Design of reflection-suppressed all-optical diode based on asymmetric L-shaped nonlinear photonic crystal cavity

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A simple design method for suppressing the reflection of the all-optical diode based on the L-shaped photonic crystal (PC) cavity is proposed. Analyzing the linear resonant characteristics of the PC cavity and using the nonlinear coupled-mode theory (NL-CMT), the strategy for obtaining reflection-suppressed structure is illustrated. Based on the design rule, “reflection-suppressed all-optical diode” is presented and numerical solutions indicate that the designed structure has very small reflection as well as extremely large nonreciprocal transmission ratio (NTR).

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1. INTRODUCTION

Nonreciprocal optical devices, such as optical isolators and optical diodes, are crucial components in optical communication systems. For example, an optical isolator protects lasers from back reflections. An optical diode is a nonreciprocal two-port device that transmits forward propagating light and does not transmit backward propagating light. To realize the nonreciprocal diode operation, it is necessary to break the Lorentz reciprocity [1–3]. Although magneto-optical materials, including garnet, have nonreciprocal property, such materials are unfortunately incompatible with a silicon-on-insulator (SOI) platform [4,5]. Another approach to realize the diode operation is to use nonlinear optical effects, such as optical Kerr-effect, free-carrier effect, thermo-optic effect, four-wave mixing, stimulated Brillouin scattering, and so on [6–15]. These optical diodes based on the optical nonlinear effects have “dynamic reciprocity”, and hence the diode operation can be observed under certain input conditions [2,3]. To achieve the diode operation based on the intensity-dependent refractive index, the structural asymmetry is indispensable for the symmetry breaking of the transmission characteristics. So far, two types of all-optical nonlinear diodes based on the intensity-dependent refractive index have been proposed. In [11–13], the nonlinear optical diodes are composed of two different resonators, and the transmission is estimated by the product of two nonlinear transmission spectra. However, the overall structure is relatively large. On the other hand, the nonlinear optical diodes composed of a single photonic crystal (PC) cavity that has two asymmetric input and output ports are superior in terms of structural miniaturization [7–9,15]. Recently, it was reported that the asymmetric PC cavity can exhibit the nonreciprocal transmission by its nonlinear mode coupling and perform the nonreciprocal transmission ratio (NTR) of 50 dB [15]. NTR is defined as the ratio of the backward transmission to forward transmission and used to evaluate the optical nonreciprocity. In the ideal case, the forward transmission of 100 % and backward transmission of 0 % are desired as shown in Fig. 1 (a) and (b), resulting in NTR of +∞. To the best of our knowledge, the theoretically and experimentally reported maximum values of NTR are 50 dB [15] and 40 dB [16], respectively. Although it was not fully pointed out in previous

(a) Ideal Forward Propagation

(b) Ideal Backward Propagation

(c) Undesired Back Reflection for Forward Propagation

Fig. 1. Illustration of the optical diode. (a) Ideal forward propagation. (b) Ideal backward propagation. (c) Nonideal forward propagation due to the existence of back reflection.
In this paper, the design rule for suppressing the reflection of two-port nonlinear optical diode based on the asymmetric L-shaped nonlinear PC cavity is presented. By using the combination of the nonlinear-coupled-mode theory (NL-CMT) [15,19,20] and FETD-BPM [17], it is possible to specify the reflection-suppressed structure. The brief formulation of both methods is provided in Section 2. In the following section, the design method to suppress the reflection on the diode operation is illustrated and it is shown that only two parameters are crucial to determine the reflection characteristics. And then, the nonlinear characteristics of the designed all-optical diode are examined rigorously. However, a lot of calculation time is required for specifying the reflection-suppressed structure and input conditions by trial and error. Hence, the strategy for suppressing the reflection is essential for designing the all-optical diode based on the L-shaped nonlinear PC cavity.

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2. NONLINEAR ANALYTICAL METHODS

A. Coupled-Mode Theory

Figure 2(a) shows the schematics of the all-optical diode based on the asymmetric L-shaped square lattice PC cavity, which is composed of the input and output ports and the nonlinear defect rod cavity (filled orange circle). The defect cavity supports two eigenfield distributions, called as "dipole modes". The forward lightwave propagating in the input port can couple to one dipole mode (labeled as "1st cavity mode") and the backward lightwave propagating in the output port can similarly couple to another dipole mode (labeled as "2nd cavity mode") as shown in Fig. 2(b). In the linear regime, two dipole modes can hardly couple each other due to their orthogonal eigenfield distributions. However, the Kerr-nonlinearity in the defect rod enables to break the orthogonality, resulting in the nonlinear coupling of the dipole modes. In order to break the symmetry in the transmission characteristics, the three and four rods are placed adjacent to the nonlinear cavity rod for the input and output ports, respectively. Figure 2(b) shows the illustration of the principle of the diode operation. In the nonlinear regime, if the input power and frequency are properly set, forward lightwave propagating in the input port can pass through the defect rod cavity due to the nonlinear coupling between two dipole modes.

From the coupled-mode theory for the nonlinear dipole mode [15,19,20], we obtain the following equations

\[
\frac{dA_1(t)}{dt} = -j\left(\omega_1 - V_{11} - \frac{j}{\tau_1}\right)A_1 + jV_{12}A_2 + \frac{2}{\tau_1}E_{in}e^{-j\omega_1 t}, \quad (1)
\]

\[
\frac{dA_2(t)}{dt} = -j\left(\omega_2 - V_{22} - \frac{j}{\tau_2}\right)A_2 + jV_{21}A_1 + \frac{2}{\tau_2}E_{in}e^{-j\omega_2 t}, \quad (2)
\]

where \(A_i (i=1, 2)\) is the amplitude of \(i\)-th cavity mode, \(\omega_i (i=1, 2)\) is the normalized resonant frequency of \(i\)-th cavity mode, \(\alpha_i \) is the normalized resonant frequency of input lightwave, \(\tau_i (i=1, 2)\) is the normalized decay rate of \(i\)-th cavity mode, and \(E_{in}\) and \(E_{out}\) are the amplitudes of the electric field entering from the input and output ports, respectively. \(\omega_i \) and \(\tau_i \) are given by

\[
\omega_i = \frac{a}{L_i}, \quad (3)
\]

\[
\tau_i = \frac{2Q_i}{\omega_i}, \quad (4)
\]

where \(a\) is the lattice constant, \(L_i (i=1, 2)\) is the resonant wavelength of \(i\)-th cavity mode, and \(Q_i \) is \(Q\)-value of \(i\)-th cavity mode. The \(Q\)-value is defined as a ratio of resonant frequency to full width at half maximum. \(V_{nm} (m, n, 1, 2)\) is the parameter of nonlinear contribution given by

\[
V_{nm} = \frac{(\omega_n + \omega_m)}{4N_m} \int \int \delta E_n(x, y) E_m(x, y) dx dy, \quad (5)
\]
where $N_n$ is the normalization constant, $\delta\varepsilon$ is the instantaneous Kerr change of the dielectric constant, and $E_i$ is the electric field distribution of $i$-th cavity mode. $N_n$ and $\delta\varepsilon$ are defined as

$$N_n = \int \int n_0^m \delta\varepsilon \mathbf{E}_n(x,y) \mathbf{E}_n^*(x,y) dxdy = \frac{a^2}{cn_2}, \quad (6)$$

$$\delta\varepsilon = n_0cn_2 \left[ A_E(1) + A_E(1) \right]. \quad (7)$$

where $n_0$ is the linear refractive index, $n_m$ is the nonlinear index, $a$ is the lattice constant, and $c$ is the speed of light in vacuum. After substitution of Eqs. (5), (6), and (7) and $A_m = A_{m+1} e^{-j\kappa x}$ into Eqs. (1) and (2), the stationary CMT can be derived as

$$\left( \omega - \omega_1 + \kappa_1 |A_1|^2 + \kappa_2 |A_2|^2 + \frac{j}{\tau_1} \right) A_1$$

$$- 2\kappa_2 \text{Re} \left[ A_1^* A_2 \right] A_2 = j \sqrt{2 \tau_1} E_{in}, \quad (8)$$

$$\left( \omega - \omega_2 + \kappa_2 |A_1|^2 + \kappa_1 |A_2|^2 + \frac{j}{\tau_2} \right) A_2$$

$$- 2\kappa_2 \text{Re} \left[ A_1 A_2^* \right] A_1 = j \sqrt{2 \tau_2} E_{in}, \quad (9)$$

where $\kappa_{mn}$ is the nonlinear coupling coefficient for stationary CMT given by

$$\kappa_{mn} = \frac{(\omega_m + \omega_n)n_0 c^2}{16\pi a^2} \int \int n_0^m n_m^m (x,y) E_n^*(x,y) E_m^*(x,y) dxdy. \quad (10)$$

The transmission and back reflection of forward propagation (assuming $E_{in} = 0$) light can be expressed as

$$T_{in} = \sqrt{\frac{2}{\tau_2} A_2}$$

$$\left[ \frac{2}{\tau_1} A_1 + \frac{2}{\tau_2} A_2 - E_{in} \right] \right]^2, \quad (11)$$

$$R_{in} = \sqrt{\frac{2}{\tau_1} A_1}$$

$$\left[ \frac{2}{\tau_1} A_1 - E_{in} \right] \right]^2, \quad (12)$$

and those of backward propagation can be expressed as well [15,19,20].

**B. Finite-Element Time-Domain BPM**

For the analysis of the linear resonant characteristics of the PC cavity, the time-domain beam-propagation method (TD-BPM) based on the finite-element method (FEM), called as FETD-BPM, is used. FETD-BPM scheme is powerful numerical method to investigate the light propagation in the waveguides [17]. According to the procedures in FETD-BPM, the time evolution of light propagation can be obtained by solving the following simultaneous linear equation

$$\left[ A(\mathbf{r}) \right] \left[ \phi \right]_{j+1} = \left[ B(\mathbf{r}) \right] \left[ \phi \right]_j, \quad (13)$$

where $[A(\mathbf{r})]$ and $[B(\mathbf{r})]$ are the finite-element matrices related to refractive index distribution $n(\mathbf{r})$, and $[\phi]$ and $[\phi]_{j+1}$ are electric or magnetic field vectors on $j$-th and $(j+1)$-th step.

By including the Kerr nonlinearity in the material, we can solve nonlinear propagating problem as well. In the nonlinear regime, the refractive index can be expressed as

$$n(\mathbf{r}, |\mathbf{E}|^2) = n_0 \left( 1 + \frac{1}{2} \varepsilon_0 c n_2 |\mathbf{E}|^2 \right), \quad (14)$$

where $\varepsilon_0$ is permittivity in vacuum, and $|\mathbf{E}|$ is the electric field vector. Substituting Eq. (14) to Eq. (13), the simultaneous equation for the nonlinear optical propagation analysis is derived as [18]

$$\left[ A \left( n(\mathbf{r}, |\mathbf{E}|^2) \right) \right] \left[ \phi \right]_{j+1} = \left[ B \left( n(\mathbf{r}, |\mathbf{E}|^2) \right) \right] \left[ \phi \right]_j, \quad (15)$$

where $[A(n)]$ and $[B(n)]$ are $i$-th finite-element matrices whose nonlinear refractive index distribution is calculated by $[\phi]$. This method directly solves nonlinear Maxwell’s equations rigorously and precise solutions can be obtained. Therefore, we use the results obtained by NL-FETD-BPM to verify the validity of our proposed design.

### 3. NUMERICAL ANALYSIS

**A. Design and Basic Characteristics**

We assume the lattice constant $a = 400$ nm, the radius of dielectric rod $r = a/4$, and the radius of defect rod $r_{nl} = 5a/12$ [21]. This PC structure has a photonic band gap (PBG) for the transverse-magnetic (TM) modes, which have electric field parallel to the rods, between $\omega_{bnl,\text{min}} = 0.244$ and $\omega_{bnl,\text{max}} = 0.292$. To introduce asymmetry, 3 and 4 rods are placed adjacent to the nonlinear rod for input and output ports. As seen in Fig. 2(a), the radii of two rods located at the edge of the input port and output port are assumed as $r_1$ and $r_2$ to control the resonant characteristics. The linear refractive indices of rods and background material are $n_{rod} = 3.5$ and $n_{back} = 1.5$, assuming the GaAs dielectric rods and silica glass, respectively [22]. We consider that the nonlinear index of defect rod is $n_2 = 1.5 \times 10^{-17} \text{m}^2/\text{W}$ [23] and that of other materials is zero. The decay rate and resonant frequency of cavity modes are obtained by solving Eq. (13) [17].

We let start the case of $r_1 = r_2 = r$. Figure 3 shows the transmission spectra of 1st and 2nd cavity mode with $r_1 = r_2 = r$ and the relationship between resonant frequency and detuning parameter $\delta$ defined as $(\omega - \omega_0)\tau_1/2$ [21]. Calculated resonant frequency and Q-value of 1st and 2nd cavity modes are $\omega_1 = 0.262719$ and $Q_1 = 1280$, and $\omega_2 = 0.262244$ and $Q_2 = 5489$, respectively. Calculated nonlinear coupling coefficients are $\kappa_1 = 1.03 \times 10^{-4}$, $\kappa_2 = 3.75 \times 10^{-5}$, $\kappa_3 = 3.75 \times 10^{-5}$, and $\kappa_4 = 1.03 \times 10^{-5}$.
1.5 × 10⁻⁴. Figures 4 (a) and (b) show the resonant characteristics of the all-optical diode with \( r_1 = r_2 = r \) for the forward propagation (assuming \( E_{in} = 0 \)) and the detuning parameter is \( \delta = 2.39 \) (\( \omega = 0.26194 \)). The green dashed and blue solid lines in Fig 4(a) represent the solutions of \(|A_1|^2\) and \(|A_2|^2\) calculated by the Eqs. (8) and (9). The closed and opened circles represent steady solutions of \(|A_1|^2\) for increasing and decreasing power, respectively. Although the nonlinear interaction between dipole modes do not appear if input amplitude is sufficiently low, both solutions \((|A_1|, |A_2|)\) are simultaneously observed for the higher power due to the nonlinear coupling. As shown in Fig. 4(b), the minimum reflection for this structure is \( R_{min} = 28\% \) (−6 dB). Figures 5 (a) and (b) show the resonant characteristics of the all-optical diode with \( r_1 = r_2 = r \) for the backward propagation (assuming \( E_{in} = 0 \)). The threshold intensity for the propagation and backward propagation is \(|E_{o1}^n|^2 = 1.5 \times 10^{-4}\) and \(|E_{o2}^n|^2 = 5.1 \times 10^{-4}\). Therefore, the diode operation can be observed within the range of \(|E_{o1}^n|^2\) from \(1.5 \times 10^{-4}\) to \(5.1 \times 10^{-4}\) and \(E_{o1}^n < E_{o2}^n\) is necessary for the diode operation. Figure 6 shows the map of the reflection of the all-optical diode with \( r_1 = r_2 = r \) as a function of input intensity \( |E_{in}|^2 \) and the detuning parameter \( \delta \). In the area labeled as "Diode operation is impossible", the backward transmission is permitted when the lightwave enter from output port for the input power of \( E_{in} \). The minimum reflection is \( R_{min} = 28\% \) when \(|E_{in}|^2 = 1.5 \times 10^{-4}\) and \( \delta = 2.39 \) for the all-optical diode with \( r_1 = r_2 = r \). To reduce the reflection further, the appropriate design method is required.

### B. Strategy for Suppressing the Reflection

The resonant characteristics are determined by the following parameters: \( \omega_1, \omega_2, \tau_1, \tau_2, k_{11}, k_{12}, k_{21}, \) and \( k_{22} \). Except for \( \omega_1 \), other parameters \((\omega_2, \tau_1, \text{and } k_{mn})\) are fixed by the structural parameters. Among these parameters, \( \omega_2 \) and \( \tau_1 \) can be easily controlled by \( r_1 \), whereas \( k_{mn} \), mainly defined by the eigenmode distribution, is hardly affected by \( r_1 \). Hence, we should focus on the following parameters: \( \omega, \omega_1, \omega_2, \tau_1, \tau_2 \).

Here, we consider following parameter transformation:

\[
(\omega_1, \omega_2, \tau_1, \tau_2) \rightarrow (\omega_1, \omega_2, \tau_1, \tau_2, k_{11}, k_{12}, k_{21}, k_{22}) \rightarrow (\omega_1, \omega_2, \tau_1, \tau_2, \delta, p_1, p_2) \text{.}
\]

where \( p_1 = \tau_2 / \tau_1 \) and \( p_2 = \tau_1 (\omega_2 - \omega_1) \). Equations (8), (9), and (12) can be rewritten as

\[
-\delta \frac{\alpha_1}{Q_1} - \kappa_1 |A_1|^2 - \kappa_2 |A_2|^2 + \frac{j \alpha_1}{2 Q_1} A_1 \nonumber (16)
\]

\[
-2 \kappa_{12} \text{Re}[A_1^* A_2] A_2 = j \frac{\alpha_2}{Q_1} E_{in} \nonumber \text{,}
\]

\[
-\delta \frac{\alpha_1}{Q_1} - \frac{p_2}{2} \frac{\alpha_1}{Q_1} - \kappa_2 |A_2|^2 - \kappa_{21} |A_1|^2 + \frac{j \alpha_2}{2 p_2 Q_1} A_2 \nonumber \text{,}
\]

\[
-2 \kappa_{21} \text{Re}[A_1^* A_2] A_1 = j \frac{1}{p_2} \frac{\alpha_1}{Q_1} E_{in} \nonumber \text{.}
\]
If we define $\tilde{\alpha}_1, \tilde{Q}_1$ as
\[
\tilde{\alpha}_1 = \frac{\alpha_1}{\tilde{Q}_1}, \quad \tilde{Q}_1 = \frac{Q_1}{\alpha_1},
\]
with $\alpha_1$ being an arbitrary number, Eqs. (16) and (17) can be rewritten as
\[
\begin{align*}
\left(-\delta \frac{\tilde{\alpha}_1}{\tilde{Q}_1} - \kappa_{11} |\tilde{A}_1|^2 - \kappa_{12} |\tilde{A}_2|^2 + j \frac{\tilde{\alpha}_1}{2 \tilde{Q}_1}\right) \tilde{A}_1 \\
-2 \kappa_{12} \text{Re} \left[\tilde{A}_1 \tilde{A}^*_2\right] \tilde{A}_2 = j \frac{\tilde{\alpha}_1}{\tilde{Q}_1} \tilde{E}_{\text{fw}},
\end{align*}
\]
and
\[
\begin{align*}
\left(-\delta \frac{\tilde{\alpha}_2}{\tilde{Q}_2} - \frac{p_2}{2 \tilde{Q}_2} - \kappa_{22} |\tilde{A}_2|^2 - \kappa_{21} |\tilde{A}_1|^2 + j \frac{\tilde{\alpha}_2}{2 \tilde{Q}_2}\right) \tilde{A}_2 \\
-2 \kappa_{21} \text{Re} \left[\tilde{A}_2 \tilde{A}^*_1\right] \tilde{A}_1 = j \frac{\tilde{\alpha}_2}{\tilde{Q}_2} \tilde{E}_{\text{fw}},
\end{align*}
\]
where
\[
\tilde{E}_{\text{fw}} = E_{\text{fw}} \alpha_1,
\]
\[
\tilde{E}_{\text{bw}} = E_{\text{bw}} \alpha_2,
\]
\[
\tilde{A}_1 = A_1 \sqrt{\alpha_1},
\]
\[
\tilde{A}_2 = A_2 \sqrt{\alpha_2}.
\]
The reflection $\tilde{R}_{\text{fw}}$ of the structure having $\tilde{\alpha}_1$ and $\tilde{Q}_1$ as a function of $\tilde{E}_{\text{fw}}$ is given by
\[
\tilde{R}_{\text{fw}}(\tilde{E}_{\text{fw}}) = R_{\text{fw}}(E_{\text{fw}}).
\]

Equation (26) implies that, regardless of the ratio of $\alpha_1$ and $Q_1$, the reflection is constant as long as $p_1, p_2$, and $\delta$ are fixed. Therefore, for searching the structure having minimum reflection, only $p_1, p_2$, and $\delta$ have to be considered. We can also choose $\alpha_2$ and $Q_2$ instead of $\alpha_1$ and $Q_1$. We call $p_1$ and $p_2$ as "asymmetry parameters" because the condition of $p_1 = 1$ and $p_2 = 0$ indicates the perfect symmetry of two cavities. Since $\kappa_{21}$ is less dependent on the cavity structure, it is effective to tune the asymmetry parameters for minimizing the reflection. Hereafter, we will show how to choose the structural parameters (and hence, $p_1$ and $p_2$) for minimizing the reflection.

Figure 7 shows the map of the reflection of the all-optical diode as a function of two asymmetry parameters obtained by NL-CMT for $\kappa_1 = 1.03 \times 10^{-4}, \kappa_2 = 3.75 \times 10^{-5}, \alpha_1 = 3.75 \times 10^{-5}$, and $\alpha_2 = 1.03 \times 10^{-4}$. The same value of the structure of Fig. 4.) In Fig. 7, for each point $(p_1, p_2)$, we search the value of $\delta$ which gives the minimum reflection. The reflection is minimum and can be suppressed up to $\sim -120$ dB when the asymmetry parameters are
\[
p_1 = \frac{\tau_2}{\tau_1} = 1.90,
\]
\[
p_2 = \frac{\tau_1 (\alpha_2 - \alpha_1)}{\alpha_2} = 0.58,
\]

Fig. 8. (a) Structural dependence of resonant frequency and $Q$-value of 1st and 2nd cavity modes when changing $r_1$ and $r_2$ between 0.15$a$ and 0.30$a$. The green and blue solid lines correspond to 1st and 2nd cavity modes, respectively. (b) Optimal resonant frequency and $Q$-value of 1st and 2nd cavity modes. The dotted lines are the same with Fig. 8 (a). The green and blue solid lines represents optimal values calculated by Eqs. (27) and (28).
The green and blue solid lines in Fig. 8 (b) show the resonant frequency and minimum reflection of \( |E_{\text{in}}|^2 \) at the input intensity of \( 0.2355a \) and \( 0.1517a \), respectively. The green solid line is obtained as well. Therefore, if we set \( r_1 = 0.30a \) for the 1st cavity, \( Q_1 \) and \( \alpha_1 \) are obtained from green dotted lines (left hand side point in this figure). From these \( Q_1 \) and \( \alpha_1 \), \( Q_2 \) and \( \alpha_2 \) are calculated from the relationship of Eqs. (27) and (28) (the condition for minimizing reflection obtained by NL-CMT) and these are denoted by blue solid line. The green solid line is obtained as well. Therefore, if we set \( r_1 \), corresponding \( Q_1 \) and \( \alpha_1 \) are obtained, an optimum \( Q_2 \) and \( \alpha_2 \) can be calculated from Eqs. (27) and (28). The next task is to determine another cavity structure having optimum \( Q_2 \) and \( \alpha_2 \). The 2nd cavity structure for obtaining optimum \( Q_2 \) and \( \alpha_2 \) is given by the cross point of blue dotted and solid lines. Since the dotted line represents physically possible cavity structure for given geometry (in this case, one \( r_1 \) and two \( r \) rods for 1st cavity and one \( r_2 \) and three \( r \) rods for 2nd cavity). Therefore, \( r_1 = 0.2355a \) and \( r_2 = 0.1517a \) are the optimum structure for minimizing the reflection (open red circle in Fig. 6(b)), and the corresponding resonant frequency and \( Q \)-value are \( \omega_0 = 0.26222 \), \( \omega_2 \approx 0.26228 \), \( Q_1 = 1271 \) and \( Q_2 \approx 2414 \). Calculated nonlinear coupling coefficients of this structure are \( \kappa_1 = 1.03 \times 10^{-4} \), \( \kappa_{12} = 3.76 \times 10^{-5} \), \( \kappa_{21} = 3.76 \times 10^{-5} \), and \( \kappa_{22} = 1.03 \times 10^{-4} \).

Figures 9 (a) and (b) show the resonant characteristics of the all-optical diode with \( r_1 = 0.2355a \) and \( r_2 = 0.1517a \) for the forward propagation. The minimum reflection of \( R_{\text{min}} \approx 0 \% \) (\( \approx 10^{-6} \)) can be obtained in Fig. 9(b) at the input intensity of \( |E_{\text{in}}|^2 = 3.1 \times 10^{-4} \). Figures 10 (a) and (b) show the resonant characteristics of the all-optical diode with \( r_1 = 0.2355a \) and \( r_2 = 0.1517a \) for the backward propagation. The diode operation is possible within the range of \( |E_{\text{in}}|^2 \) from \( 3.1 \times 10^{-4} \) to \( 6.4 \times 10^{-4} \).

Next, we search the structural parameters \( r_1 \) and \( r_2 \) for obtaining the condition of Eqs. (27) and (28). Figure 8(a) shows the structural dependence of the resonant frequency and \( Q \)-value of 1st and 2nd cavity modes when changing \( r_1 \) and \( r_2 \) from 0.15a to 0.30a. The green and blue solid lines correspond to 1st and 2nd cavity modes, respectively. The green and blue solid lines in Fig. 8(b) shows the resonant frequency and \( Q \)-value of 1st and 2nd cavity modes satisfying Eqs. (27) and (28). The dotted lines are the same with Fig. 8(a). The solid lines are obtained as follows. For example, if we set \( r_1 = 0.30a \) for the 1st cavity, \( Q_1 \) and \( \alpha_1 \) are obtained from green dotted lines (left hand side point in this figure). From these \( Q_1 \) and \( \alpha_1 \), \( Q_2 \) and \( \alpha_2 \) are calculated from the relationship of Eqs. (27) and (28) (the condition for minimizing reflection obtained by NL-CMT) and these are denoted by blue solid line. The green solid line is obtained as well. Therefore, if we set \( r_1 \), corresponding \( Q_1 \) and \( \alpha_1 \) are obtained, and optimum \( Q_2 \) and \( \alpha_2 \) can be calculated from Eqs. (27) and (28). The next task is to determine another cavity structure having optimum \( Q_2 \) and \( \alpha_2 \). The 2nd cavity structure for obtaining optimum \( Q_2 \) and \( \alpha_2 \) is given by the cross point of blue dotted and solid lines. Since the dotted line represents physically possible cavity structure for given geometry (in this case, one \( r_1 \) and two \( r \) rods for 1st cavity and one \( r_2 \) and three \( r \) rods for 2nd cavity). Therefore, \( r_1 = 0.2355a \) and \( r_2 = 0.1517a \) are the optimum structure for minimizing the reflection (open red circle in Fig. 6(b)), and the corresponding resonant frequency and \( Q \)-value are \( \omega_0 = 0.26222 \), \( \omega_2 \approx 0.26228 \), \( Q_1 = 1271 \) and \( Q_2 \approx 2414 \). Calculated nonlinear coupling coefficients of this structure are \( \kappa_1 = 1.03 \times 10^{-4} \), \( \kappa_{12} = 3.76 \times 10^{-5} \), \( \kappa_{21} = 3.76 \times 10^{-5} \), and \( \kappa_{22} = 1.03 \times 10^{-4} \).

Figures 9 (a) and (b) show the resonant characteristics of the all-optical diode with \( r_1 = 0.2355a \) and \( r_2 = 0.1517a \) for the forward propagation and the detuning parameter is \( \delta = \left( \omega_1 - \omega_0 \right) / 2 = 3.31 \).

Figures 10 (a) and (b) show the resonant characteristics of the all-optical diode with \( r_1 = 0.2355a \) and \( r_2 = 0.1517a \) for the backward propagation. The drastic reduction of the reflection can be seen compared with Fig. 6. Also, since the area of the low-reflection region is very wide, the designed structure has wide operation range, leading to strong tolerance to the operation condition.

**C. Nonlinear Analysis of NL-FETD-BPM**

Here, we confirm the validity of the discussion by comparing the results obtained by NL-CMT and rigorous numerical NL-FETD-BPM.

Figures 12 (a) and (b) show the reflection characteristics as a function of the frequency and power of input lightwave obtained by NL-CMT (dashed line) and NL-FETD-BPM (dots). For Fig 12 (a), the input power is constant (0.270 W/μm), for (b), the wavelength is constant (1527.46 nm). The input power of 0.270 W/μm in the NL-FETD-BPM scheme is corresponding to \( |E_{\text{in}}|^2 = 3.2 \times 10^{-4} \) in the NL-CMT scheme. If the height of the rod is \( a (= 400 \text{ nm}) \), input power and power density are...
approximately 0.108 W (= 0.270μ) and 0.675 W/μm² (= 0.270 / ω). The solution of NL-FETD-BPM agrees well with that of NL-CMT except for the small difference in threshold power between two solutions. The reflection of ~65 dB is obtained for NL-FETD-BPM, showing the effectiveness of the proposed strategy for suppressing the reflection calculated by NL-CMT. Under this input condition, the maximum refractive index change is approximately 0.01.

Figures 13 (a) and (b) show the forward and backward transmission as a function of (a) frequency and (b) power of the input lightwave calculated by NL-FETD-BPM. The diode operation is possible within the range of input wavelength from 1526.91 nm to 1527.49 nm for input power of 0.270 W/μm as shown in Fig. 13 (a), and the range of input power from 0.26 W/μm to 0.53 W/μm for input wavelength of 1527.46 nm as shown in Fig. 13 (b). We can see that the NTR is almost 90 dB in the diode operation region, which is extremely large value.

4. CONCLUSION
The all-optical diode based on the asymmetric L-shaped nonlinear PC cavity is investigated by the NL-CMT and the design method to suppress the reflection in the diode operation is illustrated. For designing the reflection-suppressed all-optical diode, only two “asymmetry parameters” are crucial values. In our proposed strategy, it is required to specify the asymmetry parameters by using NL-CMT and calculate the resonant frequency and Q-value by using FETD-BPM. This simple procedure enables to design the reflection-suppressed structure in a very short calculation time. Furthermore, we confirm that the designed all-optical diode achieves the very small reflection (~65 dB) as well as extremely large NTR (90 dB).

REFERENCES