%	9%	9%	9%
		S	18%	18%	28%	18%	18%	18%	18%	18%	18%	18%
		A	19%	19%	46%	19%	19%	19%	19%	19%	19%	19%
Intake												
Number	Female	J										
		S										
		A										
	Male	J										
		S										
		A										

Figure 3: *Part "Parameters" of dynmod_proj.xls – Demography.*

Production			Year 1	2	3	4	5	6	7	8	9	10
Milk												
	Offtake per lactation		180	180	180	180	180	180	180	180	180	180
Live weight (at beginning of age group)												
	Female	J	30	30	30	30	30	30	30	30	30	30
		S	50	50	50	50	50	50	50	50	50	50
		A	250	250	250	250	250	250	250	250	250	250
	Male	J	30	30	30	30	30	30	30	30	30	30
		S	70	70	70	70	70	70	70	70	70	70
		A	300	300	300	300	300	300	300	300	300	300
Meat												
	Carcass yield		0.47									
Financial value												
	Offtake Female	J	43.00	43.00	28.00	43.00	43.00	43.00	43.00	43.00	43.00	43.00
		S	75.00	75.00	49.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00
		A	100.00	100.00	65.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	Male	J	43.00	43.00	28.00	43.00	43.00	43.00	43.00	43.00	43.00	43.00
		S	85.00	85.00	55.00	85.00	85.00	85.00	85.00	85.00	85.00	85.00
		A	130.00	130.00	84.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00
	Intake Female	J										
		S										
		A										
	Male	J										
		S										
		A										
Skin and hides												
	Female	J										
		S										
		A										
	Male	J										
		S										
		A										
Wool												
		J										
		S										
		A										
Manure per day												
		J										
		S										
		A										

Figure 4: *Part "Parameters" of dynmod_proj.xls – Production.*

Feeding			Year 1	2	3	4	5	6	7	8	9	10
Daily requirements (DM per day and kg LW)												
	Female	J	0.00025									
		S	0.00025									
		A	0.00025									
	Male	J	0.00025									
		S	0.00025									
		A	0.00025									

Figure 5: *Part "Parameters" of dynmod_proj.xls – Feeding.*

Size and structure			t 0	1	2	3	4	5	6	7	8	9	10
Population size													
	Female	J	9.00	9.66	10.00	5.75	6.60	7.05	7.34	7.57	7.78	7.98	8.18
		S	19.70	20.16	20.81	16.90	15.97	15.79	15.93	16.22	16.57	16.95	17.36
		A	37.70	38.42	39.20	26.01	27.22	28.13	28.94	29.70	30.46	31.22	31.99
	Male	J	8.80	9.46	9.80	5.65	6.48	6.91	7.20	7.42	7.62	7.82	8.02
		S	15.20	15.57	16.12	12.19	11.64	11.71	11.99	12.33	12.69	13.04	13.39
		A	9.60	9.80	10.03	5.19	5.95	6.47	6.88	7.23	7.56	7.85	8.14
	Total	F	66.40	68.24	70.01	48.66	49.79	50.98	52.21	53.49	54.80	56.15	57.53
		M	33.60	34.83	35.95	23.03	24.07	25.09	26.07	26.99	27.86	28.71	29.54
		T	100.00	103.07	105.96	71.69	73.86	76.07	78.28	80.47	82.67	84.86	87.07
Population structure													
Global	Female	J	9%	9%	9%	8%	9%	9%	9%	9%	9%	9%	9%
		S	20%	20%	20%	24%	22%	21%	20%	20%	20%	20%	20%
		A	38%	37%	37%	36%	37%	37%	37%	37%	37%	37%	37%
	Male	J	9%	9%	9%	8%	9%	9%	9%	9%	9%	9%	9%
		S	15%	15%	15%	17%	16%	15%	15%	15%	15%	15%	15%
		A	10%	10%	9%	7%	8%	9%	9%	9%	9%	9%	9%
	Total	F	66%	66%	66%	68%	67%	67%	67%	66%	66%	66%	66%
		M	34%	34%	34%	32%	33%	33%	33%	34%	34%	34%	34%
		T	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Intra-sex	Female	J	14%	14%	14%	12%	13%	14%	14%	14%	14%	14%	14%
		S	30%	30%	30%	35%	32%	31%	31%	30%	30%	30%	30%
		A	57%	56%	56%	53%	55%	55%	55%	56%	56%	56%	56%
	Male	J	26%	27%	27%	25%	27%	28%	28%	27%	27%	27%	27%
		S	45%	45%	45%	53%	48%	47%	46%	46%	46%	45%	45%
		A	29%	28%	28%	23%	25%	26%	26%	27%	27%	27%	28%

Figure 6: *Part "Results" of dynmod_proj.xls – Size and structure.*

Production		Year 1	2	3	4	5	6	7	8	9	10
Number/Initial size											
Stock variation (growth rate)	F	2.8%	2.6%	-30.5%	2.3%	2.4%	2.4%	2.4%	2.5%	2.5%	2.5%
	M	3.7%	3.2%	-35.9%	4.5%	4.2%	3.9%	3.5%	3.3%	3.0%	2.9%
	T	3.1%	2.8%	-32.3%	3.0%	3.0%	2.9%	2.8%	2.7%	2.7%	2.6%
Offtake	F	7%	7%	18%	7%	7%	7%	7%	7%	7%	7%
	M	20%	20%	31%	20%	20%	20%	20%	20%	20%	20%
	T	11%	11%	22%	11%	11%	11%	11%	11%	11%	11%
Intake	F	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	T	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total production (SV + O - I)	F	10%	10%	-12%	9%	9%	9%	9%	9%	9%	9%
	M	24%	23%	-5%	24%	24%	24%	23%	23%	23%	23%
	T	14%	14%	-10%	14%	14%	14%	14%	14%	14%	14%

Figure 7: *Part "Results" of dynmod_proj.xls – Production (Number/Initial size).*

Number											
Average living stock	F	J	9.33	9.83	7.87	6.17	6.83	7.19	7.46	7.67	7.88	8.08
		S	19.93	20.49	18.86	16.44	15.88	15.86	16.08	16.39	16.76	17.16
		A	38.06	38.81	32.60	26.61	27.67	28.54	29.32	30.08	30.84	31.60
	M	J	9.13	9.63	7.72	6.06	6.70	7.05	7.31	7.52	7.72	7.92
		S	15.39	15.85	14.16	11.92	11.68	11.85	12.16	12.51	12.86	13.21
		A	9.70	9.91	7.61	5.57	6.21	6.67	7.06	7.39	7.70	7.99
Total	F		67.32	69.13	59.34	49.22	50.38	51.59	52.85	54.15	55.48	56.84
	M		34.22	35.39	29.49	23.55	24.58	25.58	26.53	27.42	28.29	29.13
	T		101.54	104.52	88.82	72.77	74.96	77.17	79.38	81.57	83.76	85.97
Stock variation	F		1.84	1.78	-21.36	1.13	1.19	1.24	1.28	1.31	1.35	1.38
	M		1.23	1.11	-12.92	1.04	1.02	0.97	0.92	0.88	0.85	0.83
	T		3.07	2.89	-34.27	2.17	2.21	2.21	2.20	2.19	2.20	2.21
Offtake	F		4.63	4.74	12.72	3.29	3.41	3.51	3.60	3.70	3.79	3.88
	M		6.73	6.94	10.98	4.56	4.76	4.96	5.16	5.35	5.52	5.70
	T		11.36	11.68	23.71	7.86	8.17	8.48	8.76	9.04	9.31	9.58
Intake	F		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	T		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total production (SV + O - I)	F		6.47	6.52	-8.63	4.42	4.60	4.75	4.88	5.01	5.14	5.26
	M		7.97	8.05	-1.93	5.61	5.78	5.94	6.08	6.22	6.37	6.52
	T		14.43	14.57	-10.56	10.03	10.38	10.68	10.96	11.23	11.51	11.79
%variation				0.9%	-172.5%	194.9%	3.6%	2.9%	2.6%	2.5%	2.4%	2.4%

Figure 8: *Part "Results" of dynmod_proj.xls – Production (Number).*

Live weight equivalent										
Average living stock	19,089.72	19,555.03	16,582.76	13,544.73	13,931.90	14,348.55	14,771.56	15,194.08	15,615.30	16,036.46
Stock variation	435.44	495.18	-6,439.72	363.67	410.65	422.65	423.37	421.68	420.75	421.58
Offtake	2,389.20	2,447.43	5,312.56	1,620.22	1,691.30	1,758.63	1,822.62	1,883.88	1,943.05	2,000.76
Intake	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total production (SV + O - I)	2,824.64	2,942.62	-1,127.16	1,983.90	2,101.95	2,181.28	2,246.00	2,305.56	2,363.81	2,422.34
%variation		4.2%	-138.3%	276.0%	6.0%	3.8%	3.0%	2.7%	2.5%	2.5%
Meat equivalent										
Average living stock	8,972.17	9,190.86	7,793.90	6,366.02	6,547.99	6,743.82	6,942.63	7,141.22	7,339.19	7,537.14
Stock variation	204.65	232.74	-3,026.67	170.93	193.01	198.64	198.99	198.19	197.75	198.14
Offtake	1,122.92	1,150.29	2,496.91	761.50	794.91	826.56	856.63	885.42	913.24	940.36
Intake	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total production (SV + O - I)	1,327.58	1,383.03	-529.77	932.43	987.92	1,025.20	1,055.62	1,083.61	1,110.99	1,138.50
%variation		4.2%	-138.3%	276.0%	6.0%	3.8%	3.0%	2.7%	2.5%	2.5%
Financial equivalent										
Average living stock	8,663.04	8,889.70	4,897.85	6,157.38	6,339.83	6,531.00	6,723.48	6,915.26	7,106.35	7,297.47
Stock variation	220.28	233.05	-1,906.71	175.25	189.64	192.70	192.26	191.30	190.89	191.34
Offtake	1,065.40	1,092.55	1,495.21	723.75	755.79	785.90	814.42	841.70	868.04	893.73
Intake	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total production (SV + O - I)	1,285.68	1,325.60	-411.49	899.00	945.44	978.60	1,006.68	1,033.00	1,058.93	1,085.07
%variation		3.1%	-131.0%	318.5%	5.2%	3.5%	2.9%	2.6%	2.5%	2.5%
Milk										
Avg. per reprod. fem. and year	108.00	108.00	72.00	108.00	108.00	108.00	108.00	108.00	108.00	108.00
Total per year	4,110.24	4,191.20	2,347.51	2,874.14	2,988.82	3,081.79	3,166.43	3,248.45	3,330.36	3,413.31
%variation		2.0%	-44.0%	22.4%	4.0%	3.1%	2.7%	2.6%	2.5%	2.5%
Skin & hides										
Total per year	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
%variation										
Wool										
Total per year	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
%variation										
Manure										
Total per year	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
%variation										
Productivity measures										
Nb. new sub-adult/adult F	49%	49%	22%	49%	49%	49%	49%	49%	49%	49%
Nb. new adult/adult F	40%	40%	14%	40%	40%	40%	40%	40%	40%	40%

Figure 9: *Part "Results" of dynmod_proj.xls – Production (Live weight, Meat, etc.).*

Feed	Year 1	2	3	4	5	6	7	8	9	10
Feed requirements										
DM	1,741.94	1,784.40	1,513.18	1,235.96	1,271.29	1,309.30	1,347.90	1,386.46	1,424.90	1,463.33
%variation		2.4%	-15.2%	-18.3%	2.9%	3.0%	2.9%	2.9%	2.8%	2.7%

Figure 10: *Part "Results" of dynmod_proj.xls – Feed.*

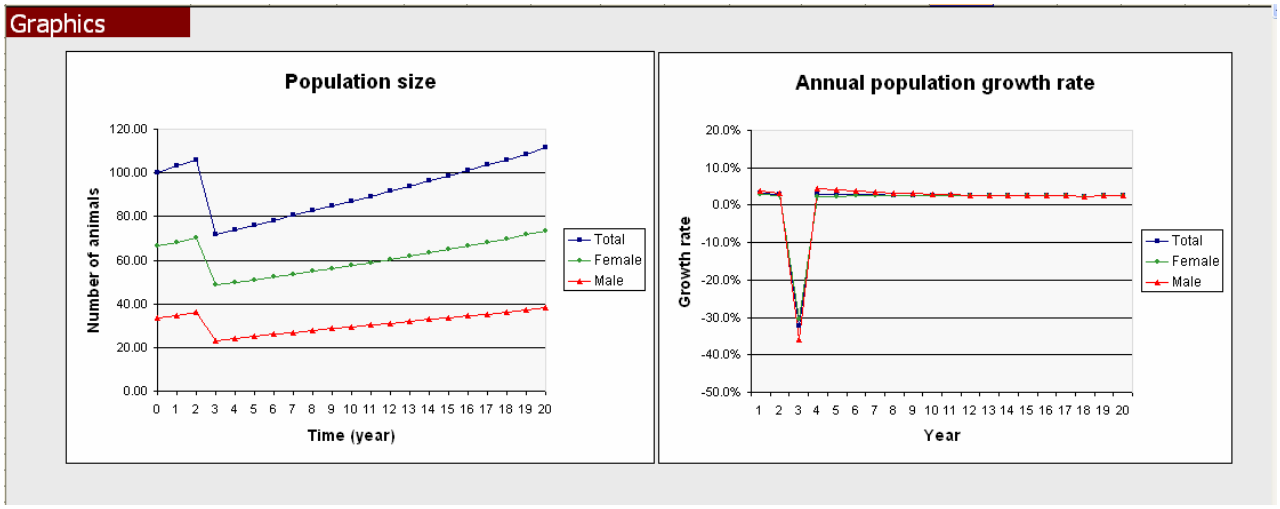

Figure 11: *Part "Graphics" of dynmod_proj.xls.*

2. Dynmod_steady.xls

The Excel interface *dynmod_steady.xls* is similar to *dynmod_proj.xls*, but calculations only concern a 1-year period and the livestock population is assumed to be at a steady state (i.e., the sex-by-age structure is stable and the annual population growth rate =1).

The interface consists of the sheet *Steady state model* (Figure 12), which is composed of two parts: "Parameters" and "Results". Sheet "Tmp Steady state" contains all the intermediary calculus.

Parameters and results are the same as for *dynmod_proj.xls*, except the following items:

- Concerning the population size, the user fixes the number of adults (by sex). Numbers of juveniles and sub-adults are automatically calculated from the demographic rates and the steady state equations (appendix 3). DynMod calculates each month the adult surplus (column "Surplus" in the sheet) on the base of the specified constant adult sizes. When positive, the surplus is removed from the population and added to the offtake (and inversely if the surplus is negative);

- No intake number can be entered in the sheet, which simplifies some of the results.

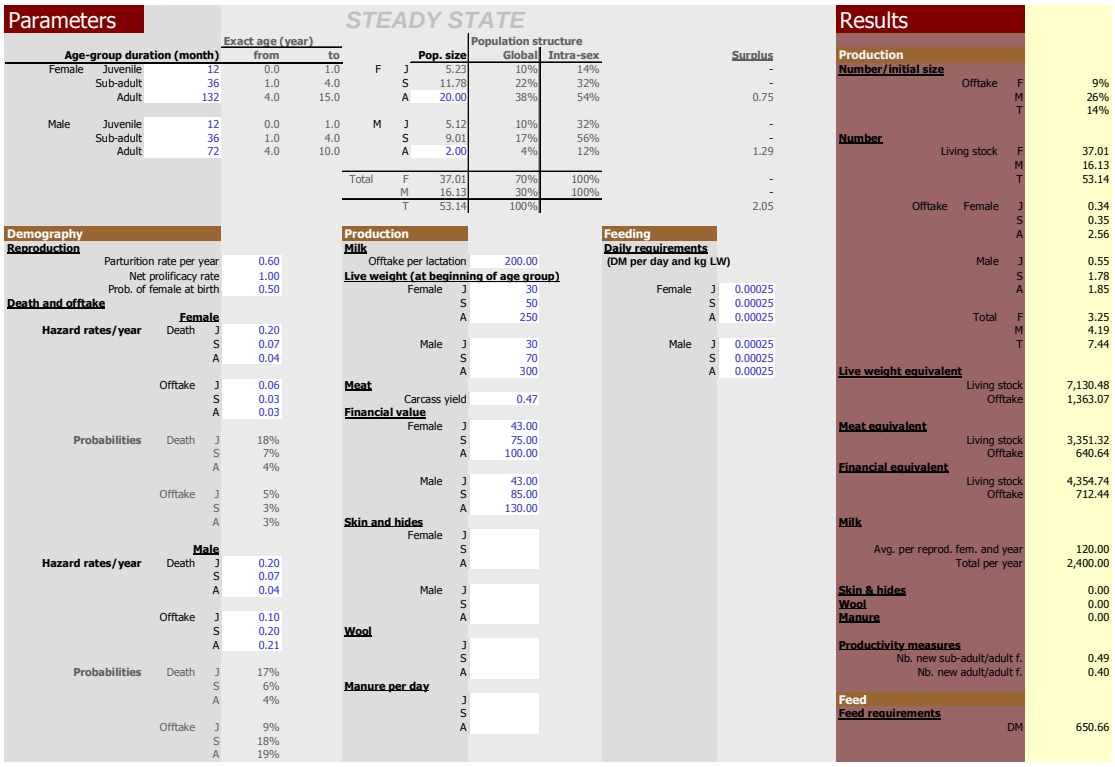

Figure 12: *Interface dynmod_steady.xls.*

Chapter 3. Numerical examples

All the examples developed in this chapter were simplified for facilitating the presentation.

1. Drought in cattle population

In this example, we used *dynmod_proj.xls* for simulating the impact of a drought on the dynamics of a cattle population in the Sahel.

Parameters were assumed to be constant over the projection except during the year of drought. Parameters for normal years (i.e., without drought) were defined from literature. For the drought, we assumed that a field survey was implemented for evaluate the impact of the shock (e.g., using a cross-sectional 12-month retrospective method; Lesnoff et al., 2007) and has provided the demographic rates. These survey was assumed to demonstrate a general increase in the mortality rates (particularly for young and old animals) and in offtake rates (particularly for adult males), and a drop in the parturition rate.

Parameters and results are those presented in Figures 2 to 11 of Chapter 2 (interface *dynmod_proj.xls*).

1.1. Parameters

1.1.1. General characteristics

We carried out a projection over 20 years, assuming that the drought occurred in year "3".

Female age group durations were set to 12, 36 and 132 months for juveniles, sub-adults and adults, respectively. Male age group durations were set to 12, 36 and 72 months. Corresponding age groups (in exact ages) are "0 to 1 year", ">1 to 4 years" and ">4 to 15 years" for females and "0 to 1 year", ">1 to 4 years" and ">4 to 10 years" for males.

We standardized the initial total population size to 100. The assumed population sex-by-age structure is presented in Figure 2 (females represented 66% of the initial stock).

1.1.2. Demography

Parturition rate was set to 0.60 $year^{-1}$ on normal years (corresponding to an average interval between parturitions of 1.67 years) and to 0.40 $year^{-1}$ in the drought year. Net prolificacy rate was unchanged during the projection and set to 1. Proportion of females at birth was 50%.

No massive restocking was assumed after the drought. Offtake rates represented "net offtake" (balance between offtake and intake). Mortality and offtake rates used in the projection are presented in Table 1.

1.1.3. Production

The milk offtake per lactation was set to 180 l (e.g., 4 months of lactation x 30 days x 1.5 l/day). Live weights and net offtake financial values by sex and age group in normal and drought years are presented in Table 2 (they are expressed in 1000-FCFA; 1 Euro = 657 FCFA). We assumed a drop of 35% of the selling prices during the drought.

The carcass yield was set to 47% and we assumed that the daily feed requirements in dry matter were 2.5% of the live weight for all sexes and age groups (entered value in DynMod was "0.00025", for expressing feed requirements in Ton; Figure 5).

Table 1: *Mortality and offtake instantaneous hazard rates (and corresponding probabilities) used in the projection.*

Sex	Age group	Hazard rates ($year^{-1}$)				Corresponding probabilities (%)			
		Normal year		Drought		Normal year		Drought	
		Death	Offtake	Death	Offtake	Death	Offtake	Death	Offtake
Female	Juvenile	0.20	0.06	0.60	0.08	18	5	44	6
	Sub-adult	0.07	0.03	0.14	0.13	7	3	12	11
	Adult	0.04	0.03	0.24	0.30	4	3	19	23
Male	Juvenile	0.20	0.10	0.60	0.10	17	9	43	7
	Sub-adult	0.07	0.20	0.14	0.36	6	18	11	28
	Adult	0.04	0.21	0.24	0.71	4	19	15	46

Table 2: *Live weights and financial values (net offtake) by sex and age used in the projection. Live weights correspond to the beginning of the age groups.*

Sex	Age group	Live weight (kg)		Financial value (net offtake; 1000-CFA)	
		Normal	Drought	Normal	Drought
Female	Juvenile	30	30	43	28
	Sub-adult	50	50	75	49
	Adult	250	250	100	65
Male	Juvenile	30	30	43	28
	Sub-adult	70	70	85	55
	Adult	300	300	130	84

1.2. Results

The drought generated a drop of -30.5, -35.9 and -32.3% in female, male and total population sizes.

After the drought, the annual population growth rate fluctuated between 2.4 and 3.0% and converged to almost 2.5%. The stock was fully reconstituted during the year "16" (Figure 11), i.e. 13 years after the drought. In normal years, the total production (in number of animals per year) represented almost 9, 22 and 14% of the initial female, male and total populations.

Dynamics of the other productions and of feed requirements are presented in Figure 7 to Figure 10. For a more complete evaluation of the drought impact, the user should carry out a projection without the drought and compare productions with those of the present projection.

2. De-worming impact in sheep herd

In this example, we used *dynmod_steady.xls* for simulating the impact of deworming of an ovine herd, assuming a steady state (see Lesnoff et al., 2000, 2001 for other examples with matrix models).

We compared productions of two herds: a "control" herd (without deworming) and a "treated" herd (with deworming). We also estimated the (approximate) benefit/cost ratio of the treatment that can be expected by the farmer.

In treated herds, all animals of age >3 months were assumed to be dewormed. The cost of treatment was calculated assuming that the unique expense for farmer was the purchase of medicines. The treatment cost per animal (>3 months) and year was set to 450 FCFA (Lesnoff et al., 2000).

We carried out a simulation for each herd (control and treated) in files *dynmod_steady_ctrl.xls* and *dynmod_steady_treated.xls*, respectively.

2.1. Parameters

2.1.1. General characteristics

Juvenile age group duration was set to 3 months (Figure 13), for being able to get the number of animals of exact age >3 months at steady state and to calculate the cost of deworming.

Other age group durations were set to 9 (sub-adults) and 120 (adults) months for females, and to 9 and 24 for males.

The numbers of female and male adults were set to 30 and 2, respectively.

2.1.2. Demography

In the control herd, parturition rate was set to 0.90 $year^{-1}$ (corresponding to an average interval between parturitions of 1.11 years) and net prolificacy rate to 1.01 (Figure 14). Proportion of females at birth was 50%.

Mortality and offtake rates are presented in Table 1. Lamb <=3 months old were assumed not commercially used by farmers. Adult offtake rates were also set to 0: therefore adult offtake corresponded to the surplus compared to the constant adult population sizes.

Deworming was assumed to reduce the mortality of 30% in all sexes and age groups, and to increase the parturition and prolificacy rates of 5% (Figure 14).

2.1.3. Production

For the purpose of comparing the productions of the control and treated herds, we only considered the offtake in animal numbers and in financial values (in 1000-FCFA; 1 Euro = 657 FCFA) (Figure 15). Living capital (quite similar between the two herds, see below) was not considered.

2.2. Results

Results are presented in Figure 16. The total living population sizes were 54.7 animals in the control herd and 58.2 animals in the treated herd.

Total annual productions (offtake) were 17.2 animals (297.0 1000-FCFA) in the control herd and 22.3 animals (378.1 1000-FCFA) in the treated herd. This corresponds to an increase of 22.3/17.2 = 29.6% of the production in animal number and of 378.1/297.0 = 27.3% in financial value.

The cost of deworming was calculated from the population steady state vector (sub-adults and adults):

$$C = [(9.6 + 30.0) + (9.3 + 2)] * 0.45 = 22.9 \text{ 1000-FCFA.}$$

In this scenario, the benefit/cost ratio of deworming was then:

$$B/C = (378.1 - 297.0)/22.9 = 3.5.$$

(a)

Parameters — STEADY STATE

Age-group duration (month)			Exact age (year) from	to			Pop. size	Population structure Global	Intra-sex
Female	Juvenile	3	0.0	0.3	F	J	3.25	6%	8%
	Sub-adult	9	0.3	1.0		S	8.23	15%	20%
	Adult	120	1.0	11.0		A	30.00	55%	72%
Male	Juvenile	3	0.0	0.3	M	J	3.25	6%	25%
	Sub-adult	9	0.3	1.0		S	7.98	15%	60%
	Adult	24	1.0	3.0		A	2.00	4%	15%
					Total	F	41.49	76%	100%
						M	13.23	24%	100%
						T	54.72	100%	

(b)

Parameters — STEADY STATE

Age-group duration (month)			Exact age (year) from	to			Pop. size	Population structure Global	Intra-sex
Female	Juvenile	3	0.0	0.3	F	J	3.64	6%	8%
	Sub-adult	9	0.3	1.0		S	9.59	16%	22%
	Adult	120	1.0	11.0		A	30.00	52%	69%
Male	Juvenile	3	0.0	0.3	M	J	3.64	6%	24%
	Sub-adult	9	0.3	1.0		S	9.29	16%	62%
	Adult	24	1.0	3.0		A	2.00	3%	13%
					Total	F	43.23	74%	100%
						M	14.93	26%	100%
						T	58.15	100%	

Figure 13: *Parameters of dynmod_proj.xls – General characteristics (deworming example). (a): Control herd, (b): Treated herd.*

(a)

Demography			
Reproduction			
Parturition rate per year			0.90
Net prolificacy rate			1.01
Prob. of female at birth			0.50
Death and offtake			
		Female	
Hazard rates/year	Death	J	0.28
		S	0.28
		A	0.10
	Offtake	J	0.00
		S	0.10
		A	0.00
Probabilities	Death	J	7%
		S	18%
		A	10%
	Offtake	J	0%
		S	7%
		A	0%
		Male	
Hazard rates/year	Death	J	0.28
		S	0.28
		A	0.10
	Offtake	J	0.00
		S	0.20
		A	0.00
Probabilities	Death	J	7%
		S	18%
		A	10%
	Offtake	J	0%
		S	13%
		A	0%

(b)

Demography			
Reproduction			
Parturition rate per year			0.95
Net prolificacy rate			1.06
Prob. of female at birth			0.50
Death and offtake			
		Female	
Hazard rates/year	Death	J	0.20
		S	0.20
		A	0.07
	Offtake	J	0.00
		S	0.10
		A	0.00
Probabilities	Death	J	5%
		S	13%
		A	7%
	Offtake	J	0%
		S	7%
		A	0%
		Male	
Hazard rates/year	Death	J	0.20
		S	0.20
		A	0.07
	Offtake	J	0.00
		S	0.20
		A	0.00
Probabilities	Death	J	5%
		S	13%
		A	7%
	Offtake	J	0%
		S	13%
		A	0%

Figure 14: *Parameters of dynmod_proj.xls – Demography (deworming example). (a): Control herd, (b): Treated herd.*

(a)

Production		
Milk		
Offtake per lactation		
Live weight (at beginning of age group)		
Female	J	
	S	
	A	
Male	J	
	S	
	A	
Meat		
Carcass yield		
Financial value		
Female	J	5.00
	S	9.00
	A	12.00
Male	J	5.00
	S	12.00
	A	23.00

(b)

Production		
Milk		
Offtake per lactation		
Live weight (at beginning of age group)		
Female	J	
	S	
	A	
Male	J	
	S	
	A	
Meat		
Carcass yield		
Financial value		
Female	J	5.00
	S	9.00
	A	12.00
Male	J	5.00
	S	12.00
	A	23.00

Figure 15: *Parameters of dynmod_proj.xls – Production (deworming example). (a): Control herd, (b): Treated herd.*

(a)

Results			
Production			
Number/initial size			
	Offtake	F	17%
		M	76%
		T	31%
Number			
	Living stock	F	41.49
		M	13.23
		T	54.72
	Offtake Female	J	0.00
		S	0.81
		A	6.35
	Male	J	0.00
		S	1.56
		A	8.47
	Total	F	7.16
		M	10.03
		T	17.19
Live weight equivalent			
	Living stock		0.00
	Offtake		0.00
Meat equivalent			
	Living stock		0.00
	Offtake		0.00
Financial equivalent			
	Living stock		608.41
	Offtake		297.02

(b)

Results			
Production			
Number/initial size			
	Offtake	F	23%
		M	81%
		T	38%
Number			
	Living stock	F	43.23
		M	14.93
		T	58.15
	Offtake Female	J	0.00
		S	0.95
		A	9.18
	Male	J	0.00
		S	1.83
		A	10.32
	Total	F	10.13
		M	12.15
		T	22.28
Live weight equivalent			
	Living stock		0.00
	Offtake		0.00
Meat equivalent			
	Living stock		0.00
	Offtake		0.00
Financial equivalent			
	Living stock		640.14
	Offtake		378.12

Figure 16: *Results of dynmod_proj.xls (deworming example). (a): Control herd, (b): Treated herd.*

Appendix

1. Instantaneous hazard rates

Mortality and offtake rates entered in DynMod are “instantaneous hazard rates”, referred as h in the present handbook.

Instantaneous hazard rates are common parameters in statistical survival methods (see for example Chiang, 1984; Kalbfleisch and Prentice, 1980; Lee, 1992). When h is constant in a given period, typical hazard rates estimates are $h = m/T$ or $h = m/\overline{n}$, where m is the number of events (e.g. deaths) observed in the period, T the total time of presence of the animals in the period ("time at risk" in epidemiology) and $\overline{n}$ the average number of animals in the period ($\overline{n}$ is an approximate of T). Hazard rates have different properties than probabilities. For example, hazard rates are expressed in time^{-1} unit and can have values >1. In contrast, probabilities have no unit and are <=1.

Under some hypotheses, probabilities can be calculated from hazard rates using simple exponential functions. For instance, assuming constant death and offtake rates (h_{deatk} and $h_{offtake}$) in a given period and no other causes of removal from the population, death and offtake probabilities over the period are:

$$p_{death} = \frac{h_{death}}{h_{death} + h_{offtake}} \left[1 - \exp\left(-\left(h_{death} + h_{offtake}\right)\right)\right],$$

$$p_{offtake} = \frac{h_{offtake}}{h_{death} + h_{offtake}} \left[1 - \exp\left(-\left(h_{death} + h_{offtake}\right)\right)\right].$$

For example, the death rate h_{death} = 0.50 year^{-1} corresponds to an annual death probability p_{deatk} = 0.39 when there is no offtake ($h_{offtake}$ = 0), and to p_{death} = 0.33 when $h_{offtake}$ = 0.40 year^{-1}. In the first case, 39% of the animals will die in average over the year (and not 50%) and 33% in the second case.

In this example, the decrease from p_{death} = 39% to p_{death} = 33% is due to the competing risk phenomenon: for the same intrinsic mortality (h_{death}), the observed mortality will decrease when offtake will increase (mortality is then referred as apparent). This is due to the fact that animals removed as offtake will escape to the daily natural death risk (which here is h_{death}/365) in the population.

Competing risks concern all the demographic events (e.g. parturitions can be affected by animal removals) and should always be taken into account in dynamics models.

In DynMod, the competing risks problem is controlled by using hazard rates (instead of directly using probabilities) and by decomposing the years in months (biases due to competing risks become negligible when the duration of the time-interval used for computations is sufficiently small).

Hazard rates entered by the user must always be expressed in year^{-1} (even if the age group duration is < or >1 year). The entered rates are then automatically transformed to monthly

rates (e.g., h_{death} = 0.50 year^{-1} = 0.0417 month^{-1}), transformed to corresponding monthly probabilities, and finally successively applied to months of the considered year.

2. The demographic model

DynMod uses a discrete-time matrix model (for theory of matrix models, see Caswell, 1989; Tuljapurkar, 1990; Tuljapurkar and Caswell, 1997; Caswell, 2001). A matrix model consists on a system of mathematical equations whose parameters are the demographic rates (fecundity, mortality, etc). Such a system simulates the evolution of the population size (by sex and age group) on regular and successive time intervals. It can be written in a matrix form:

$$x_t = A_{t-1} * x_{t-1},$$

where:

- x_{t-1} and x_t represent vectors ("population state vectors") whose components are the numbers of the animals present respectively at time t - 1 and time t in the different sexes and age groups;

- A_{t-1} represents a projection matrix, containing the demographic rates for the time interval between t – 1 and t.

Numerous applications of matrix models can be found in literature on tropical livestock populations (e.g., Tacher, 1975; Landais, 1986; Baptist, 1988; Cossins and Upton, 1988; Upton, 1989; von Kaufmann, 1990; Baptist, 1992; Itty, 1995; Lesnoff, 1999; 2000; Mill et al., 2004; Moulin et al., 2004; Texeira and Paruelo, 2006; Hary, 2004).

3. Demographic equations

This section presents the mathematical equations used in DynMod for simulating the livestock population dynamics over a given month. Notations differ slightly from Chapter 1.

3.1. Notations for this section

- The considered month represents period (t - 1, t)

- Indexes
  - Sex: s = F or M (females and males)
  - Age group: i = J, S or A (juveniles, sub-adults and adults)

- $x_{t, s, i}$: Number of animals present at time t in sex s and age group i.
  Vector ($x_{t, F, J}$, $x_{t, F, S}$, $x_{t, F, A}$, $x_{t, M, J}$, $x_{t, M, S}$, $x_{t, M, A}$) is the "population state vector"

- $h_{parturition}$: Monthly parturition rate for reproductive females (month^{-1}) (average number of parturitions per month and reproductive female present all the month in the population), calculated from the annual value entered in DynMod: $h_{parturition}$ = $h_{parturition, year}$/12

- *prol*: Net prolificay rate (average number of born alive per parturition)

- $p_{birth, F}$: Proportion of females at birth

- $h_{death, s, i}$: Monthly natural death hazard rate (month^{-1}), calculated from the annual value entered in DynMod: $h_{death, s, i} = h_{death, year, s, i}/12$

- $h_{offtake, s, i}$: Monthly offtake hazard rate (month^{-1}), calculated from the annual value entered in DynMod: $h_{death, s, i} = h_{death, year, s, i}/12$

- $p_{death, s, i}$ and $p_{offtake, s, i}$: Monthly probabilities corresponding to $h_{death, s, i}$ and $h_{offtake, s, i}$

$$p_{death} = \frac{h_{death}}{h_{death} + h_{offtake}}\left[1 - exp\left(-\left(h_{death} + h_{offtake}\right)\right)\right]$$

$$p_{offtake} = \frac{h_{offtake}}{h_{death} + h_{offtake}}\left[1 - exp\left(-\left(h_{death} + h_{offtake}\right)\right)\right]$$

- $\gamma_{s, i}$: Proportion of animals of sex *s* and age group *i* entering in group *i* + 1 (or culled if *i* represents the adult age group *A*) after surviving until the end of the month. Denoting $surv = 1 - p_{death} - p_{offtake}$ the monthly probabilty of survival in the population and *d* the age group duration (month), this proportion is calculated by (Caswell, 2001):

$$\gamma_{s,i} = \frac{\left(surv_{s,i}\right)^{d_{s,i}-1} - \left(surv_{s,i}\right)^{d_{s,i}}}{1 - \left(surv_{s,i}\right)^{d_{s,i}}}$$

3.2. Projection model

3.2.1. Reproduction

Parturitions are assumed occurring at beginning of the month. Numbers of animals born alive by sex in the month are:

$$B_F = p_{birth, F} * prol * h_{parturition} * x_{t-1, F, A},$$

$$B_M = (1 - p_{birth, F}) * prol * h_{parturition} * x_{t-1, F, A}.$$

3.2.2. Survival and transition between age groups

Natural renewal

The number of born alive of sex *s* surviving from *t* - 1 to *t* (and therefore entering in the juvenile age group *J*) is:

$$(1 - (p_{death, s, J} + p_{offtake, s, J})) * B_s.$$

Dynamics of the population state vector

The number of animals of sex *s* and age group *i* surviving from $t - 1$ to t is:

$$surv_{s,i} * x_{t-1,s,i}.$$

At the end of the month, the proportion $(1 - \gamma_{s,i})$ of these survivals stays in age group *i* and the remaining proportion $\gamma_{s,i}$ enters in age group $i + 1$ (or is culled if *i* represents the adult age group; this represents the final culling).

3.2.3. Natural deaths and offtake

The number of natural deaths of sex *s* and age group *i* in the month is:

$$n_{death,s,i} = p_{death,s,i} * x_{t-1,s,i}.$$

The number of offtake of sexe *s* and age group *i* in the month is calculated as follows. A first value is calculated as:

$$n_{offtake,s,i} = p_{offtake,s,i} * x_{t-1,s,i}.$$

Then, if a maximum population size is specified and if $x_{t,s,i} > x_{max,s,i}$, the surplus is:

$$x_{t,s,i} - x_{max,s,i},$$

and is added to $n_{offtake,s,i}$.

Finally, if *i* represents the adult age group, the final culling is calculated by:

$$\gamma_{s,A} * surv_{s,A} * x_{t-1,s,A}$$

and is added to $n_{offtake,s,A}$.

3.3. Steady-state model

Equations are the same as for projection model, except that numbers of juveniles and sub-adults at steady state (equilibrium) are constant and calculated by:

Juveniles: $$x_{s,J} = \frac{\left(surv_{s,J}\right)^{d_{s,J}} - 1}{surv_{s,J} - 1} * surv_{s,J} * B_s$$

Sub-adults: $$x_{s,S} = \frac{\left(surv_{s,S}\right)^{d_{s,S}} - 1}{surv_{s,S} - 1} * \left(surv_{s,J}\right)^{d_{s,J}} * surv_{s,J} * B_s$$

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