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Aerodynamical and structural analysis of operationally used turbine blades

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#### Abstract

This paper presents an integrated methodology for the analysis of operationally-used turbine blades, incorporating aerodynamic and multiple structural simulations. In jet engines, blade rubbing and erosion lead to deviations of the blade geometry. The presented functional simulations are conducted in order to predict the influence of wear on the performance of turbine blades based on these geometric variations.

A numerical simulation of the investigated turbine blades using CFD show the change of aerodynamic performance and the flow field due to wear. Additionally, the deviations of the blade geometry lead to a different pressure and temperature distribution on the blade surface, which is used as input for the structural simulations.

The change in geometry, surface pressure and temperature lead to a change in vibration behavior of the blade. Particularly the eigenfrequencies and excitation are affected. This is incorporated into the analysis by performing a structural vibration simulation of a complete bladed disk, using component mode synthesis and wave base substructuring. The mistuning effects are analyzed statistically using the Monte Carlo method.

The change in vibration amplitudes influences crack opening and closing for a single blade under thermo-mechanical load. These processes, including thermal expansion, are investigated using the extended finite element method. Two real turbine blades are used to compare the characteristics of a new and a used blade.

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# 1. Introduction

The regeneration of jet engines is of high interest for aircraft companies because it generates 8% of the operating cost of an airplane due to maintenance and overhaul of engine parts. The main cost factor of the overhaul is caused by the airfoils which incur approximately 50% of the cost. The replacement of used blades of the high pressure turbine are the main cost factor for the airfoils [1]. These blades are one of the most highly loaded parts in jet engines.

The goal of the project area "Variance of Production and Material Properties in Regeneration" in the collaborative research center (CRC) 871 "Regeneration of Complex Capital Goods" is to estimate the influence of geometric variances on the functional properties of regenerated and used capital goods, in particular high pressure turbine blades. The geometric variances of these highly loaded parts occur due to operation and overhaul [2]. Hence, the main objective of this study is to compare the functional properties of a new and used turbine blade. Therefore, the influence of the geometric variance on the aerodynamic performance and aeroelastic behavior is investigated, as well as the crack opening and closing due to the change of vibration amplitudes and thermo-mechanical loads.

The influence of geometric variances in upstream blade rows on the aerodynamic excitation of turbine blades is numerically investigated in several studies [3–8]. These results have been confirmed by numerical and experimental investigations in [7, 9]. The main conclusion of these studies is that the influence of different kinds of variations in an upstream blade row has to be considered in the design process. The main excitation source of the turbine blades are the potential effect and the wake excitation. However, it was also shown in numerous studies that the tip leakage flow has also an impact on the aeroelastic behavior of rotor blades, [10–14] and therefore the deviations in the tip geometry. As in [15,16] was shown is that the tip geometry of turbine rotor blades also has an influence on the aerodynamic performance of turbine cascades and on the aerothermal effect of turbine blades.

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The changes in blade geometry due to regeneration or operation processes alter the structural properties of the blade. This can introduce mistuning and affect the sensitivity to additional mistuning from other sources. Mistuning in general describes differences of the structural dynamic properties among sectors of a bladed disk, and leads to increased vibration amplitudes. A comprehensive overview of the phenomenon and modelling approaches are presented in [18]. To calculate the forced response, finite-element-based reduced-order models (ROMs) are commonly employed. The component mode synthesis (CMS) is slower than other methods, but allows the assembly of arbitrary blades with completely independent geometry. Most methods can only accommodate small changes in the stiffness or mass matrix for each sector with a fixed number of degrees of freedom, limiting the possible geometric variances considerably. Sinha [20] uses proper orthogonal decomposition of the geometric variances to enrich the ROM and allow larger geometric variations. Madden et al. [19] show a way to allow an arbitrary finite-element mesh for one "rogue" sector only.

Additionally, geometric variances can change the mechanical response and the temperature distribution of turbine blades. Effective forces are changed at the same operating point due to a different mass distribution. The heat flux is also modified due to changes in the geometry. Additionally, defects like cracks can alter the displacement field and the temperature field. Changes and sensitivities are analyzed by numerical simulations.

### 2. Outline

In this paper a new and a used blade from the first stage of a high pressure turbine are compared based on their geometric differences. Other properties such as the material, are assumed to be constant. The numerical approach which is used in this study is presented in the following sections. It consists of aerodynamic, structural dynamic and crack analyses.

# 3. Aerodynamic Analysis

#### 3.1. Numerical Setup

The numerical investigations of the influence of a geometric variation on the aerodynamics were conducted comparing a used and a new turbine blade. Both blade geometries were scanned using a commercial structured-light 3D scanner. Geometric differences are present all over the blade, and especially at the blade tip, where considerable wear is observed (see Fig. 2). The results of the aerodynamic simulations are used as initial boundary condition in the following sections to investigate the influence of the geometric variances on the structural behavior.

The setup depicted in Fig. 1 was used for the aerodynamic analysis. The operating point used is typical for cruising altitude in jet engines. At the inlet of the first stage of the high pressure turbine, the mass flow rate and the total temperature were specified. The temperature is an averaged temperature resulting from investigations of the combustion chamber [17]. The turbulence intensity at the inlet was set to 20%, as the turbulence intensity is higher in high pressure turbine stages compared to standard turbomachines. At the outlet of the



Fig. 1: Setup for the aerodynamic analysis of the first high pressure turbine stage



Fig. 2: Total pressure and temperature distribution on the new and used turbine blade

first turbine stage, the static pressure is specified. The steadystate simulations were performed with a frozen rotor interface between the stator and rotor. Both domains have a 45 degree segment, in order to ensure an equal pitch of the domains for unsteady simulations. The first stage of the HPT consists of 40 vanes and 64 rotor blades. The numerical model was discretized with 6.33 million mesh nodes for the case of the new part. The numerical model with the used turbine blade was discretized with around 11 million mesh nodes. A higher mesh resolution was necessary to ensure a good mesh quality because of the special blade geometry. The simulations for the aerodynamic analysis were performed with ANSYS CFX 15.0. The mesh quality of the numerical model was sufficient, according to the recommendations by Ansys.

Mode	New blade [kHz]	Used blade [kHz]
1	2.360	2.619
2	3.655	4.187
3	5.487	5.707
4	8.040	8.408
5	10.72	11.01
6	12.99	13.01
7	14.50	14.43
8	15.64	15.54

Table 1: Blade eigenfrequencies

#### 3.2. Aerodynamic Results

The influence of the geometric variances on the aerodynamic performance of both turbine blades are investigated in this section. The specific stage work Y is defined as

$$Y = u_2 c_{u2} - u_1 c_{u1}.$$
 (1)

with u as circumferential velocity at the leading edge  $(u_1)$ , and trailing edge  $(u_2)$  of the blade tip and  $c_u$  as the circumferential component of the absolute velocity. The specific stage work indicates the mechanical loading of the blades. In the case of the HPT, the used turbine blade delivers 15.5% more work than the new turbine blade. This leads to a higher mechanical and dynamical loading of the blade. This can also be observed in the pressure and temperature distribution on the new blade and used blade, depicted in Fig. 2. The pressure side of the used blade has a higher mechanical and thermal loading due to higher pressure and temperature distribution in comparison to the new blade. This higher mechanical loading influences the structural behavior, which is analyzed in detail in the next sections. The temperature and pressure distribution of the blades is used as initial boundary condition to investigate the functional properties by the structural simulations.

### 4. Structural Dynamics

### 4.1. Setup

To investigate the influence of wear on the structural dynamics, the complete bladed disk is considered. A scanned blade surface geometry is complemented by fictional geometries for the blade cooling channels and the disk, loosely approximating the real geometry. The blade is assumed to be an isotropic nickel superalloy with the room-temperature Youngs modulus adjusted to match the first three free-free eigenfrequencies obtained via FEM to the measured eigenfrequencies of the used blade. For simplicity, the same material is used for the disk and 1% structural damping is assumed. Underplatform dampers are omitted, and the blades are fixed to the disk at the contact surfaces. The results are valid for the simulated bladed disk, and show the influence of varying blade geometry caused by wear.

For both blade types, the same disk model was used and one sector was considered. The analyses were performed independently for each blade geometry. First the temperature distribution inside the blade is calculated, with the surface temperature from the aerodynamic analysis and an assumed



Fig. 3: Interference diagram of the tuned bladed disk

temperature on the surface of the cooling channels. A static analysis of one sector is then performed at nominal speed with the pressure distribution mapped on the blade surface. For the forced response, the same pressure distribution is used as stimulus with an amplitude of 5% and an engine order 40 excitation, corresponding to the number of stator vanes.

### 4.2. Reduced Order Model

To account for mistuning, it is not sufficient to consider only a single sector with cyclic boundary conditions. Instead the full bladed disk is simulated using the component mode synthesis reduced-order modeling technique as presented in [21]. Using this substructuring approach, the blade and disk matrices are reduced independently with the well known Craig-Bampton [22] and Craig-Martinez [23] reduction methods, respectively. To reduce the interface DOFs, the wave-based substructuring method [24] is used. For each forced-response analysis the bladed disk is assembled with stiffness mistuning:

$$\mathbf{K}_{\text{blade},i} = (1 + \delta_i) \, \mathbf{K}_{\text{blade},\text{tuned}} \tag{2}$$

The stiffness matrix of each blade *i* is calculated from the nominal matrix and the scalar value  $\delta_i$ , representing the mistuning amount. A secondary modal analysis of the assembled system is used to reduce the size further and decouple the equations of motion, in order to enable the short calculation times necessary for the Monte Carlo method.

### 4.3. Results

The different geometry changes eigenfrequencies of a single blade and the whole assembly. Table 1 shows the eigenfrequencies of a single blade, fixed at the fir tree. The eigenfrequencies of the first blade modes increase for the used blade due to the lost mass at the blade tip. The biggest change is observed for mode 2, whereas the eigenfrequencies of the modes 6 and higher change very little. Fig.3 shows the eigenfrequencies of the whole bladed disk and the engine order excitation at nominal speed.

The different pressure distribution on the blade surface leads to an increase in excitation force for the used blade. Therefore,



Fig. 4: Comparison of modal excitation



Fig. 5: Comparison of mistuning sensitivity (mode 4)

the relative modal excitation of the first blade mode increases as shown in Fig. 4. Other modes are affected in a seemingly random way, with big changes in some cases. The fourth mode in particular, which is closest to the excitation frequency at nominal operation, experiences has an increase of 34%.

As a result of the changes in structure and excitation, the forced-response amplitude is affected as well. The amplitude is measured at the same point near trailing edge tip for both blade geometries. At resonance, the amplitudes of first two modes increase by 13.8% and 6.8% respectively. The third mode's (1T) amplitude decreases significantly, in accordance with the reduced excitation. A small increase in vibration amplitude at resonance is observed for the fourth mode (2F), but the resonance frequency increase (see Fig. 3) leads to an increase in amplitude by 72 % at nominal excitation.

In addition to the excitation, two factors explain the change of resonance amplitudes: The modal damping and the mode shapes change. The aerodynamic damping is the biggest contributor to the overall damping in real bladed disks (in absence of added friction dampers) [25]. It is not considered in the used simplified model, but it is expected to change significantly with wear, considering the change in pressure distribution on the blades surface. To eliminate the impact of the change in mode shape on the results, other measures for the amplitude could be used instead of the maximum displacement of a fixed point. The vibration energy, the maximum stress or the maximum displacement of any blade node are possible alternatives.

To evaluate the sensitivity to random mistuning, the Monte Carlo method is used. Fig. 5 shows the amplitude magnification of the fourth blade mode for varying levels of independent and identically distributed random Gaussian mistuning of the blade stiffness. A value of 1 represents the tuned resonance amplitude for each geometry. 1000 random configurations are analyzed for each value of the standard deviation. A three parameter weibull distribution is fitted to the results [28] using the modified maximum product of spacings method [29]. The increase in Amplitude due to mistuning is very similar for both blade geometries. Nonetheless, considering the tuned forced response results, a significant increase of the vibration amplitudes due to wear was found.

#### 5. Crack analysis

### 5.1. Problem definition and numerical solution

The mechanical response of blades due to thermomechanical loads is also investigated. Typical defects are cracks of the size of several centimeters, down to 40-70 microns. In addition to the high centrifugal forces, the temperature near the surface can reach up to 1000°C. The combination of high mechanical and thermal loading can lead to high mechanical stresses and/ or plastic deformation.

The standard FEM with Lagrange polynomials is not able to resolve the stress singularity and heat flux singularity of the analytical solution in the vicinity of the crack tip. Increasing the mesh density does not lead to convergence to the analytical solution.

To avoid this problem, the eXtended Finite Element Method (XFEM)[27] can be used. In XFEM, cracks are considered at the element level, and discontinuities in the displacement and temperature field are allowed. The method allows the acurrate determination of the crack opening displacement and crack tip stress near field, by extending the default approach by additional degrees of freedom:

$$\mathbf{u} = \sum_{I} N_{I} [\mathbf{u}_{I} + H\mathbf{a}_{I} + \sum_{l}^{4} f_{l} \mathbf{b}_{ll}]$$
(3)

 $HN_I \mathbf{a_I}$  is the so-called jump enrichment and  $N_I f_I \mathbf{b}_{II}$  is the front enrichment. The temperature field has the same behaviour as the displacement field, and for this reason the same approach (but only with one enrichment function  $f_1$ ) is selected [26]:

$$T = \sum_{I} N_{I} [T_{I} + Ha_{I} + f_{1}b_{I}]$$

$$\tag{4}$$

Since the current implementation of the XFEM allows only for one crack front inside a single element, the cracks should spread over several finite elements.

### 5.2. Numerical setup and results

The mechanical and thermal behaviour of a used turbine blade under quasi-static conditions is numerically investigated.



Fig. 6: Displacement distribution on the used turbine blade in area of crack



Fig. 7: Temperature distribution on used turbine blade in area of crack

A crack is put at the front edge of a turbine blade. In contrast, the new turbine blades are assumed to be free of defects. The turbine blade containing a crack is meshed with 10-Node-Tetrahedral-Elements with quadratic shape functions using XFEM. The results of the aerodynamic and vibration analysis are used as boundary conditions for the XFEM-computation.

Fig. 6 and Fig. 7 show the simulation results for the displacement field and temperature field. Both fields exhibit the expected jump over the crack surface.

# 6. Conclusions

Multiple simulations were performed, comparing the performance of a new and a used high-pressure turbine blade. The blade surface geometry was scanned from real blades. The aerodynamic analyses show a significant change in the pressure and temperature distributions on the blade surface, resulting in an increase of work delivered by the blade. The change in pressure distribution leads to a different relative excitation of the blade modes, and an increased loading overall for the used blade geometry. The structural properties differ significantly as well. Additionally, a change in mistuning sensitivity and forced response amplitudes is shown. Finally, the simulation of the influence of a crack on the leading edge is demonstrated. By combining different simulations as presented, the turbine blades can be analyzed more thoroughly to gain comprehensive insights into the functional properties of turbine blades with different geometry due to wear. This is a step towards informing decisions during maintenance and design of these blades.

## 7. Outlook

To achieve the goal of predicting the functional properties of turbine blades during use and after repair with good accuracy Further investigations have to be done as to extend the fluidstructure interaction to include aerodynamic damping. The distribution of real cracks will be measured or modeled. In future regenerated blades will be analyzed, as well as the combination of blades in different parts of their life cycle on the same disk, so as to achieve a good comparison to the designed blade.

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#### References

- Rupp O., Instandhaltungskosten bei zivilen Strahltriebwerken, Deutscher Luft- und Raumfahrtkongress 2001, Hamburg DGLR-2001-008; 2001.
- [2] Aschenbruck, J., Adamczuk, R., and Seume, J., 2014, Recent Progress in Turbine Blade and Compressor Blisk Regeneration. Proceedings of 3rd International Conference on Through-life Engineering Services, November 4-5 2014, Cranfield, England.
- [3] Vahdati, M., Sayma, A., and Imegrun, M., 2000, An Integrated Nonlinear Approach for Turbomachinery Forced Response Prediction. Part II: Case Studies. Journal of Fluids and Structures, Vol. 14(1), pp. 103125.
- [4] Brard, C., Green, J., and Imregun, M., 2003, Low-Engine-Order Excitation Mechanisms in Axial-Flow Turbomachinery. Journal of Propulsion and Power, Vol. 2003(19), pp. 704712.
- [5] Di Mare, L., Imregun, M., Smith, A., and Elliott, R., 2007, A Numerical Study of High Pressure Turbine Forced Response in the Presence of Damaged Nozzle Guide Vanes. Aeronautical Journal, Vol. 111 / 3177, pp. 751757.
- [6] Meyer, M., Parchem, R., and Davison, P., 2011, Prediction of Turbine Rotor Blade Forcing due to in-service Stator Vane Trailing Edge Damage. Proceedings of ASME Turbo Expo, June 6-10 2011, Vancouver, British Columbia, Canada, GT2011-45204.
- [7] Petrov, E., Di Mare, L., Hennings, H., and Elliott, R., 2010, Forced Response of Mistuned Bladed Disks in Gas Flow: A Comparative Study of Predictions and Full-Scale Experimental Results. Journal of Engineering for Gas Turbines and Power, Vol. 132(5) / 052504.
- [8] Aschenbruck, J., Meinzer, C., Pohle, L., Panning-von Scheidt, L., and Seume, J., 2013, Regeneration-induced Forced Response in Axial Turbines. Proceedings of ASME Turbo Expo, June 3-7 2013, San Antonio, Texas, USA, GT2013-95431.
- [9] Aschenbruck, J., and Seume, J., 2015, Experimentally Verified Study of Regeneration-Induced Forced Response in Axial Turbines. ASME Journal of Turbomachinery, Vol. 137(3) / 031006.

- [10] Vo, H., 2006, Role of Tip Clearance Flow in the Generation of Non-Synchronous Vibrations. 44th AIAA Aerospace Sciences Meething and Exhibit, January 9-12 2006, Reno, Nevada, USA, AIAA 2006-629.
- [11] Thomassin, J., Vo, H., and Mureithi, N., 2009, Blade Tip Clearance Flow and Compressor Nonsynchronous Vibrations: The Jet Core Feedback Theory as the Coupling Mechanism. ASME Journal of Turbomachinery, Vol. 131(1) / 011013.
- [12] Drolet, M., Vo, H., and Mureithi, N., 2011, Effect of Tip Clearance on the Prediction of Non-Synchronous Vibrations in Axial Compressors. Proceedings of ASME Turbo Expo, June 6-10 2011, Vancouver, British Columbia, Canada, GT2011-45392.
- [13] Im, H.-S., and Zha, G.-C., 2012, Simulation of Non-Synchronous Blade Vibration of an Axial Compressor Using a Fully Coupled Fluid/Structure Interaction. Proceedings of ASME Turbo Expo, June 11-15 2012, Copenhagen, Denmark, GT2012-68150.
- [14] Juengst, M., Holzinger, F., Wartzek, F., Brandstetter, C., Mller, D., and Schiffer, H.-P., 2015, Analysis of Blade Vibrations in a 1.5-Stage Transonic Compressor. Proceedings of the 14th International Symposium of Unsteady Aerodynamics, Aeroacoustics Aeroelasticity of Turbomachines, ISUAAAT14, Stockholm, Sweden, 114-S9-1.
- [15] Heyes, F. J. G., Hodson, H. P., and Dailey, G. M., 1991, The Effect of Blade Tip Geometry on the Tip Leakage Flow in Axial Turbine Cascades. Proceedings of ASME Turbo Expo, June 3-6 1991, Orlando, USA, 91-GT-135.
- [16] Krishnababu, S. K., Newton, P. J., Dawes, W. N., Lock, G. D., Hodson, H. P., Hannis, J., and Whitney, C., 2007, Aero-Thermal Investigations of Tip Leakage Flow in Axial Flow Turbines Part i Effect of Tip Geometry and Tip Clearance Gap. Proceedings of ASME Turbo Expo, May 14-17 2007, Montreal, Canada, GT2007-27954.
- [17] Hauptmann, T., Aschenbruck, J., Christ, P., Hennecke, C., Dinkelacker, F., and Seume, J., 2015, Influence of Combustion Chamber Defects on the Forced Response Behavior of Turbine Blades Proceedings of the 14th Internation Symposium on Unsteady Aerodynamics Aeroacoustics & Aeroelasticity of Turbomachines, September 8-11 2015, Stockholm, Sweden, 114-56-2.
- [18] Castanier M. P., and Pierre C., 2006, Modeling and Analysis of Mistuned Bladed Disk Vibration: Current Status and Emerging Directions, Journal

of Propulsion and Power, 22(2), pp. 384396.

- [19] Madden A., Epureanu B. I., and Filippi S., 2012, Reduced-Order Modeling Approach for Blisks with Large Mass, Stiffness, and Geometric Mistuning, AIAA Journal, 50(2), pp. 366-374.
- [20] Sinha A., 2009, Reduced-Order Model of a Bladed Rotor With Geometric Mistuning, J. Turbomach., 131(3), p. 31007.
- [21] Hohl A., Siewert C., Panning L., and Wallaschek J., A Substructure Based Reduced Order Model for Mistuned Bladed Disks, ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, pp. 899-906.
- [22] Bampton M. C. C., and Craig, JR., R. R., 1968, Coupling of substructures for dynamic analyses, AIAA Journal, 6(7), pp. 1313-1319.
- [23] Martinez D. R., Carne T. G., Gregory D. L., and Miller A. K., 1984, Combined experimental/analytical modeling using component mode synthesis, 25th Structures, Structural Dynamics and Materials Conference, pp. 140/152.
- [24] Donders S., Pluymers B., Ragnarsson P., Hadjit R., and Desmet W., 2010, The wave-based substructuring approach for the efficient description of interface dynamics in substructuring, Journal of Sound and Vibration, 329(8), pp. 1062-1080.
- [25] Kielb J. J., and Abhari R. S., 2003, Experimental Study of Aerodynamic and Structural Damping in a Full-Scale Rotating Turbine, J. Eng. Gas Turbines Power, 125(1), p. 102.
- [26] Duflot M., 2008, The extended finite element method in thermoelastic fracture mechanics, Int. Journal for Numerical Methods in Engineering
- [27] Sukumar N., Moös N., Moran B., Belytschko T., 2000, Extended finite element method for three-dimensional crack modeling, Int. Journal for Numerical Methods in Engineering
- [28] Castanier M. P., and Pierre C., 1997, Consideration on the benefits of intentional blade mistuning for the forced response of turbomachinery rotors, Analysis and design issues for modern aerospace vehicles- 1997, pp. 419-425.
- [29] Renyan Jiang, 2013, A modified MPS method for fitting the 3-parameter Weibull distribution, International Conference on Quality, Reliability, Risk, Maintenance and Safety Engineering (QR2MSE), pp. 983-985.