A Combined Wavelet and ARIMA Approach to Predicting Financial Time Series

Ken Bailey

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Ken Bailey

Dublin City University

School of Computing

Supervisor: Dr. Mark Roantree, School of Computing

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Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Master of Science is entirely my own work, and that I have exercised reasonable care to ensure that the work is original, and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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Abstract

Agri-data analysis is growing rapidly with many parts of the agri-sector using analytics as part of their decision making process. In Ireland, the agri-food sector contributes significant income to the economy and agri-data analytics will become increasingly important in terms of both protecting and expanding this market. However, without a high degree of accuracy, predictions are unusable. Online data for use in analytics has been shown to have significant advantages, mainly due to frequency of updates and to the low cost of data instances. However, agri decision makers must properly interpret fluctuations in data when, for example, they use data mining to forecast prices for their products in the short and medium term. In this work, we present a data mining approach which includes wavelet analysis to provide more accurate analysis of when events which may appear to be outliers, are instead patterns representing events that may occur over the duration of the data stream and then are used for predictions by an ARIMA modelling approach. Our evaluation shows an improvement over more established uses of wavelet analysis in conjunction with ARIMA as we attempt to predict prices using agri-data.

Chapter 1

Introduction

A well-known parlour game is to ask people if they were a superhero what ability would they like to possess. Often the two main contenders are the power of invisibility or the power of flight. They both have clear attractions and applications but we all know they are impossible. However, one ability which we would all like to possess is the ability to tell the future. Moreover, it is an ability that we think we can possess, if only we understood how events in the past created the future. The question is therefore: how do we analyse the events of the past and quantify how they affected the future so that if those events recur, we can have a predictor for a likely future event.

In this dissertation we apply the technique of wavelet analysis to pre-process a signal of financial time series data into constituent series, as in the work of Bailey et al. 2017. [5]. These constituent series are modelled with ARIMA methods to produce predictions of future values of the constituent series. Predicted values of the original time series are created from recombining the predictions of the constituent series using an inverse wavelet transform process. Similar approaches have been applied in [10] and [44] among others, which are detailed in chapter 2.

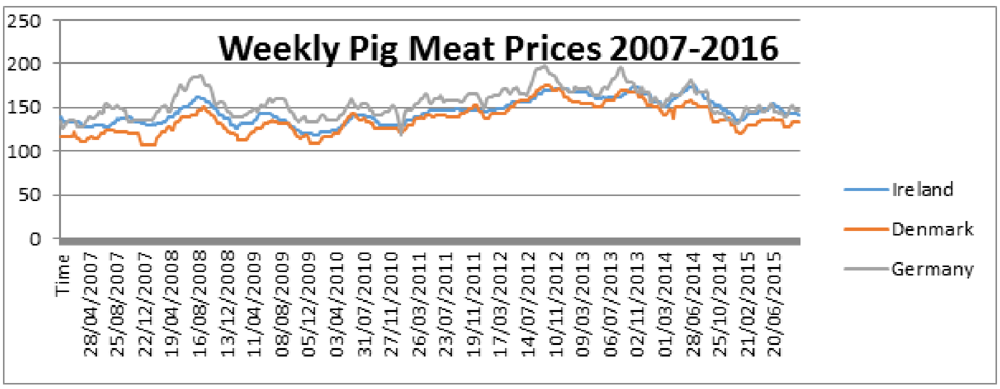
In the opening chapter to this dissertation, we will provide background to this research in §1.1 with an introduction of the domain area; in §1.2, we examine those aspects of time series predictions that present specific problems; we then present our research goals and hypothesis in §1.3, before summarising and outlining the remainder of the dissertation in §1.4.

1.1 Background

In all walks of life we wish to make good, well-informed decisions. It is difficult to make good decisions but probably even harder to make well-informed ones. To be well informed we need information. Information about our areas of interest is constructed by processing raw data into a usable format relating to this area of interest. The process of gathering together and extracting information from raw data is *Data Mining*. In this work, we are exploring and combining two distinct approaches to data mining to extract information from our dataset, *wavelet analysis* and *ARIMA modelling and prediction*.

Machine Learning is an approach within the field of data mining [41]. It is the process of using software to analyse data from a given dataset and identify patterns within it. From these patterns predictions of the future behaviour of the dataset can be produced. We make use of machine learning to find patterns in historical pigmeat prices to predict the future behaviour of these prices.

Figure 1.1: Graph of Irish, German and Danish Pigmeat Prices



This dissertation uses datasets from agricultural sectors from around Europe. Figure 1.1 is a line graph representation of the pigmeat prices, expressed in Euro per tonne, in Ireland, Germany and Denmark for the period from January 2007 to October 2015. This data has been sampled weekly from on-line sources.

From this graphical representation of our dataset we can intuitively see some of the characteristics of typical time series data. There appear to be some indications of regular patterns with similar rising and falling occurring, but there are also sudden movements, shifting the patterns of the series upward or downward. There is undoubtedly overlapping of patterns, in effect cancelling one another out. If we can separate and isolate this regular and irregular behaviour in the series, we can model and predict each of them separately and produce a combined model that replicates the overall series behaviour into the future.

A characteristic of this dataset, which is of particular relevance to our research, is that the period of the time series is reasonably long, almost 9 years, but there are only 459 values for each series. This low frequency characteristic is a distinguishing factor of the dataset and is one which we explore in our experiments and contrast against other high frequency datasets.

In the June 2011 Central Statistics Office Livestock Survey [12], there were 1.56 million pigs in Ireland, which represents an increase of over 2% on prior year levels. In 2011, Ireland exported an estimated 168,000 tonnes worth approximately Eur395 million. In 2011, the UK was the main market for Irish pig meat taking over 46% of our total exports. Continental EU markets accounted for 28% of our pig meat exports while the remaining 26% went to international markets.

Future prices are regularly used to construct agricultural commodity forecast prices. Both grain elevators and livestock packer buyers forward price off the board, generally using a number of formulas, [36]. These methods are largely used in the United States where well traded future prices exist. Forecasts can be made on commodity future prices, however, long term accuracy and reliability are often found to be weak.

Additionally, European markets do not possess high volume trading meat or dairy futures and while prices from the US do correlate with European commodity prices the relationship is not strong enough to be relied upon on a commercial basis. In many cases, this is caused by market access to individual countries and irregularities within individual markets.

Typically, production of animals for a particular product is very much dependent on individual regulations within a country. Much of this knowledge can be found in on-line data sources. A recent study [11] looked at using on-line prices to create their own database (or index) for analysis and research purposes. This research highlighted some interesting advantages in using on-line data in this manner. The obvious advantage is the low cost per data element (or observation). There is a cost to harvesting data from websites but this is far cheaper than visiting factories or paying for commercial databases. Further advantages are the speed of access to frequently changing data and the high volumes which make it easier to detect errors in the data.

The access to this data provides us with an input to produce future predictions. The key is to identify a process which can use this data to produce accurate future prices. This is the aim of our dissertation.

1.2 Time Series Analysis

There have been many approaches to time series analysis and prediction such as exponential smoothing [26], Autoregressive Integrated Moving Average (ARIMA) modelling [8], state space models [35], non-linerar models [58] etc. A comprehensive survey of time series prediction methods is presented in [17]. To be able to predict a time series, you first need to model the behaviour of previous values of the series. When a good fit is found future values can be extrapolated from the existing time series. ARIMA is a widely used modelling and prediction technique for time series since the 1970s.

A model is affected by the input data used to formulate it. Typically, there will be jumps and discontinuities in the data which make it difficult to model the underlying behaviour. Wavelets provide a mechanism to decompose a time series into a set of smoother series, isolating regular patterns. This is crucial to enabling accurate modelling. ARIMA models are more effective on such better behaved series.

1.2.1 ARIMA: Autoregressive Integrated Moving Average

The ARIMA approach to model and enable prediction of time series evolved out of earlier work on *exponential smoothing*. These approaches recognised that modelling could not be achieved purely through deterministic approaches but that time series should be regarded as a stochastic process where modelling a time series is based on a probabilistic model.

The research presented in [8] is an updated version of the of the work first presented by [9] which described how to determine an appropriate ARIMA model for a given time series. The ARIMA model has gained popularity in the field of time series prediction and has been applied widely in prediction in fields as diverse as monthly tourism demand [21] and commercial property rental values [55].

The parameters associated with an ARIMA model are introduced later in this work in §3.1. The identification of these parameters is key to specifying an accurate model for a time series and thus facilitating the production of accurate prediction values. Our approach is to use *wavelets* to help improve the accuracy of the prediction by creating simpler time series from the original series that are more amenable to ARIMA modelling.

A typical example of the application of ARIMA models to financial time series forecasting is in [2]. The author uses historical gold bullion prices in Malaysia to predict future prices. They chose this method as they state that it is the most widely used forecasting method for time series. Their method involved trial and error to identify the parameters for the model. The experiments resulted in a Mean Absolute Percentage Error (MAPE) of less than 10% with which they found acceptable.

The authors point out a limitation of ARIMA models, which is very relevant for our work, in that ARIMA modelling requires a reasonably large sample to produce accurate forecasts. We will see in our dissertation that a Discrete Wavelet Transform (DWT) reduces the number of input values during decomposition which may affect modelling from the decomposed series. Our approach using a Maximal Overlap Discrete Wavelet Transform (MODWT) provides a constant and sufficient number of input values to an ARIMA model.

1.2.2 An Approach Using Wavelets

Wavelets are an approach to signal analysis which identify patterns of frequencies at different time scales occurring in a signal. The widely regarded seminal work on wavelets from 1992 is [16]. The wavelet approach applies a well-defined mathematical transform to observed values to produce details of frequencies within the dataset while retaining the location of the frequencies in the time domain. In an application where the signal can be observed with a very large number of samples, such as a sound wave, a very fine resolution form of the wavelet transform, the Continuous Wavelet Transform (CWT) can be applied. In [20] a CWT is used on a signal with over 100,000 data points in the field of peak detection in mass spectrometry. Where the observations are more sparse such as in a financial time series, measured hourly, daily or weekly, the DWT is more appropriately applied [15] as there are invariably too few observations to use with a CWT.

The DWT uses two filters which are applied to the data. Fundamentally, the filters are a pair of orthogonal vectors. Wavelet decomposition of a signal represents the signal in the space defined by the filter. When chosen appropriately, the filter represents the signal in terms of *high* frequency and *low* frequency components. This representation is in the form of a set of coefficients. The low frequency coefficients can be decomposed repeatedly in the same fashion by the wavelet filter to the level of resolution required. The most commonly used wavelet filters are the Haar [28] filter and Daubechies filters [16].

This processing of a signal identifies and isolates the patterns in the signal at different scales, removing them from the series to leave a smoother underlying trend series. These component series can then be manipulated with a method appropriate to the research before being reconstructed to a new version of the signal to compare against the original signal.

Each iteration or level of the DWT on N values produces a set of $N/2$ values, producing a *decimated* transform. Therefore, the limitation on the number of levels L that can be produced is $2^L \leq N$. In addition, coefficients at each level are out of phase with respect to each other in the DWT with the result that decomposed coefficients are different depending on the starting point chosen within the time series.

When dealing with time series analysis, these limitations can be overcome by using the Maximal Overlap Discrete Wavelet Transform (MODWT) [15]. MODWT came out of the work of [48]. This is a non-decimated transform which produces N coefficients at each level of decomposition. The MODWT uses the same parameters as the DWT to specify the filter being used and the number of levels of decomposition required.

The MODWT addresses some of the issues associated with the DWT which are of particular relevance to time series prediction. Having a constant high number of coefficients makes each of these component series amenable to time series analysis individually with features in the data remaining synchronised across the decomposition levels. The resolution at each scale remains constant making it easier to identify the patterns in the data at each scale [15]. The requirement to have a sufficient number of values for ARIMA modelling is more adequately met by the non-decimated MODWT than the decimated DWT.

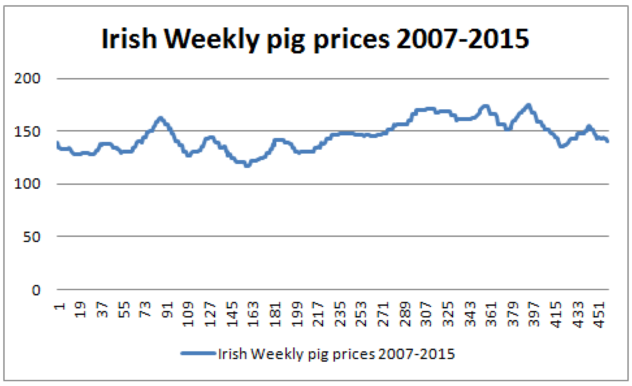
We have seen that ARIMA models are a widely-used method for modelling time series and provide a means to predict future values. We have also seen that wavelets decompose a signal into series in different frequency ranges. We will now see in §1.2.3 how combining these two techniques can lead to more robust predictions.

1.2.3 Wavelets with ARIMA

We have seen that ARIMA methods and Wavelet Transforms have been developed independently in the 1970s and 1990s. The combination of wavelets and ARIMA uses the features of these two techniques to enhance the accuracy of predictions for time series.

The seemingly complicated movements in a series as in Figure 1.2 can be simplified by *wavelet analysis* decomposing the series into simpler series each representing frequencies at different scales. These simpler series make it easier to identify the parameters for ARIMA models at each of these levels of decomposition. Predictions from the ARIMA models at each level of decomposition can be recombined by the use of the *inverse wavelet transform* to provide predictions for the original series. This provides us with more accurate predictions for the original series than using an ARIMA on the original series alone.

Figure 1.2: Graph of Irish Pig Meat Prices



The use of an MODWT provides more robust input for ARIMA modelling than the DWT by providing a consistent number of values to construct the ARIMA models. Shorter prediction windows, which are more reliable from ARIMA models, are facilitated by this feature of MODWT by providing more predicted coefficients at the deepest level to feed into the inverse transform. The DWT is limited for predictive purposes for use with an ARIMA. The levels of of decomposition due to its decimated property, reduces the number of coefficients by a factor of 2 each time, resulting in fewer input values to an ARIMA model, limiting the accuracy of the model for describing the series and providing predictions.

1.3 Hypothesis and Research Goals

The ability to predict financial time series has very clear benefits in that it enables decision makers in many and disparate markets to predict how their product will behave in the future. Our area of interest is the agri-sector. As an example of this sector we have taken samples of data relating to the weekly prices of pigmeat in Ireland, Germany and Denmark in a period from 2007-2015. We are seeking to explore these time series to find patterns in them and predict future pigmeat prices for a period of 8 weeks.

The hypothesis for this research is that a combination of ARIMA modelling with Wavelet Analysis provides a more robust prediction methodology than ARIMA modelling alone for univariate time series. In order to evaluate our hypothesis, it will be necessary deliver on a number of research goals:

- To interpret and present modelling of a time series and to predict future values with ARIMA.
- To highlight the particular difficulties of producing an ARIMA model for non-stationary data.
- To demonstrate how Wavelets partially resolve some of the difficulties in ARIMA modelling.

- To compare and contrast the MODWT and DWT approaches to time series prediction with ARIMA.
- To develop our own approach to combining MODWT and ARIMA for time series data prediction.
- To evaluate our approach with a direct comparison with other approaches.

Approach Our approach is to explore research into financial time series prediction which combines two disparate techniques. We use *wavelet analysis* to find sets of patterns in the past values of a time series. We model these patterns using a widely-used approach of *ARIMA modelling* which can then produce predictions based on these historical patterns. Wavelets provide us with a facility to recombine these predictions to produce a combined prediction from these patterns for future values of our initial financial time series. We propose that preprocessing data using Wavelets improves the predictive power of ARIMA models of time series. Further, we investigate whether the Discrete Wavelet Transform (DWT) or the Maximal Overlap Discrete Wavelet Transform (MODWT) is the better preprocessor of the data. These two transforms are compared on low frequency and high frequency datasets to investigate their relative effectiveness with each.

In order to do this, we investigate the technique introduced by [10] which used a high frequency dataset for forecasting electricity day ahead prices using the DWT. We then apply this technique to a lower frequency dataset, weekly pig prices, with a prediction window of 8 weeks.

At this point, we can compare the DWT method with our own method, which uses the non-decimated MODWT. We apply the two techniques to the two different datasets with a view to comparing the methods to identify a reliable forecasting method for low frequency data such as weekly price data and high frequency data such as hourly price data.

1.4 Summary and Approach

The goal of predicting accurate future values of financial time series is a desirable one. The readily accessible historical data for commodities provides the inputs needed to identify patterns in the past which may repeat into the future.

We combine a technique to identify the patterns in a dataset with a model to predict the future of these patterns. We investigate this combined approach and apply it to low and high frequency datasets.

The remainder of the dissertation is structured as follows:

- In chapter 2, we provide a discussion on the use of wavelets and their application in predicting financial time series.

- In chapter 3 we introduce and explain the concepts and terminology of ARIMA and Wavelets that underlie this research.

- In chapter 4, we present our method to create predicted prices from a time series and by describing a Wavelet/ARIMA model generated from Maximal Overlap Discrete Wavelet Transform combined with ARIMA modelling to generate predicted prices.

- In chapter 5 we present the experiments that were conducted to compare the predictive power of the ARIMA models produced by the pre-processing of the data by the Discrete Wavelet Transform and Maximal Overlap Discrete Wavelet Transform methods, against each other and against ARIMA predictions alone.

- Finally in chapter 6, we conclude the dissertation and offer our proposals for future work.

Chapter 2

Related Research

This chapter discusses research which uses ARIMA modelling and Wavelet Analysis techniques. For those who are unfamiliar with these techniques, chapter 3 provides an introduction to them.

The formal definition and application of Autoregressive Integrated Moving Averages (ARIMA) models is credited to the authors of [9] in 1970 who built on earlier work in linear forecasting in [37]. ARIMA has become widely used in itself and has spawned many variants such as ARARMA [47] for univariate series, VARIMA for multivariate applications [6] and transfer models when there is more than one input variable [22].

The rise of the popularity of the use of wavelets to analyse signals is usually credited to the work of Daubechies [16]. Wavelets have been applied in applications as diverse as video compression [30] using the Discrete Wavelet Transform (DWT) and medical imaging [49] and seismology [13] using the Continuous Wavelet Transform (CWT). The application of wavelet analysis to financial applications has been popularised by the work in [48] in 2000 and has been applied to many types of financial series since then.

Wavelets are used to isolate frequencies, identify lags within and between series. To use wavelets for prediction requires the combination with time series forecasting techniques. Ramsey [51] states that wavelets have the potential to improve the results from time series prediction methods. In [54], Shafie et al. maintain that the technique of combining wavelets with an ARIMA can be more accurate than predictions from an ARIMA model on the original series alone as the subseries generated from the wavelet decomposition have a more stable variance and no outliers.

Schleicher in [52] provides a non-mathematical introduction to wavelets with a more applied focus on financial time series. Their view is that the benefit of wavelets is to reveal features in the original time series at different scales which are possibly hidden in the original time series. The decomposed levels themselves which are in the form of sets of coefficients form time series which are then more amenable to techniques such as ARIMA.

Research undertaken into the use of wavelets for time series forecasting follow a basic underlying technique. Each series is decomposed using a DWT or a Maximal Overlap Discrete Wavelet Transform (MODWT). The decomposed levels of the series are then manipulated according to the approach undertaken by the researchers to produce forecasts for each decomposition level. The forecasts are then recombined to provide a forecast model for the original series.

The various research approaches differ on 4 parameters which distinguish them: type of dataset used; transform and the filter used; how the decomposed levels are manipulated and predictions created; and the reconstruction process. It is necessary for us to consider different combinations of models and transforms in an attempt to understand the method most suited to our requirements and dataset.

In §2.1, we discuss research that combines ARIMA with the Discrete Wavelet Transform; we then discuss the usage of ARIMA with Maximal Overlap Discrete Wavelet Transform in §2.2; and finally in §2.3, we examine other prediction methods used with wavelet transforms.

2.1 Discrete Wavelet Transform with ARIMA

In this section, we consider research that applies the DWT and ARIMA models to time series prediction. ARIMA models provide a means to model a time series. Using an ARIMA model, predictions for future values of the series can be constructed.

In [10], Conejo et al. have proposed a method for forecasting day ahead electricity prices in the Spanish market. At the time of the paper, the Spanish market was a duopoly with one dominant player whose behaviour primarily dictated the price movements in the market.

The rationale for their methodology is that the decomposition of the historical electricity price series using wavelets results in a less volatile set of representations of the behaviour of the original time series at several scales. ARIMA models of these constituent series are proposed to be more stable for forecasting techniques.

The authors used the hourly market prices from the year 2002. The experiments sought to predict the day ahead hourly prices for each of 4 weeks evenly spaced across the course of the year as representative of the price behaviour of the price time series. The weeks were labelled Spring, Summer, Autumn(Fall) and Winter.

Each day of the target weeks was predicted from the preceding 48 days of actual hourly prices. There were thus 1152 hourly prices in the training set to predict the 24 prices of the next day. The training set was a sliding window across the testing week. The training dataset was thus a high frequency dataset. The predicted prices for each hour of a particular day were compared with the actual hourly prices for that day. Two measures were used for the comparison - a daily error and a weekly error which the authors defined.

The prediction generated from their wavelet/ARIMA model were compared against 2 other prediction mechanisms. The first used predictions generated from the training set using an ARIMA model of the training set. The other comparator was termed a naive predictor which was the corresponding hourly price from the previous week.

The authors used the DWT to decompose the original series. They made use of a Daubechies [16] filter of width 10, that is, it samples 10 values from the series at a time. They declare their reasons for choosing this particular filter is that Daubechies is a commonly used filter. The width of the filter was chosen as a balance between the smoothness of the filter, as its width increases, against the loss of detail from a filter sampling a large number of values each time.

Having decomposed the series into its detail and trend coefficients to 3 levels of decomposition, the authors apply a 'suitable' ARIMA to model each series and produce the predicted values for each decomposed series using the related model. The series is then reconstituted using the inverse wavelet transform. The authors do not describe how the inverse transform is used with the predictions, which is a crucial component of reconstructing predictions. It is stated that there are implementations of the transforms widely available in software, but they do not specify which they used for their method.

To measure the accuracy of their predictions, they devised their own metrics in terms of a *daily error* and a *weekly error* by comparing their predictions for hourly prices to the actual hourly prices for the prediction window and compared these with the errors from the ARIMA only method and the naive method.

The authors recognise that the prediction window should not be too wide as the prediction error is noted to be greater the further into the prediction window - the prediction for hour 24 has a greater error than for hour 1. However, they do not specify the limitations of the DWT in respect of the prediction window where the number of predictions that can be produced is limited by the number of decomposition levels and width of the filter.

Overall, their results are encouraging and show that a wavelet/ARIMA approach can provide better predictions than an ARIMA prediction approach alone. However the lack of detail of the process in terms of the DWT Inverse Transform method and the construction of the ARIMA models for the original series and decomposed series makes replicating the experiment difficult.

In [39], Kumar et al. investigated the time series of prices in the energy sector. In particular, they focus on 2 companies Gas Authority of India Limited (GAIL) and Oil and Natural Gas Corporation Limited (ONGC). GAIL is the largest natural gas processing and distribution company in India and ONGC is an Indian multinational oil and gas company. The dataset is drawn from the daily closing prices for each company's products over a period from January 2012 to January 2015. The training set is the series of closing prices from January 2012 to December 2013 and the testing set is January 2014. It is not clear how many values are in each set, but there is likely to be approximately 520 values in the training set and 22 in the testing set assuming there are closing prices on 5 days of the week.

The authors use the DWT transform using the MATLAB package. They repeated their experiments using a series of Daubechies wavelet filters *db2, db3, db4, db5, db7* to find the optimal filter which they deduced was a db2 filter with 3 levels of decomposition.

The authors describe the process for identifying an ARIMA model manually, but it is not clear if this is followed or a MATLAB function is used. The reconstruction using the DWT inverse transform is used, but again it is not specified how the prediction values are fed into the transform. Experiments were run on the 2 sets of training data, one from each company, and were measured against the 2 testing sets using 3 measures: Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). The results are presented for the db2 filter with 3 levels of decomposition which the authors identified as producing the optimal results.

There are only 2 test runs for this process which limits the ability to measure its overall effectiveness. Furthermore, it is unclear how the DWT and ARIMA are actually applied, whether MATLAB functions alone or some manual process was used.

In [1], the area of interest of Ababneh et al. is the stock market prices in Jordan, using the historical data of daily returns from April 1993 to December 2009. One of their concerns is to ensure that their dataset is *dyadic*, that is the cardinality of the dataset is a power of 2. In this case they use 4096 historical values. The dyadic restriction is confined to the DWT, but is really only a concern when decomposing to a number of levels approaching the DWT limit of $2^L \leq N$ where L is the number of levels and N is the cardinality of the dataset as introduced in §1.2.2.

In this work, the authors use both a Haar and a Daubechies filter to decompose their data but do not indicate which width of Daubechies filter they use. The authors decompose the data to 3 levels of decomposition but it is unclear how they use the 2 filters as there is only one set of experiments.

The technique to choose an ARIMA to apply to the original series and the decomposed series is to use trial and error of ARIMA models for each series with p,d,q parameters (explained in 3.1) chosen for each combination between (0,0,0) and (2,2,2). They select the optimum ARIMA model for each series by calculating the RMSE for each model and choosing the model generating the smallest RMSE. Their conclusion is that the DWT with ARIMA provides improved predictions over those generated by an ARIMA model alone but there is a lack of detail for the process and they use only one experiment. The authors measure their predictions using one experiment and make their comparison with ARIMA predictions on the training set using the RMSE and the Mean Absolute Percentage Error (MAPE). However, in terms of reusing this work or reconstructing their experiments, it is unclear what form of inverse wavelet transform is used to reconstruct the series used for the final prediction set.

In [53], Seo et al. use a slight variation on the process of combining a DWT and ARIMA for prediction. One of the other uses of wavelets is to de-noise a signal. This is achieved by removing the coefficients of the decomposed series with values outside a certain range. The authors use this as part of their method.

The dataset for their paper is concerned with short-term wind speed volatility which may affect the production of electrical power from wind energy. They assert as in other work in [40] that physical methods which take into account local geography and climate conditions are used to model long term wind patterns but that statistical approaches are more suited to short term low frequency data.

The authors use the Haar filter based on the findings of other similar forecasting studies. They decompose the data to 3 levels of decomposition. The novelty of their approach is that for each level of decomposition they de-noise the data in each of the detail series before fitting the series with an ARIMA model and making predictions. This is done by excluding detail coefficients which are outside dynamically constructed intervals for each detail series.

Their ARIMA(p,d,q) models are derived through a process of trial and error on the parameters p,d,q. The set of parameters which produced the model with the least error as measured by the Residual Sum of Squares is chosen. The residuals are the differences between the actual values and the values which the model produces.

The dataset consisted of 300 values although the time interval for the values is not specified. The data was split into a 200 values training set and a 100 values testing set. The authors cite an improvement of 10% in the error of their method and an ARIMA only method.

2.1.1 Summary

Overall, each of the papers reviewed which use a technique of combining the prediction of ARIMA models of a time series with the Discrete Wavelet Transform to provide smoother series for the ARIMA process, report positive results for the technique. The use of measures appropriate to their own datasets makes it difficult to cross compare the efficacy of their implementations. None of the papers provide a detailed description at a low level of the method used, making their method difficult to replicate.

The method to reconstruct the final set of predictions from the decomposed series' predictions using the inverse wavelet transform is not specified. In some papers, there appears to be an approach that decomposed levels can simply be added together to reconstruct a signal. This method can only be implemented with the Haar filter. In contrast, we will provide a detailed description of our method and show in detail how the formal process of reconstructing a series must be followed and the steps required to achieve it.

2.2 Maximal Overlap Discrete Wavelet Transform with ARIMA

In this section, we review research that has been undertaken using the MODWT and an ARIMA for modelling and prediction.

Nguyen et al. [44] point out the limitations of using the DWT for forecasting. They make use of a non-decimated form of the discrete wavelet transform, the MODWT, or Redundant Wavelet Transform as they refer to it. They highlight that the decimated form of transform, DWT, halves the number of coefficients produced at each subsequent decomposition. The non-decimated MODWT results in the same number of coefficients as the original series at each level of decomposition to feed into a predictive model.

Kriechbaumer et al. in [38] state that the DWT is restricted to time series with a length that is a power of 2, that is, dyadic. This requirement only becomes relevant when a researcher wishes to fully decompose a series. From the point of view of prediction, this is usually not the case as in the case of Conejo et al. [10] who use a series of length 1152, which is not dyadic, with a DWT.

Gencay et al. in [27] show that the MODWT transform can handle a series of any length. Jammazi et al. in [33] state that the MODWT produces smoother approximations to the original time series.

Nguyen et al. in [44] and Jin et al. in [34], also advocate the use of the Haar filter over other filters such as Daubechies [16], Symlet (a symmetrical form of Daubechies filter) or Coiflet [57]. They point out that other filters use succeeding values within a series whereas the Haar filter uses only the current value and preceding values. This feature makes the Haar more appropriate to prediction as the filter does not require estimated values beyond the end of the original series which the other filters achieve by reflecting the original series or treating the series as periodic and using the values from the beginning of the series.

In [40], Liu et al. provide an interesting variation on the method to produce the predictions for a decomposed series from a wavelet transform. The authors do not state that they use an MODWT but the decomposed levels which are presented seem to show the same number of coefficients as the originating series, which occurs in the MODWT but not the DWT. The authors dataset concerns prediction of wind speed. The dataset is a set of 400 values of actual samples in time. There appear to be many of these sets collected from multiple recording sites, although only one set of experiments is reported. The experiments appear to be repeated in a sliding window using 150 values in the training set. There are 3 prediction sets produced for each training set consisting of 3, 5 and 10 prediction values. A Daubechies 6 filter is used with the MODWT.

The novelty of the method is in the use of an *improved time series method* (ITSM) to produce the predictions for the constituent decomposed series. A training set of 150 values is decomposed by the wavelet transform. An ARIMA model is used to predict the first predicted value from the 150 decomposed coefficient series. This value is appended to the decomposed coefficient series, the ARIMA is remodelled for this extended series of 151 values and then used to predict the next term. This process is repeated to get the set of 10 prediction values. After producing 10 predictions for the original series from the first 150 values, the experiment is repeated starting from the second position in the series. The results of four methods are compared. The methods are Wavelets with ITSM/ARIMA, ITSM/ARIMA on the original series, ARIMA on the original series and an Artificial Neural Network (ANN) method. The methods are compared using MAE, Mean Square Error (MSE) and MAPE. The author's method produces a lower error for each measure and for each prediction window.

What is missing from this research is a comparison with a Wavelet and ARIMA model to assess whether the considerable extra effort to produce each prediction value using an ITSM improves the predictions generated from a wavelet decomposition.

Kriechbaumer et al. [38] use the MODWT and ARIMA predictions from the decomposed series. They employed a series of experiments to identify the choice of filter and level of decomposition that provided the least error with their dataset. The authors point out that in the case of the DWT, the choice of filter had a greater effect on the sensitivity of the results than the MODWT.

Their dataset concerns monthly metal prices for Copper, Zinc, Lead and Aluminium. The dataset used 628 monthly prices from 1960 to 2012. The authors sought to identify which filter from a set of 25 filters applied with the DWT and MODWT to each of 1 to 9 levels of decomposition produced the least error as measured by the MAPE and RMSE. The prediction periods were tested from 1 to 12 months. From this process they sought to identify the most appropriate transform, filter and decomposition level. The experiments were repeated for each of the 4 metals.

The results of their tests show that there is no single combination of wavelet parameters which consistently produce the best predictions across the different time series. This 'one size does not fit all' feature of wavelet transformation is a common theme with different authors citing best results with different filters and decomposition levels such as 3 levels in [10] with a Daubechies 10 filter, 5 levels of decomposition with Daubechies 7 filter in [59] and 7 levels with a symlet s8 filter in [24].

However, the authors identify the MODWT as consistently more appropriate for their decomposition than the DWT. The method of reconstruction in [38] of the series after prediction of the decomposition levels does not make use of the inverse wavelet transform for the filter which created the decomposition. Instead they make use of a feature of the most basic filter, Haar, which uniquely, allows reconstruction of a series by simply summing corresponding decomposed coefficients. This feature does not apply to any other filter.

2.2.1 Summary

In this section, we have shown the widespread application of the MODWT transform to analysing financial data. The application of the MODWT has shown itself to provide an improved input to ARIMA models in the reviewed research and the authors of these works have attested to this. From the timeline of the papers looked at in the past two sections, the DWT was initially the transform used in this research area, but the MODWT has become more prevalent in more recent research.

In trying to identify the parameters of wavelet filter and level of decomposition to use with the MODWT and optimum prediction windows, more recent studies have used the approach of applying a large range of the available options for these parameters, possibly due to the speed of modern hardware. The authors have sought out the combination of parameters which results in the least error. No one combination of parameters is optimum for all datasets. This opens a line of investigation for us to find the filter with the *least* error for predictions in our approach.

2.3 Other Prediction Methods with Wavelet Transforms

ARIMA models are widely used for prediction and we have seen many applications of the DWT and MODWT to create more stable series for an ARIMA approach to produce a data model. Other authors have sought to enhance these methods. We briefly review two of the techniques applied including Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) and Artificial Neural Networks.

Some time series can be more volatile than others, that is, at certain times there can be an unusually dense grouping of large increases and decreases in the values of the series. In [23], Engle labels this as *volatility clustering*. This can be assessed using the error terms in a model. Homoskedasticity is the characteristic that the variances of the error terms in a model of a time series are the same at any point. Heteroskedasticity is the term for when this is not the case and ARCH/GARCH modelling is used to model this volatility in the series by modelling the errors.

In [61], Zhongfu et al. use a similar approach to Conejo et al. [10] to forecast day ahead electricity prices. The same dataset, Spanish hourly electricity prices, is used with which to compare their own method. They use a DWT, but use a slightly different filter, Daubechies 4, without stating their reason. They decompose to 3 levels. The authors note that the dataset contains very rapid changes and postulate that the error terms in any modelling of the series may benefit from being modelled with a GARCH process.

Their method takes the method in [10] and examines each of the decomposition levels. Thus GARCH models are applied to model the errors in the ARIMA models for each decomposed series and predictions produced as a result of the combined process. The wavelet reconstruction series created from these amended wavelet subseries are used for prediction as before. Using the same datasets used as those in [10], the authors cite improved results.

The high frequency nature of the data used in [10] and [61] of hourly electricity prices lends itself to the application of GARCH models to the errors of the ARIMA models rather than in the case of our low frequency data as this high frequency data contains dense grouping of large increases and decreases in the values of the series which ours does not.

Artificial Neural Networks (ANN) [7] mimic the functioning of the neural network of a brain which continually changes its structure, which is based on old inputs, as it learns from new inputs. An ANN can be used to model data and produce predictions which can change as new inputs are added to the model, self-correcting itself. The individual predictions of an ANN model are not necessarily based on the entire dataset but use samples of the data.

In [4], the research of Amjady et al. concerns predicting day ahead electricity prices. The approach applied takes into account the influence of exogenous variables on the decomposed levels of the wavelet transformation using the inputs of price, load and available generation capacity using an ANN with an evolutionary algorithm.

The authors make use of the Discrete Wavelet Transform to decompose the original time series to 3 levels of decomposition, the filter is not specified. The process combines lagged values of the wavelet subseries and time domain features of the subseries to include lags from the original time series and the exogenous variables.

The approach further deviates from the models we have reviewed so far, in that ARIMA is not used to model the data and produce predictions for each decomposed level. Their process, instead, includes selecting the most appropriate variable by a feature selection technique based on the data model. The prediction values for the next 24 hours are produced by a set of 24 cascaded forecasters. Multiple candidate forecast series are created. They are differentiated by validation error. The authors report improved results compared to using ARIMA models alone but have not compared it to the wavelet transform combined with ARIMA alone.

The authors explicitly state that they make use of the inverse wavelet transform applied to produce the overall forecast series from the predictions for each decomposed level.

2.3.1 Summary

In this section, we have seen some of the variations that authors have applied to the technique of wavelet transform combined with ARIMA models. The key to their use is the dataset being used and whether, for example, in the case of GARCH the dataset exhibits the volatility for which a GARCH model is appropriate or whether the identification of exogenous variables might support an ANN approach.

2.4 Summary

In this chapter, we have shown that ARIMA models and Wavelet Transforms have solid foundations in research literature and in practical applications. The combination of techniques from these two research streams have been widely used in different application areas and we have seen many examples applied to financial time series.

The research has shown an evolution from using the DWT to using the MODWT, especially when authors have been using datasets with a lower frequency, sampled weekly or monthly. Throughout, the tailoring of the techniques to the particular dataset through the choice of wavelet filter and decomposition levels has been based on a trial and error to find the best fit to the particular application. Similar domains such as metal prices analysed in Kreichbaumer et al. [38] require different choices of filter and decomposition to get the best predictive results.

We have identified that most of the research has not been specific in how it has applied ARIMA predictions to the decomposed series and none have given details of how they have reconstructed the original series. In some cases, this lack of clarity, leads us to suppose that the authors have not applied the inverse transform to reconstruct the series. We will provide detailed methods for combining the wavelet transform with ARIMA modelling to produce predictions in Chapter 4. But first, in the next chapter, we will provide some of the technical background to ARIMA modelling, the wavelet transform and demonstrate with an example our reasons for selecting the MODWT as our transform.

Chapter 3

Background to our Approach

In this chapter, we present explanations of the techniques underpinning our methodology for incorporating wavelet analysis into predictive algorithms. We have chosen a process that uses ARIMA models for prediction as they are widely used and respected - we explain their features in this chapter. We have also chosen to use an MODWT as the wavelet transform as it is more suited to decomposing a time series for use with a prediction model than a DWT and we explain why in this chapter.

An *Autoregressive Integrated Moving Average (ARIMA)* model provides us with a description of the relationship of values in the time series to previous values. This facilitates the predictions of future unknown values for the time series.

A Maximal Overlap Discrete Wavelet Transform (MODWT) analysis of a signal splits the signal into several better behaved constituent series which can then each be modelled with a specific ARIMA model and predictions generated for each constituent series. Recombining these predictions using an *inverse transform* produces predictions for the original time series.

We begin by covering some background work necessary to understand the ARIMA model in §3.1. The Fourier Transform is a widely used technique for identifying frequencies in signals. It is introduced in §3.2 along with its limitations. The evolution of the DWT and MODWT to meet the limitations of the Fourier Transform approach is introduced in §3.3. We provide a detailed description of the decomposition of a series using an extensive example of DWT and MODWT applied to a simple series which illustrates what happens at each level of decomposition and why we have chosen an MODWT approach. Finally a brief summary of the chapter is provided in §3.4.

3.1 ARIMA Fundamentals

A time series consists of a set of observations related to some particular measurement which are equally spaced in time. A univariate Autoregressive Integrated Moving Average(ARIMA) model [8] represents a time series in the form of an *algebraic statement* describing how values of the variable are statistically related to past values of the same variable [46].

A value $\hat{y_t}$ is related to previous values of the series as described in Def.1 for a stationary series. A *stationary series* is one whose mean, variance, autocorrelations etc. are all constant in time [19]. A stationary series is more amenable to modelling.

Definition 1 *ARIMA Model for Stationary series*

$\hat{y_t} = \mu + \phi_1 y_{t-1} + ... + \phi_p y_{t-p} - \theta_1 e_{t-1} - ... - \theta_q e_{t-q}$

where μ is the mean, ϕ are the auto-regressive parameters, θ are the moving average parameters and e are the error terms.

The ARIMA model requires the specification of the 3 parameters termed *(p,d,q)*: p, the number of past observations required to define the auto regression; d, the levels of differencing to produce a stationary form of the series; and q, the number of past observations required in the moving average. There should be at least 50 past values in a series to use an ARIMA model for prediction of future series values [46].

Once the methods are chosen for identifying the most appropriate values for *(p,d,q)* e.g. in [8] and [46], the ARIMA model can be used to forecast future values with the general forecast equation for a ARIMA(p,0,q) as shown in Def. 1.

The estimation of the p, d, and q parameters is crucial to producing an accurate model for the series. The Auto-Correlation Function (ACF) measures the correlation between a value in the time series and its preceding values at successively higher lags between terms. The Partial Auto-Correlation Function measures the correlation between a value in the time series and a preceding value while adjusting for the correlation from intervening terms, that is, the PACF between a term at x_t and a term at x_{t-3} is the correlation that is not explained by their common correlations with the intervening terms x_{t-1} and x_{t-2}.

Normally, the correct amount of differencing, d, is the lowest order of differencing that yields a time series which fluctuates around a well-defined mean value and whose autocorrelation function (ACF) decays fairly rapidly to zero [43].

When a series is made stationary by differencing, the p and q parameters can be estimated by balancing the effect of amending each on the ACF and PACF of the subsequent series. [43] gives a set of 13 rules to identify the most appropriate combination of values for p, d, and q.

Trial and error combined with domain knowledge is required to specify an ARIMA model. The ARIMA functions in *R* [50] test combinations of values to identify an optimum ARIMA model.

The ARIMA modelling of a series produces a new series with a set of values matching the original series as closely as possible. the differences between the original series and the ARIMA model values are termed *residuals*.

The ARIMA model can be used to predict future values of the series. Each predicted value is calculated using the specific model parameters discovered in the modelling process applied to preceding values. This process is repeated for further future values using the previous predicted values. [43] warns about the potential inaccuracy of predicting too far ahead using ARIMA models.

In *R*, a *forecast* or *predict* function can be used to predict future values using an ARIMA model of the time series as input.

ARIMA models provide a means to model a time series and predict future values. In order to do this the parameters of the ARIMA model need to be accurately identified. Having a smooth series makes the identification of the parameters more readily achievable.

3.2 Identifying Frequencies

A technique in mathematics which has many applications is to represent a function in terms of other functions. The purpose of this transformation is to represent the initial function *f(t)* in terms of more useful or easily managed functions $\phi(t)$ called basis functions. Formally, we can represent this as equation 3.1.

$$f(t) = \sum_{i=1}^{n} c_i \phi_i(t) \tag{3.1}$$

The important values then become the coefficients c_i in 3.1 from which we hope to be able to learn some characteristics of the data or manipulate them more easily than the original series. The characteristics of the basis functions may vary depending on the application, but we specifically want them to be easily invertible and be well-defined over the domain of the functions *f*.

The functions we are dealing with in this dissertation represent a sequence (or signal) from a particular domain which have values at regular intervals. In our case it is a time series of prices of a commodity sampled hourly, daily, weekly etc. So the order of the values in the time series is important and this order is identified by the time at which the values occur.

The technique of wavelet analysis can be considered as a progression from earlier work such as Fourier Analysis which identifies the frequencies present in the signal. Wavelet analysis has sought to overcome some of the shortcomings of this earlier work by providing the means to locate frequencies in the time domain.

3.2.1 Fourier Transform

A widely-used representation of a signal using basis functions is the Fourier Transform [25] which is of the form of Definition 2.

Definition 2 *Fourier Transform* $F(\xi) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i\xi t}dt$

where $F(\xi)$ is the Fourier transform representation of the original signal in terms of the frequencies in the signal ξ from a projection onto the basis functions $e^{-2\pi i\xi t}$. with the inverse transform in Definition 3

Definition 3 *Inverse Fourier Transform* $f(t) = \int_{-\infty}^{\infty} F(\xi)e^{2\pi i\xi t}d\xi$

The Fourier Transform represents a function in terms of sine and cosine functions. Any signal can be represented by a linear combination of sine functions of different frequencies. A Fourier Transform representation can model even the most discontinuous of functions by using a potentially infinite number of frequencies to model the discontinuities.

The Fourier Transform has had countless applications in, for example, the fields of Image Compression, Optics, Seismology, Crystallography etc. However, when the field of interest is in the location of frequencies at a particular point in time, the Fourier Transform by itself cannot identify both the frequency and the time location. This is because a *sine* wave is defined over all possible time, that is its *support* is infinite.

3.2.2 Short Term Fourier Transform

If we need to locate frequencies in time, we can use a technique called Short Term Fourier Transform to transform a signal using a Fourier Transform in small windows.

This identifies the frequencies in the window. However, as the window gets smaller, there are fewer samples from the signal to determine the frequencies. The possible frequencies (ω) increase in number as the location in time (τ) becomes more accurate. This trade-off is an example of the Heisenberg Inequality shown in equation 3.2 where $\Delta\tau$ represents a small change in time and $\Delta\omega$ represents a small change in frequency.

$$\Delta\tau\Delta\omega > 1 \tag{3.2}$$

We cannot get a precise representation of the frequencies at a particular point in time with a Short Term Fourier Transform.

As we have seen, the Fourier Transform is a widely used technique which identifies the frequencies contained in a signal. However it cannot identify the locations of the frequencies in time. When analysing a signal in which we want to identify frequencies localised in time we want to represent the signal in terms of basis functions which are compactly supported, that is have a value in a small window and 0 elsewhere, but can vary in width to capture sufficient samples to identify the frequencies in the signal. The Wavelet Transform enables us to do this.

3.3 The Wavelet Transform

The Continuous Wavelet Transform (CWT) is a transform that is applied to signals modelling a continuous process, such as a seismological tremor, with a very large number of data points.

For discrete signals, such as a financial time series, there are two primary techniques for applying a wavelet transform - the Discrete Wavelet Transform (DWT) and the Maximal Overlap Discrete Wavelet Transform (MODWT). Although very similar, the two transforms differ on some key features which are especially relevant to financial time series prediction [15]. This section introduces some of the foundations of wavelets and provides a detailed example which illustrates the key differences between the DWT and MODWT which are key to why we chose the MODWT for our predictions.

3.3.1 DWT and MODWT

A dataset whether in the form of a continuous signal such as sound or a set of discrete points in a financial time series contains a number of underlying frequencies. Frequencies can extract crucial information that is hidden in the aggregated data. The development of *wavelets* provides a mechanism to identify the frequencies in a signal at different times. A wavelet transform takes a signal and *magnifies* the high frequencies at shorter intervals while *smoothing* the remaining low frequencies. This process is repeated on the low frequency component to the desired number of levels which are particular to the domain from which the data set is drawn, with each level representing the frequencies at higher scales.

Definition 4 *DWT Father & Mother wavelets*
$\int \phi(x) = 1$ *and* $\int \psi(x) = 0$

Definition 5 *DWT Father & Mother functions*
$\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$
$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$

In the DWT case, the transform consists of two functions, a father ϕ and mother ψ wavelet as shown in Def. 4 with a more formal specification shown in Def. 5, where j = 1 . . . J in a J-level decomposition is the *scale* parameter and k is the *shift* parameter.

The functions are applied to the data at successively smaller scales, typically reducing the detail by 50% each time. At each level of decomposition, the mother wavelet captures the high frequency components in a zero mean series while the father wavelet captures the remaining smoother components.

In this dissertation, we mostly make use of the Haar filter [28] and to a lesser extent the Daubechies filter [16], to extract different frequency levels together with their location in time.

As an illustration, the mother and father wavelets of the Haar transform are represented in Defs. 6 and 7 respectively.

Definition 6 *Haar-Mother-Transform*

$$\psi(x) = \begin{cases} 1 & for \quad x \in [0, 0.5] \\ -1 & for \quad x \in (0.5, 1] \\ 0 & for \quad x \notin [0, 1] \end{cases}$$

Definition 7 *Haar-Father-Transform*

$$\phi(x) = \begin{cases} 1 & for \quad x \in [0, 1] \\ 0 & for \quad x \notin [0, 1] \end{cases}$$

The result of a DWT decomposition is 2 vectors representing the wavelet coefficients for the high frequencies (detail) and scaling coefficients for the low frequencies (trend) of the scale. The coefficients relating to the high frequencies at level 1 are denoted by D_1 and the low frequency coefficients by S_1. The S_1 coefficients are further decomposed into D_2 and S_2. The trend series S_2 is further decomposed into D_3 and S_3 and so on. The resulting set of coefficients for a J-level decomposition is known as a crystal, as in Def. 8.

Definition 8 *Decomposition Crystal*

$$\{S_J, D_J, D_{J-1}, D_{J-2}, ..., D_1\}$$

The DWT representation of a discrete signal f(t) is shown in Def. 9, where $s_{j,k}$ and $d_{j,k}$ are the coefficients from S_J and D_J respectively. The Haar wavelet samples data points 2 at a time and the coefficient is generated from a sample value and its preceding value. In the case of the first value in the time series, the previous value is taken from the end of the series (periodicity) or from a reflection of the series. The number of coefficients affected by this *boundary problem* increases at each level of decomposition. In our case, the transform used, MODWT, locates the affected boundary coefficients at the beginning of the series and thus, do not affect the coefficients at the end of the series which are primarily required for prediction.

Definition 9 *DWT of a Discrete Signal*
$f(t) = \sum_k s_{j,k}\phi_{j,k}(t) + \sum_k d_{j,k}\psi_{j,k}(t) + ... \sum_k d_{1,k}\psi_{1,k}(t)$

where $j = 1, 2...J$ and
$s_{j,k} \approx \int x(t)\phi_{j,k}(t)dt$ and $d_{j,k} \approx \int x(t)\psi_{j,k}(t)dt$

The MODWT addresses some of the issues with the DWT but is a non-orthogonal redundant transform [48]. The MODWT is a non-decimated transform meaning that the number of coefficients produced at each level has the same cardinality as the original time series, providing for more robust ARIMA modelling. The DWT coefficients contain enough information to reconstruct a series, while the MODWT contains this information but it is repeated as each neighbouring coefficient is constructed from overlapping time series values as we will see in our next section.

3.3.2 Example of DWT and MODWT

To more clearly understand the process of the decomposition of a series and the distinctions between the DWT and MODWT, we will step through a simple illustrative example. In this section, we work through applying the DWT and MODWT to a simple series to illustrate the coefficients that are produced at each stage of a wavelet transform decomposition using a simple set of sample values.

DWT Example

Consider the simple series of values:
$X(t) = \{4, 6, 5, 8, 9, 6, 7, 8, 5, 4, 6, 9, 8, 10, 11, 12, 14, 12, 11, 13\}$

The Haar father wavelet is $(1, 1)/2^{1/2}$ and the mother wavelet is $(1, -1)/2^{1/2}$. Multiplying the father wavelet by the first and second element of the series produces the first low frequency coefficient. Similarly multiplying the mother wavelet by the first and second element of the series produces the first high frequency coefficient. The next coefficient is produced from the third and fourth element of the series etc.

Applying the DWT function with the Haar filter to the whole series X to 1 level of decomposition, we get the coefficients in 3.1. The alignment of the coefficients to the original values comes from the decomposition by the *R dwt* function.

Table 3.1: HAAR DWT 1 Level Decomposition

t	1	2	3	4	5	...	17	18	19	20
X(t)	4	6	5	8	9	...	14	12	11	13
S1 coefficients		7.07		9.19		...		18.38		16.97
D1 coefficients	1.41		2.12		-2.12	...	-1.41		1.41	

Using the Haar filter, the father wavelet gives us the average of successive pairs of terms at each scale which is the low frequency trend. The mother wavelet yields the difference between successive terms which is the high frequency detail.

As can be seen, the number of coefficients produced for the trend and detail parts of the signal is half the number of elements in the original series. If we repeat the process to 2 levels, the coefficients produced are in Table 3.2.

Table 3.2: HAAR DWT 2 Level Decomposition

t	1	2	3	4	5	...	17	18	19	20
X(t)	4	6	5	8	9	...	14	12	11	13
S2 coefficients				11.5		...				20
D2 coefficients		1.5				...		-1		
D1 coefficients	1.41		2.12		-2.12	...	-1.41		1.41	

These coefficients are produced by decomposing the trend part of the signal *S1* from the first level decomposition using the Haar filter. As before this halves the number of coefficients, from the input signal, in the output signal.

MODWT Example

Consider again the same initial sample series

$X(t) = \{4, 6, 5, 8, 9, 6, 7, 8, 5, 4, 6, 9, 8, 10, 11, 12, 14, 12, 11, 13\}$

The first level decompositions of the signal using the MODWT with the Haar filter are as in Table 3.3. Each set of decomposed coefficients has the same number of values as the original series. This continues through each level of decomposition.

Table 3.3: HAAR MODWT 1 Level Decomposition

t	1	2	3	4	5	...	17	18	19	20
X(t)	4	6	5	8	9	...	14	12	11	13
S1 coefficients	8.5	5	5.5	6.5	8.5	...	13	13	11.5	12
D1 coefficients	-4.5	1	-0.5	1.5	0.5	...	1	-1	-0.5	1

The second level coefficients of the MODWT with the Haar filter are as in Table 3.4.

Table 3.4: HAAR MODWT 2 Level Decomposition

t	1	2	3	4	5	...	17	18	19	20
X(t)	4	6	5	8	9	...	14	12	11	13
S2 coefficients	10	8.5	7	5.75	7	...	11.75	12.25	12.25	12.5
D2 coefficients	-1.5	-3.5	-1.5	0.75	1.5	...	1.25	0.75	-0.75	-0.5
D1 coefficients	-4.5	1	-0.5	1.5	0.5	...	1	-1	-0.5	1

The coefficients are produced by multiplying the filter by the first and second element, then multiplying the filter by the second and third element and so on. There are therefore the same number of coefficients as there are values in the input series.

The first coefficient in each decomposed level is obtained in one of two ways as there is no preceding value to which to apply the filter along with the first value of the series. The two techniques most commonly used involve multiplying the filter by a vector consisting of the final element of the series and the first element (periodicity) or by multiplying the filter by a vector consisting of the first element and the first element again (reflection). We use periodicity here.

As can readily be seen, at each level of decomposition, the original series can be retrieved by adding the corresponding detail coefficients and the trend coefficient.

This can only be done with the Haar filter.

For example, in table 3.5, with a Daubechies 4 filter the MODWT coefficients for 1 level of decomposition are presented.

Table 3.5: Daubechies 4 MODWT 1 Level Decomposition

t	1	2	3	4	5	...	17	18	19	20
X(t)	4	6	5	8	9	...	14	12	11	13
S1 coefficients	9.70	5.47	4.70	6.27	8.05	...	12.71	13.27	11.98	11.57
D1 coefficients	-0.02	2.75	-3.48	0.66	-1.18	...	-0.09	0.02	1.27	-0.62

From table, 3.5, it can clearly be seen that for the Daubechies filter, the original series cannot be recovered by a simple addition of corresponding coefficients. This is the case for every filter other than Haar.

In the general case, the original series is recovered by multiplying the trend and detail coefficients at the deepest level of decomposition by the inverse matrix of the wavelet filter to produce the trend signal at the previous level of decomposition and repeating through the decomposition levels with the detail coefficients and the corresponding reconstructed trend coefficients. Our methods cover both of these cases.

For an orthogonal matrix H, as in this case, the inverse of the matrix H^{-1} is the same as the transpose of the matrix H^T. The inverse transformation uses the inverse of the original filter matrix to reconstruct the series.

The process of decomposition of a series using wavelets produces a number of sets of detail and trend coefficients. Each of these forms a distinct series which isolate the frequencies at different scales or time periods. For example, in a 6-level decomposition the detail series represent the frequencies in table 3.6.

Table 3.6: Frequencies captured by Wavelet Detail Coefficient Series

Detail Coefficients	Frequencies	For example: weekly prices
D1	2-4	2-4 weeks
D2	4-8	1-2 months
D3	8-16	2-4 months
D4	16-32	4-7 months
D5	32-64	7-15 months
D6	64-128	15-29 months

The level of decomposition is chosen to reflect the frequencies of most interest.

The decomposition of a signal extracts the high frequency details of the frequencies from the original signal as a series of the detail coefficients and the low frequency trend as the trend (scale) coefficients series. The trend series is repeatedly decomposed.

For example, using the Irish pig meat prices, a decomposition of the series using a MODWT transform with a Daubechies 4 filter to 6 levels of decomposition is represented in Figure 3.1. This example was chosen to highlight the smoothing feature of the wavelet transform over repeated decomposition.

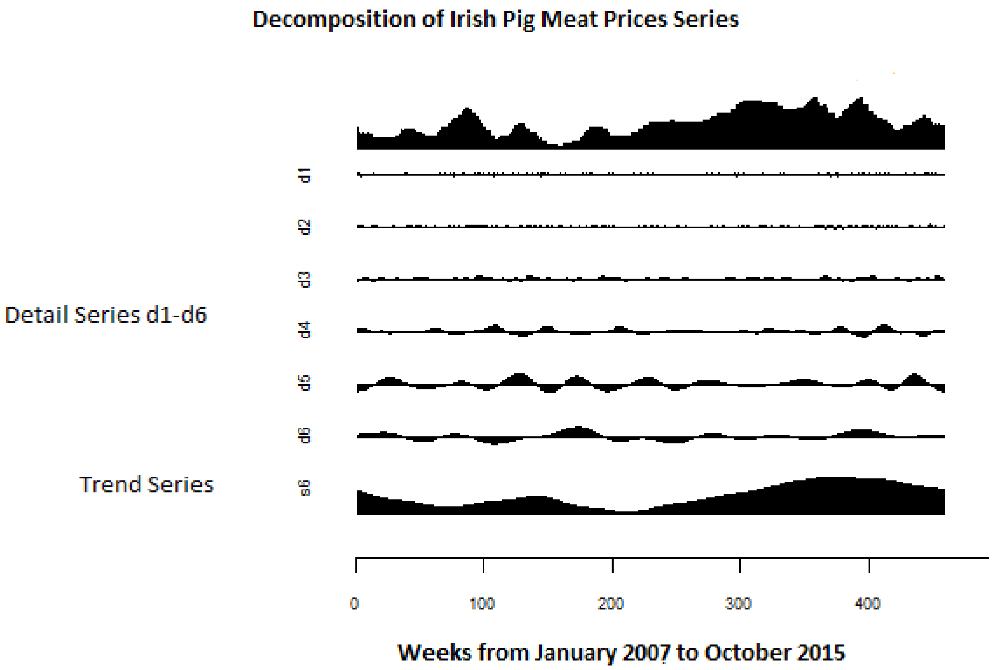
The original series is represented at the top of the figure. Each of the sets of detail coefficients at increasing scale is graphed as an individual time series for each scale. The remaining trend series, the last graph in the figure, is a smooth representation of the original series.

3.4 Summary

In this chapter we introduced the techniques of ARIMA modelling and Wavelet Decomposition of discrete time series.

An Auto Regressive Integrated Moving Average (ARIMA) is a widely used tech-

Figure 3.1: Graphs of detail and trend series in a 6 level D4 decomposition of Irish Pigmeat Prices



nique for modelling a time series. The model seeks to identify parameters which reproduce each successive term from previous ones. The 3 parameters which are denoted by p, d, q describe the number of terms p involved in the *autoregression*, the number of *differencing* operations d required to make the series *stationary* and the number of terms q involved in the *moving average* component of the model. The optimal combination of p, d, q can be then used to produce future values of the series.

A Fourier Transform can identify frequencies in a signal but loses the time domain information.

For time series we need to retain the time domain information and identify the frequencies and to do this we can use *Wavelets*. They overcome the limitations of earlier work on identifying frequencies by identifying the location of frequencies in the time domain. This is achieved by decomposing the signal into a number of constituent series at increasing scales which identify the patterns or frequencies in the data over different time intervals.

When using wavelet transforms with time series we must make several choices. We can use a DWT or MODWT. We can use one of a range of filters. We must decide the number of level of decomposition, each level extracting different frequencies.

The DWT uses a pair of orthogonal filters to separate a signal into its high frequency *detail* series and its low frequency *trend* series, each with half the number of values of the originating series. The trend series can be repeatedly decomposed to a desired level of decomposition to identify patterns at intervals appropriate to the particular time series being investigated.

The MODWT is similar to the DWT. Its main benefit for time series prediction is that each decomposition level series has the same cardinality allowing for alignment of decomposed coefficients more readily and better accuracy for modelling with ARIMA due to having more coefficients at each decomposed level than a DWT for input to the ARIMA model.

A range of filters such as Haar and Daubechies have been developed. They differ in the detail they can model and their width. The width of the filter specifies the number of values from a series which are sampled by the filter in the production of each decomposed coefficient. One filter may be more appropriate to a particular time series and trial and error may be required to identify the most appropriate filter.

The approach of using wavelets to decompose a series into a number of constituent series before applying ARIMA models results in the identification of the ARIMA parameters more readily for the constituent series than for the original series making the prediction values more reliable.

The MODWT is a more robust wavelet transform than the DWT when dealing with generating predictions from an ARIMA as it maintains the cardinality of the original series in each of the decomposed series it generates and locates boundary coefficients at the beginning of the decomposed series thereby not affecting the values at the end of the series that are used for prediction. Our approach using an inverse transform on the predicted values of the decomposed series to generate the predictions for the original time series is the more appropriate process to generate predictions in a time series.

We are now ready to construct our method using an MODWT transform to provide smoother input values for ARIMA modelling and correctly reconstruct our predictions from the predicted decomposed values using the Inverse Transform. This is our MAM method which is described in the next chapter.

Chapter 4

Building the MAM method

In this chapter, we describe in detail our MODWT ARIMA Method (MAM) for producing predictions of future values for a time series. In §4.1 we give an outline of the reasoning and functioning of the MAM process in general and describe the particular agri-data dataset which forms the basis of our implementation of the process and how we divided it into our Training and Testing sets. In section §4.2 we give a step by step guide to how our process works along with a detailed walkthrough using a subset of our dataset to show exactly what is happening at each stage of the method. Finally, we summarise this chapter in §4.3.

4.1 The MAM Process Introduction

In this section, we provide a more detailed account of the MAM process. We have seen that an ARIMA can be used to model a time series and produce predictions. We have also shown how an MODWT can decompose a series into constituent series and can be used to reconstruct a series from decomposed levels. Our process combines these two processes to produce more accurate predictions and is described in detail both here and in [5].

4.1.1 Outline of the Process

The modelling of time series using ARIMA models is a standard time series analysis tool for modelling non-stationary data. It can capture auto-regressive behaviour between terms and lags in the series which can be used predict future values of the series.

The approach of using wavelets is to separate out the patterns which can occur at certain intervals or frequencies. These patterns may help to explain data items which might otherwise be considered as outliers. This is done by extracting each of the constituent series of details at increasing scales and a smoother remainder series of the trend of the series. Each of these constituent series is more readily modelled by an ARIMA model.

Our method, called MODWT ARIMA Model (MAM), uses the Maximal Overlap Discrete Wavelet Transform (MODWT) to decompose a signal, in our specific case, weekly pig meat prices, into multiple series capturing its constituent frequencies to a desired level of decomposition. The decomposition is represented as sets of coefficients for each of the detail series and the trend series. The number of detail series is the level of decomposition.

Using the wavelet decomposition coefficient series, each of the detail and the remainder trend series is modelled separately using an ARIMA model specific to each series. The ARIMA model can be used to predict the future values of the constituent series using the known existing series values. The initial prediction can subsequently be used to predict future values recursively.

The *inverse wavelet transform* reconstructs a signal using the inverse of the filter matrix applied to the trend series and the corresponding detail series from the lowest level of decomposition to produce the trend series for the previous level of decomposition. This process continues until the first level detail and trend series reproduce the original series. If the decomposed series have been altered, the inverse transform produces a signal which would have produced the amended decomposition levels. In this way you can create a new signal from the amended decomposed levels.

When a signal is decomposed with a particular wavelet filter and specified number of levels of decomposition, it must be reconstructed using the inverse wavelet transform which is specific to the filter used for the decomposition. As we saw in §3 uniquely in the case of the *Haar filter*, the reconstruction can be achieved by adding the corresponding coefficients from the trend series and each of the detail series. In all other cases the inverse transform for the filter must be used.

In the MAM process, we model each of the detail series and the trend series by a specific ARIMA model for each series. We use each of these ARIMA models to produce predictions and append them to the decomposed constituent series. We apply the inverse transform to these extended constituent series. The inverse transform applied to these extended series produces a signal which contains a set of values beyond the end of the original decomposed series. This set of values are our MAM predicted values for the original series.

As is typical in Data Mining experiments, our original series of prices is divided into a training set and a testing set. The training set is decomposed using the MODWT with an appropriate filter. Each of the series produced by the decomposition is modelled separately with an appropriate ARIMA model and predictions for each constituent series produced. The number of predicted values matches the cardinality of the testing set. The constituent series and its prediction set constructs an extended constituent series.

The extended constituent series are passed through the *Inverse Wavelet Transform* to produce one new series with the same cardinality as the original series. This new series consist of the training set, amended by the wavelet process, which has been extended by a set of MAM predicted values.

The training set from the original series is modelled with an ARIMA model. The ARIMA model is used to produce a Prediction set with the same cardinality as the Testing set.

The predicted values output from the MAM process and the predicted values from an ARIMA model of the original series are each measured against the Testing set to investigate whether there is a difference in accuracy between the MAM process and ARIMA modelling alone.

4.1.2 Sample Data to Illustrate the MAM process

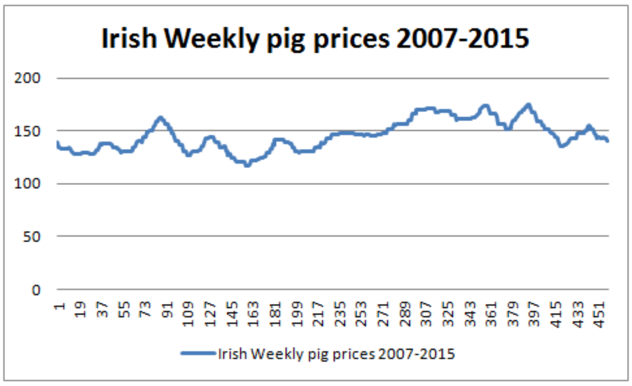
In order to explain fully the MAM process we will walk through the process using a sample of the data used in the experiments in chapter 5.
The datasets which we used to test our MAM process are the weekly pig meat prices from Ireland, Germany and Denmark from January 2007 to October 2015, a total of 459 weeks. A sample from the Irish dataset contains the values in Table 4.1. This shows the pigmeat price at the end of the sample weeks specified in the table to give an indication of the behaviour of the series.

Table 4.1: Sample of Irish Pigmeat Prices per Tonne

Week	1	2	3	...	229	230	231	...	457	458	459
Price	139.8	134.6	134.2	...	146.7	146.3	146.6	...	143	142.9	140.7

As a reminder for the reader, figure 1.2 is reproduced here as figure 4.1. This represents the weekly Irish pigmeat prices per tonne from January 2007 to October 2015. The movement in the prices is clearly not uniform but contains some clear cyclical patterns. There are sudden movements upwards and downwards with a general move upwards in the middle of the data.

Figure 4.1: Graph of Irish Pig Meat Prices



We used the dataset of 459 values in evaluating our process. We used time series cross validation to sample this data with replacement into 21 training sets of 299 values and 21 testing sets of the next 8 values for each training set. We chose a prediction window of 8 values as a useful period of prediction for the dataset that could test the effectiveness of our process and be beneficial to typical users of the dataset.

Each training set is modelled with an ARIMA which is specific to the training set. The ARIMA models for each training set are used to produce predictions for that training set. These are one set of outputs from the experiments which are compared to the actual prices contained in the Testing set. These will form a comparison against the prediction values produced by the MAM process itself.

The MAM process takes each training set in turn. A training set is decomposed using the MODWT with an appropriate filter to the desired level of decomposition. Each of the series of detail coefficients and the final trend series produced are modelled by an ARIMA specific to each of these series. Future values are predicted for each series using the ARIMA model. The number of future values, in this case 8, matches the cardinality of the testing set.

Each constituent series is extended by appending its predicted values to the end of the constituent series. The inverse MODWT is applied to these extended series to produce a MAM series. The MAM series has a cardinality greater than the training set. The MAM prediction for the future prices are these extra values located at the end of the MAM series. So, a training set of 291 values is decomposed, 8 values are predicted for each decomposition and the reconstructed series from the decomposed series extended by the predictions produces a new series of 299 values. The final 8 values are the MAM predictions.

The two sets of predictions from the training set ARIMA and the MAM process are compared against the testing set using a root mean square error (RMSE) test in Definition 10.

As an example, the first Training set is presented in Table 4.2. The first 291 values from weeks 1 to 291 form the training set. The values from weeks 292-299 are the testing set. For the second experiment the training set is from weeks 9 to 299 and the testing set is from weeks 300 to 307. This is repeated 21 times.

Table 4.2: Sample of Training Set and Testing Set

Training Set								
Week	1	2	...	100	101	...	290	291
Values	139.8	134.6	...	140.6	140.6	...	156	156.1
Testing Set								
Week	292	293	294	295	296	297	298	299
Values	156.2	160.2	160.1	160.1	166	166	166.4	166.1

This section has described the MAM process in detail and given some samples from the dataset to indicate how the process works in practice. We will now specify the steps of the MAM process in a more formal way.

4.2 Specification of MAM Steps

This section presents a formal specification of the steps in the MAM process. The first set of steps represent the simpler case using the Haar filter. This is presented first to remove some of the detail from the process related to the more generic case, so that not too much detail is provided in one process. The substitute steps for the generic case are presented separately in the hope of making the process easier to follow.

We make repeated use of the Root Mean Square Error (RMSE) which when dealing with two time series is defined to be the error between the two sets of values in this case the Forecasted values $Z_f = \{z_{f_1}, z_{f_2} ... z_{f_n}\}$ and the Observed values $Z_o = \{z_{o_1}, z_{o_2} ... z_{o_n}\}$.

Definition 10 $RMSE_{fo} = \sqrt{\sum_{i=1}^{N} \frac{(z_{f_i} - z_{o_i})^2}{N}}$

4.2.1 Method 1 Special Case(Haar)

1. Select dataset $X = \{x_1, x_2, .., x_n\}$
2. Select prediction period of width k
3. Divide the data into a training set $T = \{x_1, x_2, ..., x_{n-k}\}$ and a testing set $Q = \{x_{n-k+1}, ...x_n\}$
4. Use a wavelet filter to transform T using a MODWT function to output a crystal $C = \{S_J, D_J, D_{J-1}, D_{J-2}, ...D_1\}$, each element consisting of $n - k$ coefficients
5. Construct an ARIMA for the trend and each detail coefficient series to produce a new crystal $C^a = \{S^a_J, D^a_J, D^a_{J-1}, D^a_{J-2}, ...D^a_1\}$
6. Generate k predictions for each decomposition $P_{sJ}, P_{dJ}...P_{d1}$ from the ARIMA crystal C^a.
7. Reconstruct the MAM prediction series W_p by adding the corresponding terms from $P_{sJ}, P_{dJ}...P_{d1}$
8. Construct ARIMA model for T and predict forward a set of k values T_p. Append T_p to the end of T to produce series P of n values.

9. Compare the Root Mean Square Error (RMSE) of the MAM prediction W_p against the testing set Q and the RMSE of the ARIMA prediction set T_p against the testing set Q.

In Method 1, we formally presented our MAM method for time series prediction, which follows the approach introduced earlier in this chapter. We now explain each of these steps in less formal terms before proceeding to an example.

In Steps 1-3, the dataset of cardinality n is selected and referred to as X, the number of elements, k, to be predicted is decided and the dataset is split into a training set labelled T and testing set Q.

In step 4, the MODWT transform using an appropriate filter, in this case Haar, is applied to the training set T to the level of decomposition required J to produce a crystal C consisting of a series of coefficients the same length as T.

In step 5, a separate ARIMA model is produced for each series of coefficients in the crystal C. Step 6 uses a forecast function for ARIMA models to predict the k future values for each series in the crystal C.

Step 7 produces k predictions W_p for the training set T. In the case of the Haar filter, this can simply be done by adding the corresponding predicted coefficients from the P_{dj} j = 1..J and the corresponding predicted coefficients from P_{sJ}. In the case of filters other than Haar the steps in Method 2 replace this step.

Step 8 constructs an ARIMA model for the training set T and produces a predicted set of values T_p from the ARIMA model.

Step 9 uses the Root Mean Square Error (RMSE) from def. 10 to compare the MAM predictions W_p against the testing set Q and the ARIMA predictions T_p against the training set Q.

4.2.2 Example of Method 1

As an example of Method 1, we use the data from Table 4.2 as an our dataset and work through it for each step of the method.

1. Select dataset

   We use the data from Table 4.2, that is, a set of n = *299* values:

   $X = \{139.8, 134.6, ..., 156, 156.1, 156.2, 160.2, 160.1, 160.1, 166, 166, 166.4, 166.1\}$

2. Select prediction period of width k

   We wish to predict 8 values, so *k=8*.

3. Divide the data into Training set and Testing Set

   We divide the data into a Training set T of cardinality *n-k*, that is *299 - 8 = 291*

   $T = \{139.8, 134.6, ..., 140.6, 140.6, ..., 156, 156.1\}$

   and a Testing set Q of cardinality k = *8*

   $Q = \{156.2, 160.2, 160.1, 160.1, 166, 166, 166.4, 166.1\}$

4. Use a wavelet filter to transform T using a $MODWT$ function

   We use a Haar wavelet with 3 levels of decomposition to produce a crystal $C = \{S_3, D_3, D_2, D_1\}$

   where the coefficients produced are in table 4.3. We can see the original Training Set series from weeks 1 to 291. The coefficients for the 3 detail series *D1, D2 and D3* are shown along with the coefficients for the trend series *S3*. It can readily be seen that the corresponding coefficients from each of the decomposed series sum to the original series value.

Table 4.3: 3 Level Haar Decomposition of Irish pig meat prices First Training set

Week	1	2	...	100	101	...	290	291
Original Series	139.8	134.6	...	140.6	140.6	...	156	156.1
Coefficients								
S3	153.975	151.3075	...	147.8425	145.9075	...	155.9625	155.9825
D3	-1.985	-4.69	...	-4.3175	-4.32	...	.065	.0825
D2	-4.065	-9.4475	...	-2.89	-.9975	...	.0275	0
D1	-8.165	-2.59	...	-.025	0	...	-.015	.025

5. Construct an ARIMA for the trend and each detail coefficient series

6. Generate k predictions for each decomposition

   Using the *forecast* [31] function in *R*, produce a set of *k=8* predictions for each ARIMA model of the coefficient series S_3, D_3, D_2, D_1 as in table 4.4

Table 4.4: 8 Prediction values for the Decomposition Coefficients

Predicted Week	292	293	294	295	296	297	298	299
Coefficients								
S3	156.00	156.02	156.04	156.05	156.07	156.08	156.09	156.10
D3	0.11	0.13	0.16	0.18	0.21	0.23	0.26	0.28
D2	0	0	0	0	0	0	0	0
D1	0.005	-0.012	-0.026	-0.037	-0.046	-0.05	-0.059	-0.063

7. Reconstruct the MAM prediction series W_p, adding the corresponding terms from $P_{sJ}, P_{dJ}...P_{d1}$

   Adding the corresponding coefficients from table 4.4 produces the prediction values for the training set T listed in table 4.5 in the MAM predictions row.

Table 4.5: Training Set and Prediction values

Week	292	293	294	295	296	297	298	299
Testing Set	156.2	160.2	160.1	160.1	166	166	166.4	166.1
MAM Predictions	156.11	156.14	156.17	156.2	156.23	156.26	156.29	156.32
ARIMA Predictions	156.1	156.11	156.11	156.12	156.13	156.13	156.13	156.14

8. Append T_p to the end of T to produce series P of n values. Using the *forecast* function in R predict 8 future values from the Training Set T. The values are presented in table 4.5 in the ARIMA predictions row.

9. Compare the Root Mean Square Error (RMSE) of the MAM prediction W_p against the testing set Q and the RMSE of the ARIMA prediction set T_p against the testing set Q.

   The MAM prediction set and the Testing set Q are contained in table 4.5. The *RMSE* for the MAM predictions is *7.38*.

   The ARIMA prediction values and the Testing set Q are presented in table 4.5. The *RMSE* for the ARIMA predictions is *7.49*.

4.2.3 Method 2 - Generic Case

If a filter other than Haar is used in step 4 in Method 1, replace step 7 in Method 1 with the following called Method 2

- Apply the wavelet transform to the original series X to provide a container for the wavelet prediction W_a, where each decomposed level has n coefficients
- Append each prediction set to the corresponding decomposition e.g. $S_{Jp} = S_J \& P_{sJ}$ to produce a crystal $C_p = \{S_{Jp}, D_{Jp}, ...D_{1p}\}$
- Swap C_p into the container W_a
- Apply the inverse transform to W_a to produce a MAM version of the original series X_M where the last k coefficients are the MAM prediction set W_p.

Method 2 is a modified version of step 7 in Method 1. This is used in the inverse transform procedure for experiments where a non-Haar filter such as a Daubechies filter [16] is used in step 4 of Method 1.

Method 2 uses the Inverse MODWT Transform to construct the predicted series. The MODWT is applied to the complete series X to create a container W_a to hold the coefficients for the inverse transform. The coefficients in the container are replaced at each level j with the corresponding ARIMA model and predictions C_p to produce an amended container W_a. The inverse transform is applied to the amended container W_a to generate the MAM prediction set Q.

4.2.4 Example of Method 2

To illustrate the steps in Method 2 we will use the dataset from the example of Method 1. The filter in this case is a Daubechies filter *d4*, again with 3 levels of decomposition.

Recall that the dataset X of cardinality n=*299* is
$X = \{139.8, 134.6, ..., 156, 156.1, 156.2, 160.2, 160.1, 160.1, 166, 166, 166.4, 166.1\}$
which is divided into a training set T of cardinality m=*291*
$T = \{139.8, 134.6, ..., 140.6, 140.6, ..., 156, 156.1\}$
and a Testing set Q of cardinality $k = 8$
$Q = \{156.2, 160.2, 160.1, 160.1, 166, 166, 166.4, 166.1\}$

The steps in Method 1 are followed up to Step 6 to produce the coefficients in Table 4.6.

Table 4.6: 8 Prediction values for the d4 Decomposition Coefficients

Week	292	293	294	295	296	297	298	299
Coefficients								
S3	156.33	156.37	156.4	156.44	156.47	156.51	156.55	156.58
D3	0.74	0.84	0.92	0.99	1.04	1.08	1.11	1.14
D2	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
D1	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

To enable the use of the Inverse Transform function in *R*, to reconstruct a series of length n=*299* a series of the same length must be decomposed to create a wavelet transform *object* in *R*. This object W forms a *container* which has the required pa-

Table 4.7: Daubechies d4 decomposition in W with the original series

Week	D1	D2	D3	S3	Original
1	2.636	0.445	-1.302	161.118	139.76
2	6.955	4.004	-0.764	160.293	134.58
3	-7.671	6.234	0.158	158.736	134.19
4	-1.566	7.478	1.398	156.478	133.04
5	0.126	2.874	3.469	153.323	133.35
	...	...	...	...	...
159	0.771	0.353	-0.862	119.579	117.5
160	-0.968	0.384	-1.242	119.331	117.59
161	-0.045	0.522	-1.055	119.166	117.84
162	-0.4	-0.002	-0.788	119.051	121.86
	...	...	...	...	...
290	0.041	-0.018	0.459	156.258	156.04
291	0.006	-0.011	0.614	156.294	156.09
292	-0.029	0	1.083	156.292	156.16
293	-0.373	-0.096	1.151	156.488	160.23
294	-0.978	-0.326	0.89	156.76	160.06
295	1.432	-0.636	0.464	157.052	160.06
296	-0.597	-1.133	-0.242	157.595	165.95
297	-1.479	-0.685	-0.69	158.43	166.02
298	1.956	0.335	-1.008	159.496	166.44
299	-0.052	-0.362	-1.305	160.765	166.12

rameters to enable the use of the inverse transform to rebuild a series from a wavelet decomposition. We need only change the coefficients in the container object W to produce W_a before applying the inverse transform to W_a to construct a predicted series.

Table 4.7 consists of a sample of the coefficients S_3, D_3, D_2, D_1 from the Daubechies 4 filter with 3 levels of decomposition of the training set. It can be seen that the coefficients at the beginning of the trend series *S3* are noticeably different from the original series. This is due to the boundary problem where the coefficients are calculated using values from the end of the series.

The 8 predicted coefficients are appended to the ARIMA models of each of the decomposed coefficients series of length *n=291* from the training set T to form a crystal C_p with the component series of length *n=299*. This value is the same as the cardinality of the original series and the same as the cardinality of the decomposition levels derived from the original series in the *container object W*.

Each of the coefficient series in C_p is swapped into the corresponding decomposition series in W_a.

Table 4.8: ARIMA of Decomposed Levels extended with predictions and the original series

Week	D1	D2	D3	S3	Original
1	0	0.515	0.799	155.594	139.76
2	0	0.513	1.235	154.329	134.58
3	0.001	1.594	2.093	153.031	134.19
4	0	2.996	2.272	150.716	133.04
5	0	4.769	2.536	147.801	133.35
...	...	...	...	...	...
159	0	0.246	-0.654	119.627	117.5
160	0	0.352	-1.01	119.273	117.59
161	0	0.384	-1.541	119.083	117.84
162	0	0.522	-0.927	119.001	121.86
...	...	...	...	...	...
290	0	-0.119	0.327	156.462	156.04
291	0	-0.018	0.611	156.455	156.09
292	0	-0.011	0.74	156.33	156.16
293	0	-0.011	0.841	156.366	160.23
294	0	-0.011	0.921	156.403	160.06
295	0	-0.011	0.986	156.439	160.06
296	0	-0.011	1.037	156.475	165.95
297	0	-0.011	1.079	156.511	166.02
298	0	-0.011	1.112	156.547	166.44
299	0	-0.011	1.138	156.583	166.12

The inverse transform for the filter which was used in the initial decomposition, in this case the *Daubechies d4* filter is applied to produce a MAM version X_M of the initial series. The last 8 elements of the series X_M are the prediction set W_p.

Table 4.9: Testing Set and MAM Prediction values

Week	292	293	294	295	296	297	298	299
Testing Set	156.2	160.2	160.1	160.1	166	166	166.4	166.1
MAM Predictions	156.71	156.32	156.17	156.22	156.03	155.64	155.25	154.12

Steps 8 and 9 from Method 1 are then proceeded with. The MAM Method 2 prediction set and the Testing set are presented in Table 4.9 with an *RMSE* of *8.07*. The *RMSE* for the ARIMA predictions is *7.49*, as before.

4.3 Summary

This chapter described in detail our MODWT ARIMA Method (MAM) process. The purpose of the process is to produce a set of predictions from a time series dataset. The process is compared to predictions arising from an ARIMA model for the same time series.

The process uses a Maximal Overlap Discrete Wavelet Transform (MODWT) to decompose a dataset into several constituent series capturing the detail and the underlying trend components of the series.

The dataset we are using which illustrates the process is the weekly pig meat prices in the Irish market captured from January 2007 to October 2015. To test the process with this dataset we used overlapping samples to sample with replacement in order to allow for multiple tests of the process.

A detailed description of the main implementation of the method (Method 1) using a Haar wavelet filter was presented. To illustrate each of the steps in the method a sample run taken from our dataset was presented in detail with sample values to show what was happening at each step. The method of assessing the relative accuracy of the MAM process compared to the ARIMA prediction method is to use the Root Mean Square Error (RMSE) of each process measured against the real observed values in the Testing set.

The alternative generic case method (Method 2) was presented. Using a Daubechies d4 filter, the extra steps required in this method were illustrated using the same dataset as the example for Method 1 to make the differences in the processes clear.

We have described in detail the steps in the MAM process and what they do with the help of a single run of the process. In the next chapter we will present the experiments we used to test out the two methods in the MAM process on our datasets and provide an analysis of the results of these experiments.

Chapter 5

Experiments and Analysis

In this chapter, we describe our experiments designed to evaluate the MAM process and provide a discussion on the results from each series of experiments. The chapter begins in §5.1 with a description of the structure of experiments from a physical and logical point of view. The 4 categories of experiments are described in detail in section §5.2 along with the results from each of the experiments. An analysis of these results is presented in §5.3.

5.1 Experimental Setup

The overall focus of our experiments was to evaluate our MAM process using a Maximal Overlap Discrete Wavelet Transform (MODWT). Other work in [10] and elsewhere had used a Discrete wavelet Transform (DWT) as presented in Chapter 2. Another difference between our work and others was in the type of data being used. Our dataset was a low frequency dataset compared to the high frequency dataset of [10], for example.

Our experiments were thus divided into four distinct experimental types comparing the MAM approach and the DWT/ARIMA approach on low and high frequency datasets against using an ARIMA prediction model alone.

Data The DATAS [32] project collects agricultural price data from around the world at the frequencies with which the data is published. For this dissertation, we used the time series of market price of pigmeat in Ireland, Denmark and Germany. This data, sampled weekly, is available on-line. We used a dataset of 459 values from week 1 in 2007 to week 42 in 2015. All initial investigations were carried out with the Irish dataset.

The experiments in [10] seek to forecast hourly electricity prices over a period of 1 day ahead for the Spanish Electricity market. The data used in [10] provides a contrast to our dataset in that the data has a high frequency of samples, 24 per day, and the historical data is known for a long period relative to the prediction window. Our data uses weekly pigmeat prices from markets in Ireland, Germany and Denmark and so has a much lower frequency.

Software Experiments were developed using *R* [50] in the RStudio development environment with libraries *forecast* [31] and *wmtsa* [14] providing the necessary functionality. The *wavModwt* and *dwt* functions were used with the Haar and Daubechies wavelets with varying filter lengths and levels of decomposition.

Hardware The machine used was a Dell Optiplex 7020 3.6GHz, 16GB RAM, running Microsoft Windows 7 Professional.

5.2 Experiment Specification and Results

Similar to the work we presented in [5], four separate experiment classes were carried out as part of our evaluation of the MAM process. The four classes were:

- Experiment 1: Replicating the experiments in [10] as a starting point for our evaluation using the Spanish Electricity prices dataset with a DWT/ARIMA. This enabled us to establish a known baseline for our tests to compare our MAM predictions against a high frequency dataset. We could then re-use this DWT/ARIMA method for comparative purposes in Experiment 4.

- Experiment 2: Using our MAM method with the high frequency Spanish Electricity prices dataset from [10] to provide baseline results for comparative purposes with the DWT/ARIMA approach.
- Experiment 3: Using our MAM method on our low frequency pigmeat prices dataset. This was to compare our MAM approach against an ARIMA only approach on low frequency data.
- Experiment 4: Using the algorithm from Experiment 1 for a DWT/ARIMA approach on our low frequency pigmeat prices dataset. This experiment allowed us to compare the results of our MODWT with a known valid DWT/ARIMA approach on low frequency data.

5.2.1 Experiment 1 - DWT/ARIMA for High Frequency Dataset

An established approach to applying wavelets and ARIMA to time series is to use a DWT as in [10]. We initially sought to duplicate their original experiment using their Spanish Electricity Hourly prices high frequency dataset to act as a comparison for our own approach.

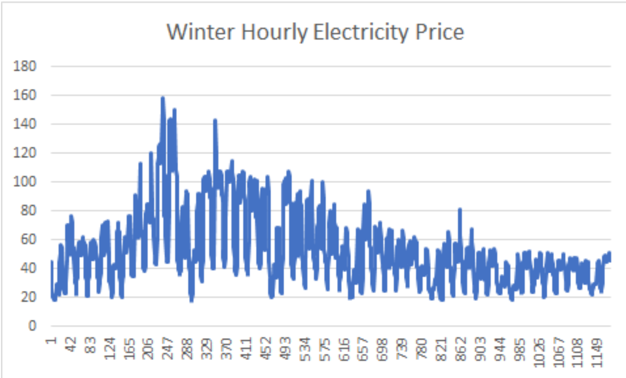
Their method used the DWT with a Daubechies 5 wavelet [16] to 3 levels of decomposition. They predicted 24 values representing the hourly prices for a day ahead. Their training dataset used the previous 48 days of hourly prices, that is a training dataset of 1152 instances of hourly values. As the authors did not provide precise details of their DWT implementation nor the ARIMA parameters, it was not possible to deliver an exact replica of their experiments but we achieved similar results.

The electricity data was accessed from the Spanish Electricity Wholesale Managment company website [45]. The electricity wholesale hourly prices are available from there for each day for the target year, 2002.

Figure 5.1 displays the training and testing dataset for predicting the Monday prices of the Winter Week Spanish hourly wholesale electricity prices. This period covers

49 days consisting of the prices from the preceding 48 days in the training set and the actual price from the Monday used as the testing set.

Figure 5.1: Graph of Winter Electricity Dataset



The experiments in [10] produced predictions for hourly prices for each of the 7 days from 4 test weeks - labelled Winter, Spring, Summer and Autumn(Fall) weeks - with a total of 28 experiments. In [10], the predictions from the ARIMA and Wavelet/ARIMA are measured and compared by a *daily error* presented in Def. 11.

Definition 11 *Error Definition* $\frac{1}{24}\sum_{h=1}^{24}\frac{|p_h^{true}-p_h^{est}|}{\bar{p}_{day}^{true}}$

where

$\bar{p}_{day}^{true} = \frac{1}{24}\sum_{h=1}^{24} p_h^{true}$

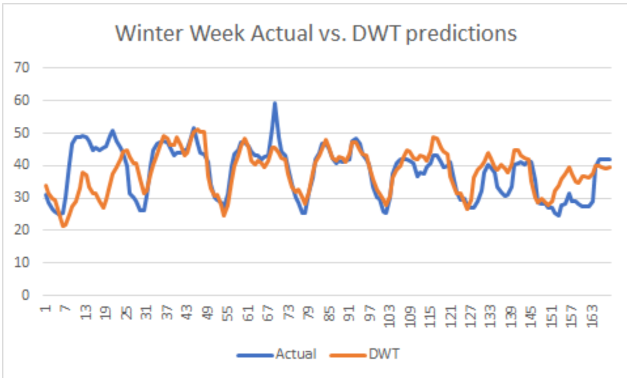
In Def. 11 the p_h^{true} is the actual hourly electricity price for a particular hour and p_h^{est} is the estimated hourly price, produced by their method, for the same hour.

Our experiments used the parameters specified in [10] for a DWT transform with a Daubechies 5 filter to 3 levels of decomposition. We used the *forecast* function in *R* to produce the predictions from ARIMA models. We used the Inverse Wavelet Transform to reconstruct the predictions for the day ahead prices. The error was measured using the daily error from Definition 11.

For this series of experiments, the results from [10] show that the DWT/ARIMA produced a lower daily error rate than the ARIMA predictions on 20 days of the 28 days predicted (Experiment 1a in Table 5.5). When we replicated the experiment, our results produced a lower daily error rate than ARIMA predictions on 22 days of the 28 days predicted which we felt was a satisfactory reproduction (Experiment 1b in Table 5.5).

The winter week actual prices and predicted prices using the DWT method are displayed in figure 5.2 which by visual inspection suggest a good match. This is verified by the daily error in Table 5.1.

Figure 5.2: Winter Week Actual and Predicted Prices - Experiments 1-7



5.2.2 Experiment 2 - MAM on High Frequency Dataset

We applied our MAM approach using the same Spanish electricity data as used in Experiment 1. We implemented the MAM method using both the Haar filter and the Daubechies 5 filter used in [10]. For both experiments, we used the daily error defined in Def. 11 to estimate the errors for each filter against the error of an ARIMA approach.

There were thus 28 experiments grouped into 4 weeks for each of the 2 filters. The daily errors for the MAM process with the Haar filter, the MAM process with the Daubechies 5 filter and the ARIMA model for each of the 4 weeks of experiments are displayed in the Table 5.1.

Table 5.1: Daily Errors for MAM with Electricity Dataset

Season	Experiment	Day	MAM/Haar	MAM/Daubechies	ARIMA
Winter	1	Monday	3.86	3.47	4.64
Winter	2	Tuesday	2.64	0.99	2.46
Winter	3	Wednesday	2.36	0.81	2.6
Winter	4	Thursday	2.25	1.23	2.17
Winter	5	Friday	1.95	1.54	1.82
Winter	6	Saturday	2.21	2.72	2.76
Winter	7	Sunday	3.68	2.75	4.86
Spring	8	Monday	5.66	5.75	5.31
Spring	9	Tuesday	6.14	6.91	8.34
Spring	10	Wednesday	3.46	6.96	4.09
Spring	11	Thursday	2.69	8.14	1.91
Spring	12	Friday	3.02	5.17	2.92
Spring	13	Saturday	4.34	7.78	3.49
Spring	14	Sunday	6.7	9.34	4.95
Summer	15	Monday	3.86	11.94	6.52
Summer	16	Tuesday	2.64	3.99	6.61
Summer	17	Wednesday	2.36	5.83	9.34
Summer	18	Thursday	2.25	4.55	8.56
Summer	19	Friday	1.95	8	5.56
Summer	20	Saturday	2.21	11.55	16.97
Summer	21	Sunday	3.68	3.19	3.96
Autumn	22	Monday	6.29	10.01	10.29
Autumn	23	Tuesday	4.82	3.45	5.76
Autumn	24	Wednesday	4.97	2.52	10.71
Autumn	25	Thursday	5.43	3.75	6.89
Autumn	26	Friday	6.15	7.19	4.65
Autumn	27	Saturday	4.81	5.97	3.06
Autumn	28	Sunday	10.92	3.99	11.86

In this series of 28 experiments for each filter, the MAM approach using a Haar filter generated lower daily errors in 18 tests compared to an ARIMA which produced lower daily errors in 10 tests (Experiment 2a in Table 5.5). The MAM approach using a Daubechies filter produced lower daily errors in 18 tests compared to an ARIMA which produced lower daily errors in 10 tests (Experiment 2b in Table 5.5). The Haar filter produced lower daily errors in 17 tests compared to 11 tests with lower daily errors using a Daubechies filter.

When the results of our tests using the Haar filter were compared against the

DWT/ARIMA approach in [10], the MAM approach generated lower daily errors in 12 tests compared to lower daily errors in 16 tests using a DWT/ARIMA approach (Experiment 2c in Table 5.5). When using the MAM with a Daubechies filter 13 tests produced a lower daily error than the experiments using the DWT/ARIMA approach with the latter producing lower daily errors in 15 tests (Experiment 2d in Table 5.5).

5.2.3 Experiment 3 - MAM on Low Frequency Datasets

A key consideration when applying a wavelet transform is the choice of filter and the number of levels of decomposition to be used.

Prior to applying the MAM method to our dataset, we performed experiments aimed at determining an optimum prediction period for a Haar wavelet and Daubechies wavelets of lengths 4, 6 and 8. The experiments were repeated for each of 3 to 8 levels of decomposition. The proportion of the data given over to the testing set was varied from 4 to 56 weeks for each combination of filter and decomposition. The remainder was used as the training set. The purpose of these initial experiments was to try to identify the most appropriate filter for a specific prediction period.

The test runs were compared by calculating the Root Mean Square Error (RMSE) of the predicted values against the actual values. The Haar filter produced the lowest RMSE for the prediction period of 8 weeks. We therefore used a Haar filter for our experiments. We felt 8 weeks gave a good balance between useful predictions and acceptable accuracy.

The main focus of our experiments was to look at a prediction period of 8 weeks for pigmeat prices from the Irish, German and Danish markets. Our datasets consisted of 459 values of weekly prices from January 2007 to October 2015. This provided a maximum of 451 data points in the training set with 8 points to validate the predictions. Using both the suggestions in [44] and our own empirical evidence, we selected a Haar wavelet with 3 levels of decomposition and the non-decimated

MODWT.

We split the training set of 451 values into 21 sliding subsets of 299 values, sliding 8 values at a time. This provided 291 values in the training set and 8 testing set values to compare against our predictions for each of the 21 experiments for each of the 3 datasets.

We then compared the RMSE of the MAM predictions to the ARIMA predictions for each of the 21 experiments for each of the 3 datasets. Table 5.2 gives a breakdown of the weeks from the original dataset that formed each experimental dataset in the 21 experimental runs for the 3 datasets.

Table 5.2: MAM Process Applied to Low Frequency Data

Experiment	Dataset Weeks	Training Set Weeks	Testing Set Weeks
1	1-299	1-291	292-299
2	9-307	9-299	300-307
3	17-315	17-307	308-315
4	25-323	25-315	316-323
5	33-331	33-323	324-331
6	41-339	41-331	332-339
7	49-347	49-339	340-347
8	57-355	57-347	348-355
9	65-363	65-355	356-363
10	73-371	73-363	364-371
11	81-379	81-371	372-379
12	89-387	89-379	380-387
13	97-395	97-387	388-395
14	105-403	105-395	396-403
15	113-411	113-403	404-411
16	121-419	121-411	412-419
17	129-427	129-419	420-427
18	137-435	137-427	428-435
19	145-443	145-435	436-443
20	153-451	153-443	444-451
21	161-459	161-451	452-459

The MAM experiments were run using the datasets specified in table 5.2 for each of the 3 pig meat price datasets for Ireland Germany and Denmark. The RMSE for the results were calculated for the MAM and ARIMA predictions. Table 5.3 shows the RMSE for each dataset in each of the 21 experiments for each of the 3 datasets.

Table 5.3: RMSE for MAM vs. ARIMA in Irish, German and Danish Datasets

	Ireland		Germany		Denmark	
Experiment	MAM	ARIMA	MAM	ARIMA	MAM	ARIMA
1	7.38	7.49	21.67	12.03	9.2	8.99
2	3.73	2	14.6	10.83	8.33	4.87
3	1.21	0.6	7.72	12.64	2.93	3.43
4	3.69	3.7	3.06	3.35	2.79	2.9
5	1.46	2.42	3.91	2.34	1	2.34
6	1.13	1.51	11.02	17.47	3.54	3.64
7	0.65	0.97	2.65	11.33	2.66	2.65
8	5.28	5.81	18.55	23.9	7.02	2.79
9	8.99	7.95	5.24	8.55	4.42	3.96
10	4.22	4.54	3.92	8.72	9.2	6.94
11	5.95	2.09	4.89	12	3.01	4.97
12	7.63	7.4	9.69	3.57	4.29	4.44
13	1.93	3.02	7.18	9.27	3.28	2.6
14	13.54	8.05	17.49	7.9	2.51	3.51
15	3.51	5.53	9.89	3.9	17.38	16.22
16	4.27	4.53	6.16	7.53	3.4	4.36
17	6.06	5.01	14.81	19.22	5.19	5.46
18	1.48	1.24	14.25	4.99	8.08	1.91
19	2.23	2.33	3.44	10.73	3.41	1.63
20	14.44	10.25	3.21	10.36	6.53	6.16
21	5.86	1.74	10.93	7.53	11.1	6.8

The MAM approach produced a lower RMSE in 11 tests and the tests using ARIMA alone produced a lower RMSE in 10 of the 21 tests on the Irish pigmeat prices data. MAM produced a lower RMSE in 13 tests and ARIMA alone alone produced a lower RMSE in 8 tests for the German pigmeat prices data. Using the Danish pigmeat prices data, MAM produced a lower RMSE in 9 tests and ARIMA alone produced a lower RMSE in 12 tests.

5.2.4 Experiment 4 - DWT/ARIMA on Low Frequency Datasets.

For this experiment, we applied the DWT/ARIMA approach on the pigmeat prices datasets from Ireland, Germany and Denmark. The experiments used 21 subsets of the data using a sliding window, to produce 8 predicted values for each subset in 21 tests for each dataset. The same training and testing datasets as specified in table 5.2 were used. This was repeated for each of these markets for a total of 63 tests.

The results of each of the tests using DWT/ARIMA were compared with the MAM approach using the Root Mean Square Error (RMSE) on the predictions of each approach compared to the testing set. The results of the tests are produced in Table 5.4.

Table 5.4: RMSE for DWT/ARIMA vs. MAM in Irish, German & Danish Datasets

	Ireland		Germany		Denmark	
Experiment	DWT/ARIMA	MAM	DWT/ARIMA	MAM	DWT/ARIMA	MAM
1	20.59	7.38	59.44	21.67	30.95	9.2
2	30.53	3.73	33.19	14.6	29.42	8.33
3	4.48	1.21	66.94	7.72	25.22	2.93
4	7.98	3.69	22.59	3.06	32.49	2.79
5	9.37	1.46	9.21	3.91	12.67	1
6	17.63	1.13	18.21	11.02	10.36	3.54
7	2.66	0.65	48.78	2.65	26.18	2.66
8	21.02	5.28	28.99	18.55	29.53	7.02
9	19.08	8.99	55.54	5.24	16.01	4.42
10	36.12	4.22	30.48	3.92	38.84	9.2
11	28.87	5.95	28.18	4.89	29.45	3.01
12	28.89	7.63	31.75	9.69	14.42	4.29
13	40	1.93	36.61	7.18	28.61	3.28
14	27.71	13.54	21.64	17.49	14.71	2.51
15	43.61	3.51	68.32	9.89	44.84	17.38
16	28.56	4.27	45.24	6.16	24.33	3.4
17	26.86	6.06	22.68	14.81	27.94	5.19
18	20.31	1.48	33.54	14.25	16.06	8.08
19	23.5	2.23	10.21	3.44	22.97	3.41
20	12.92	14.44	14.17	3.21	12.23	6.53
21	24.47	5.86	8.32	10.93	13.41	11.1

The DWT/ARIMA approach performed extremely poorly against the MAM approach in the 63 experiments carried out here. Only 1 of the 21 test runs of the DWT/ARIMA produced a lower RMSE in each of the Irish and German markets with 0 test runs recording a lower RMSE for DWT in the Danish market.

5.3 Analysis of Results

The results of our experiments detailed above are summarised in Table 5.5 which shows the number of each type of test which produced the better result in each test in each of the 4 categories. We present an analysis of experiments 2 to 4 which relate to our MAM process.

Table 5.5: Summary of Experiment Results

Exp	Tests	Dataset	Size	Period	Transform	Filter	Results	
							DWT/AR	ARIMA
1a	28	Electricity	1152	24	DWT	D5	20	8
1b	28	Electricity	1152	24	DWT	D5	22	8
							MAM	ARIMA
2a	28	Electricity	1152	24	MODWT	Haar	18	10
2b	28	Electricity	1152	24	MODWT	D5	18	10
							MAM	DWT/AR
2c	28	Electricity	1152	24		Haar	12	16
2d	28	Electricity	1152	24		D5	13	15
							MAM	ARIMA
3a	21	Irish	299	8	MODWT	Haar	11	10
3b	21	German	299	8	MODWT	Haar	13	8
3c	21	Danish	299	8	MODWT	Haar	9	12
							MAM	DWT/AR
4a	21	Irish	299	8		Haar	20	1
4b	21	German	299	8		Haar	20	1
4c	21	Danish	299	8		Haar	21	0

The column headings have the following meaning:

- **Exp** identifies the experiment (1-4 described in sec. 5.2) being undertaken.
- **Tests** is the number of individual tests run for each experiment.
- **Dataset** is either *Electricity* for Spanish Electricity Prices or *Irish*, *German* or *Danish* for pigmeat prices.
- **Size** is the cardinality of the dataset used.
- **Period** is the number of predictions in the prediction period.

- **Transform** is either the Discrete Wavelet Transform (DWT) or Maximal Overlap Discrete Wavelet Transform (MODWT). In experiments 2c, 2d, 4a, 4b and 4c the MODWT is the transform in the MAM process and the DWT is the transform in the DWT/AR process.
- **Filter** is the wavelet used: the Haar filter or D5 to represent the Daubechies 5 filter. In all cases 3 levels of decomposition were used.
- **Results** summarises the number of tests for which the specified method was more successful. DWT/AR is the DWT with ARIMA.

5.3.1 Analysis of Experiment 2 Results

In our experiments, we have seen that wavelet analysis with ARIMA in both MAM and the process in [10] can produce different results from different subsets of the same dataset.

In Table 5.1, the test results are presented for each of 4 weeks - Winter, Spring, Summer and Autumn(Fall). The results indicate that there is a noticeable difference in results between the weeks. For example, the results using either the Haar or Daubechies 5 filter with either the DWT or MODWT produce significantly different results for each of the 4 weeks.

Experiments 2a and 2b

Experiments 2a and 2b use the MAM process with the Haar filter and the MAM process with the Daubechies filter. From Table 5.1 for experiments 2a and 2b there are statistically different results for each of the 4 weeks for each filter. These differences are summarised in Table 5.6 in terms of the number of each of the 7 experiments per week with a lower daily error for each method.

Table 5.6: Summary of Experiment Results 2a and 2b

Season	Haar	ARIMA		Daubechies	ARIMA
Winter	4	3		7	0
Spring	2	5		1	6
Summer	7	0		5	2
Autumn	5	2		5	2

Paired t-tests at the 95% confidence level were carried out using Excel on each of the sets of results for the 4 weeks in Table 5.6 to compare the daily errors for the MAM with the Haar filter and MAM with the Daubechies filter against the daily errors for the ARIMA predictions.

For the Winter week, the results for the MAM approach using the Haar filter compared with the ARIMA results gave a result of a lower error for MAM with p=0.07, t=1.7, showing no statistical difference in the results. The results for the MAM approach using the Daubechies filter results compared with the ARIMA results gave a lower error for MAM with p=0.004, t =3.898. The MAM approach gave errors that were significantly lower using the Daubechies filter.

For the Spring week, the MAM approach using the Haar filter results compared with the ARIMA results gave an inconclusive result with a p=0.39, t=.292 indicating that the two processes produced no statistical difference in results. However, for the MAM approach the results using the Daubechies filter compared with the ARIMA results, the t-test gave a lower error for MAM with p=0.016, t=2.78. The MAM approach gave errors that were significantly lower for the Daubechies filter.

For the Summer week, the results of the t-tests were different for both filters also. On this occasion the MAM approach using the Haar filter results compared with the ARIMA results gave a clear statistical result of a lower error for the MAM process filter with p=0.01, t=3.14. The MAM approach using the Daubechies filter showed no statistical difference compared with the ARIMA with p=0.22, t=0.827.

For the Autumn week, the results of both tests were inconclusive with the MAM approach using the Haar filter results compared with ARIMA results giving p =0.11, t=1.369 and the MAM approach using the Daubechies filter results compared with the ARIMA results giving p=0.1, t=1.44.

Looking at all these results together, the MAM process with either filter gave a statistically better result than ARIMA or statistically equivalent result to ARIMA.

The variation in the results suggests that further investigation to identify characteristics of the data from each of these weeks could potentially point to factors which could optimise the benefits of wavelet analysis of this time series.

Experiments 2c and 2d

The results of the experiments in 2c and 2d also show statistically different results between MAM with the Haar and Daubechies 5 filters against the results using the DWT/ARIMA method for different weeks. Paired t-tests were carried out to compare the results of the MAM and DWT/ARIMA methods using the daily error measurement in Def. 11 from [10] for each of the 4 weeks Winter, Spring, Summer and Autumn.

The Winter week showed no significance between MAM with the Haar filter and DWT/ARIMA with p=0.419, t=0.214 but a significant improvement using MAM with Daubechies filter with p=0.032, t=2.27.

The Spring week showed the daily error for DWT/ARIMA to be significantly lower than either MAM with the Haar filter p=0.012, t=3.0 and MAM with Daubechies filter p=0.002, t=4.52.

The Summer week showed different results for the different MAM filters. The MAM with the Haar filter showed no significant difference to DWT/ARIMA with p=0.418, t=0.216 whereas DWT/ARIMA had a lower error than MAM with Daubechies filter with p=0.005, t=3.71.

The Autumn week showed that there was no statistical difference between the MAM method with the Haar filter and the DWT/ARIMA, giving p=0.089, t=1.525 nor the MAM method with the Daubechies 5 filter and the DWT/ARIMA giving p=0.121, t=1.298.

5.3.2 Analysis of Experiment 3 Results

The third set of experiments show that there is statistically no significant difference between applying ARIMA and the MAM process in a long term dataset with low frequency.

21 datasets were extracted for usage as test data for each of the Irish, German and Danish markets giving a total of 63 experiments. Paired t-tests were carried out between the RMSE for ARIMA and MAM methods, with no statistical differences *(p=0.25)* found between the ARIMA and the MAM methods with regard to the mean square error.

This result shows that the MAM method is as robust as the established ARIMA only prediction method.

5.3.3 Analysis of Experiment 4 Results

In the fourth set of experiments, the results of applying a DWT/ARIMA to the pigmeat prices dataset show a statistically different result compared to using an MODWT. Paired t-tests between the MAM and DWT/ARIMA results were carried out for the 63 tests. The t-test score was $p < 0.001$, with the mean values being significantly lower for MAM.

The poorer performance of the DWT/ARIMA when applied to low frequency data is likely due to the cardinality of the pig meat dataset being small. The DWT being a decimated transform reduces the cardinality at each decomposition level by a factor of 2 resulting in a set of series of low cardinality which is not conducive to ARIMA modelling.

In addition, the length of the prediction window for a DWT significantly affects the number of coefficients which are predicted from each decomposition level. In the case of a requirement for 8 predictions, as in our case, this means that at the level 3 decomposition, there is only 1 predicted value, at level 2 decomposition there are 2 predicted values and at level 1 there are 4 predicted values to feed into the inverse transform. Clearly, this would make the predictions for the original series very sensitive to the value of only 1 prediction at level 3.

The MAM approach which produces the same number of coefficients at each level of decomposition is more suited to a smaller, infrequent dataset. Using an MODWT also means that there are the same number of coefficients at each decomposition level to feed into an ARIMA model and from there into the inverse transform. Having a constant number of prediction values at each decomposition level provides more input values for the reconstruction using the inverse wavelet transform resulting in more robust predictions less influenced by a small number of values as in the DWT approach.

5.4 Summary

In this chapter we described 4 sets of experiments which were designed to compare the MAM process to 2 other processes for time series prediction, namely an ARIMA only approach and a DWT with ARIMA approach. The structure of the datasets and the results produced were documented. We provided detailed analysis of the results of the experiments.

ARIMA modelling requires significant domain knowledge together with trial and error, to find an appropriate model for a time series before producing reliable predictions.

The advantage of the MAM process is that by decomposing a signal, using an MODWT, into statistically better behaved series with a constant number of values at each decomposition level, ARIMA models can be more readily produced on these decomposed levels.

For high frequency data, the MAM process produced results that were significantly better than an ARIMA prediction process. The DWT/ARIMA and MAM methods showed no statistical difference on high frequency data.

We have seen that the series produced from recombining the predictions from the decomposed series for a low frequency dataset is as statistically accurate as predictions from an ARIMA method applied to the original series.

For low frequency datasets with a short prediction window, the MAM method produces better results than a DWT/ARIMA method.

Overall our MAM process provides a better set of results for low frequency data than DWT and similar results for high frequency data. The MAM process is at least as good as ARIMA for predictions with either low or high frequency data and shows itself to be a highly robust process with this wide range of results.

Chapter 6

Conclusions

In this dissertation, we set out to devise a method to predict future values of a financial time series using a combined wavelet and ARIMA approach. We introduced the domain of this research and the proposed methods in Chapter 1. In Chapter 2 we showed that there is a wide body of research on the application of wavelets to time series analysis. In Chapter 3 we introduced the technical aspects of ARIMA and MODWT that we use in our method and showed what advantages and functionality they provide us with and why we chose the MODWT approach over the DWT approach. Our method, MODWT ARIMA Method (MAM), which combines the MODWT and ARIMA approaches was presented in extensive detail in Chapter 4. Our results and analysis of our experiments in Chapter 5 showed that our method was consistently as accurate or better in producing predictions for low and high frequency datasets than ARIMA and DWT/ARIMA approaches.

6.1 Predictions using Wavelets

The goal of predicting financial time series has obvious attractions, but is clearly difficult to achieve. Approaches to time series prediction assume that the series knows about itself, that is, it contains information within the values in the series which determine other values in the series. By identifying relationships between series values a researcher hopes to provide a model which can be applied to the known series values to predict future values.

Pricing predictions from agri datasets are very difficult to achieve mainly due to *events* occurring at different timelines that are present within the datasets. This requires more complex algorithms for prediction than the traditional and well-respected ARIMA approach. Modelling a time series with an ARIMA specifies parameters linking a value to a combination of previous values, thereby allowing prediction of future values from known ones.

Wavelet analysis of any signal, including time series, identifies and locates frequency patterns in the signal. Wavelets decompose a signal into multiple components which are more amenable to modelling with a technique such as ARIMA. Wavelets also provide a mechanism to reconstruct a signal from the amended component series.

While a number of different wavelet approaches have been tested elsewhere, our approach is to use both the MODWT and ARIMA methods in combination for better predictions in time series data and to present a detailed method to apply to other datasets. As a baseline for our results, we used the approach and similar evaluation as [10] to determine what is currently possible using a wavelet approach to time series data. Our evaluation used 4 sets of experiments to try to determine if our approach was better than a DWT/ARIMA method.

The MODWT in conjunction with the ARIMA method was comparable with the DWT combined with ARIMA method proposed by [10] for the high frequency Spanish electricity price data. However, our MAM approach showed superior results to a DWT when attempting forecasts for low frequency data streams such as the pigmeat price dataset.

For low frequency data, MAM and ARIMA demonstrated equivalent results, but importantly, MAM demonstrated a resilience and robustness that neither of the other 2 methods demonstrated as it achieved equally good results for both low and high frequency data.

In ARIMA methods, it is difficult to find the correct parameter settings when dealing with complex combinations of Moving Average and Auto Regressive components. Implementing a MODWT decomposes the complexity of the series and allows the researcher a much higher chance of identifying simpler ARIMA models for each level of the decomposed series. Thus, MAM gives the researcher a level of comfort knowing that it will produce predictions that are robust to the frequency components of the data.

The choice of wavelet filter is very much dependent on the characteristics of the data. A suitable wavelet and decomposition level might only be identifiable by trial and error. However, in all uses of wavelets and in particular for prediction, the issue of coefficients affected by the boundary problem becomes more intrusive, the wider the wavelet and the deeper the decomposition. This is of particular concern when using the DWT wavelet transform as opposed to MODWT.

The size of prediction windows imposes constraints on the type of transform that can be used. The limitations of the decimation property of the DWT limits the size of the prediction window as few coefficients at deeper levels of decomposition will result in prediction values generated from these few prediction coefficients.

When using wavelet analysis for prediction purposes a large high frequency dataset may suit a DWT, but for a low frequency lower cardinality dataset or for a short prediction window a MODWT is a more appropriate tool.

6.2 Future Work

The results of our experiments demonstrated that MAM was sufficiently robust when attempting to predict the high frequency Spanish electricity prices or the low frequency Irish, German and Danish pig prices. Having a method that manages a wide range of data characteristics enables the researcher to have a degree of confidence in their results. By its nature, breaking a complicated time series into simpler series avoids over complicating the orders of ARIMA or GARCH models. However, it is clear from our experiments that there are other factors in the data which affect the results. We now consider some possibilities for further research in this area.

6.2.1 External Factors

In particular, when dealing with the Spanish Electricity data there were very different results for the prediction weeks with 2 of the 4 weeks being more accurately predicted than the remaining 2 weeks. This occurred for both the DWT/ARIMA and MAM approaches. This suggests that there is some condition being satisfied in some datasets than in others which makes the wavelet analysis with ARIMA better than ARIMA alone. Further research is necessary to reveal the factors which are influencing this.

6.2.2 Using MAM with Volatile Datasets

Our research has suggested that DWT/ARIMA is suited to high frequency datasets with reasonably long prediction windows to allow several levels of decomposition while not being as effective for short prediction windows. MAM is effective for each scenario. It would be interesting to see if this held true for other datasets which included heteroskedastic data. Certainly it would be enlightening to examine the MAM approach with data that demonstrated a high degree of volatility.

6.2.3 Preprocessing Survey

Additionally, one must be cognisant of the fact that not all datasets have "Big Data" characteristics. In fact, smaller datasets which typically occur from monthly or weekly data do not lend themselves to machine learning techniques such as neural networks, RNN or Deep Learning.

However, a survey to compare the performance of techniques such as MAM with some of the proposed machine learning algorithms for smaller datasets of monthly or weekly commodity prices could potentially yield very useful results. For this type of survey, an analysis of the complexity of the solution versus the predictive power of each method is required.

6.2.4 A More Formal Approach to Wavelet Filter Selection

Typically, in commodity price forecasting the practitioner does not expect to get an exact result from their predictive algorithm. However, they require the algorithm to provide reasonable predictive boundaries. Using ARIMA or GARCH as the base method give extremely wide confidence bands [8] when moving forward into the future. This is predominantly caused by the ARIMA or GARCH method inheriting the error from the previous time point. An incorporation of Bayesian estimation techniques with exogenous variables may allow the practitioner to introduce expert knowledge in their predictive algorithms and thus, avoid unrealistic predictions.

The choice of wavelet filter can be found by trial and error. However, research using a large number of datasets of different characteristics might help to identify the characteristics appropriate to a particular wavelet. This research should also examine or recommend the correct number of decomposition levels required in any given analysis as the time required to determine the level of granularity in any given analysis is extensive.

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