MEMBRANE STRUCTURES WITH
THE ISOGEOMETRIC B-REP ANALYSIS (IBRA)

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Summary. The present contribution gives an introduction to the design and analysis of
structural membranes with the Isogeometric B-Rep Analysis (IBRA) which aims on a new
approach for a consistent design-through-analysis workflow. Therefore the concept of IBRA
is briefly introduced. For the analysis of structural membranes the necessary elements are
presented, where the cable element is formulated following the new paradigm of embedded
B-Rep edge elements. Benchmarks and application of non-linear analyses and form-finding of
structures highlight the potential of the developed framework and the applicability to the
design and analysis of structural membranes.

1 INTRODUCTION

Architectural membranes provide minimal use of material combined with an attractive and
impressive language of shapes. These shapes are directly mechanically motivated: based on
the chosen level of pre-stress and the boundary conditions, form-finding analysis is used to
determine the shape of equilibrium which allows the membrane to act in pure tension. This
mechanical background usually leads to an iterative design procedure, where the mechanical
form-finding and the modification of boundary conditions as design handles mutually interact
until a solution that is desirable from a structural as well as from an esthetical point of view is
found.

Classically the architectural part of a membrane structure’s design is performed within a
CAD environment, whereas the form-finding and analysis are performed within a FE-code.
The separation of these models requires considerable amounts of time [10] and is obviously
rather error-prone. Recently, the Isogeometric B-Rep Analysis (IBRA) has been proposed as a
consequent generalization of IGA with the aim of directly using the CAD-model – enriched
by mechanical information – for the analysis of structures [1].

In the present contribution, the realization of the IBRA-framework for the form-finding
and analysis of membrane structures is presented. The contribution is outlined as follows:
Section 2 gives an introduction to the Isogeometric B-Rep Analysis. The development of an
IBRA membrane and a cable element are briefly sketched in Section 3. Section 4 presents
some benchmarks for the analysis and form-finding with these elements. In Section 5 an
application example is presented in order to demonstrate the capability and potential of the
derived methods for structural membranes. Finally Section 6 gives a conclusion and an outlook on future work.

2 THE ISOGEOMETRIC B-REP ANALYSIS (IBRA)

The Isogeometric B-Rep Analysis (IBRA) [1] is a recent development in the field of finite element analysis. Its main objective is to allow for structural analyses on full CAD geometries, i.e. in general trimmed and coupled NURBS-surfaces, without the need for creating a separate analysis mesh.

In order to present the IBRA especially compared to the well-established Isogeometric Analysis (IGA), a brief introduction to the geometry representation in CAD is necessary.

2.1 Boundary Representation (B-Rep)

In today’s CAD systems, the Boundary-Representation (B-Rep) is a technique used to describe arbitrary geometrical entities by their boundaries. Following the B-Rep approach, for a three dimensional object a set of adjacent bounded surface elements called faces describes the "skin" of the object and thus the object itself. These faces at their turn are bounded by sets of edges which are curves lying on the surface of the faces. Several edges meet in points that are called vertices.

![Figure 1: Boundary-Representation (B-Rep) description of a schematic trimmed free form surface [2].](image)

The B-Rep approach intrinsically incorporates trimmed surfaces: Here the trimming curves become part of the boundaries in inner resp. outer trimming loops, see Figure 1.
As basis functions for the B-Rep description, commonly non-uniform rational B-splines (NURBS) are used. The NURBS-based B-Rep description of geometries has become the standard for geometry description in modern CAD-systems.

### 2.2 The Isogeometric B-Rep Analysis (IBRA)

The *Isogeometric B-Rep Analysis* (IBRA) [1] is a generalization of the *Isogeometric Analysis* (IGA) introduced by Hughes *et al.* [3]. The main concept in IGA is to use the same basis functions for the computation of the discretized system that are used for the geometry representation in CAD, *i.e.* commonly NURBS.

While IGA in its pure form is restricted to complete patches, IBRA at its turn refers to the full B-Rep description from the CAD system, *i.e.* including trimmed patches (see Fig. 1). Moreover, the concept of IBRA permits enforcing various conditions to the B-Rep entities, *e.g.* coupling or support conditions along the trimming edges or mechanical entities like a geometrically non-linear shell element [1] or the membrane and cable-element [4,5] that will be presented in Section 3. Thus the pure geometrical CAD model can be augmented to an analysis suitable model.

### 2.3 The Analysis in CAD (AiCAD)-approach

The IBRA serves as mechanical framework for the *Analysis in CAD* (AiCAD)-approach which follows the philosophy of basing the structural analysis directly on the CAD model, thus omitting the need for a separate, specialized model [1,2,5,6], and providing a smooth design-through-analysis workflow.

### 3 DEVELOPMENT OF MEMBRANE AND CABLE ELEMENT FOR IBRA

For the analysis of structural membranes, the most important element types are membrane and cable elements. The formulation of these elements is shown in detail in [5]. In the following, some aspects in their development shall be pointed out.

#### 3.1 Membrane element formulation

The prestressed membrane element is formulated according to classical membrane theory, using non-linear kinematics. For the application of oriented anisotropic prestress a projection scheme, according to the one presented in [7] is used (see Fig. 2).

Basically the membrane element’s formulation is straightforward with these assumptions. For trimmed surfaces, the integration procedure is adapted in order to account for the “active” parts, *i.e.* figuratively spoken the parts that are left after the trimming operation (see Fig. 1). This integration is realized with the *Nested Jacobian Approach* (NEJA) [1].
3.2 Cable element formulation

Cables are of crucial importance for structural membranes, since they allow the formation of edge, ridge and valley cables that transfer forces and help to accentuate the desired shape. In order to comply with the developed membrane elements within the IBRA process, cable elements are formulated as embedded B-Rep edge elements (see Fig. 3).

This new paradigm, presented in [1,5] entails the embedding of the B-Rep edge element in the parameter space of the NURBS-patch it is embedded in, here in the NURBS-patch of the membrane element. From that NURBS-patch, the cable element extracts its basis functions (see Fig. 3(d)). Through the integration rule of the NURBS patch, the extracted patches are correctly considered in the numerical integration of the respective mechanical contributions (stiffness, load, etc.), see Fig. 3(c).
Figure 3: Formulation of a cable element as a B-Rep edge element consistently embedded within the underlying membrane’s parameter space (from [5]).

For the sake of illustration for this “embedded” integration procedure one might consider the determination of the length $L$ of a B-Rep edge cable in the reference configuration as illustrated in Fig. 3 (a). Here the length is computed as

$$
|L| = \int_{\Gamma_e} dl = \int \left\| \tilde{\mathbf{G}}_1 \right\| d\tilde{\theta}^i = \int \left\| \frac{d\mathbf{X}_{\text{curve}}}{d\tilde{\theta}^i} \right\| \tilde{d}\tilde{\theta}^i,
$$

(1)

where $\tilde{\mathbf{G}}_1$ is the local base vector of the B-Rep element corresponding to the curvilinear coordinate $\tilde{\theta}^i$ along the curve, derived from the curve’s position vector $\mathbf{X}_{\text{curve}}$, see Fig. 3(a). Through the implicit description of the curve’s geometry $\mathbf{X}_{\text{curve}}$ the expression is referred to the position vector of the surface along the B-Rep edge,

$$
\mathbf{X}_{\text{curve}}(\tilde{\theta}^i) = \mathbf{X}_{\text{surf}}(\tilde{\theta}^i(\tilde{\theta}^i), \tilde{\theta}^i(\tilde{\theta}^i)).
$$

(2)

Thus Equ. (1) can be expressed through the surface’s coordinates:
To perform this integration, the basis functions of the NURBS surface along the B-Rep edge are extracted as illustrated in Fig. 3(c) and (d).

The cable element is then derived according to classical cable mechanics, assuming a constant cross-section $A_{\text{cable}}$ along the cable and a homogeneous stress distribution in the cross section. For a detailed description of the element formulation, see [5].

As mentioned above, the presented cable element is formulated as a B-Rep edge element along a trimming curve, i.e. along the boundary of a considered patch. Since several B-Rep contributions can be applied to one B-Rep edge simultaneously [1,5], situations like ridge or valley cables can be simulated with the present approach, as is illustrated in Fig. 4: At the intersection between two strips, i.e. two trimmed surfaces that geometrically share one edge, a cable is assigned to one of the B-Rep edges. Additionally, the two B-Rep edges are coupled according to the coupling approach presented in [1].

Hence, all scenarios for the application of cable elements in the context of architectural
membranes can be treated as is exemplarily illustrated in Section 5.

4 BENCHMARKS FOR STRUCTURAL ANALYSIS AND FORM-FINDING

Benchmark examples are used to demonstrate the capability of the developed methods and to highlight its potential in the form-finding of structural membranes.

4.1 Stretched quarter circle with integrated edge cable

The first benchmark example (from [5]) is used to demonstrate the correct behavior of the developed elements in a non-linear analysis for various configurations, including trimming, coupling and the application of edge cables as illustrated in Fig. 5.

![Different configurations for the stretched quarter circle](image)

For this example a quarter circle with symmetry conditions is radially loaded by a load $p$ of varying magnitude. The expected radial extension for the configuration in Fig. 6 is

$$ \frac{p}{E_{\text{mem}} \cdot t + E_{\text{cab}} \cdot A_{\text{cab}} / R} = \frac{1}{2} \frac{d^3}{R^3} + \frac{3}{2} \frac{d^2}{R^2} + \frac{d}{R}, $$

where $p$ is the radial load, $R$ is the initial radius, $E_{\text{mem}}$ and $t$ are the Young’s modulus and the thickness of the membrane, and $E_{\text{cab}}$ and $A_{\text{cab}}$ are the Young’s modulus and the cross section of the cable, respectively, whereas $d$ is the unknown radial extension of the edge. For the case of the pure membrane example where the cable is not activated, the corresponding term vanishes.
Fig. 6 shows the corresponding deformed structures for the configurations from Fig. 6. As can be seen from the deformed shapes, the different modelling approaches yield the same results, although the parametrizations differ substantially. From this it can be concluded that the presented approach and element provide accurate results for non-linear analyses, independent of trimmed or coupled patches or including the application of edge cables.

4.2 Form-finding of minimal surface examples

As an example for the form-finding of minimal surfaces a catenoid and Costa’s minimal surfaces are presented. As form-finding approach, the Updated Reference Strategy [8] is
applied, as discussed in [5]. The form-finding of the catenoid in Fig. 8 is used to present the quality of the geometry approximation of a NURBS-parametrization. From the error in the computed surface area compared to the analytical solution it can be concluded that NURBS-surfaces are not intrinsically capable of catching the cosh-shape of the catenoid. Nevertheless an arbitrary quality of the approximation can be achieved for an increasing number of control points respectively a higher polynomial degree.

![Form-finding of a catenoid](image1)

Figure 8: Form-finding of a catenoid: Representation and discretization scheme of the problem (left); Error in the surface area for various polynomial degrees and different refinements in meridian direction (right) (from [5]).

The form-finding of Costa’s minimal surface [11], illustrated in Fig. 9, highlights the robustness of the developed elements: Although in the collapsed version (right), the elements are degenerated to a line, no convergence problems are observed up to that point.

![Form-finding of Costa’s minimal surface](image2)

Figure 9: Form-finding of Costa’s minimal surface. Starting configuration (left), intermediate stable solution (middle), and “collapsed” final solution (right) [4].
5 APPLICATION EXAMPLE – FORM-FINDING OF AN EYE-CABLE TENT

As final example the form-finding of an eye-cable tent is used (example from [6]). The tent is modelled in CAD (Fig. 10, top) out of four trimmed patches, the respective mechanical conditions (support conditions, edge and ridge cables, eye cable, prestressed membrane, coupling conditions, etc.) are applied as illustrated in Fig. 10, bottom.

Figure 10: Tent with an eye-cable: Geometry modeling in perspective (top) and application of structural properties in top-view (bottom). The starting geometry is modeled by four NURBS-patches that are trimmed by a circle. The patches are coupled along the common edges, edge cables are applied along the outer bounds, an elastic edge cable is applied at the inner ring. Supports are applied at the vertices and the inner ring (from [6]).
Fig. 11 shows the resulting membrane surface. Note that the form-found shape still is provided as a complete CAD-description and can directly be processed in a CAD environment, \textit{e.g.} for purposes of rendering or further geometry modifications that can lead to follow-up analyses as discussed in [2].

Figure 11: Form-found tent with eye cable. The form-found shape is a CAD-geometry and can thus be consistently treated in further steps of design and manufacturing (from [6]).

The presented form-finding examples highlight the capabilities of the presented methods. Especially the concept of the embedded cable elements and their applicability to the various types of cables in structural membranes has shown to be very promising. The existence of the resulting form-found shape as a CAD-description gives maximum freedom for further analyses and processing.

6 CONCLUSIONS AND OUTLOOK

In the present contribution, the necessary developments for the design and analysis of structural membranes with the Isogeometric B-Rep Analysis (IBRA) have been introduced. Besides a brief introduction to the IBRA-concept and the development of a non-linear, prestressed membrane element, the formulation of a cable element has been presented to some extent: The element is formulated as an embedded B-Rep edge element, \textit{i.e.} applied along arbitrary trimming curves, which perfectly aligns with the aim of using an enriched version of the CAD-model as basis for the analysis.

Several benchmark applications from non-linear analyses in different modelling configurations to the form-finding of minimal surfaces have shown the robustness and accuracy of the derived approaches and elements. A real-life example has finally demonstrated the applicability of the developed environment – realized as a fully integrated
plugin to the CAD-software Rhinoceros – to the design and analysis of structural membranes.

Future research will focus on a completion of the design cycle for structural membranes, where especially the determination of cutting patterns and the integration of these in the mounting analysis are challenging and promising aspects. The introduction of staged analysis – which is in the scope of the design-through-analysis workflow with the Analysis in CAD (AiCAD)-approach – opens the way towards a reliable and close-to-design analysis of hybrid structures, i.e. tensile structures (usually form-found) that include elastic, bending-active elements [9].

REFERENCES


